A not so fine modal version of generality relativism

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ABSTRACT: The generality relativist has been accused of holding a self-defeating thesis. Kit Fine proposed a modal version of generality relativism that tries to resist this claim. We discuss his proposal and argue that one of its formulations is self-defeating.

Keywords: absolute generality, generality relativism, indefinite extensibility, expansionism.

1. *Russell's paradox and absolute generality*

Russell's paradox, as standardly interpreted, shows that there cannot be a universal set, that is, a set containing all sets as its elements. This result raises several philosophically interesting questions. For example, given that there is no universal set, it becomes necessary to have some other conception of the collection of all sets and, related to it, an explanation for why is it that there cannot be a set of all sets. The standard set-theoretical reply is to argue that the collection of all sets could not be a set because somehow it is too big to be one. Collections of such kind are usually called *proper classes*.

It can be further argued that Russell's paradox also shows that when we try to generalize over everything, we are not generalizing over the members of a set. If someone puts forward a set U that allegedly corresponds to the collection of absolutely everything, by a reasoning similar to the one employed in the paradox, it will be possible to prove the existence of an object not belonging to U (namely, the set of all non-self-membered sets in U). But what then are we quantifying over?

Independently of what kind of objects belong to the domain of quantification, philosophically speaking, claiming that we are quantifying over the members of a proper class does not seem very satisfactory. Firstly, because it is not clear in what sense might a collection be too big to be a set. Secondly, because the only reason to accept the existence of proper classes is the threat of paradox. Thus, even if appealing to the existence of proper classes somehow avoids the paradox, it clearly fails to explain it.

In the set-theoretical case, a possible move on the face of this difficulty is to argue that Russell's paradox actually shows that there cannot be a completed collection containing all sets: if someone puts forward a collection that purports to be the collection of all sets, it will be possible to prove the existence of a set not belonging to it. Following Michael Dummett (1991), one can then claim that the set-theoretical universe is indefinitely extensible. One can also consider applying this strategy for the absolutely general case. Since an absolutely general domain would need to range over everything, in particular, it would need to range over every set. Thus, it could be claimed that we fail when trying to quantify over absolutely everything because the domain of quantification can always be extended. Let us call the proponent of this claim the *generality relativist*.

David Lewis (1991), Vann McGee (2000) and Timothy Williamson (2003) have put forward similar versions of an argument for the claim that generality relativism is a self-defeating thesis. The problem, as presented by the latter, can be put in the following way. In order to assume that it is not possible to quantify over absolutely everything, one must be committed with the claim that there must be something that is not in the range of the (allegedly) absolutely general quantifier. But then the generality relativist is not only committed with the objects within the range of the quantifier, but also with those lying beyond its reach. In other words, in order to state his thesis, the generality relativist appears to be himself quantifying over absolutely everything.

2. *Fine's version of the extensibility argument*

Our goal in the rest of this paper is to discuss a reformulation of the generality relativist thesis that has been proposed by Kit Fine (2006). For that purpose, we will adopt Fine's terminology, calling *universalist* to the person arguing in favor of the intelligibility of quantifiers ranging over absolutely all sets and *limitativist* to the person arguing against it. Adopting Fine's notation, use I, J,... as variables for interpretations of the quantifiers, and I_0, J_0,... as constants for particular interpretations. Moreover, use $\exists_I x\varphi(x)$, with I as a subscript of the quantifiers, to indicate that there is some x under interpretation I for which $\varphi(x)$.[1] We begin by reviewing the indefinite extensibility argument according to this notation.

The universalist claims to have an absolutely general understanding of quantification, so let us assume that his intended use of the quantifiers conforms to a particular interpretation I_0 of quantification. He may then express his claim as,

1. $\forall x\exists_{I_0} y\,(y = x) \wedge \forall_{I_0} y\exists x\,(x = y)$.

Now, from

2. $\exists_{J_0} y\forall_{I_0} x\,(x \in y \leftrightarrow x \notin x)$

by a Russell style reasoning, we may derive

3. $\exists_{J_0} y\forall_{I_0} x\,(x \neq y)$.

