On the reality and meaning of the wave function

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1 Introduction

The physical meaning of the wave function is an important interpretative problem of quantum mechanics. Notwithstanding nearly ninety years of development of the theory, it is still an unsolved issue. During recent years, more and more research have been done on the ontological status and meaning of the wave function (e.g. Monton 2002; Lewis 2004; Gao 2011a, 2011b; Pusey, Barrett and Rudolph 2012; Ney and Albert 2013). In particular, Pusey, Barrett and Rudolph (2012) demonstrated that, under certain nontrivial assumptions such as preparation independence assumption, the wave function of a quantum system is a representation of the physical state of the system[1]. This poses a further question, namely whether the reality of the wave function can be argued without resorting to nontrivial assumptions . Moreover, a harder problem is to determine the ontological meaning of the wave function, which is still a hot topic of debate in the alternatives to quantum mechanics such as the de Broglie-Bohm theory (Belot 2012).

In this article, we will first give a clearer argument for the reality of the wave function in terms of protective measurements, which does not depend on nontrivial assumptions and can also overcome existing objections. Next, based on an analysis of the mass and charge properties of a quantum system, we will propose a new ontological interpretation of the wave function. According to this interpretation, the wave function of an N-body system represents the state of motion of N particles. Moreover, the motion of particles is discontinuous and random in nature, and the modulus squared of the wave function gives the probability density that the particles appear in certain positions in space.

2 On the reality of the wave function

The meaning of the wave function in quantum mechanics is usually analyzed in the context of conventional impulsive measurements. Although the wave function of a quantum system is in general extended over space, an ideal position

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[1]For more discussions of this result, see Colbeck and Renner (2012); Lewis et al (2012); Schlosshauer and Fine (2012, 2013); Leifer and Maroney (2013); Patra, Pironio, and Massar (2013); Wallden (2013).

measurement will inevitably collapse the wave function and can only detect the system in a random position in space. Thus it seems natural to assume that the wave function is only related to the probabilities of these random measurement results as in the standard probability interpretation. However, it has been known that the wave function of a single quantum system can be protectively measured (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009). During a protective measurement, the measured state is protected by an appropriate procedure (e.g. via the quantum Zeno effect) so that it neither changes nor becomes entangled with the state of the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system, even if the system is initially not in an eigenstate of the measured observable, and in particular, the wave function of the system can also be measured as expectation values of certain observables. It is expected that protective measurement will be realized in the near future with the rapid development of quantum technologies.

What are the physical implications of protective measurements for the ontological status of the wave function? Several authors, including the discoverers of protective measurements, have given some analyses of this question (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Anandan 1993; Dickson 1995). However, these analyses have been neglected by most researchers, and they are also subject to some objections (Dass and Qureshi 1999; Gao 2013a; Lewis 2014; Schlosshauer and Claringbold 2014). Here we will first present a clearer argument for the reality of the wave function in terms of protective measurements, and then answer these objections.

According to quantum mechanics, we can prepare a single measured system whose wave function is $\psi(t)$ at a given instant t. The question is whether the wave function refers directly to the physical state of the system or merely to the state of an ensemble of identically prepared systems (which is also called state of knowledge). This question can hardly be answered by analyzing a non-protective impulsive measurement of the system, by which one obtains one of the eigenvalues of the measured observable, and the expectation value of the observable can only be obtained by calculating the statistical average of the eigenvalues for an ensemble of identically prepared systems. Now, by a protective measurement on the measured system, we can directly obtain the expectation value of the measured observable. Moreover, by a series of protective measurements of certain observables on this system, we can also obtain the value of $\psi(t)$. Since we can measure the wave function *only* from a single prepared system by protective measurements, the wave function cannot refer only to the state of an ensemble of identically prepared systems, but must directly represent the physical state of a single system. Similarly, expectation values of observables are also properties of a single system.

There are two possible objections to the above conclusion that protective measurements support the reality of the wave function. The first is based on the requirement that an unknown state can be measured. It claims that since an unknown state cannot be protectively measured, protective measurements do not have implications for the ontological status of the wave function. However, this requirement is no doubt too strong. If it were true, then no argument for the reality of the wave function including the PBR theorem could exist, because it is a well-known result of quantum mechanics that an unknown quantum state cannot be measured. On the other hand, it is also worth noting that protective measurements alone cannot imply the reality of the wave function. In both the PBR theorem and the above arguments, a realist view on the relationship between theory and reality is implicitly assumed, according to which the theoretical terms expressed in the language of mathematics connect to the entities existing in the physical world. Under this assumption, when preparing a physical system in a given wave function, the wave function refers either to the physical state of the system or to the state of an ensemble of identically prepared systems. As argued above, it is here that protective measurements can help determine which interpretation is true.

