Special Theory of Relativity and the Lorentz Force

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Abstract

For the Special Theory of Relativity (STR) to be valid the form of the Lorentz force expression (say, its Y-component) after Lorentz transformations should be $Y' = \left(Y - \frac{v}{c}N\right)$. However, the form of what is claimed to be the Lorentz force after the application of the Lorentz transformations is, in fact, approximately $Y' = \left(Y - \frac{v}{c}N + Y\frac{1}{2}\frac{v^2}{c^2}\right)$. Obviously, for $\frac{v}{c}N \leq Y\frac{1}{2}\frac{v^2}{c^2}$ the latter expression does not have the same form as the required $Y' = \left(Y - \frac{v}{c}N\right)$.

Effort to demonstrate that the application of Lorentz transformations leads to the form $Y' = \left(Y - \frac{v}{c}N\right)$ (at least to the leading orders of approximation), which has been elevated to the criterion for the validity of STR , has been the main point in the founding paper on Special Theory of Relativity (STR) [1]. The criterion for the validity of STR set forth in [1] is not fulfilled, despite the implication in [1] that it is. The text that follows gives details of this finding.

If one is concerned with the validity of the Special Theory of Relativity (STR) one must consider in the first place the founding paper [1]. The derivations, formulae and concepts in [1] are used by those who apply STR nowadays in exactly the same form as it has been at its inception, almost one hundred years ago. Despite all the developments during the last century and nowadays, paper [1] still contains everything one needs to know about the fundamentals of the Special Theory of Relativity (STR) and a problem found in [1] cannot be ignored, especially by lightly considering that [1] is already obsolete.

It can be shown that the criterion for the simultaneity of two events developed in [1] (cf. ”Definition of Simultaneity” on p.38 of [1]) lacks physical meaning and that the only outcome from that exercise in [1] has been to actually demonstrate that the light-ray device used therein to explore simultaneity is not appropriate for such exploration. The incorrectness of the said criterion for simultaneity has numerous far-reaching consequences.

Most importantly, it leads to the conclusion that the claim for the relativity of time is untenable. These problems are discussed elsewhere.

One other instance where the flaws in the criterion for simultaneity in [1] play a role for the physical inconsistency of conclusions is the existence and application of the Lorentz transformations. Although, as will be shown below, the problems with the results from application of the Lorentz transformations stand in their own right, on a deeper level the incorrectness of the criterion for simultaneity developed in [1] is the ultimate generator of the errors (the derivation of the Lorentz transformations is based on the criterion for the simultaneity of two events proposed in [1]).

The text that follows explores, in agreement with [1] (p. 54), the requirement that the expression for the Lorentz force transformed using the first postulate (the “Principle of Relativity”) must be equal (at least to the leading orders of approximation) to the expression of the Lorentz force transformed using the second postulate (using the Lorentz transformations derived on the basis of the second postulate to obtain the Lorentz force expressed with the components of the laboratory frame as functions of the co-moving frame coordinates).

It is shown below, that, contrary to the conclusion expressed in [1], the above-mentioned mandatory requirement (especially note, mandatory even from the point of view of the author of [1] who has elevated it to be the central point of his paper and to be a criterion for the validity of the theory) for the equality of the two force expressions cannot be fulfilled through the application of the Lorentz transformations derived therein.

In the laboratory frame the Lorentz force on a unit test charge q = 1 which moves along the x-axis at a velocity **v** versus the source of electric field (source of electric field is in the laboratory frame) is:

$$F = 1E + 1\frac{\mathbf{v}\times\mathbf{B}}{c} \qquad (1)$$

which can be represented in component form as:

$$F_x = 1X$$

$$F_y = 1\left(Y - \frac{v_x}{c}N\right) \qquad (2)$$

$$F_z = 1\left(Y + \frac{v_x}{c}N\right)$$

where (X,Y,Z) denotes the vector of electric force and (L,M,N) that of a magnetic force.

According to the “Principle of Relativity” [1] if one wants to use the electric and magnetic field **E’** and **B’** measured from the standpoint of an observer in the co-moving frame (at the locality of the same test charge) the Lorentz force should be given by an equation of the same form as eq.(1) (respectively eq.(2)) but with the fields primed, i.e.

$$\mathbf{F}' = 1\mathbf{E}' + 1\frac{\mathbf{v} \times \mathbf{B}'}{c} \qquad (3)$$

which in component form is

$$F'_x = X'$$

$$F'_y = \left(Y' - \frac{v_x}{c}N'\right) \qquad (4)$$

$$F'_z = \left(Y' + \frac{v_x}{c}M'\right)$$

Next we will obtain the Lorentz force expressed with the parameters (field components) of the laboratory frame as functions of the coordinates of the co-moving frame.