[1] Fine acknowledges that it would be more appropriate to use of a meta-linguistic form of expression like *the sentence* $\exists x\varphi(x)$ *is true under interpretation* I rather than $\exists_I x\varphi(x)$. However, as there will be nothing in the following that conflicts with that convention, in the interest of presentation he adopts the more straightforward notation. We will follow his example on this.

Defining $I \subseteq J$ as $\forall_I x \exists_J y \, (x = y)$, [3] can be rewritten as

3'. $\neg \, (J_0 \subseteq I_0)$.

Let $UR\,(I)$ be I *is absolutely unrestricted.* The universalist claims that

4. $UR\,(I_0) \rightarrow J_0 \subseteq I_0$.

Finally, from [3'] and [4] it follows

5. $\neg UR\,(I_0)$.

The argument from [1] to [5] has shown that the interpretation I_0 is not absolutely unrestricted. But to make its case fully convincing, the limitativist still needs to make a claim about all interpretations. That is, what the limitativist has done up to this point was to show that given an interpretation like I_0 (that is, an interpretation put forward as being absolutely general) it is possible to extend it. However, he wants to make a stronger claim, namely, one concerning the possibility of extending every interpretation. Some further argument thus seems to be needed. Given that the previous argument did not contain any special assumptions about I_0, it might seem that a generalization of that argument would do the trick. So, let the first premise of the new argument be a generalized version of [2].

2G. $\forall I \exists J \exists_J y \forall_I x \, (x \in y \leftrightarrow x \notin x)$.

[2G] seems plausible since it simply says that according to some interpretation of quantification there is the set of all the non-self-membered sets that exist according to some other interpretation of quantification. Then, by reasoning in a similar way to Russell's Paradox, it is possible to show that the range of quantification corresponding to the latter interpretation cannot be contained in the range associated with the former. That is,

3G. $\forall I \exists J \, (\neg \, (J \subseteq I))$.

Define an interpretation I to be *maximal,* $\max\,(I)$*,* if $\forall J \, (J \subseteq I)$. It then becomes possible to express [3G] as

3G'. $\forall I \neg \max\,(I)$.

Generalizing the claim made in [4] we obtain

4G. $\forall I \, (UR\,(I) \rightarrow \max\,(I))$.

And finally, from [3G'] and [4G]

5G. $\forall I \neg UR\,(I)$.

This concludes our reformulation of the extensibility argument in Fine's notation. At first sight [5G] might appear to be the conclusion that the limitativist was after, for it says that there is no interpretation of quantification which is absolutely unrestricted. However, a more careful look will reveal that just as the universalist use of the quantifier over objects was shown not to be absolutely unrestricted, it may also be shown that the quantifier over interpretations is not absolutely unrestricted. In the rest of this section we present Fine's argument to show how could someone try to argue that generality relativism is self-defeating within the framework provided by his notation.

Remember that the limitativist wanted to make a claim concerning the possibility of extending every interpretation. Thus, the quantifier in [5G] needs to range over every interpretation. Thus, assume that there is an interpretation M_0 to which the current interpretation of the quantifier over interpretations conforms:

6. $\forall I \exists_{M_0} J\,(J = I) \wedge \forall_{M_0} J \exists I\,(I = J)$.

Among the several "first-order" interpretations (of quantification over objects) that may be associated with a "second-order interpretation" M (of quantification over interpretations), there is an interpretation I that is maximal.[2] Call the later *the sum interpretation of* M. It will then be the case that if $\exists_M J \exists_J x \varphi\,(x)$ then $\exists_I x \varphi\,(x)$.[3] Now if M_0 is