The second objection concerns realistic protective measurements (Dass and Qureshi 1999; Schlosshauer and Claringbold 2014). A realistic protective measurement can never be performed on a single quantum system with absolute certainty. For example, for a realistic protective measurement of an observable A on a non-degenerate energy eigenstate whose measurement interval T is finite, there is always a tiny probability proportional to $1/T^2$ to obtain a different result $\langle A \rangle_\perp$, where $\perp$ refers to a normalized state in the subspace normal to the measured state as picked out by the first order perturbation theory[2]. It thus claims that this precludes an ontological status for the wave function. However, this objection is not valid either. On the one hand, the probability of obtaining a different result can be made arbitrarily small in principle when T approaches infinity. Our above argument is based only on the existence of this limit, in which an ideal protective measurement can be performed on a single quantum system with absolute certainty, and it is not influenced by the uncertainty of realistic protective measurements. On the other hand, it can be argued that even realistic protective measurements also support the reality of the wave function. When a realistic protective measurement obtains the right result, namely the expectation value of the measured observable in the measured state, the state of the whole system including the measured system and the measuring device collapses to the state corresponding to this result. In this result state, the states of the measured system and the measuring device are correlated, and the state of the device reflects the state of the measured system. Since the state of the system in this result state is still the original measured state, this result of measurement will reflect the original state of the measured system. This means that realistic protective measurements *can* also measure expectation values of observables as well as the wave function from a single system. Therefore, although the probability of a realistic protective measurement obtaining a right result is smaller than one, these measurements have the same efficiency to derive the reality of the wave function as ideal protective measurements.

Interestingly, we can also give another argument for ψ-ontology in terms of protective measurements, which is similar to the argument used by the PBR theorem (Pusey, Barrett and Rudolph 2012). For two arbitrary nonorthogonal states of a quantum system, select an observable whose expectation values in these two states are different. Then the results of protective measurements of the observable on these two states are different with probability that can be arbitrarily close to one (e.g. when the measurement interval T approaches infinity). If there exists a non-zero probability p that these two nonorthogonal states

[2]After obtaining the result $\langle A \rangle_\perp$, the measured state collapses to the state $\perp$ according to standard quantum mechanics. In this case, the result of the protective measurement does not reflect the original measured state, but reflects the resulting state.

correspond to the same physical state λ, then when assuming λ determines the probability distribution of measurement results as the PBR theorem assumes, the results of protective measurements of the above observable on these two states will be the same with probability not smaller than p, or in other words, these results will be different with probability not larger than $1 - p$. Since p is a determinate number, this leads to a contradiction. This argument, like the above one, only considers a single quantum system, and thus avoids the preparation independence assumption used by the PBR theorem.

Finally, we note that there might also exist other components of the underlying physical state, which are not measureable by protective measurements and not described by the wave function, e.g. the positions of the Bohmian particles in the de Broglie-Bohm theory. In this case, the wave function is still uniquely determined by the underlying physical state, though it is not a complete representation of the physical state. Certainly, the wave function also plays an epistemic role by giving the probability distribution of measurement results according to the Born rule. However, this role will be secondary and determined by the complete quantum dynamics that describes the measurement process, e.g. the collapse dynamics in dynamical collapse theories.

3 Meaning of the wave function

If the wave function represents the physical state of a single system, then what physical state does it represent? In this section, we will further investigate the ontological meaning of the wave function. We will first analyze one-body systems and then analyze many-body systems.

3.1 One-body systems

For a one-body quantum system, its spatial wave function in position x at instant t, $\psi(x,t)$, represents the physical state of the system in position x at instant t. This means that for a one-body system, there is a physical entity spreading out over a region of space where the spatial wave function of the system is not zero. In the following, we will analyze the existing form of the physical entity. The analysis may provide an important clue to the ontological meaning of the wave function.

First of all, we will argue that for a one-body quantum system with mass m and charge Q, the corresponding physical entity described by its wave function, $\psi(x,t)$, is massive and charged, and the effective mass and charge density in each position x is $|\psi(x,t)|^2 m$ and $|\psi(x,t)|^2 Q$, respectively.

The existence of effective mass and charge distributions can be seen from the Schrödinger equation that governs the evolution of the system. The Schrödinger equation for the system in an external electrostatic potential $\varphi(x)$ is

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = [-\frac{\hbar^2}{2m}\nabla^2 + Q\varphi(x)]\psi(x,t), \tag{1}$$

The electrostatic interaction term $Q\varphi(x)\psi(x,t)$ in the equation indicates that the physical entity described by $\psi(x,t)$ has electrostatic interaction with the external potential in all regions where $\psi(x,t)$ is nonzero. The existence of electrostatic interaction with an external potential in a given region means that there exists electric charge distribution in the region, which has efficiency to interact with the potential and is responsible for the interaction. Therefore, the physical entity described by $\psi(x,t)$ is charged in all regions where $\psi(x,t)$ is nonzero. In other words, for a charged one-body quantum system, the corresponding physical entity described by its wave function has effective charge distribution in space. Similarly, the existence of effective mass distribution can be seen from the Schrödinger equation for a one-body quantum system in an external gravitational potential:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = [-\frac{\hbar^2}{2m}\nabla^2 + mV_G]\psi(x,t). \tag{2}$$

The gravitational interaction term $mV_G\psi(x,t)$ in the equation indicates that the (passive gravitational) mass of the system distributes throughout the whole region where its wave function $\psi(x,t)$ is nonzero. In other words, the physical entity described by the wave function also has effective mass distribution.

