Suppes' Methodology of Economics

Adolfo GARCÍA DE LA SIENRA

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ABSTRACT: Even though Patrick Suppes has many important contributions in the logic, methodology, and philosophy of economics (not to mention economics itself) his work in this area is, surprisingly, not well known within the community of economic methodologists. The aim of the present paper is to contribute to fill this lacuna. After presenting his general views on philosophy of science, the paper discusses at length his important notion of model of data, in connection with his famous learning experiments. His views on fundamental measurement are also discussed in connection with such experiments. The article contains a comprehensive bibliography of Suppes on the subject.

Keywords: Patrick Suppes, economic methodology, models of data, representational theory of measurement, philosophy of science.

RESUMEN: *Aunque las contribuciones de Patrick Suppes en el ámbito de la lógica, la metodología y la filosofía de la economía (por no mencionar a la propia economía) hayan sido importantes, los metodólogos de la economía apenas las conocen. El propósito de este artículo es cubrir este vacío. Presentaremos su concepción general de la filosofía de la ciencia y analizaremos después con detalle su noción de modelos de datos, surgida de sus famosos experimentos sobre aprendizaje. Discutiremos también su concepción de la medición fundamental en relación con tales experimentos. El artículo incluye además una amplia bibliografía de los trabajos de Suppes sobre el tema.*

Descriptores: *Patrick Suppes, metodología económica, modelos de datos, teoría representacional de la medida, filosofía de la ciencia.*

1. Introduction

There is no doubt that Patrick Suppes is one of the eminent figures in the history of the philosophy of science. For no less than fifty years Suppes has produced not only a very influential methodology for the empirical sciences but, on top of his influential views on the nature of scientific theory and fundamental measurement, he has made scientific or philosophical contributions to probability theory, space and time theory, physics, language theory, psychology, robotics and economics. His book *Representation and Invariance of Scientific Structures* (Suppes 2002) presents an overview of many of these contributions but for some reason omits his work on economics, even though his work in this field is no less impressive than in the others. The present paper intends to present an outline of Suppes' contributions to economics and its methodology in their systematic interaction. I will rely heavily on (Suppes 2002), since most of the philosophical views expressed in his many papers are summarized there.

A bibliography of Suppes' contributions to economics or its methodology (approved by him and included here) contains no less than fifty six items, which can be classified as follows:

1. General Methodology and Philosophy of Science
2. Fundamental Measurement
3. Utility and Expected Utility
4. Rational Choice Theory
5. Experimental Economics
6. Welfare Economics
7. Economics of Science
8. Theory of Consumer Demand.

Since his production is concentrated in the other topics, I shall not deal with 6 and 7 here; for these topics the reader is referred to the bibliography. The other themes except 8 can be included in 1 and 2. After considering his general methodology and philosophy of science, I will attempt to present a systematic reconstruction of his theory on consumer demand.

2. General Methodology and Philosophy of Science

Even though Suppes is a serious practitioner of statistics, and has published his philosophical reflections on the different interpretations of the concept of probability, his methodology proceeds almost always in connection with scientific theory. Theories are the product of creative scientific work, as well as the reference point of any further scientific work: measurement, experimentation, testing, application, analysis. For Suppes statistical, in particular econometric hypotheses, presuppose some theoretical model, even if implicitly. This, of course, raises the question of what a scientific theory is.

In the Introduction of (Suppes 2002) this question is addressed but, in contradistinction to questions for which we can expect a clear and definite answer, like "What is a rational number?" or "What is a nectarine?", Suppes warns that this question

> fits neither one of these patterns. Scientific theories are not like rational numbers or nectarines. Certainly they are not like nectarines, for they are not physical objects. They are like rational numbers in not being physical objects, but they are totally unlike rational numbers in that scientific theories cannot be defined simply or directly in terms of other nonphysical, abstract objects. (Suppes 2002, 2)

Since the question we are considering cannot be answered directly in simple terms, philosophers have intended to address it from different angles. One approach has been what Suppes calls "the traditional sketch" (also known as the "statement view"), according to which

> a scientific theory consists of two parts. One part is an abstract logical calculus, which includes the vocabulary of logic and the primitive symbols of the theory. The logical structure of the theory is fixed by stating the axioms or postulates of the theory in terms of its primitive symbols. For many theories the primitive symbols will be thought of as theoretical terms like 'electron' or 'particle', which cannot be related in any simple way to observable phenomena.
> The second part of the theory is a set of rules that assign an empirical content to the logical calculus by providing what are usually called 'co-ordinating definitions' or 'empirical interpretations' for at

> least some of the primitive and defined symbols of the calculus. It is always emphasized that the first part alone is not sufficient to define a scientific theory; for without a systematic specification of the intended empirical interpretation of the theory, it is not possible in any sense to evaluate the theory as a part of science, although it can be studied simply as a piece of pure mathematics. (Suppes 2002, 2-3)

Observe here Suppes' use of the term 'science': it is not pure mathematics, but "we mean by science, as opposed to mathematics, the development of theory and the confronting of theory with quantitative data" (Suppes 1968a, 651)

Suppes finds the traditional sketch so sketchy that it "makes it possible to omit both important properties of theories and significant distinctions that may be introduced between different theories" (Suppes 1967a, 57). This calls for a different way of conceiving scientific theories.

Even though the question "What is a scientific theory?" cannot be answered in a straightforward way, a very common practice has been to think that any particular theory can be identified through a set of sentences formulated in a formal language. But it is easy to see that formulation of real-life scientific theories in such a way is utterly impractical, particularly when the theory under analysis presupposes more than first-order logic (for in such a case it becomes necessary, in order to formulate the theory, to include first-order formulations of all those presupposed theories: set theory, real analysis, etc.). Since complex scientific theories "are similar to the theories studied in pure mathematics in their degree of complexity",

> in such contexts it is very much simpler to assert things about the models of the theory rather than to talk directly and explicitly about the sentences of the theory, perhaps the main reason for this being that the notion of sentence of the theory is not well defined when the theory is not given in standard formalization. (Suppes 1967a, 58)

Here the relevant notion of model is precisely the one that Bourbaki (1968) defined under the label 'mathematical structure'. These structures are introduced by Suppes through the definition of a set-theoretic predicate, like '$\mathfrak{A}$ *is a topological space*' or '$\mathfrak{A}$ *is a game*'.[1]

Suppes proposes to characterize or identify any scientific theory through a certain class of structures since, even if it appears formulated intrinsically — by means of a certain set of statements (not necessarily capable of being formulated as sentences of first order logic) —, the question of whether such a formulation is adequate, or whether a formulation in first order logic is feasible, can only be answered after an extrinsic characterization of the same is given. Even though Suppes thinks that it is not important to provide precise definitions of the concept of scientific theory in terms of necessary and sufficient conditions, of the form "X is a scientific theory if and only if so-and-so", and has a certain tendency to shy away from grand schemes about scientific theories and their relations, he recognizes that an essential ingredient of any sophisticated scientific discipline is a hierarchy of theories, starting with models of data and culminating with what he calls a fundamental theory. Before attempting a general,

[1] For a rigorous formulation of Suppes' notion of set-theoretic predicate, see Da Costa and Chuaqui (1988).

abstract characterization of such a hierarchy, and in order to illustrate Suppes' view of a "sophisticated" scientific discipline, I shall reconstruct here the hierarchy he presented in his famous paper "Models of Data" (Suppes 1962b). The "fundamental" theory is linear response theory (LRT), whose potential models can be described as follows.