[2] The following remark lends support to this last claim. If you claim to have a quantifier that allegedly quantifies over absolutely everything, you also have a first-order interpretation of quantification I_n associated with it. As we have seen, by a Russell style reasoning, we are able to extend the range of that quantifier and obtain a new range, encompassing one more object than the previous. Notice that by doing this, we also obtain a new first-order interpretation I_{n+1}. This new range may momentarily be understood as being absolutely general, however by a Russell style reasoning it will also be possible to extend it. This procedure can be iterated indefinitely and can never be seen as completed. We could now describe a quantifier that ranges over the first-order interpretations, our understanding of it corresponding to a second-order interpretation. It seems that its range can also be extended indefinitely. Actually, these extensions are achieved by the exact same procedure that extends the range of a first-order interpretation: expanding a range of quantification presented as being absolutely general amounts to arriving at a new understanding of quantification, that is, at a new first-order interpretation I_{n+1}; in turn, this new interpretation expands the range of quantification over interpretations and allows us to arrive at a new second-order interpretation M_{n+1}. Therefore, to the initial first-order interpretation I_0 corresponds a second-order interpretation M_0, to I_1 corresponds a new second-order interpretation M_1 (notice that the range of quantification corresponding to the second-order interpretation M_1 contains two first-order interpretations I_0 and I_1 while the range corresponding to M_0 only contains I_0) and so on and so forth. Now, Fine claims that among the several first-order interpretations that may be associated with a second-order interpretation, there is a first-order interpretation that is maximal. The justification for this claim seems to be the following. Suppose that we stop the extension process of the range of absolutely general quantification at a certain point. When this process stops, so does the process of extending the range of quantification over interpretations. So, given a second-order interpretation M_n, there will be a first-order interpretation I_n that corresponds to the range that the (allegedly) absolutely general quantifier has when the extension process stopped. It is then clear that I_n will be the maximal first-order interpretation associated with M_n. For instance, the maximal interpretation associated with M_0 is I_0, the maximal interpretation associated M_1 is I_1 and so forth and so on.

[3] Take $\exists_{M_0} J \exists_J x \varphi\,(x)$. Then $\exists_{J_0} x \varphi\,(x)$. Let I_0 be the sum interpretation of M_0. By definition, $J_0 \subseteq I_0$, therefore $\exists_{I_0} x \varphi\,(x)$.

absolutely unrestricted, its sum interpretation will be maximal with respected to every other interpretation. That is

7. $UR(M_0) \rightarrow \exists_{M_0} I \forall_M (J \subseteq I)$

and by definition

8. $UR(M_0) \rightarrow \exists I (\max(I))$.

But given [3G']

9. $\neg UR(M_0)$.

The previous argument thus raises a particularly troubling difficulty for the limitativist. For it appears to have shown that it follows from the truth of its own claim that the interpretation of the quantifier in it cannot be absolutely unrestricted. Remember that for [5G] to have its intended import, the quantifier in it needs to range over every first-order interpretation. In [6] we assumed that M_0 corresponded to the second-order interpretation of that quantifier but [9] says that M_0 is not absolutely unrestricted. It follows from the truth of [9] that there are first-order interpretations lying outside the range of the quantifier in [5G] and so the latter appears to be self-defeating.

3. Fine's modal version of generality relativism

Fine begins his attempt to deal with the issue of absolute generality by proposing a modal formulation of the limitativist position. Use $I \subset J$ for J *(properly) extends* I and say that I *is extensible* — in symbols, $E(I)$ — if possibly some interpretation extends it, i.e. $\Diamond \exists J (I \subset J)$. Limitativism is then characterized in terms of the two following claims:

(L) $\forall I E(I)$

and

(L)$^+$ $\Box \forall I E(I)$ (i.e. $\Box \forall I \Diamond \exists J (I \subset J)$).

The modal limitativist says that the concept delimiting the quantifier is *extensible* if (L) holds and that it is *indefinitely extensible* if (L)$^+$ holds.