The effective mass and charge distributions manifest more directly during a protective measurement, which can measure the actual physical state of a single quantum system. Consider an ideal protective measurement of the charge of a quantum system with charge Q in an infinitesimal spatial region dv around x_n. This is equivalent to measuring the following observable:

$$A = \begin{cases} Q, & \text{if } x \in dv, \\ 0, & \text{if } x \notin dv. \end{cases} \tag{3}$$

During the measurement, the wave function of the measuring system, $\phi(x,t)$, will obey the following Schrödinger equation:

$$i\hbar\frac{\partial\phi(x,t)}{\partial t} = -\frac{\hbar^2}{2M}\nabla^2\phi(x,t) + k\frac{e\cdot|\psi(x_n,t)|^2 dvQ}{|x-x_n|}\phi(x,t), \tag{4}$$

where M and e are the mass and charge of the measuring system, respectively, and k is the Coulomb constant. From this equation, it can be seen that the property of the measured system in the measured position x_n that has efficiency to influence the measuring system is $|\psi(x_n,t)|^2 dvQ$, the effective charge there[3]. This is also the result of the protective measurement, $\langle A\rangle = |\psi(x_n,t)|^2 dvQ$. When divided by the volume element, it gives the effective charge density $|\psi(x,t)|^2 Q$[4].

Next, we will analyze the physical origin of effective charge distribution[5]. What kind of entity or process generates the effective charge distribution in

[3]Note that even in standard quantum mechanics, it is also assumed that the above interaction term indicates that there is a charge $|\psi(x_n,t)|^2 dvQ$ in the region dv.

[4]Similarly, we can protectively measure another observable $B = \frac{\hbar}{2mi}(A\nabla + \nabla A)$. The measurements will give the electric flux density $j_Q(x,t) = \frac{\hbar Q}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*)$ everywhere in space (Aharonov and Vaidman 1993).

[5]Historically, the charge density interpretation for electrons was originally suggested by Schrödinger in his fourth paper on wave mechanics (Schrödinger 1926). Schrödinger clearly realized that the charge density cannot be classical because his equation does not include the usual classical interaction between the densities. Presumably since people thought that the charge density could not be measured and also lacked a consistent physical picture, this interpretation was soon rejected and replaced by Born's probability interpretation. Now protective measurements help re-endow the effective charge distribution of an electron with reality. The question is then how to find a consistent physical explanation for it. Our following analysis may be regarded as a further development of Schrödinger's original idea to some extent. For more discussions see Bacciagaluppi and Valentini (2009) and Gao (2013b).

space or the physical efficiency of the quantity $|\psi(x,t)|^2 dvQ$? It can be expected that the answer will help understand the meaning of $|\psi(x,t)|^2$ and the wave function itself.

There are two possibilities: the effective charge distribution of a one-body system can be generated by either (1) a continuous charge distribution with density $|\psi(x,t)|^2 Q$ or (2) the motion of a discrete point charge Q with spending time $|\psi(x,t)|^2 dvdt$ in the infinitesimal spatial volume dv around x in the infinitesimal time interval $[t, t+dt]$[6]. Correspondingly, the underlying physical entity is either a continuous entity or a discrete particle. For the first possibility, the charge distribution exists throughout space at the same time, while for the second possibility, at every instant there is only a localized, point-like particle with the total charge of the system, and its motion during an infinitesimal time interval forms the effective charge distribution. Concretely speaking, at a particular instant the charge density of the particle in each position is either zero (if the particle is not there) or singular (if the particle is there), while the time average of the density during an infinitesimal time interval gives the effective charge density. Moreover, the motion of the particle is ergodic in the sense that the integral of the formed charge density in any region is required to be equal to the expectation value of the total charge in the region.

In the following, we will argue that the existence of a continuous charge distribution may lead to inconsistency. If the charge distribution is continuous and exists throughout space at the same time, then any two parts of the distribution, like two electrons, will arguably have electrostatic interaction described by the interaction potential term in the Schrödinger equation. However, the existence of such electrostatic self-interaction for a quantum system contradicts the superposition principle of quantum mechanics (at least for microscopic systems such as electrons). Moreover, the existence of the electrostatic self-interaction for the effective charge distribution of an electron is incompatible with experimental observations either. For example, for the electron in the hydrogen atom, since the potential of the electrostatic self-interaction is of the same order as the Coulomb potential produced by the nucleus, the energy levels of hydrogen atoms would be remarkably different from those predicted by quantum mechanics and confirmed by experiments if there existed such electrostatic self-interaction. By contrast, if there is only a localized particle at every instant, it is understandable that there exists no electrostatic self-interaction of the effective charge distribution formed by the motion of the particle. This is consistent with the superposition principle of quantum mechanics and experimental observations.