Definition 1. $\mathfrak{A}$ *is a* possible model of LRT *iff there exist* A, E, X, $\mathscr{F}$, P *and* θ *such that* A, E *are sets each with two elements and*

0. $\mathfrak{A} = \langle X, \mathscr{F}, P, \theta \rangle$;
1. $X = \times_{n=1}^{\infty} X_n$, *where* X_n *is a replica of* $A \times E$ *for each* $n < \omega$;
2. $\mathscr{F}$ *is the smallest Borel field containing the cylinder sets of* X;
3. P *is a a probability measure over* $\mathscr{F}$;
4. θ *is a real number in the interval* $[0, 1]$.

Thus, a possible model of LRT is a structure of the form $\langle \mathfrak{B}, \theta \rangle$, where $\mathfrak{B}$ is a probability space $\langle X, \mathscr{F}, P \rangle$ as described. A possible interpretation of such an abstract structure is as follows. A is the set of possible responses a_1 or a_2 of an ideal subject who wants to anticipate in which of two panels in front of him a light will appear. Pressing the left button (a_1) means that the subject expects the light will appear in the left panel; pushing the other (a_2) that it will appear in the one at the right. The appearing of the light in the left is reinforcing event e_1; the other is e_2. The subject sits there forever, responding again and again to the reinforcements given on succesive trials. $a_{i,n}$ will denote the event of response a_i on trial n; $e_{j,n}$ the event of reinforcement e_j on trial n, where $i, j = 1, 2$. If we let X_n be $A \times E$ for each $n < \omega$, the measure space $\langle X, \mathscr{F} \rangle$ can be built as the product of the replicas $\langle X_n, \mathscr{F}_n \rangle$. X is defined just as the Cartesian product of the X_n, namely the set of all sequences $\mathbf{x} = (x_1, x_2, \ldots)$ where $x_n \in A \times E$, and $\mathscr{F}$ is the σ-algebra generated by the cylinder subsets of X. θ is the learning parameter and $\mathbf{x}_n$ denotes the equivalence class of all sequences in X which are identical with $\mathbf{x}$ through trial n. Clearly, x_n is a pair of the form (a_i, e_j) $(i, j = 1, 2)$ for each n.

The fundamental laws of LRT assert that the probability measure P and the learning parameter θ are related in a special way. The triple $(\mathbf{x}_{n-1}, a_{i,n}, e_{j,n})$ denotes a sequence (a possible history) in which $x_n = (a_{i,n}, e_{j,n})$ occurs on trial n; i.e. in which event $e_{j,n}$ takes place after $a_{i,n}$. If $j = i$, response $a_{i,n}$ is reinforced at that trial; otherwise, a different response is reinforced. A history $\mathbf{x} \in X$ is *feasible* if $P(\mathbf{x}_{n-1}, a_{i,n}, e_{j,n})$ is positive for each n, where $P(\mathbf{x}_{n-1}, a_{i,n}, e_{j,n})$ is the probability of cylinder set

$$\{\mathbf{x} \in X | \mathfrak{a}_n(\mathbf{x}) = (x_1, \ldots, x_{n-1}, (a_{i,n}, e_{j,n}))\} \in \mathscr{F}$$

$\mathfrak{a}_n(\mathbf{x})$ being the initial history of $\mathbf{x}$, up to trial n. If $(\mathbf{x}_{n-1}, a_{i,n}, e_{j,n})$ is feasible, the laws of LRT cover two cases. If $j = i$, response $a_{i,n}$ is reinforced; if $j \neq i$, then the other response, $a_{j,n}$, is reinforced. The first law asserts that, if initial history (or path) $(\mathbf{x}_{n-1}, a_{i,n}, e_{i,n})$ is feasible for $i = 1, 2$, and response $a_{i,n}$ is reinforced, then the probability of making that response on the next trial is increased by a simple linear transformation. The second law asserts that, if $(\mathbf{x}_{n-1}, a_{i,n}, e_{j,n})$ is feasible for $i, j = 1, 2$, and response $a_{i,n}$ is not reinforced, then the probability of making that response on the next trial is decreased by a second linear transformation. Through the definition of a set-theoretic predicate characterizing the models of LRT these laws can be formulated as follows.

Definition 2. $\mathfrak{A}$ *is a* model *of LRT iff* $\mathfrak{A}$ *is a possible model of LRT and the following two axioms are realized in* $\mathfrak{A}$*:*

1. *If* $(\mathbf{x}_{n-1}, a_{i,n}, e_{i,n})$ *is feasible for* $i = 1, 2$*, then*
$$P(a_{i,n+1}|(\mathbf{x}_{n-1}, a_{i,n}, e_{i,n})) = (1-\theta)P(a_{i,n}|\mathbf{x}_{n-1}) + \theta;$$
2. *if* $(\mathbf{x}_{n-1}, a_{i,n}, e_{j,n})$ *is feasible for* $i, j = 1, 2$ *and* $i \neq j$*, then*
$$P(a_{i,n+1}|(\mathbf{x}_{n-1}, a_{i,n}, e_{j,n})) = (1-\theta)P(a_{i,n}|\mathbf{x}_{n-1}).$$

As I have argued elsewhere (García de la Sienra 2009a) axioms like 0–4 of Definition 1, and 1, 2 of Definition 2, make no empirical claim whatsoever, but rather can be used to make empirical claims. In (Suppes 1962b) Suppes only considers (implicitly) empirical claims that can be made through controlled experiments. The empirical claim would be something like: "under certain controlled situations, healthy agents with a certain education and intelligence, but not necessarily statistically sophisticated, will act in such a way as to approximately obey the laws". For instance, a particular class of experiments to which LRT was applied was one in which simple contingent reinforcement schedules were introduced. This means that the reinforcement mechanism is designed in such a way that if in any trial response a_1 is made then the probability of an e_1 reinforcement is π_1, while if it is a_2 the probability of reinforcement e_2 is π_2, in both cases independently of the previous history. Hence, the probabilities of reinforcements are characterized by means of the following two equations:

$$P(e_{1,n}|a_{1,n}) = \pi_1 = 1 - P(e_{2,n}|a_{1,n}),$$
$$P(e_{2,n}|a_{2,n}) = \pi_2 = 1 - P(e_{1,n}|a_{2,n}).$$

This is a first step toward a testing of the theory. But "the first step down from the abstract level of the linear response theory itself" is the theory of the experiment. Supposing that the experimenter has decided to run 600 trials for each subject, the experiment will consider only initial histories of the sequences in X up to trial 600, namely the set $X' = \mathfrak{a}_{600}(X)$. Since X' is finite, the corresponding family of events $\mathscr{F}'$ is just the power set of X'. A possible realization will require also a probability measure P' on $\mathscr{F}'$. Thus, formally,

Definition 3. $\mathfrak{B}'$ *is a* possible realization of the theory of the experiment *iff there exist* X'*,* $\mathscr{F}'$ *and* P' *such that*

0. $\mathfrak{B}' = \langle X', \mathscr{F}', P' \rangle$*;*
1. $X' = \mathfrak{a}_{600}(X)$*;*
2. $\mathscr{F}'$ *is the set of all subsets of* X'*;*
3. P' *is a probability measure over* $\mathscr{F}'$*.*