Call the notions of possibility and necessity relevant to the formulation of (L) and (L)$^+$ *postulational modalities*. Here is the basic idea behind such notions:

> For suppose someone proposes an interpretation of the quantifier and I then attempt to do a 'Russell' on him. Everyone can agree that if I succeed in coming up with a broader interpretation, then it shows the original interpretation not to have been absolutely unrestricted. Suppose now that no one in fact does a Russell on him. Does that mean that his interpretation was unrestricted after all? Clearly not. All that matters is that the interpretation should be possible. But the relevant

> notion of possibility is then the one we were after; it bears directly on the issue of unrestricted quantification, without regard for the empirical vicissitudes of actual interpretation. (Fine (2006), p. 35)

4. *Interlude*

Before going into the details of the modal formulation, we think that it is worth making a couple of remarks concerning the dialectics of the discussion so far. Remember that Williamson's argument tries to establish that the limitativist position concerning absolute generality is self-defeating. However, since Fine's discussion is particularly concerned with sets (remember his definitions of *universalist* and *limitativist* presented at the beginning of section 2), it seems that he might not be addressing the challenge that Williamson puts forward. On the other hand, it also seems that although the limitativist has difficulties in the absolutely general case, his case still holds within the set theoretical framework. I will now try to show that both of these claims correspond to misinterpretations of the discussion.

Concerning the latter, just consider a variation of Williamson's argument whose first premise would be: "It is impossible to quantify over absolutely every set". One can plausibly assume that this sentence expresses a claim that the limitativist would endorse. Then, by following a similar line of argument, it can easily be seen that we will end up with the following contradictory claim: "Some set over which the limitativist is quantifying at t_0 is not being quantified over by the limitativist at t_0". Thus, by means of a variant of Williamson's argument, the legitimacy of the limitativist position within the set-theoretical framework can also be called into question.

In order to see how Fine actually addresses Williamson's challenge, remember the limitativist strategy to deny the coherence of the notion of *absolutely general domain*: once the universalist puts forward a particular domain purporting to be absolutely general, the limitativist goes on to show that there is a set not belonging to that domain. Williamson then claims that in order to put forward his thesis, the limitativist not only needs to be quantifying over this set but also over all the other sets which are eventually put forward as counter-examples to the universalist claim. The problem raised by this is that quantifying over all these other sets while also quantifying over what the universalist puts forward as being an absolutely general domain of quantification seems to equate with quantifying over absolutely everything (even according to the limitativist's own standards). But then, even if Fine restricts his discussion of the debate between universalism and limitativism to a set-theoretical framework, he ends up addressing Williamson's challenge. That is, if it is possible to show that those other sets are not being quantified over by the limitativist when he puts forward his thesis, the universalist would no longer be able to accuse him of holding a self-defeating thesis.

5. *Restrictionism and expansionism*

We now go back to Fine's proposed version of generality-relativism. According to his terminology, a *postulational possibility* consists in the possibility of reinterpreting the domain of quantification and a proposition (like the one expressed by "There are more sets.") is said to be *postulationally possible* if its truth value depends on the possibility of reinterpreting the domain of quantification. It thus seems that a better understanding of the notion of *postulational modality* can be achieved by trying to grasp how Fine understands the process by means of which the reinterpretation of a quantifier occurs.

One model that he considers, consists in understanding the interpretation of a quantifier as being given by a predicate that restricts the range of the quantifier. Call this view *restrictionism*. According to Fine, a crucial difficulty for restrictionism arises when it tries to give an account of the move that he calls *the Russell jump*. To see this, begin by assuming an initial understanding of quantification represented by $\forall_{I_0}$ and $\exists_{I_0}$. Now form the set R_{I_0} of all the non-self-membered sets in the range of $\forall_{I_0}$. According to limitativism, by a Russell style reasoning we conclude that R_{I_0} could not be in the range of $\forall_{I_0}$ which allows us to (Russell) jump and arrive at a new understanding of quantification (represented by $\forall_{I_1}$ and $\exists_{I_1}$) and according to which $\exists_{I_1} x \forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$. The condition $\forall y\,(y \in x \leftrightarrow y \notin y)$ seems to play a crucial role in the jump in the sense that it is apparently by means of it that the new understanding is given. But how exactly?