Here is a further clarification of this argument. It can be seen that the non-existence of self-interaction of the charge distribution poses a puzzle. According to quantum mechanics, two charge distributions such as two electrons, which exist in space at the same time, have electrostatic interaction described

[6]Note that the expectation value of an observable at a given instant such as $\langle A \rangle = |\psi(x_n,t)|^2 dvQ$ is either the physical property of a quantum system at the precise instant (like the position of a classical particle) or the limit of the time-averaged property of the system at the instant (like the standard velocity of a classical particle). These two interpretations correspond to the above two possibilities. For the later, the observable assumes an eigenvalue at each instant, and its value spreads all eigenvalues during an infinitesimal time interval. Moreover, the spending time in each eigenvalue is proportional to the modulus squared of the wave function of the system there. In this way, such ergodic motion generates the expectation value of the observable in an infinitesimal time interval (cf. Aharonov and Cohen 2014). We will discuss later whether this picture of motion applies to properties other than position.

by the interaction potential term in the Schrödinger equation, but for the effective charge distribution of an electron, any two parts of the distribution have no such electrostatic interaction. Facing this puzzle one may have two choices. The first one is simply admitting that the non-existence of self-interaction of the effective charge distribution is a distinct feature of the laws of quantum mechanics, but insisting that the laws are what they are and no further explanation is needed. However, this choice seems to beg the question and be unsatisfactory in the final analysis. A more reasonable choice is to try to explain this puzzling feature, e.g. by analyzing its relationship with the existent form of the effective charge distribution. The effective charge distribution has two possible origins or existing forms after all. On the one hand, the non-existence of self-interaction of the distribution may help determine which possible form is the actual one. For example, one possible form is inconsistent with this distinct feature, while the other possible form is consistent with it. On the other hand, the actual existent form of the effective charge distribution may also help explain the non-existence of self-interaction of the distribution. This is just what the above argument has done. The analysis establishes a connection between the non-existence of self-interaction of the effective charge distribution and the actual existent form of the distribution. The reason why two wavepackets of an electron, each of which has part of the electron's charge in efficiency, have no electrostatic interaction is that these two wavepackets do not exist at the same time, and their effective charges are formed by the motion of a localized particle with the total charge of the electron. Since there is only a localized particle at every instant, it is understandable that there exists no electrostatic self-interaction of the effective charge distribution formed by the motion of the particle. By contrast, if the two wavepackets with charges, like two electrons, existed at the same time, then they would also have the same form of electrostatic interaction as that between two electrons[7].

To sum up, we have argued that for a one-body system, the physical entity described by its wave function is a discrete, localized particle. At every instant there is a particle with the mass and charge of the system, while during an infinitesimal time interval the ergodic motion of the particle forms the effective mass and charge distributions measurable by protective measurements, and the spending time of the particle around each position in space is proportional to the modulus squared of the wave function of the system there.

3.2 Many-body systems

In this section, we will analyze many-body systems, and present further arguments supporting particle ontology in interpreting the wave function.

For an N-body system, its wave function is defined in a 3N-dimensional configuration space. If the wave function describes a continuous entity, then this entity exists in the 3N-dimensional configuration space. It has density and flux density in the configuration space. This view is usually called wave function realism or configuration space realism (Albert 1996), and it has at least two problems, the so-called "problem of perception" and "problem of lacking invari-

[7]Note that this argument does not assume that charges which exist at the same time are classical charges and they have classical interaction. By contrast, the Schrödinger-Newton equation, which was proposed by Diósi (1984) and Penrose (1998), treats the mass distribution of a quantum system as classical.

ances" (Monton 2002; Lewis 2004; Sol 2013). The first problem is that this view needs to explain the manifest three-dimensional character of our perception. The second problem is that the dynamical symmetries of the Schrödinger equation for an N-body system include translations and rotations in three independent spatial dimensions - not 3N, and this rich structure of configuration space is in want of a reasonable explanation. Similar to the case of one-body systems, the wave function of an N-body system may also describe a discrete particle moving in the 3N-dimensional configuration space, and its motion forms the density and flux density in the configuration space. For example, the density $|\psi(x_1, x_2, ...x_N, t)|^2$ is formed by the motion of the particle with spending time $|\psi(x_1, x_2, ...x_N, t)|^2 dV dt$ in an infinitesimal volume dV around $(x_1, x_2, ...x_N)$ in the infinitesimal time interval $[t, t + dt]$. This view is another form of configuration space realism, and it also has the above two problems.

In the following, we will argue that what the wave function of an N-body system describes is not a physical entity, either a continuous entity or a discrete particle, in the 3N-dimensional configuration space, but N physical entities in 3-dimensional space, and these entities are not continuous entities but discrete particles. First of all, in the Schrödinger equation for an N-body system, there are N mass parameters m_1, m_2, ... m_N (as well as N charge parameters etc). These parameters are not natural constants, but properties of the system; they may be different for different systems. Moreover, it is arguably that different mass parameters represent the same mass property of different physical entities. If a system has N mass parameters, then it will contain N physical entities. Therefore, an N-body system contains N physical entities, and the wave function of the system describes the state of these physical entities[8]. Next, these N entities exist in 3-dimensional space, not in a 3N-dimensional configuration space. The reason is that in the Schrödinger equation for an N-body system, each mass parameter m_i is only correlated with each group of three coordinates (x_i, y_i, z_i) of the 3N coordinates in configuration space. Thirdly, these N entities cannot be continuous entities, which are completely described by density and flux density. The reason is that the density and flux density of N continuous entities, which are defined in 3-dimensional space, are not enough to constitute the (entangled) wave function defined in 3N-dimensional configuration space.