The specification of P in terms of simple contingent reinforcement schedules yields a special class of models of LRT which can be seen as potential games of one personal player responding to natural states that may occur with a certain probability, in which the following, specific behavioral strategy profile $\left(\{b_{0t}\}_{t=1}^{600}, \{b_{1t}\}_{t=1}^{600}\right)$ has been specified. At $\mathbf{o}$, the beginning, the personal player is conditioned to one of the two responses, say $a_{1,1}$, a fact that can be conceptualized in terms of the player choosing a pure strategy; i.e. strategy $a_{1,1}$ with probability $b_{11}(1, \mathbf{o}) = 1$. At node $a_{1,n}$, for any n, nature may present reinforcement $e_{1,n}$ with probability $b_{0n}(1, (\mathbf{x}_{n-1}, a_{1,n})) = \pi_1$, or reinforcement $e_{2,n}$ with probability $b_{0n}(2, (\mathbf{x}_{n-1}, a_{1,n})) = 1 - \pi_1$; at node $a_{2,n}$ nature may present reinforcement $e_{2,n}$ with probability $b_{0n}(2, (\mathbf{x}_{n-1}, a_{2,n},)) = \pi_2$, or reinforcement $e_{1,n}$ with probability $b_{0n}(1, (\mathbf{x}_{n-1}, a_{2,n})) = 1 - \pi_2$. The probability with which the personal player chooses his strategies at node e_{in}, for $n \neq 1$, depends upon the previous history. Given partial history $(\mathbf{x}_{n-1}, (a_{i,n}, e_{j,n}))$, the agent chooses action $a_{1,n+1}$ with probability $b_{1n+1}(1, (\mathbf{x}_{n-1}, (a_{i,n}, e_{j,n}))) = (1-\theta)P(a_{i,n}|\mathbf{x}_{n-1}) + \theta$ if $j = i = 1$; with probability $b_{1n+1}(1, (\mathbf{x}_{n-1}, (a_{i,n}, e_{j,n}))) = (\theta - 1)P(a_{i,n}|\mathbf{x}_{n-1}) + (1-\theta)$ if $j \neq i = 1$. He chooses action $a_{2,n+1}$ with probability $b_{2n+1}(2, (\mathbf{x}_{n-1}, (a_{i,n}, e_{j,n}))) = (1-\theta)P(a_{i,n}|\mathbf{x}_{n-1}) + \theta$ if $j = i = 2$; with probability $b_{1n+1}(1, (\mathbf{x}_{n-1}, (a_{i,n}, e_{j,n}))) = (\theta - 1)P(a_{i,n}|\mathbf{x}_{n-1}) + (1-\theta)$ if $j \neq i = 2$. This means that there are certain profiles $(\{b_{0t}\}_{t=1}^{\infty}, \{b_{1t}\}_{t=1}^{\infty})$ of behavioral strategies (for expediency, I think of nature's actions as a given in behavioral "strategies", even though nature has no utility function) that induce the probability measure P (for details, see García de la Sienra 2009b). Thus, a model of the theory of the experiment is a potential game (potential because no utility function has been explicitly assumed for the personal player) in which the probability measure P' is induced by the profile $\left(\{b_{0t}\}_{t=1}^{600}, \{b_{1t}\}_{t=1}^{600}\right)$ of behavioral strategies. The experiments devised by Suppes can be seen as methods of determining this profile through the determination of the learning parameter, θ.

In order to perform these experiments it should be noted that an experiment with even a large number of trials will produce a number of sequences small compared to that of the model of the experiment, even though the models of the theory of the experiment are finite:

> The finite sequences that are elements of X' may indeed be used to represent any possible experimental outcome, but in an experiment with, say, 40 subjects, the observed 40 sequences are an insignificant part of the 4^{600} sequences in X'. Consequently, a model closer to the actual situation is needed to represent the actual conditional relative frequencies of reinforcement used. (Suppes 1962b, 255)

Before passing to consider a model still closer to the actual situation, it is important to point out that the relation between the theory of the experiment and the fundamental theory can be characterized in terms of embeddings of partial models: structure $\mathfrak{B}' = \langle X', \mathscr{F}', P' \rangle$ is embedded in structure $\mathfrak{B} = \langle X, \mathscr{F}, P \rangle$. The embedding is given by the correspondence $\phi\colon X' \to X$, which maps event $(x_1, \ldots, x_{600}) \in X'$ into cylinder set

$$A = \{\mathbf{x} \in X | \mathfrak{a}_{600}(\mathbf{x}) = (x_1, \ldots, x_{600})\} \subseteq X,$$

with $P(A) = P'(\{(x_1, \ldots, x_{600})\})$. This implies that laws 1 and 2 of Definition 2 are also satisfied by structure $\mathfrak{A}' = \langle \mathfrak{B}', \theta \rangle$, and so one wonders why Suppes does not introduce also parameter θ in the definition of the theory of the experiment. Doing so would maintain structures $\mathfrak{A} = \langle X, \mathscr{F}, P, \theta \rangle$ and $\mathfrak{A}'$ as being of the same similarity type, making easier and more elegant the characterization of their relationship (namely a lawful embedding of $\mathfrak{A}'$ into $\mathfrak{A}$).

A possible realization of the model of the data is quite another story, since it is just the record of the trials of the experiment for all the subjects, numbered N (40 in the previous example). Thus, it consists of an N-tuple

$$\mathfrak{b} = (x^1, \ldots, x^N)$$

of elements from X', so that (in the example),

$$x^p = (x_1^p, \ldots, x_{600}^p).$$

These sequences are to be used to determine a statistical value of parameter θ and, in this way, of probability measures P' and P.

> The next question is, When is a possible realization of the data a model of the data? The complete answer, as I see it, requires a detailed statistical theory of goodness of fit. Roughly speaking, an N-tuple realization is a model of the data if the conditional relative frequencies of e_1 and e_2 reinforcements fit closely enough the probabilistic measure P of the model of the experiment. (Suppes 1962b, 256)

More precisely,

Definition 4. $\mathfrak{b}$ *is an* N-fold model of the data for experiment $\mathfrak{B}'$ *iff* $\mathfrak{B}' = \langle X', \mathscr{F}', P' \rangle$ *is a model of the theory of the experiment, and* $\mathfrak{b}$ *is an* N*-tuple of elements of* X'*, and* $\mathfrak{b}$ *satisfies the statistical tests of homogeneity (the conditional relative frequencies of reinforcements (CRF) are approximately* π *or* $1 - \pi$*, as the case may be), stationarity (the CRF of reinforcements are constant over trials) and order (the CRF of reinforcements are independent of preceding reinforcements and responses).*

Suppes and R.C. Atkinson present and discuss a large number of methods to determine parameter θ in (Suppes and Atkinson 1960, Chapter 2). The method consists, roughly, of the following: given parameters π_1 and π_2, which are controlled by the experimenter, and a model of the data, the problem is to find a "linear response model with learning parameter $\hat{\theta}$, which will maximize the probability of the observed data, as given in the model of the data" (Suppes 1962b, 259). In order to determine the conditions of homogeneity, stationarity and order, Suppes uses a methodology based upon χ^2 tests. The way he uses in general this methodology is described in (Suppes 2007). Applications of the same to economic situations are provided in the just mentioned book as well as in "Experimental Analysis of a Duopoly Situation from the Standpoint of Mathematical Learning Theory", written with J.M. Carlsmith (Suppes and Carlsmith 1962). The book considers partial games of the following sorts: zero-sum, bipersonal; nonzero-sum, bipersonal; symmetric, nonzero; bipersonal, of communication and discrimination; three-personal, with majority; with displayed payoff matrix; with monetary payoffs and noncontingent reinforcement schedules.