According to Fine, the only answer the restrictionist is able to provide is that that condition is used to relax the restriction on the quantifier that is already in play. Thus suppose that $\forall_{I_0}$ is restricted to objects satisfying the predicate $\theta\,(x)$. Hence, $\forall_{I_0} x \varphi\,(x)$ is tantamount to saying $\forall_{I_0} x\,[\theta\,(x) : \varphi\,(x)]$ (to be read: every θ-set is a φ-set). The effect of considering the condition $\forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$ is a weakening of the initial restriction to $\theta\,(x) \vee \forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$. According to this understanding $\forall_{I_1} x \varphi\,(x)$ is tantamount to saying $\forall x\,[\theta\,(x) \vee \forall_{I_0} y\,(y \in x \leftrightarrow y \notin y) : \varphi\,(x)]$ (to be read: every θ-set or set that contains all the non-self-membered sets in the range of $\forall_{I_0}$, is a φ set). Notice however that this proposal does not deliver the right results. We wanted the quantifier $\forall_{I_0}$ to include one new set in its domain, namely, the set of all (and only) the non-self-membered sets in the range of $\forall_{I_0}$. But the restriction $\forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$ not only picks out that set but also all those other sets that have all of the non-self-membered sets in the range of $\forall_{I_0}$ as members. For example, by iteration of the Russell jump we can go on to obtain an understanding of quantification — represented by $\forall_{I_2}$ and $\exists_{I_2}$ — and such that in the range of $\forall_{I_2}$ there is a set R_{I_1} of all the non-self-membered sets in the range of $\forall_{I_1}$. The set R_{I_1} also contains all the non-self-membered sets in the range of $\forall_{I_0}$ (and moreover it contains R_{I_0}) however the first Russell jump clearly does lead us to an understanding of quantification according to which R_{I_1} falls in the range of $\forall_{I_1}$.

We now move on to Fine's account of the Russell's jump. Instead of understanding the condition $\forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$ as defining a new predicate by which the quantifier is to be restricted, he proposes that we should understand it as indicating how the range of the quantifier is to be extended. Fine claims that there is an instruction or procedural postulate associated with the condition $\forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$, requiring us to introduce a set R_{I_0} whose members are all the non-self-membered sets in the range of $\forall_{I_0}$. By introducing R_{I_0} one immediately arrives at a new understanding of the quantifier — again, represented by $\forall_{I_1}$ and $\exists_{I_1}$ — and can reinterpret it as ranging over the domain resulting from adding x to the sets in the range of $\forall_{I_0}$.

According to Fine, this expansionist way of understanding the reinterpretation of a quantifier has several advantages over restrictionism. One of them is that through the expansionist approach, one is sure to obtain the desired extension of the domain. This is guaranteed by the nature of the postulational method: it is not possible to postulate into existence a set that stands in the membership relation to sets that do not belong to the range of the quantifier. For example, the Russell jump by which we arrive at the understanding of quantification represent by $\forall_{I_1}$ does not let in too many sets, since the condition $\forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$ only allows for the postulation of one set, namely, the set R_{I_0} containing all and only the non-self-membered sets in the range of $\forall_{I_0}$. The postulation of the set R_{I_1}, for instance, is not allowed by the condition $\forall_{I_0} y\,(y \in x \leftrightarrow y \notin y)$ because R_{I_1} stands in the membership relation with R_{I_0} (in the sense that $R_{I_0} \in R_{I_1}$) and R_{I_0} is not in the range of $\forall_{I_0}$.

Fine claims that under the restrictionist account, the old and new domains are to be understood as restrictions, but these are not to be understood as restricting some broader domain.[4] Under the expansionist account, on the other hand, the new domain is not to be understood as a new restriction but rather as an expansion. What we are provided with is a way of seeing how it might be expanded. The crucial difference between these approaches seems to be the following. As pointed out, on the restrictionist account there is the risk that the reinterpretation of the quantifier over sets by means of a new restricting predicate lets in too many sets. However, even supposing that this does not happen, belief that the domain has actually been extended and that there is a new set, is not automatically justified. For according to the restrictionist account, the Russell jump consists in relaxing the restriction imposed on the quantification over sets and subsequently adopting a new restriction according to which there might be a new set. However, the existence of this new set and actual expansion of the range of quantification are not guaranteed by reinterpretation of the quantifier. By contrast, under the expansionist account, the new understanding of quantification is arrived at by the discovery of a set that is not in the

[4] Actually, it seems hard to make sense of such a possibility. If the interpretations are not restricting some wider domain, then what are they restricting? That is, if they are not restricting some wider domain, in what sense are they restricting?

range of quantification over sets and that, as a set, should belong to it. Thus, success in the act of reinterpretation cannot be understood independently of the belief in the existence of a set that provokes the extension of the domain.