Therefore, it is arguably that the wave function of an N-body system describes the state of N discrete particles in 3-dimensional space. Concretely speaking, at a given instant, the positions of N particles in 3-dimensional space can be represented by a point in a 3N-dimensional configuration space. During an infinitesimal time interval, these particles move in the real space, and correspondingly, this point moves in the configuration space, and its motion, like the above case of a particle in configuration space, forms the density and flux density in the configuration space. This interpretation of the wave function has no problems of the configuration space realism.

It is worth noting that we can also protectively measure the charge density (and electric flux density) of a many-body system in 3-dimensional space. A protective measurement of the observable $\sum_{i=1}^{N} A_i$ on an N-body system whose wave function is $\psi(x_1, x_2, ...x_N, t)$ yields

[8]Note also that the wave function of an N-body system, which lives on a 3N-dimensional configuration space, is not a complete description of the system, as it contains no information about the masses and charges of its N sub-systems (even though one assumes that the configuration space has a rich structure that can group the 3N coordinates).

$\sum_{i=1}^{N} \langle A_i \rangle = \sum_{i=1}^{N} \int ... \int Q_i |\psi(x_1, ...x_{i-1}, x, x_{i+1}, ...x_N, t)|^2 dx_1...dx_{i-1}dx_{i+1}...dx_N dv$, where Q_i is the charge of the i-th subsystem. When divided by the volume element dv, it yields the charge density in space. Moreover, the previous analysis of electrostatic self-interaction also applies to many-body systems. Like a one-body system, the effective charge distribution of an N-body system are generated by the ergodic motion of N charged particles, where the spending time of particle 1 with charge Q_1 in an infinitesimal spatial volume dv around x_1 and particle 2 with charge Q_2 in an infinitesimal spatial volume dv around x_2 ... and particle N with charge Q_N in an infinitesimal spatial volume dv around x_N is $|\psi(x_1, x_2, ...x_N, t)|^2 (dv)^N dt$ in the infinitesimal time interval $[t, t+dt]$, or equivalently, the spending time of the N particles in an infinitesimal volume dV around each position $(x_1, x_2, ...x_N)$ in the 3N-dimensional configuration space in the infinitesimal time interval $[t, t+dt]$ is $|\psi(x_1, x_2, ...x_N, t)|^2 dV dt$. Such ergodic motion of particles contains the information about the entanglement between the sub-systems of the many-body system. Its existence also shows that all dynamical possibilities of a quantum universe can be properly represented in 3-dimensional space (cf. Albert 1996).

3.3 Ergodic motion of particles

Which sort of ergodic motion? This is a further question that needs to be answered. If the ergodic motion of particles is continuous, then it can only form the effective mass and charge distributions during a finite time interval[9]. But according to quantum mechanics, the effective mass and charge distributions at a given instant are required to be formed by the ergodic motion of particles during an infinitesimal time interval around the instant. Thus it seems that the ergodic motion of particles cannot be continuous but must be discontinuous. This is at least what the existing theory says. This conclusion can also be reached by analyzing a specific example. Consider an electron in a superposition of two energy eigenstates in two separate boxes. In this example, even if one assumes that the electron can move with infinite velocity, it cannot *continuously* move from one box to another due to the restriction of box walls. Therefore, any sort of continuous motion cannot generate the effective charge distribution that exists in both boxes[10].

Since quantum mechanics does not provide further information about the positions of the particles at each instant, the discontinuous motion of particles described by the theory is also essentially random. Moreover, the spending time of the N particles of an N-body system around N positions in 3-dimensional space being proportional to the modulus squared of the wave function of the system there means that the (objective) probability density for the particles to appear in the positions is also proportional to the modulus squared of the wave function there (and for normalized wave functions they are equal). This ensures that the motion of particles forms the right mass and charge distributions. In addition, from a logical point of view, the N particles as a whole must also have

[9]For other objections to classical ergodic models see Aharonov and Vaidman (1993) and Aharonov, Anandan and Vaidman (1993).

[10]One may object that this is merely an artifact of the idealization of infinite potential. However, even in this ideal situation, the ergodic model should also be able to generate the effective charge distribution by means of some sort of ergodic motion of the electron; otherwise it will be inconsistent with quantum mechanics.

an instantaneous property (as a probabilistic instantaneous condition) which determines the probability density for them to appear in the N positions in space; otherwise the particles would not "know" how frequently they should appear in each group of N positions in space. This property is usually called indeterministic disposition or propensity in the literature[11].

In conclusion, we have argued that the ergodic motion of the particles of a quantum system that forms its effective mass and charge distributions is discontinuous and random, and the probability density for the particles to appear in every group of positions is equal to the modulus squared of the wave function of the system there.