From the point of view of orthodox game theory, the determination of parameters π_1, π_2 and θ is tantamount to determining a particular profile of behavioral strategies $\mathbf{b} = (\{b_{0t}\}_{t=1}^{\infty}, \{b_{1t}\}_{t=1}^{\infty})$ as stated above. Thus, under very stringent experimental conditions Suppes and his collaborators actually specified a partial model of game theory. The question is whether the given profile $\mathbf{b}$ maximizes some expected utility function of the personal player, as it is standardly defined (for instance in García de la Sienra 2009b). The answer is no: Suppes' concept of game deviates from the usual notion in that its utility concept is rather different. While the expected utility function U is defined in game theory as

$$U(\mathbf{b}) = \int_F u(\mathbf{h})\, \mathrm{d}\mu[\mathbf{b}],$$

where F is the set of feasible histories, $u\colon F \to \mathbb{R}$ is the usual utility function of the player, $\mathbf{b}$ is a profile of behavioral strategies, and $\mu[\mathbf{b}]$ is the probability measure induced by $\mathbf{b}$ over the space of feasible histories of the game, Suppes defines a "stochastic" utility function "which may be used to predict the actual behavior of all but the statistically sophisticated few" (Suppes 1961a, 196). I shall discuss this concept in the next section, devoted to fundamental measurement; I just want to finish the present one discussing the relevance of Suppes experimental work on economics for the application of economic theories to economic phenomena at large. Indeed, What is the import of these experiments for economic theory? What have Suppes and his collaborators established through them? What is the point, indeed, of testing a theory intended for phenomena that cannot be controlled by experiment — the phenomena of real economic situations or processes — by means of very controlled experimental situations?

As Suppes himself points out, in particular in connection with his analysis of duopoly, the success of the theory in these situations is no guarantee that it will be succesful in real economic situations, even though "so far as the experimental situation bears resemblance to a real economic situation, the theory is directly relevant to economic theory" (Suppes and Carlsmith 1962, 60). Suppes seems to place the problem in the lack of information:

> it is very unlikely that economic data exist for a duopoly of fifty years' duration; and even if such data do exist, gross perturbations like major depressions or war would seriously affect their interpretation. (Suppes and Atkinson 1960, 266)

Even if such data existed, however, it is not clear why the fact that the experimental situation bears resemblance to a real economic situation makes the theory directly relevant to economic theory. At any rate, the intent of applying the theory to a certain situation (represented by a model of economic data) may or may not be succesful on its own right, independently of its success in experimental conditions. Perhaps Suppes is thinking that the *confidence* in the theory grows if it has experimental success. What is clear, anyway, is that such a success establishes that the domain of intended applications of the theory is not empty. To put it in Kuhnian terms: experimental success increases the confidence that the promise harbored by the theory makes worthwhile devoting time and resources to use it in order to produce knowledge of certain classes of phenomena.

3. *Fundamental Measurement*

Suppes seems to suggest that derived measurement, even in developed scientific disciplines, must be anchored, in the final analysis, to some fundamental measurements.

> In developed scientific disciplines the role of fundamental measurement is usually behind the scene, so to speak, for the fundamental measurement needed has already been established. What will be in question in a current experiment is a derived measurement, possibly to great accuracy, of a quantity that depends on other fundamental measurements. (Suppes 1988b, 27)

It is easy to imagine why this is so for instance in classical physics. We can produce fundamental measurement structures to measure the mass or length of bodies within our perceptual range that we can manipulate with our hands or tools, thus relating them to certain observable standards. Then we can use the laws of physics to calculate the masses or sizes of larger bodies like planets or even the sun, relating such masses or sizes to the fundamental measurements performed over those bodies closer to us. But it is not at all clear whether or how a similar procedure might be described for the social sciences. Rather, at least for the case of utility measurement, Suppes sounds very skeptical about the possibility of measuring utility of given agents by showing that their preference structures satisfy certain axioms. He relies instead on experimental methods that might, applying response theory, go beyond "the individual preference orderings to the environmental and constitutional conditions that produced them" (Suppes and Atkinson 1960, 233). The role of such axioms is to regulate the concepts, so to say, exhibiting the conditions that define them. The properties of preference structures defined by different sets of axioms are given mainly in *Foundations of Measurement*.

Almost any result in the three volumes of his monumental *Foundations of Measurement*, written in collaboration with R. Duncan Luce, D.H. Krantz and A. Tversky, might have a surprising and interesting application in economics. But all of Chapters 8 (volume I) and 17 (volume II) is clearly relevant, since they deal, respectively, with conditional expected utility and choice probabilities. Other results that have immediate application in economics are the proofs of the existence of numerical representations for countable simple orders (Theorem 1, vol. 1, p. 39), simple orders with countable order-dense subsets (Theorem 2, vol. 1, p. 40), positive-difference structures (Theorem 1, vol. 1, p. 147), algebraic-difference structures (Theorem 2, vol. 1, p. 151), cross-modality orderings (Theorem 4, vol. 1, p. 165), finite, equally spaced difference structures (Theorem 5, vol. 1, p. 168), absolute difference structures (Theorem 6, vol. 1, p. 173), additive conjoint structures (Theorem 2, vol. 1, p. 257), and n-component, additive conjoint structures (Theorem 13, vol. 1, p. 302).

In point of actual fact, however, in order to determine the utility of the experimental subjects, Suppes does not use directly fundamental measurement methods, but rather shows that, in the limit, the subjects behave as if they were maximizing a certain "stochastic" utility function which is computed out of the asymptotic limit of the transition probabilities. The general procedure consists of deriving this utility function, for finite sets of alternatives, from the axioms of stimulus sampling theory. Then it is shown that such function actually satisfies the axioms characterizing the concept of a stochastic utility function, which is defined as follows, if we let $p(a, b)$ denote the probability that a is chosen over b.

Definition 5. *A* stochastic utility function *for a set A of alternatives is a function $u\colon A \to \mathbb{R}$ such that*

$$u(a) - u(b) \geq u(c) - u(d) \quad \textit{iff} \quad p(a,b) \geq p(c,d)\,.$$

If set A and probabilities $p(a,b)$ satisfy the following axioms, there exists a stochastic utility function which is unique up to positive linear transformations.

- U1. $p(a,b) + p(b,a) = 1$.
- U2. $0 < p(a,b) < 1$.
- U3. *If* $p(a,b) \geq p(c,d)$ *then* $p(a,c) \geq p(b,d)$.
- U4. *There is a* c *in* A *such that* $p(a,c) = p(c,b)$.
- U5. *If* $p(c,d) > p(a,b) > 1/2$ *then there is a* e *in* A *such that* $p(c,e) > 1/2$ *and* $p(e,d) \geq p(a,b)$.
- U6. (Archimedian Axiom). *If* $p(a,b) > 1/2$ *then, for every probability* q *such that* $p(a,b) > q > 1/2$, *there is a positive integer* n *such that* $q \geq p(a,c_1) = p(c_1,c_2) = \cdots = p(c_n,b) > 1/2$.

For instance, for the class of Markovian specializations of LRT, Suppes derives a stochastic utility function in general and then, clearly, if in a particular experiment the transition probabilities are determined, the actual values of that utility function are also determined thereof. The details are as follows. Define a Markovian LRT model as a structure $\mathfrak{A}$ that, in addition to axioms 1 and 2 of Definition 2, satisfies the following two "independence of path" assumptions:

1. The probability θ that a sampled stimulus element will be conditioned to a reinforced response is independent of the trial number and the preceding pattern of events.
2. Given the set of stimulus elements available for sampling on a trial, the probability of sampling a given element is independent of the trial number and the preceding pattern of events.