To achieve this, instead of transforming <u>Maxwell’s equations</u> from the laboratory to the co-moving frame, as is done in [1], we will now undertake a procedure of direct transformation of the <u>Lorentz force itself</u> from the laboratory to the co-moving frame.

Obviously, the procedure for the transformation of eq.(1) (respectively eq.(2)) from the laboratory to the co-moving frame must be the same as the procedure used in [1] for transformation of Maxwell’s equations from the laboratory into the co-moving frame.

In other words, the components of the <u>same</u> (of the laboratory frame) electro-magnetic field X, Y, Z, L, M, and N as well as q should be observed as functions of the new coordinates, x’, y’, z’ and t’ (coordinates of the co-moving frame are also denoted by the symbols ξ, η, ς and τ ; we will use both notations which, of course, have the same meaning) and should be replaced in eq.(1). Thus, we have:

$$F_x(\xi,\eta,\varsigma,\tau) = X(\xi,\eta,\varsigma,\tau)$$

$$F_y(\xi,\eta,\varsigma,\tau) = \left(Y(\xi,\eta,\varsigma,\tau) - \frac{v_x}{c} N(\xi,\eta,\varsigma,\tau) \right) \quad (5)$$

$$F_z(\xi,\eta,\varsigma,\tau) = \left(Z(\xi,\eta,\varsigma,\tau) + \frac{v_x}{c} M(\xi,\eta,\varsigma,\tau) \right)$$

Notice that charge q = 1 remains unchanged despite the motion (an experimental fact). Also, notice that in eq.(5) velocity component $v_x \neq 0$ because we are still in the laboratory frame and the co-moving frame travels versus laboratory frame at a speed $v_x \neq 0$, despite the fact that we have represented the fields as a function of the coordinates of the co-moving frame. Because of the same reasons of symmetry as in [1] (p.53) we consider that $\psi\left(v_x\right) = 1$ and have not included it in eq.(5).

Thus, so far, we obtained two expressions for the Lorentz force in terms of components or coordinates of the co-moving frame – one, namely eq.(3) (respectively eq.(4)), according to the "Principle of Relativity", and the other, namely eq.(5), through direct transformation of the Lorentz force (notice that so far the postulate for constancy of the speed of light has not been involved).

According to the same reasoning applied in [1] (see how this is done in [1] by following the procedure applied for transformation of Maxwell's equations) we now note that the system of equations (4) and the system of equations (5) express the same thing since both systems of equations are equivalent to the Lorentz force equation for the laboratory system. Therefore, we may write

$$X' = X$$

$$Y' = 1\left(Y - \frac{v}{c} N \right) \quad (6)$$

$$Z' = 1\left(Z + \frac{v}{c} M \right)$$

As is seen from eq.(6), when the Lorentz force is transformed directly from the laboratory into the co-moving frame (again, specially notice here that during this transformation <u>Lorentz transformations were not applied</u>) the Lorentz force expressed through the components of the laboratory frame as functions of the coordinates of the co-moving frame, has exactly the same form as the form of the Lorentz force in the laboratory frame.

This exactly should be expected to be the case because the same charge residing in the same electro-magnetic circumstances should experience only one force despite the viewpoints from which this force is observed. The Lorentz force in the co-moving frame when expressed through the components of the fields of the laboratory frame should have the form of eq.(6) no matter how one chooses to derive this force. It is to be noted also that this understanding is in full agreement with the understanding in [1] regarding the form the transformed Lorentz force should have.

Everything thus far is entirely smooth and there is no way to even consider existence of internal inconsistencies or problems of physical nature.

If we now are concerned with transformations of Maxwell's equations that would lead to Lorentz force, as is the case with paper [1], it is seen that transformations have to be sought which will lead to nothing else but to an equation of the form of eq.(6), i.e. to an equation which shows that the transformed (as explained) force on the test charge is of the same form as the force on the charge expressed with the field components and the coordinates of the laboratory frame.

Surprisingly, however, when one such attempt of transforming Maxwell's equations is made ([1] (p.52-54)) with the aim to obtain Lorentz force expression of the form of eq.(6), an expression is obtained that actually has nothing to do with eq.(6) to any degree of approximation (despite some deceiving superficial similarity which has led the author of [1] to erroneously conclude that the derivation of eq.(6) has been fulfilled).