3.4 Interpreting the wave function

According to the above analysis, microscopic particles such as electrons, which are described by quantum mechanics, are indeed particles. Here the concept of particle is used in its usual sense. A particle is a small localized object with mass and charge, and it is only in one position in space at an instant. Moreover, the motion of these particles is not continuous but discontinuous and random in nature. We may say that an electron is a quantum particle in the sense that its motion is not continuous motion described by classical mechanics, but random discontinuous motion described by quantum mechanics.

We first discuss the description of the state of random discontinuous motion of a single particle. Unlike the deterministic continuous motion, the trajectory function $x(t)$ can no longer provide a useful description for random discontinuous motion. It has been shown that the strict description of random discontinuous motion of a particle can be given based on the measure theory (Gao 2013b). Loosely speaking, the random discontinuous motion of a particle forms a particle "cloud" extending throughout space (during an infinitesimal time interval), and the state of motion of the particle is represented by the density and flux density of the cloud, denoted by $\rho(x,t)$ and $j(x,t)$, respectively, which satisfy the continuity equation $\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial j(x,t)}{\partial x} = 0$. The density of the cloud, $\rho(x,t)$, represents the probability density for the particle to appear in position x at instant t, and it satisfies the normalization condition $\int_{-\infty}^{+\infty} \rho(x,t)dx = 1$.

As we have argued above, for a charged particle such as an electron, the cloud will be an electric cloud, and $\rho(x,t)$ and $j(x,t)$, when multiplied by the total charge of the particle, will be the (effective) charge density and electric flux density measurable by protective measurements, respectively. Thus we have the following relations:

$$\rho(x,t) = |\psi(x,t)|^2, \tag{5}$$

$$j(x,t) = \frac{\hbar}{2mi}[\psi^*(x,t)\frac{\partial \psi(x,t)}{\partial x} - \psi(x,t)\frac{\partial \psi^*(x,t)}{\partial x}]. \tag{6}$$

[11]Note that the propensity here denotes single case propensity. In addition, it is worth stressing that the propensities possessed by the particles relate to their objective motion, not to the measurements on them. By contrast, according to the existing propensity interpretations of quantum mechanics, the propensities a quantum system has relate only to measurements; a quantum system possesses the propensity to exhibit a particular value of an observable if the observable is measured on the system.

Correspondingly, the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for an overall phase factor):

$$\psi(x,t) = \sqrt{\rho(x,t)}e^{im\int_{-\infty}^{x}\frac{j(x',t)}{\rho(x',t)}dx'/\hbar}. \tag{7}$$

This means that the wave function $\psi(x,t)$ also provides a description of the state of random discontinuous motion of a particle.

This description of the state of motion of a single particle can be extended to the motion of many particles. The extension may explain the multi-dimensionality of the wave function. At a given instant, a quantum system of N particles can be represented by a point in a 3N-dimensional configuration space. During an infinitesimal time interval, these particles perform random discontinuous motion in 3-dimensional space, and correspondingly, this point performs random discontinuous motion in the configuration space and forms a cloud there. Then, similar to the single particle case, the state of the system is represented by the density and flux density of the cloud in the configuration space, $\rho(x_1, x_2, ...x_N, t)$ and $j(x_1, x_2, ...x_N, t)$, where the density $\rho(x_1, x_2, ...x_N, t)$ represents the probability density that particle 1 appears in position x_1 and particle 2 appears in position x_2, ..., and particle N appears in position x_N[12]. Since these two quantities are defined in the 3N-dimensional configuration space, the many-particle wave function, which is composed of these two quantities, is also defined in the 3N-dimensional configuration space.

One important point needs to be stressed here. Since the wave function in quantum mechanics is defined at a given instant, not during an infinitesimal time interval around a given instant, it should be regarded not simply as a description of the state of motion of particles, but more suitably as a description of the dispositional property of the particles that determines their random discontinuous motion at a deeper level. In particular, the modulus squared of the wave function determines the probability density that the particles appear in every group of positions in space. By contrast, the density and flux density of the particle cloud in the configuration space, which are defined during an infinitesimal time interval, are only a description of the state of the resulting random discontinuous motion of particles, and they are determined by the wave function. In this sense, we may say that the motion of particles is "guided" by their wave function in a probabilistic way.

3.5 On momentum, energy and spin

We have been discussing random discontinuous motion of particles in position space. Does the picture of random discontinuous motion exist for other dynamical variables such as momentum and energy? Since there are also wave functions of these variables in quantum mechanics, it seems tempting to assume that the above interpretation of the wave function in position space also applies to the wave functions in momentum space etc. This means that when a particle is in a superposition of the eigenstates of a variable, it also undergoes random discontinuous motion among the corresponding eigenvalues of this variable. For example, a particle in a superposition of energy eigenstates also undergoes random discontinuous motion among all energy eigenvalues. At each instant the

[12]When these N particles are independent, the density $\rho(x_1, x_2, ...x_N, t)$ can be reduced to the direct product of the density for each particle, namely $\rho(x_1, x_2, ...x_N, t) = \prod_{i=1}^{N}\rho(x_i, t)$.

energy of the particle is definite, randomly assuming one of the energy eigenvalues with probability given by the modulus squared of the wave function at this energy eigenvalue, and during an infinitesimal time interval the energy of the particle spreads throughout all energy eigenvalues. Since the values of two noncommutative variables (e.g. position and momentum) at every instant may be mutually independent, the objective value distribution of every variable can be equal to the modulus squared of its wave function and consistent with quantum mechanics[13].