In a structure that satisfies these axioms the transitions from one state to another can be depicted by means of a diagram, as shown in the figure, and the transition matrix for the Markov chain is the following:

$$\begin{array}{c|cc} & a_1 & a_2 \\ \hline a_1 & 1-\theta(1-\pi_1) & \theta(1-\pi_1) \\ a_2 & \theta\pi_2 & 1-\theta\pi_2 \end{array}$$

If $p_n(a_i)$ is the probability that the agent be in state a_i at stage n, the probability that it will be in state a_1 at $n+1$ will be

$$\begin{aligned} p_{n+1}(a_1) &= p_{11} \cdot p_n(a_1) + p_{21} \cdot [1 - p_n(a_1)] \\ &= [1 - \theta(1-\pi_1)] \cdot p_n(a_1) + \theta\pi_2 \cdot [1 - p_n(a_1)]\,. \end{aligned}$$

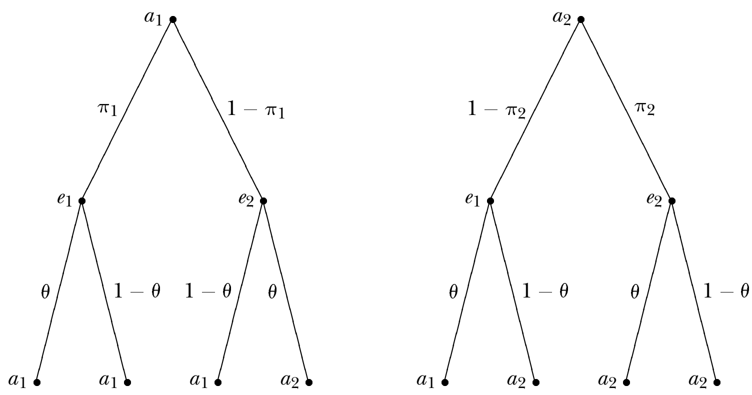

Figure 1: Typical transitions in Markov structures.

At asymptote,

$$p_{n+1}(a_1) = p_n(a_1) = p_\infty(a_1),$$

and so

$$P_\infty(a_1) = [1 - \theta(1-\pi_1)] \cdot p_\infty(a_1) + \theta\pi_2 \cdot [1 - p_\infty(a_1)]\,.$$

The solution to this linear equation is

$$p_\infty(a_1) = \frac{\pi_2}{1-\pi_1+\pi_2},$$

which is independent of parameter θ. An analogous argument shows that

$$p_\infty(a_2) = \frac{1-\pi_1}{1-\pi_1+\pi_2} = 1 - \frac{\pi_2}{1-\pi_1+\pi_2}.$$

Action a_1 (say) having a higher asymptotic probability than action a_2, $p_\infty(a_1) > p_\infty(a_2)$, indicates a "revealed" preference of a_1 over a_2; i.e. $p(a_1, a_2) > p(a_2, a_1)$. Clearly, p_∞ itself is one utility function for such a simple, trivial relationship. But utility functions satisfying axioms U1–U6 can also be obtained in less trivial cases in much the same way (for details, see (Suppes 1961a, 193–199); also (Suppes and Atkinson 1960, Chapter 11)). The stochastic utility functions so obtained are *not* like the usual expected utility functions characteristic of game theory (as defined above), but are rather characterized by the theory of choice probabilities. *Suppes own economic theory is based upon choice probabilities and not upon the typical utility function.* We shall review this theory in the final section of this paper.

4. *Suppes' Theory of Consumer Demand*

Suppes set as a scientific research program the development of an economic theory based upon the notion of stochastic probability: "It would be desirable to develop the utility theory begun in Chapter 11 into a new theory of consumer demand" (Suppes and Atkinson 1960, 265). We shall reconstruct here such a theory, as developed by Suppes himself.

Walrasian demand theory explains the observed behavior of the consumer — his demand function — in terms of utility: the consumer behaves as he does because there exists a utility function that rationalizes his choices. Any application of this theory presupposes that the consumer has consistent preferences over time (that his preferences remain fixed for some relevant interval of time) and, moreover, that he has unlimited computational powers that allow him to keep maximizing his utility all of the time. After arguing that such a theory

> is utterly mistaken as a psychological account of how any of us go about making decisions about practical problems or solving theoretical ones. (Suppes 2001, 140)

Patrick Suppes has proposed a different theory to explain the observed behavior of the consumer, one based upon the notion of free association, habits, and random utilities. According to Suppes, far from spending his time with complicated computations aimed at maximizing a rigid utility function,

> [t]he rational individual, who is choosing gladly and happily, is one who is freely associating and choosing that one of the available set that seems most attractive, because of the depth of past associations that are brought up, as can be the case in buying a house, or, in other instances, by the association to anticipated events. (Suppes 2001, 149)

The distinction between the observational *explanandum* and the theoretical (relative to the Walrasian theory) *explanans* is better seen from the point of view of the concept of choice structure. This concept will facilitate also the comparison between Walras' theory and the one proposed by Suppes.

Roughly speaking, a choice structure is a set of "situations" in which a given agent must make some choice among a certain set of possible options or "actions". The set of possible actions is designated as X and the family of situations is just a collection $\mathscr{B}$ of nonempty subsets of X. The choices made by the agent are described by a correspondence ν that assigns to each situation $B \in \mathscr{B}$ a nonempty set $\nu(B)$ of X, namely the set of those points of X any of which is fine for the agent under such a situation. For the sake of the record, the formal definition of a choice structure is as follows.

Definition 6. $\mathfrak{C}$ *is a* choice structure *iff there exist* X, $\mathscr{B}$ *and* ν *such that*

0. $\mathfrak{C} = \langle X, \mathscr{B}, \nu \rangle$*;*
1. X *is a nonempty set;*
2. $\mathscr{B}$ *is a family of nonempty sets of* X*;*
3. $\nu\colon \mathscr{B} \to X$ *is a correspondence that assigns to each* $B \in \mathscr{B}$ *a nonempty set* $\nu(B)$ *of* X*;*
4. $\forall B \in \mathscr{B}\colon \nu(B) \subseteq B$.

One important application of the concept of a choice structure is as follows. Having in sight the behavior of a consumer, X is interpreted as a subset of nonnegative vectors of $\mathbb{R}^L$ (called 'consumption menus'), the price systems that he may face are thought of as L-dimensional real positive vectors $\mathbf{p} = (p_1, \ldots, p_L)$, while the income levels that he may enjoy are seen as nonnegative real numbers w. If so, a choice structure can be used to represent the choices made by the agent under conditions of prices-income $(\mathbf{p}, w)$ if the elements B of $\mathscr{B}$ are thought of as "budget sets"; i.e. sets of the form $B_{\mathbf{p},w} = \{\mathbf{x} \in X \mid \mathbf{px} \leq w\}$. For every combination of prices and income level (wealth) $(\mathbf{p}, w)$, the choice correspondence ν describes the choice of consumption menus that the consumer actually makes in set $B_{\mathbf{p},w}$. Under this interpretation, correspondence ν is in fact a demand correspondence.

A Walrasian demand correspondence, on the other hand, is a correspondence which is derived from a given preference relation. The gist of classical demand theory is to approximate the observed demand correspondence ν by means of a Walrasian demand correspondence μ generated by a regular (connected and transitive) preference relation that satisfies certain properties, specifically those enlisted in the following definition.

Definition 7. $\mathfrak{P}$ *is a* classical preference structure *iff there exist* X *and* $\succsim$ *such that*

0. $\mathfrak{P} = \langle X, \succsim \rangle$;
1. $\langle X, \succsim \rangle$ *is a regular preference structure;*
2. $\succsim$ *is strictly convex;*
3. $\succsim$ *is locally unsatiated;*
4. $\succsim$ *is differentiable.*

These axioms guarantee the existence of a strictly quasiconcave, locally unsatiated, C^1 utility function u representing $\succsim$. But the existence of a utility function with such properties generates a Walrasian demand correspondence (structure), in the precise sense of the following theorem.