6. Relatively unrestricted quantification

Fine (2006, p. 40) endows the universalist with the view that there is absolutely unrestricted quantification and claims that this view should be understood as the conjunction of two distinct positions:

> But if I am right, the view is really a conjunction of two distinct positions, one signified by 'unrestricted' and the other by 'absolutely'. The first is the affirmation of unrestricted (i.e. completely unrestricted) quantification. The second is the rejection of any relativity in the interpretation of the quantifier beyond a restriction on its range; once the range of the quantifier has been specified by means of a suitable predicate, or even by the absence of a predicate, then there is nothing else upon which its interpretation might depend.

I must confess that I am not completely sure of having understood this characterization. My difficulty lies in the fact that according to it universalism allows for a form of absolutely unrestricted quantification that is restricted by a concept.

Remember that according to limitativism, quantification over sets is always relative to a particular interpretation and that by the Russell jump we are able to arrive at other interpretations. Let us again suppose that we are quantifying over sets. The universalist will claim that it is possible to quantify over absolutely every set, thus denying that quantification always needs to be relative to some interpretation. This corresponds to the *absolutely* part of the above characterization and to me it appears to be clear. What I find confusing is that according to the above characterization, even though the range of the quantifier is restricted to sets, this quantification might still be called *absolutely unrestricted*. Why not call it *absolutely restricted*?[5] The universalist would then reserve *absolutely unrestricted* to the case where no predicate delimits the range of the quantifier, like when he (allegedly) quantifies over absolutely everything.

I think that this same distinction might be useful for characterizing the position held by Fine. According to his version of limitativism, quantification always has to be understood as being relative to an interpretation and this is why he defends that the most general form of quantification we might come up with is a *relatively unrestricted quantification*: even if the range of the quantifier is not restricted by any concept, it is always relative to some interpretation of quantification. On the other hand, in the case of quantification over sets we have a *relatively restricted quantification*: the range is restricted in such a way

[5] It might sound odd to call quantification over absolutely all sets *absolutely restricted* but I guess that it is less confusing than calling it *absolutely unrestricted* while being restricted to sets. There might be a worry that by adopting this distinction, Fine's expansionist limitativism collapses into a form of restrictionism. I will try to explain why this is not so in the following footnote.

that nothing other than sets belongs to it; moreover, the range is always relative to some interpretation of the quantifier.[6]

7. *Expansionism and (L)*

Finally, I want to go back to expansionism and the question of how it might allow us to provide a reply to what I have called Williamson's challenge. Remember that to win this challenge, the generality relativist needs to put forward a non-self-defeating version of his thesis. I take it that so far this challenge has not been won in any decisive way and that expansionism is to be seen as an attempt at it. At least, that is what seems to be implied by Fine (2006, p. 41) in the following passage:

> The restrictionists have operated within an unduly limited model of how domain extension might be achieved; and I believe that it is only by embracing expansionism that a more adequate account of domain extension and a more viable form of opposition to universalism can be sustained.

Remember that Fine proposed characterization of limitativism was given in terms of the two following formulas:

(L) $\forall I E(I)$

and

(L)$^{+}$ $\Box \forall I E(I)$ (i.e. $\Box \forall I \Diamond \exists J (I \subset J)$.

I will now try to show that (L) is self-defeating.