However, there is also another possibility, namely that the picture of random discontinuous motion exists only for position, while momentum, energy etc do not undergo random discontinuous change among their eigenvalues. On this view, the position of a particle is an instantaneous property of the particle defined at instants, while momentum and energy are properties relating only to its state of motion (e.g. momentum and energy eigenstates), which is formed by the motion of the particle during an infinitesimal time interval[14]. This may avoid the problem of defining the momentum and energy of a particle at instants. Certainly, we can still talk about momentum and energy on this view. For example, when a particle is in an eigenstate of the momentum or energy operator, we can say that the particle has definite momentum or energy, whose value is the corresponding eigenvalue. Moreover, when a particle is in a momentum or energy superposition state and the momentum or energy branches are well separated in space, we can still say that the particle has definite momentum or energy in certain local regions.

Lastly, we note that spin is a more distinct property. Since the spin of a free particle is always definite along one direction, the spin of the particle does not undergo random discontinuous motion, though a spin eigenstate along one direction can always be decomposed into two different spin eigenstates along another direction. But if the spin state of a particle is entangled with its spatial state due to interaction and the branches of the entangled state are well separated in space, the particle in different branches will have different spin, and it will also undergo random discontinuous motion between these different spin states. This is the situation that usually happens during a spin measurement.

4 Conclusions

Quantum mechanics is basically a physical theory about the wave function and its time evolution. There are two main problems in the conceptual foundations of quantum mechanics. The first one concerns the physical meaning of the wave function in the theory. The second one is the measurement problem, which con-

[13]Note that for random discontinuous motion a property (e.g. position) of a quantum system in a superposed state of the property is indeterminate in the sense of usual hidden variables, though it does have a definite value at each instant. For this reason, the particle position should not be called hidden variable for random discontinuous motion of particles, and the resulting theory is not a hidden variable theory either. This makes the theorems that restrict hidden variables such as the Kochen-Specker theorem irrelevant. Another way to see this is to realize that random discontinuous motion of particles alone does not provide a way to solve the measurement problem, and wavefunction collapse may also be needed. For further discussions see Gao (2013b).

[14]It is worth stressing that the particle position here is different from the position property described by the position operator in quantum mechanics, and the latter is also a property relating only to the state of motion of the particle such as position eigenstates.

cerns the time evolution of the wave function during a measurement. Although the meaning of the wave function should be ranked as the first interpretative problem of quantum mechanics, it has been treated as a marginal problem, especially compared with the measurement problem. There are already several alternatives to quantum mechanics which give seemingly satisfactory solutions to the measurement problem. However, these theories at their present stages have not yet succeeded in making sense of the wave function.

In this paper, we propose a new approach for solving the problem of interpreting the wave function, which is to analyze the mass and charge properties of a quantum system. First, with the help of protective measurements, we argue that the wave function of a quantum system is a representation of the physical state of the system. The argument does not depend on nontrivial assumptions and also overcomes existing objections to the implications of protective measurements. Next, we further analyze the ontological meaning of the wave function. The key is to realize that the Schrödinger equation, which governs the evolution of a quantum system, contains more information about the system than the wave function of the system, which can help unveil the ontological meaning of the wave function. An important piece of information is the mass and charge properties of the system, which are responsible for the gravitational and electromagnetic interactions between systems. We first analyze the mass and charge distributions of a one-body quantum system. It is argued that the mass and charge of a one-body system such as an electron is distributed throughout space in efficiency, and the effective mass and charge distributions manifest more directly during a protective measurement, which indicates that the effective mass and charge density in each position is proportional to the modulus squared of the wave function of the system there. By analyzing the origin of the effective charge distribution, we further argue that the effective mass and charge distributions are formed by the ergodic motion of a localized particle with the total mass and charge of the system. Moreover, the ergodic motion of the particle is discontinuous and random, and the probability density that the particle appears in every position is equal to the modulus squared of its wave function there. We then analyze the mass and charge properties of a many-body system. It is argued that the wave function of an N-body system describes the state of N discrete particles in 3-dimensional space.

Based on these analyses, we propose a new ontological interpretation of the wave function in terms of particle ontology. According to this interpretation, quantum mechanics, like Newtonian mechanics, also deals with the motion of particles in space and time. Microscopic particles such as electrons are still particles, but they move in a discontinuous and random way. The wave function describes the state of random discontinuous motion of particles, and at a deeper level, it represents the dispositional property of the particles that determines their random discontinuous motion. Quantum mechanics, in this way, is essentially a physical theory about the laws of random discontinuous motion of particles. It is a further and also harder question what the precise laws are, e.g. whether the wave function undergoes a stochastic and nonlinear collapse evolution.

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References

[1] Aharonov, Y., Anandan, J. and Vaidman, L. (1993). Meaning of the wave function, Phys. Rev. A 47, 4616.