Theorem 8. *Let* $\mathfrak{P} = \langle X, \succsim \rangle$ *be a classical preference structure and* u *a strictly quasiconcave, locally unsatiated,* C^1 *utility function representing* $\mathfrak{C}$. *Moreover, let* $\mu\colon \mathscr{B} \to X$ *be the correspondence that assigns to each* $B_{\mathbf{p},w} \in \mathscr{B}$ *the optimum vectors with respect to* u *within set* $B_{\mathbf{p},w}$. *Then* μ *is a Walrasian demand correspondence and* $\langle X, \mathscr{B}, \mu \rangle$ *is called the Walrasian demand structure generated by* $\mathfrak{P}$.

(The proof is omitted.)

Classical demand theory takes as empirical structures, i.e. as structures given empirically and to be accounted for, demand structures $\langle X, \mathscr{B}, \nu \rangle$ obtained out of the relevant observations. It then postulates a preference relation $\succsim$ and corresponding utility function (with the properties given in the theorem) and generates with the latter a Walrasian demand structure $\langle X, \mathscr{B}, \mu \rangle$. The empirical claim is that μ "approximates" ν "fairly well". Notice that ν is obtained out of empirical observations, while μ is derived from a postulated preference relation. More concisely, the structure of the theory is given in the following definition.

Definition 9. $\mathfrak{C} = \langle X, \mathscr{B}, \nu, \succsim \rangle$ *is a* classical demand structure *iff*

1. $\langle X, \mathscr{B}, \nu \rangle$ *is a demand structure;*
2. $\langle X, \succsim \rangle$ *is a classical preference structure;*
3. $\langle X, \mathscr{B}, \mu \rangle$ *is generated by* $\langle X, \succsim \rangle$.
4. $\forall B \in \mathscr{B}$: $\nu(B) \approx \mu(B)$.

Axiom 4 expresses the *fundamental law* of classical demand theory, to wit, that the consumption choices of the agent are "approximately rationalizable", in the sense that they maximize *some* preference relation. An *empirical claim* of the theory is a sentence expressing that certain empirical data can be embedded in a demand function ν that satisfies approximately the fundamental law.

What is the difference between classical demand theory and Suppes' demand theory? Concisely stated, it seems to me that the difference is twofold. First of all, habits play a central role in Suppes' theory. Since habits function as filters that discard certain possible choices, I suggest that the concept of habit can be represented as a certain restriction upon the sets within $\mathscr{B}$: $B_{\mathbf{p},w}$ would be the set of all choices within budget set $\{\mathbf{x} \in X \mid \mathbf{px} \leq w\}$ not discarded by the habits. Second, momentary associations are more likely to influence choice behavior than a fixed, rigid preference relation over X. In order to theorize over these associations, Suppes introduces the concept of a random utility structure, which would perform a role analogous to the one played by classical preference structures in classical demand theory.

Before considering random utility structures, it may be useful to see that the empirical structures to which this theory is applied are no longer choice structures, but rather structures of choice probabilities, a concept that can be defined as follows.

Definition 10. $\mathfrak{C}$ *is a* structure of choice probabilities *iff there exist* X, $\mathscr{B}$ *and* ν *such that*

0. $\mathfrak{C} = \langle X, \mathscr{B}, \nu \rangle$;
1. X *is a nonempty set;*
2. $\mathscr{B}$ *is a family of nonempty sets of* X;
3. $\nu \colon X \times \mathscr{B} \to [0, 1]$ *is a function that assigns to each* $\mathbf{x} \in X$ *and* $B \in \mathscr{B}$ *a number* $\nu(\mathbf{x}, B)$ *such that* $\sum_{\mathbf{y} \in B} \nu(\mathbf{y}, B) = 1$, *and* $\nu(\mathbf{x}, B) = 0$ *if* $\mathbf{x} \notin B$.

Number $\nu(\mathbf{x}, B)$ is the probability that $\mathbf{x}$ be chosen if conditions B obtain. Now, just as the Walrasian demand correspondence μ, devised to approximate ν, was obtained out of a utility function, a probability function μ is obtained out of a random utility structure. In order to introduce its precise definition, consider first a finite set X with cardinal N and the set R of all possible preference rankings of elements of X. The ranking with more indifference classes contains N (when no two members of X are indifferent), and so we may take set $U = \{1, \ldots, N\}$ as the image of any utility function representing a ranking in R. In contradistinction to what happens in classical preference structures, where one element $r \in R$ is singled out as *the* preference ranking of the agent, "the subjective value of each alternative fluctuates, and the alternative with the highest momentary value is selected" (Suppes *et al.* 1989, 421). Hence, for each $\mathbf{x} \in X$ we have a random variable $\mathbf{U}_{\mathbf{x}}$ that may take any of the values in U. The formal definition is as follows.

Definition 11. $\mathfrak{R}$ *is a* random utility structure *iff there exists a nonempty set* X *and a collection* $\mathbf{U} = \{\mathbf{U}_{\mathbf{x}} \mid \mathbf{x} \in X\}$ *of jointly distributed random variables such that* $\mathfrak{R} = \langle X, \mathbf{U} \rangle$.

Just as a choice structure is generated by a regular preference structure, a structure of choice probabilities is generated by a random utility structure, if a given family of "budget sets" $\mathscr{B}$ is given. This is the gist of the following obvious theorem.

Theorem 12. *Let* $\mathfrak{R} = \langle X, \{\mathbf{U}_{\mathbf{x}} \mid \mathbf{x} \in X\} \rangle$ *be a random utility structure and* $\mathscr{B}$ *a family of nonempty sets of* X. *Let* μ *be a function such that*

$$\mu(\mathbf{x}, B) = Pr(\mathbf{U}_x = \max\{\mathbf{U}_y \mid y \in B\}).$$

Then $\langle X, \mathscr{B}, \mu \rangle$ *is a structure of choice probabilities, called* the structure of choice probabilities generated by $\mathfrak{R}$.

The theorem is obvious because, by construction, $\langle X, \mathscr{B}, \mu \rangle$ satisfies the random utility model $\mathfrak{R}$. Quite another problem is whether, given a structure of choice probabilities $\mathfrak{C} = \langle X, \mathscr{B}, \nu \rangle$, there exists a random utility structure $\mathfrak{R}$ such that $\mathfrak{C}$ is (approximately) the structure of choice probabilities generated by $\mathfrak{R}$. In such a case we would have an analogue of what is usually called a "rationalization" of a choice structure. An "observable" condition called 'nonnegativity' is necessary and sufficient for a closed structure of choice probabilities to satisfy a random utility model (cf. Definition 17.14 and Theorem 17.10 in Suppes *et al.* 1989, 423) This condition is an analogue of what in classical demand theory is known as Samuelson's strong axiom of revealed preference. Within the framework of his doctrine on freedom (see Suppes 1996, 1987b, 1997), Suppes has developed also a measure of the freedom of markets, based upon the concept of entropy. Basically, the idea is that the amount of options, as well as the degree of uncertainty in the choices of the agent, are a measure of freedom. For instance, if there are n goods and p_{ij} is the probability that any agent will change his choice from good of type i to good of type j, then a measure of the freedom of the market of these goods is $-\sum p_{ij} \log p_{ij}$. The larger this number, the freer is the market. A study of transition data observed six times for 264 buyers choosing one of eight soft-drink brands (Bass 1974) quoted in (Suppes 2001) shows that the markets of Coke and Pepsi are the most stable. The total entropy rate of the soft-drink market — the p_i weighted average of the row entropies — is 1.85 (see the table).