The failure in [1] to derive eq.(6) is due to the use of one particular set of transformations – the Lorentz transformations – for deriving of the Lorentz force expressed through the components of the electro-magnetic field in the laboratory frame observed as functions of the co-moving frame coordinates.

This problem is a clear indication of the non-physical nature of the Lorentz transformations.

Before showing explicitly this fact I will again mention, in passing, that the non-physical nature of the Lorentz transformations stems from the method of their derivation which relies on an inappropriate criterion developed in [1] for the simultaneity of two events.

In paper [1] Maxwell's equations for a free space are postulated to be of the same form, if the co-moving frame field components are used, as is the form of Maxwell's equations in the laboratory frame (in accordance with the "Principle of Relativity"); on the other hand, Lorentz transformations are applied on the Maxwell equations in the laboratory frame to obtain expressions for Maxwell's equations in terms of the laboratory components of the electro-magnetic field but observed as functions of the co-moving frame coordinates. Then, the expressions of Maxwell's equations resulting from the application of the Lorentz transformations are compared with the above-mentioned, postulated form of Maxwell's equations, expressed in terms of the field components of the co-moving frame. The comparison yields the following relations [1]

$$X' = X$$

$$Y' = \beta\left(Y - \frac{v}{c}N\right) \qquad (7)$$

$$Z' = \beta\left(Z + \frac{v}{c}M\right)$$

For the sake of clarity we will note that eq.(7) is called in [1] (p.54) "[n]ew manner of expression" of the Lorentz force components, while eq.(2) is called "[o]ld manner of expression", i.e. the latter is the known expression of the Lorentz force components as obtained in the laboratory frame. Notice, however, that if by "[n]ew manner of expression" we are to understand an expression for the transformed Lorentz force then eq.(6) should also be considered a new form of expression of this force.

Now, obviously, since the same Lorentz force is considered acting on the same charge residing in the same co-moving frame, Lorentz force formula given by eq.(7) must coincide with the Lorentz force formula given by what we derived and was numbered as eq.(6). It is claimed in [1] that eq.(7) actually coincides in form with an equation such as eq.(6) to "the first order of small quantities" (to the second or higher powers of $\frac{v}{c}$ approximation).

We will show below that this conclusion is incorrect.

Below, we will observe whether it can be asserted, as is done in [1], that these two forms (eq.(6), respectively eq.(2) on one hand, and eq.(7) on the other) are in fact practically the same to "the first order of small quantities". Is the difference between these two forms really slight ?

In a private communication G. Pellegrini [2] showed me that the latter consideration is incorrect but insisted that this is a minor point in [1] since, according to him, the Lorentz-transformed force, when expressed by the components of the laboratory frame, should neither be required to have the same form nor the same magnitude as the same force in the laboratory frame. In other words, if it happened so that eq.(6), respectively eq.(2) and eq.(7) coincide (for instance, in form), this would only be a fortuitous coincidence which is not necessarily required by the physical situation. As was seen from the above direct transformation of the Lorentz force, as well as from the views expressed in [1], such an impression is untenable.

Indeed, the author of [1] goes to great lengths (cf. p.54) to prove that the old and the new form of expression of the Lorentz force are the same. Is this some early insignificant misperception on his part (and uncovering it would only be of historical interest) or perhaps this is a fundamental requirement whose violation would mean collapse of the theory ? The answer to this question is clearly evident from the result of the transformation made above (eq.(6)) – transformed force must have the same form as the force in the laboratory frame – a theory that would produce transformed Lorentz force different from eq.(6) is incorrect.

Therefore, we should state again very clearly, in agreement with [1], that the requirement for the old manner of expression of the Lorentz force to coincide with the new manner of expression of that force (at least to the order of small quantities) is mandatory if absurdities are to be avoided.

Unfortunately, such coincidence of the two manners of expression of the Lorentz force is not demonstrated in [1] despite the assertion to the contrary.