[2] Aharonov, Y., Anandan, J. and Vaidman, L. (1996). The meaning of protective measurements, Found. Phys. 26, 117.

[3] Aharonov, Y. and Cohen, E. (2014). Protective Measurement and the Heisenberg Representation. This volume.

[4] Aharonov, Y. and Vaidman, L. (1993). Measurement of the Schrödinger wave of a single particle, Phys. Lett. A 178, 38.

[5] Albert, D. Z. (1996), Elementary Quantum Metaphysics. in J. Cushing, A. Fine and S. Goldstein (eds.), Bohmian Mechanics and Quantum Theory: An Appraisal. Dordrecht: Kluwer, 277-284.

[6] Anandan, J. (1993). Protective measurement and quantum reality. Found. Phys. Lett., 6, 503-532.

[7] Bacciagaluppi, G. and Valentini, A. (2009). Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference. Cambridge: Cambridge University Press.

[8] Belot, G. (2012). Quantum states for primitive ontologists. A case study. European Journal for Philosophy of Science 2, 67-83.

[9] Colbeck, R., and Renner, R. (2012). Is a system's wave function in one-to-one correspondence with its elements of reality? Phys. Rev. Lett., 108, 150402.

[10] Dass, N. D. H. and Qureshi, T. (1999). Critique of protective measurements. Phys. Rev. A 59, 2590.

[11] Dickson, M. (1995). An empirical reply to empiricism: protective measurement opens the door for quantum realism. Philosophy of Science 62, 122.

[12] Diósi, L. (1984). Gravitation and the quantum-mechanical localization of macro-objects. Phys. Lett. A 105, 199-202.

[13] Gao, S. (2011a). The wave function and quantum reality, in Proceedings of the International Conference on Advances in Quantum Theory, A. Khrennikov, G. Jaeger, M. Schlosshauer, G. Weihs (eds), AIP Conference Proceedings 1327, 334-338.

[14] Gao, S. (2011b). Meaning of the wave function, International Journal of Quantum Chemistry. 111, 4124-4138.

[15] Gao, S. (2013a). On Uffink's criticism of protective measurements, Studies in History and Philosophy of Modern Physics, 44, 513-518 (2013).

[16] Gao, S. (2013b). Interpreting Quantum Mechanics in Terms of Random Discontinuous Motion of Particles. http://philsci-archive.pitt.edu/9589/.

[17] Leifer, M. S. and Maroney, O. J. E. (2013). Maximally epistemic interpretations of the quantum state and contextuality. Phys. Rev. Lett., 110, 120401.

[18] Lewis, P. (2004). Life in configuration space, British Journal for the Philosophy of Science 55, 713-729.

[19] Lewis, P. (2014). Measurement and Metaphysics. This volume.

[20] Lewis, P. G., Jennings, D., Barrett, J., and Rudolph, T. (2012). Distinct quantum states can be compatible with a single state of reality. Phys. Rev. Lett. 109, 150404.

[21] Monton, B. (2002). Wave function ontology. Synthese 130, 265-277.

[22] Ney, A. and Albert, D. Z. (eds.) (2013). The Wave Function: Essays on the Metaphysics of Quantum Mechanics. Oxford: Oxford University Press.

[23] Patra, M. K., Pironio, S. and Massar, S. (2013). No-go theorems for ψ-epistemic models based on a continuity assumption. 111, 090402.

[24] Penrose, R. (1998). Quantum computation, entanglement and state reduction. Phil. Trans. R. Soc. Lond. A 356, 1927.

[25] Pusey, M., Barrett, J. and Rudolph, T. (2012). On the reality of the quantum state. Nature Phys. 8, 475-478.

[26] Schlosshauer, M., and Fine, A. (2012). Implications of the Pusey–Barrett–Rudolph quantum no-go theorem. Phys. Rev. Lett, 108, 260404.

[27] Schlosshauer, M., and Fine, A. (2013). No-go theorem for the composition of quantum systems. Eprint arXiv:1306.5805v2 [quant-ph]. To appear in Phys. Rev. Lett.

[28] Schlosshauer, M. and Claringbold, T. V. B. (2014). Entanglement, scaling, and the meaning of the wave function in protective measurement. This volume.

[29] Schrödinger, E. (1926). Quantizierung als Eigenwertproblem (Vierte Mitteilung). Ann. d. Phys. (4) 81, 109-139. English translation: Quantisation as a Problem of Proper Values. Part IV, Reprint in Schrödinger, E. (1982). Collected Papers on Wave Mechanics. New York: Chelsea Publishing Company, pp. 102-123.

[30] Solé, A. (2013). Bohmian mechanics without wave function ontology. Studies In History and Philosophy of Modern Physics 44, 365-378.

[31] Vaidman, L. (2009) Protective measurements, in Greenberger, D., Hentschel, K., and Weinert, F. (eds.), Compendium of Quantum Physics: Concepts, Experiments, History and Philosophy. Springer-Verlag, Berlin. pp.505-507.

[32] Wallden, P. (2013). Distinguishing initial state-vectors from each other in histories formulations and the PBR argument. Found. Phys., 43, 1502-1525.