Recall what was explained in footnote 2 concerning the relation between the quantifier ranging over first-order interpretations and the quantifier ranging over all second-order interpretations: the extension of the latter is achieved by the exact same procedure that extends the range of a first-order interpretation, since expanding a range presented as being absolutely general amounts to arriving at a new understanding of absolutely

[6] I now try to explain why is it that the adoption of this distinction within Fine's proposed framework does not cause his expansionist limitativism to collapse into a form of restrictionism. According to the above characterization of restrictionism, it is the concept that restricts the range of the quantifier, not allowing it to be absolute. On the other hand, according to expansionism it is the interpretation of the quantifier that restricts its range, not allowing it to be absolute. This is clear in the case of relatively unrestricted quantification, because even though there is no concept restricting the range of quantification, this is not absolute but rather relative. Now, as an illustration of what I have called *relatively restricted quantification,* take the case in which the range of the quantifier is restricted to sets. The fact that it is always possible to find sets outside the range of quantification cannot be explained by quantification being restricted to sets. After all, all of these other sets are also sets. That is, they also fall within the extension of the concept *set* . Therefore, the fact that it is always possible to find sets outside the range of quantification needs to be uniquely explained in terms of interpretations of quantification. Consequently, adoption of the distinction previously suggested does not cause expansionism to collapse into restrictionism.

general quantification, that is, at a new first-order interpretation. Thus, Fine is not only committed with,

10. $\forall IE(I)$

but also with,

11. $\forall ME(M)$.

Notice that if [10] has its intended import, it must range over absolutely every first-order interpretation. It then needs to be the case that

12. $\exists M \neg E(M)$

and by simple quantificational logic, we easily arrive at

13. $E(M_0) \wedge \neg E(M_0)$.

The original point in this argument is that [11] is said to follow from the truth of [10] by the relation between first and second-order interpretations. But actually [11] (which says that every second-order interpretation can be extended) is perfectly consistent with what Fine had already established in his argument from [6] to [9]. The latter established that assuming the existence of an absolutely unrestricted second-order interpretation is self-defeating, so it could even be said that it provides independent support for [11]. Given this, what becomes puzzling is how should the quantifier in [10] be interpreted. Its interpretation cannot be absolutely unrestricted and yet we are obliged to assume that the quantifier ranges over absolutely every first-order interpretation (for otherwise the limitativist thesis could not have its intended import). But that is what being an absolutely unrestricted second-order interpretation consists in, therefore the contradictory [13].

Should we conclude from this that (L) is self-defeating? If there is a particular second-order interpretation associated with the universal quantifier in (L), then the above argument shows that to be the case. On the other hand, someone might try to claim that the above argument is flawed because [12] does not follow from [10]. The justification for that inferential move was that [12] must be the case given [10]'s intended meaning. However, maybe it could be argued that such a justification actually illustrates a misunderstanding of [10]'s intended meaning. Since [10] says that every interpretation is indefinitely extensible, it should be understood as applying to *every* interpretation, no matter what order it might have. Remember that by extending the range of a universal quantifier having a first-order interpretation I_n associated with it, we arrive at a new first-order interpretation I_{n+1}. But that extension also implies an extension of $\forall_{M_n}$. Hence, the original extension allows us to arrive at a new second-order interpretation M_{n+1}. Now notice that the same is true for third-order interpretations, fourth-order and so forth and so on. So, by extending the range of $\forall_{I_n}$ we not only arrive at a new first-order interpretation of quantification I_{n+1} but also at a new interpretation for each higher order. Given this, it could be claimed that the universal quantifier in [10] is to be understood as ranging over all these interpretations. That being the case, it does not follow from [10] that there is a second-order interpretation M_0 that cannot be extended. It is obvious that the argument no longer runs if this inference is not allowed.

Is this line of argument available to Fine? In particular, can he make the claim that the universal quantifier in [10] is to be understood as ranging over all interpretations? Remember that Fine presents himself as a believer in relatively unrestricted quantification, that is, he does not admit the existence of an absolutely unrestricted quantifier. The difference between both understandings of quantification is precisely that in the case of the former, a domain is always relative to some interpretation of quantification and consequently, there cannot be quantification independent of some interpretation. But then this means that the reasoning developed in the previous paragraph does not allow Fine to block the argument from [10] to [13].

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