Indeed, observe one of the y'-axis components of the electric field of eq.(7) – which is in fact an equation from [1] (page 54) – (same conclusions apply also to y'-axis component of the magnetic field and z'-components of the electric and the magnetic field):

$$Y' = \beta\left(Y - \frac{v}{c}N\right) = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\left(Y - \frac{v}{c}N\right) \qquad (8)$$

and apply the expansion

$$(a + b)^n = a^n + na^{n-1}b + \frac{n(n-1)a^{n-2}}{2}b^2 + \ldots \qquad (9)$$

to obtain

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{v^2}{c^2}\right)^2}{2} + \ldots \qquad (10)$$

For the sake of this discussion, consider only the first two terms of the expansion and obtain

$$\beta = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2} \tag{11}$$

Thus, the expression $Y' = \beta\left(Y - \frac{v}{c}N\right)$ may be rewritten in the following way:

$$\begin{aligned} & Y' = \beta\left(Y - \frac{v}{c}N\right) = \\ & \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\left(Y - \frac{v}{c}N\right) \approx \\ & \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)\left(Y - \frac{v}{c}N\right) = \\ & Y - \frac{v}{c}N + Y\frac{1}{2}\frac{v^2}{c^2} - \frac{1}{2}\frac{v^3}{c^3}N \end{aligned} \tag{12}$$

Let us observe the fourth line of eq.(12) and neglect the terms of third order versus $\frac{v}{c}$:

$$Y' = Y - \frac{v}{c}N + Y\frac{1}{2}\frac{v^2}{c^2} \tag{13}$$

It is obvious from eq.(13) that when the second and the third terms are of the same order, i.e. when $\frac{v}{c}N \cong Y\frac{1}{2}\frac{v^2}{c^2}$ then ignoring them leads to

$$Y' = Y \tag{14}$$

From the above it is seen that, contrary to the conclusions made in [1], eq.(14) (which is the new manner of expression of the Lorentz force) is not equal in form to any degree of approximation to eq.(2) (which is the old manner of expression of the Lorentz force).

The fact that second and third term of eq.(13) can be made comparable to each other indicates that second term cannot be preferred over the third term and *vice versa* – neglecting one of these terms inevitably requires neglecting of the other term as well.

Thus, the seeming possibility to neglect the third term because it is in the second order of $\frac{v}{c}$ versus the second term which is not of second order versus $\frac{v}{c}$ is inapplicable.

Suppose one says, I do not care what everything else is, I just want to neglect only terms of second order over $\frac{v}{c}$ (forgetting the N itself contains a velocity over c term) and, yet, what I obtain will still be the same as eq.(6) to that order of approximation. That is incorrect. If one neglects just the terms of second order over $\frac{v}{c}$ and does not care what everything else is, then, under the above conditions ($\frac{v}{c}N \cong Y\frac{1}{2}\frac{v^2}{c^2}$) one incorrectly prefers to neglect one term over another term of the same order of magnitude. This is like claiming that, say, (1 – ½ + ½) is the same as (1 – ½) where the third term ½ had been neglected – note, at relativistic speeds, where $v \approx c$, first and third (respectively second) terms in eq.(13) become of comparable magnitude.

To neglect third over the second term is like neglecting only the third term in an instance like this: $y = A - Bx + \frac{1}{2}Ax^2$ when $Bx = \frac{1}{2}Ax^2$; expression $y = A - Bx$ can only be obtained after (incorrectly) neglecting the third term, which is unacceptable. On the other hand, transformation required the obtainment of an expression of the type $y = A - Bx$. Observe the difference between $y = A$ and the required $y = A - Bx$.

On can continue with such examples and get into really amusing paradoxes. Thus, following the logic in [1] one may, for instance, decide to claim that 1 – 1 = 1 and not that 1 – 1 = 0, because one presumably can neglect the second term on the left-hand side of the equality. It should be stated very clearly, however, that such neglecting of the second term on the left-hand side of the equality is unacceptable and it is still true that 1 – 1 = 0, as it has always been.

One may really wonder why we are discussing this at all. Does it not go without saying that when $\frac{v}{c}N \cong Y\frac{1}{2}\frac{v^2}{c^2}$ the obtained expression $Y' = Y$ most obviously differs in form from the desired expression $Y' = \left(Y - \frac{v}{c}N\right)$? That is correct, these two expressions of the force obviously differ from each other and it is unfounded from any point of view to claim their forms coincide especially under the discussed circumstances ($\frac{v}{c}N \cong Y\frac{1}{2}\frac{v^2}{c^2}$). The reason for this special discussion of the obvious is that, surprisingly, this difference in form of the expression obtained in [1] $Y' = \beta\left(Y - \frac{v}{c}N\right)$ compared to the desired form $Y' = \left(Y - \frac{v}{c}N\right)$ is not recognized in [1] and a wrong conclusion is reached that these forms coincide (tho the leading orders of approximation) and therefore the criterion for the validity of the theory has been fulfilled.