In general terms, given a stochastic process, an indexed family $\mathfrak{r} = \{\mathbf{X}_n\}$ of random variables over a set X with $n = 1, 2, 3, \ldots$, the entropy rate $H(\mathfrak{r})$ of $\mathfrak{r}$ is defined as

$$H(\mathfrak{r}) = \lim_{n \to \infty} \frac{1}{n} H(\mathbf{X}_1, \ldots, \mathbf{X}_n)$$

where $H(\mathbf{X}_1, \ldots, \mathbf{X}_n) = -\sum_{i=1}^{n} p_i \log p_i$ is the entropy of the first n random variables. The data on soft-drink choices are also experimental, but Suppes (also in Suppes 2001) includes a study on car choices taken from the U.S. car industry statistics. It is interesting that the total entropy of the national sample is 1.84, a figure quite close to the one obtained under the more controlled conditions of the soft-drink experiment! This

$n/(n+1)$	Coke	7-Up	Tab	Like	Pepsi	Sprite	D-Pep	Fresca	p_∞	Entropy
Coke	0.61	0.11	0.01	0.03	0.13	0.06	0.01	0.04	0.29	1.45
7-Up	0.19	0.45	0.00	0.06	0.14	0.10	0.01	0.05	0.18	1.82
Tab	0.08	0.12	0.16	0.36	0.08	0.04	0.08	0.08	0.03	2.38
Like	0.09	0.15	0.09	0.15	0.24	0.04	0.13	0.11	0.06	2.55
Pepsi	0.18	0.13	0.01	0.03	0.51	0.07	0.03	0.04	0.23	1.68
Sprite	0.11	0.18	0.03	0.07	0.16	0.33	0.03	0.09	0.10	2.27
D-Pep	0.09	0.05	0.18	0.09	0.12	0.09	0.26	0.12	0.04	2.52
Fresca	0.22	0.09	0.05	0.11	0.15	0.11	0.07	0.20	0.07	2.37
Total entropy										1.85

Transition matrix for soft-drink choices

brings Suppes' methodology out of the laboratory into real-life markets, suggesting that carefully planned experiments might yield results that are close to the ones that obtain in the open waters of the economy. It also suggests that the notion of random utility structure might be more plausible than the usual utility concept in the scientific analysis of consumer demand. For an interesting example of its application in real-life markets, the reader is referred to (McFadden and Train 2000).

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REFERENCES

Bass, R.M. 1974. The Theory of Stochastic Processes and Brand Switching. *Journal of Marketing Research* 11: 1–20.

Bourbaki, N. 1968. Theory of Sets. Redding, MA: Addison-Wesley.

Da Costa, N.C.A., and R. Chuaqui. 1988. On Suppes' Set-Theoretical Predicates. *Erkenntnis* 29: 95–112.

García-Bermejo, J.C. 2009. "Sobre la economía y sus métodos". *Enciclopedia Iberoamericana de Filosofía*, vol. 30. Madrid: Editorial Trotta y Consejo Superior de Investigaciones Científicas. ISBN: 978-84-9879-056-6.

García de la Sienra, A. 2009a. "La aplicación a la economía de la concepción estructuralista de las teorías". In (García-Bermejo 2009, 355–366).

García de la Sienra, A. 2009b. La estructura lógica de la teoría de los juegos. *Crítica* 41, 122: 3–27. ISSN: 0011-1503. http://critica.filosoficas.unam.mx/pg/es/numeros_detalle.php?numero=122 (accessed September 24, 2011).

McFadden, D. and K. Train. 2000. Mixed MNL Models for Discrete Response. *Journal of Applied Econometrics* 15: 447–470.

Suppes, P. 1951. A Set of Independent Axioms for Extensive Quantities. *Portugaliae Mathematica* 10: 163–172.

Suppes, P. 1954. Some Remarks on Problems and Methods in the Philosophy of Science. *Philosophy of Science* 21: 242–248.

Suppes, P. and M. Winet. 1955. An Axiomatization of Utility Based on the Notion of Utility Differences. *Journal of Management Science* 1: 259–270. Reprinted in *Mathematical Studies in Management Science*: 284–295, ed. A.F. Veinott, Jr. 1965, Stanford: Stanford University Press.

Suppes, P., D. Davidson and J.C.C. McKinsey. 1955. Outlines of a Formal Theory of Value, I. *Philosophy of Science* 22: 140–160.

Suppes, P. and H. Rubin. 1955. A Note on Two-Place Predicates and Fitting Sequences of Measure Functions. *Journal of Symbolic Logic* 20: 121–122.

Suppes, P. and D. Davidson. 1956. A Finitistic Axiomatization of Subjective Probability and Utility. *Econometrica* 24: 264–275.

Suppes, P. 1956. "The Role of Subjective Probability and Utility in Decision-Making". In *Proceedings of the Third Berkeley Symposium on Mathematical Statistics and Probability, 1954–1955* 5: 61–73.

Suppes, P., D. Davidson and S. Siegel. 1957. *Decision Making: An Experimental Approach*, 121 pp. Stanford: Stanford University Press. Reprinted as Midway Reprint, 1977. Chicago: University of Chicago Press.

Suppes, P. and R.C. Atkinson. 1958. An Analysis of Two-Person Game Situations in Terms of Statistical Learning Theory. *Journal of Experimental Psychology* 55: 369–378.

Suppes, P. and D. Scott. 1958. Foundational Aspects of Theories of Measurement. *Journal of Symbolic Logic* 23: 113–128.

Suppes, P. and K. Walsh. 1959. A Non-Linear Model for the Experimental Measurement of Utility. *Behavioral Science* 4: 204–211.

Suppes, P., H.L. Royden and K. Walsh. 1959. A Model for the Experimental Measurement of the Utility of Gambling. *Behavioral Science* 4: 11–18.

Suppes, P. 1959. "Measurement, Empirical Meaningfulness and Three-Valued Logic." In *Measurement: Definitions and theories*, ed. C.W. Churchman and P. Ratoosh 129–143. New York: Wiley.

Suppes, P. and R.C. Atkinson. 1960. *Markov Learning Models for Multiperson Interactions*. Stanford: Stanford University Press.

Suppes. P. 1960. A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences. *Synthese* 12: 287–301.

Suppes. P. 1961a. Behavioristic Foundations of Utility. *Econometrica* 29: 186–202.

Suppes. P. 1961b. The Philosophical Relevance of Decision Theory. *Journal of Philosophy* 58: 605–614.

Suppes. P. 1962a. Recent Developments in Utility Theory. In *Recent Advancements in Game Theory. Princeton University Conference*, 61–72. Princeton: Princeton University Press.

Suppes. P. 1962b. Models of Data. In *Logic, Methodology and Philosophy of Science: Proceedings of the 1960 International Congress*, ed. E. Nagel, P. Suppes, and A. Tarski, 252–261. Stanford: Stanford University Press. Czechoslovakian translation: Modely dat. In *Teorie modelu a modelovani*, ed. K. Berka and L. Tondl, 223–235. Dordrecht: Reidel, 1967. German translation: Modelle von Daten. In *Zur Logik empirischer Theorien*, ed. M. Balzer and W. Heidelberger, 191–204. Berlin: Walter de Gruyter, 1983.

Suppes, P. and J. M. Carlsmith. 1962. Experimental Analysis of a Duopoly Situation from the Standpoint of Mathematical Learning Theory. *International Economic Review* 3: 60–78.

Suppes, P. and J. L. Zinnes. 1963. Basic Measurement Theory. In *Handbook of Mathematical Psychology*, ed. R.D. Luce, R.R. Bush and E.H. Galanter, 1: 3–76. New York: Wiley.