It may happen also that $\frac{v}{c}N \ll Y\frac{1}{2}\frac{v^2}{c^2}$. Then eq.(13) becomes

$$Y' = Y + Y\frac{1}{2}\frac{v^2}{c^2} \qquad (15)$$

which being, again, the new manner of expression of the Lorentz force, also, contrary to the conclusions in [1], is not equal in form to eq.(2) (the old manner of expression of the Lorentz force).

The only condition when the old form of expression of the Lorentz force (eq.(2)) and the new form of expression of the Lorentz force (eq.(12)) coincide in form to the second and higher order of approximation over $\frac{v}{c}$ would be when $\frac{v}{c}N \gg Y\frac{1}{2}\frac{v^2}{c^2}$. However, this is only a special case and does not suffice to accept that neglecting the third term in eq.(13) is viable in general – especially at near relativistic speeds, i.e. at speeds where the effects of STR should become detectable.

It follows from the above that the theory proposed in [1] is unacceptable because it does not abide by the very same criterion its own author himself has proposed as a test for its validity.

Now we will once more note that it cannot be emphasized more strongly – according to eq.(6) the force, transformed as described, should indeed be of the same form as the form of the force expressed with the field components and the coordinates of the laboratory frame. It was shown that Lorentz transformations cannot accomplish that. Therefore, some other transformations have to be sought to substitute for eq.(13), the approximate equation that was actually derived in [1].

Conclusion

From the above discussion it is seen that following exactly the derivations and the logic in paper [1] one finds, upon careful inspection, a fatal hitherto unrecognized discrepancy between what actually had been derived in [1] and what paper [1] claims to have been derived therein. The seriousness of this discrepancy cannot be overestimated because it concerns the criterion for the validity of the theory presented in [1].

Regarding the significance of the above finding, one should note that the ideas in paper [1] are by no means ideas of the past and it is incorrect to assume that nowadays some modified form of the STR is used that does not contain the above-discussed problems. The ideas in [1] and the expressions derived therein lie at the very foundations of contemporary hard sciences exactly in the form we discussed them here. Despite all the developments, paper [1] still contains everything one needs to know about the fundamentals of the Special Theory of Relativity (STR). Thus, a potential problem that might exist in [1], as it does as shown, cannot be ignored, especially by lightly considering that [1] is already obsolete.

Further, the validity of the theory in [1] should be assessed based on its own merits, based on the merits the text and the derivations in the text [1] itself offer. Recall that paper [1] is the result of work only with "a pencil and a paper". It should not be considered unexpected then, that arguments and analysis of [1] only the result of work with "a pencil and a paper" should be considered powerful enough to even question its validity. Furthermore, it is hardly an appropriate methodology for refuting a "thought analysis", such as the one presented herewith, through using claims that experimental proofs for the theory in [1] exist. Moreover, it is well known that conclusions of STR either concern phenomena that cannot be observed by simple means or various adjustment and approximations (beyond the realm of this discussion) are applied if discrepancies are observed.

On the other hand, undoubtedly, as a separate exercise, unrelated to the conclusion herein for the validity of STR, it would be interesting to explore what the basis for claims for experimental confirmations of STR really is. Nevertheless, it is to be noted once again that no experimental finding can undo the inherent lack of physical meaning of the theory in [1] and if some coincidence between the latter and experiment is found it has to be considered a fortuitous coincidence at best.

For one reason or another, modern approach, however, is to never question the validity of STR even though efforts are now under way to test the validity of the General Theory of Relativity (GTR). It should be pointed out that problems in the Special Theory in Relativity (STR) inevitably lead to problems in the General Theory of Relativity (GTR) and any analysis of the validity of GTR should necessarily be preceded by an analysis of the validity of STR.

Unfortunately, experience shows that the specialized terminology used when discussing the problem with the mathematical "machinery" such as that in [1], as it was done in the present text, greatly obscures the problem in question for a more general readership. Therefore, a way has been sought [3] to demonstrate conclusively the problems in STR by designing a *'gedanken'* experiment which can be described and analyzed without using a single mathematical formula. Such *'gedanken'* experiment is presented in [3] whereby two simultaneous events (from the point of view of an observer in a train) trigger an explosion which destroys a train. A stationary observer applying the discussed theory [1] to determine simultaneity of the two events in the train concludes that no simultaneous events have occurred on the train, hence the train must be intact. The conclusion the stationary observer makes is incorrect because it is based on STR as a method to determine simultaneity of two events.

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References

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