Suppes, P. and R.D. Luce. 1965. Preference, Utility and Subjective Probability. In *Handbook of Mathematical Psychology*, ed. R.D. Luce, R.R. Bush and E.H. Galanter, 3, 249–410. New York: Wiley.

Suppes, P. 1965. Logics Appropriate to Empirical Theories. In *Theory of Models*, ed. J.W. Addison, L. Henkin and A. Tarski, 364–375. Amsterdam: North-Holland. Reprinted in *The Logico-algebraic Approach to Quantum Mechanics*, ed. C.A. Hooker, 329–340. Dordrecht: Reidel, 1975.

Suppes, P. 1966. Some Formal Models of Grading Principles. *Synthese* 16: 284–306.

Suppes, P. 1967a. What is a Scientific Theory? In *Philosophy of Science Today*, ed. S. Morgenbesser, 55–67. New York: Basic Books.

Suppes, P. 1967b. Decision Theory. In *Encyclopedia of Philosophy* 2: 310–314. New York: Macmillan and Free Press.

Suppes, P. 1968a. The Desirability of Formalization in Science. *Journal of Philosophy* 65: 651–664.

Suppes, P. 1968b. Information Processing and Choice Behavior. In *Problems in the Philosophy of Science*, ed. I. Lakatos and A. Musgrave, 278–299. Amsterdam: North-Holland.

Suppes, P., D.H. Krantz, R.D. Luce and A. Tversky. 1971. *Foundations of Measurement I: Additive and Polynomial Representations*, 577 pp. New York: Academic Press. Reprint Mineola, NY: Dover, 2007.

Suppes, P. 1972a. Measurement: Problems of Theory and Application. In *Mathematics in the Social Sciences in Australia*, 613–622. Canberra: Australian Government Publishing Service.

Suppes, P. 1972b. Finite Equal-Interval Measurement Structures. *Theoria* 38: 45–63.

Suppes, P. and R. D. Luce. 1974. Theory of Measurement. In *Encyclopaedia Britannica* 11, 15th edition 11, 739–745.

Suppes, P. 1974a. The Measurement of Belief. *Journal of the Royal Statistical Society* 36: 160–191 (Series B).

Suppes, P. 1974b. The Axiomatic Method in the Empirical Sciences. In *Proceedings of the Tarski Symposium 1971*, ed. L. Henkin, J. Addison, W. Craig, C.C. Chang, D. Scott and R. Vaught, 465–479. Providence, RI: American Mathematical Society.

Suppes, P. 1974c. The Essential but Implicit Role of Modal Concepts in Science. In *PSA 1972*, ed. K.F. Schaffner and R.S. Cohen, 305–314. Dordrecht: Reidel.

Suppes, P. 1974d. The Structure of Theories and the Analysis of Data. In *The Structure of Scientific Theories*, ed. F. Suppe 2nd edition, 266–283. Urbana, IL: University of Illinois Press.

Suppes, P. 1976. Testing Theories and the Foundations of Statistics. In *Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science*, ed. W.L. Harper and C.A. Hooker, 2: 437–455. Dordrecht: Reidel.

Suppes, P. 1977. The Distributive Justice of Income Inequality. *Erkenntnis* 11: 233–250. Reprinted in *Decision Theory and Social Ethics*, ed. H.W. Gottinger and W. Leinfellner, 303–320. Dordrecht: Reidel, 1977.

Suppes, P. 1979a. The Role of Formal Methods in the Philosophy of Science. In *Current Research in Philosophy of Science*, ed. P.D. Asquith and H.E. Kyburg, Jr., 16–27. East Lansing, MI: Philosophy of Science Association.

Suppes, P. 1979b. El estudio de las revoluciones científicas: teoría y metodología. In *La filosofía y las revoluciones científicas: teoría y praxis*, 295–306. México: Editorial Grijalbo.

Suppes, P. 1982. Rational Allocation of Resources to Scientific Research. In *Logic, Methodology and Philosophy of Science, VI*, ed. L.J. Cohen, J. Los, H. Pfeiffer and K.-P. Podewski, 773–789. Amsterdam: North-Holland.

Suppes, P. 1987a. Axiomatic Theories. In *The New Palgrave: A Dictionary of Economics*, ed. J. Eatwell, M. Milgate and P. Newman, 1: 163–165. New York: Stockton Press.

Suppes, P. 1987b. Maximizing Freedom of Decision. In *Arrow and the Foundations of Economic Policy*, ed. G.R. Feiwel, 243–254. New York: New York University Press.

Suppes, P. 1988a. Representation Theory and the Analysis of Structure. *Philosophia Naturalis* 25: 254–268.

Suppes, P. 1988b. Empirical Structures. In *The Role of Experience in Science, Proceedings of 1986 Conference of the Académie Internationale de Philosophie des Sciences (Bruxelles)*, ed. Erhard Scheibe, 23–33. New York: Walter de Gruyter.

Suppes, P. 1988c. Philosophical Implications of Tarski's Work. *Journal of Symbolic Logic* 53: 80–91.

Suppes, P. 1988d. Lorenz Curves for Various Processes: A Pluralistic Approach to Equity. *Social Choice and Welfare* 5, 89–101. Reprinted in *Distributive Justice and Inequality*, ed. W. Gaertner and P.K. Pattanaik, 1–13. Berlin Heidelberg: Springer-Verlag.

Suppes, P., D.H. Krantz, R.D. Luce and A. Tversky. 1989. *Foundations of Measurement II: Geometrical, Threshold, and Probabilistic Representations*, New York: Academic Press, 493 pp. New edition Mineola, NY: Dover, 2007.

Suppes, P., D.H. Krantz, R.D. Luce and A. Tversky. 1990. *Foundations of Measurement III: Representation, Axiomatization, and Invariance*. New York: Academic Press, 356 pp. New edition Mineola, NY: Dover, 2007.

Suppes, P. 1992. Axiomatic Methods in Science. In *Nature, Cognition and System II*, ed. Marc E. Carvallo, 205–232. Dordrecht: Kluwer Academic Publishers.

Suppes, P. 1993. *Models and Methods in the Philosophy of Science: Selected Essays*. Dordrecht: Kluwer Academic Publishers, 525 pp.

Suppes, P. 1996. The Nature and Measurement of Freedom. *Social Choice and Welfare* 13: 183–200.

Suppes, P. 1997. Freedom and Uncertainty. In *Uncertainty: Models and Measurement, Essays in Honor of R. Duncan Luce*, ed. H.G. Natke and Y. Ben-Haim, 103–109. Mahwah, NJ: Lawrence Erlbaum.

Suppes, P. 2001. Rationality, Habits and Freedom. In *Cognitive Processes and Economic Behavior*, ed. N. Dimitri, M. Basili and I. Gilboa, 137–167. New York: Routledge.

Suppes, P. 2002. *Representation and Invariance of Scientific Structures*. Stanford: CSLI Publications, 636 pp.

Suppes, P. 2007. Statistical Concepts in Philosophy of Science. *Synthese* 154: 485–496.

Adolfo García de la Sienra is research professor at the Instituto de Filosofía of the Universidad Veracruzana. His main area of research is philosophy of economics. He is author of *The Logical Foundations of the Marxian Theory of Value* (Kluwer, Dordrecht) and several papers published in journals like *Erkenntnis, Crítica, Diánoia, Axiomathes* and *Metatheoria*.

Address: Instituto de Filosofía, Universidad Veracruzana, Xalapa, Veracruz, Mexico.
Email: asienrag@gmail.com.