On Leggett Theories: A Reply

Federico Laudisa

Department of Human Sciences, University of Milan-Bicocca

Piazza dell'Ateneo Nuovo 1, 20126 Milan – Italy

federico.laudisa@unimib.it

Abstract

In his 2013 *Foundations of Physics* paper Mathias Egg claims to show that my critical arguments toward the foundational significance of Leggett's non-local theories are misguided. The main motivation is that my argument would connect too strongly the Leggett original motivation for introducing this new class of theories with the foundational significance of these theories *per se*. Egg basically aims to show that, although it can be conceded that the Leggett original motivation relies on a mistaken view of the original Bell theorem, the investigation on the Leggett theories does have a foundational meaning that can be disassociated from the view that *Leggett himself* has of of them. As a reply to Egg, I would like to argue here that, even if we assume to disentangle the Leggett *view* from the fate of the Leggett *theories*, there is still room to dispute the foundational significance of the Leggett 'non-local realistic' research program.

1 Introduction

If we evaluate a scientific result not only in terms of its immediate meaning and perhaps its technological applications, but also in terms of the breadth of discussions that it gives rise to, we may see why as early as 1977 the Bell theorem had been defined as “the most profound discovery of science” (Stapp 1977, p. 173). One of the many routes that researchers decided to investigate starting from the Bell theorem has been the issue of how wide is the class of theories that the Bell theorem itself is supposed to rule out. It is in this vein that in 2003 Anthony Leggett introduced a new class of theories that, albeit being assumed to be non-local in a suitable sense – and hence not immediately ruled out by the Bell theorem – allowed to derive a new inequality (Leggett inequality): this inequality turned out to be violated both theoretically and experimentally (Leggett 2003, Gröblacher et al. 2007).

In a paper published in this journal in 2008, I questioned the implications that had been drawn from the Leggett result (Laudisa 2008), by claiming that in fact the foundational import of such implications was far less relevant than it was thought to be and that, moreover, the original motivation underlying the result itself was based on a mistaken interpretation of the Bell theorem (basically, an interpretation in which the bulk of the Bell theorem is the rejection not just of locality but of a much more controversial assumption, usually termed *local realism*). The main claim of this paper has been radically criticized by a recent paper of Mathias Egg, published in this journal, along the following lines: even if we concede (as Egg does) that the ‘local-realistic’ reading of the Bell theorem is wrong, this need not undermine the foundational significance of the Leggett result. The latter consists in showing that not even non-local theories are able to recover a minimal sense of ‘realism’ for quantum systems (where by ‘minimal’ we refer to the generalization of the Leggett result by Branciard et al. 2008, more details below) and it is in this sense, then, that the Leggett result *does* teach us a useful foundational lesson (Egg 2013).

In the present note, I would like to reply by arguing why, even conceding that the Leggett research program can be detached from the motivation that Leggett himself defended for his framework, the foundational import of his result is still limited[1]. In section 2 I will briefly rehearse the state of the art whereas, in the subsequent section, I will re-evaluate the scope of the Leggett framework and I will argue why – all of the Egg criticisms notwithstanding – it is still reasonable to hold that the implications are much less illuminating than what has been claimed.

[1] The limitations I will refer to are analogous to several other limitations occurring in the now long sequence of no-go results for non-relativistic quantum mechanics: for a sample of related, recent work I refer to Laudisa 2014.

2 Leggett theories: the state of the art

According to the Bell theorem in its most general formulation, no theory that agrees with the statistical predictions of quantum mechanics can be local, *whatever* its further details concerning the way of describing states or properties of the involved systems might be. In the Leggett approach, even accepting all the implications of the above reading of the Bell theorem (including that there is no local *realism* involved), the latter does not prevent from introducing a new class of theories that might be a bit ‘more non-local’, so as to avoid to be immediately ruled out by the Bell theorem. In this sense, the question arises whether it can be shown that also these theories are ruled out: the question is answered in the affirmative by proving the derivation of a new inequality (the Leggett inequality), that turns out to be violated both theoretically and experimentally by the predictions of quantum mechanics (Leggett 2003, Gröblacher et al. 2007). Therefore – so the argument goes – the Leggett result establishes a ‘finer’ point with respect to the Bell theorem, a point that justifies a high foundational evaluation of the violation of the Leggett inequality: namely, not even within a suitably, partially non-local theory it is possible to ascribe a certain degree of definiteness to properties of subsystems in a typical EPR-like experiment. As it is claimed in the more general formulation contained in Branciard et al. 2008: “The falsification of Leggett’s model proves that it is impossible to reconstruct quantum correlations from hypothetical, more elementary correlations in which individual properties would be sharply defined. […] a much stronger statement holds, namely, that individual properties cannot be even partially defined” (Branciard *et al*. 2008, p. 683).
So, is there any room for questioning even this more general formulation of the Leggett research program concerning his new class of non-local hidden variable theories? In what follows I will argue that, *pace* Egg 2013, the answer is yes.

The theories in the Leggett class are supposed to account for the results obtained in a general experimental framework, in which some polarization measurements are performed on pairs of photons emitted by atoms in a cascade process, with measurement settings denoted as usual by parameters **a** and **b**. Since this framework encompasses, after the emission, a number of detection processes involving a pair of spatially separated detectors (let us call them $\mathbf{D}_1$ and $\mathbf{D}_2$), attention is focused as usual on correlations between the counts: clearly, the aim is to compare the predictions for a given function of such correlations as prescribed by quantum mechanics on the one hand and the Leggett-type of theory on the other (Leggett 2003, p. 1471 ff).

The general conditions that the Leggett-type of theories are assumed to satisfy are such that each pair of photons emitted in the cascade of a given single atom is characterized by a unique value for a (hidden) variable denoted by λ and a pair of polarization directions **u** and **v**. On the other hand, the Leggett-type of theory is so designed in principle as to go *beyond* quantum mechanics, in that the two parameters **u** and **v** characterize *uniquely* the polarization states of the photons in each pair: namely, the ensemble of the photon pairs is assumed to be the disjoint union of two subsensembles in which each member has a definite polarization. It is in this sense that it is reasonable to call the Leggett-type of theory a 'hidden-variable' theory, although – within a given subensemble – photon pairs can have different λ, with a statistical distribution $\rho_{\mathbf{uv}}(\lambda)$: such distribution is assumed to be *independent* of the polarizer settings parameters **a** and **b** and detection processes. (Egg 2013, p. 874). Moreover, the Leggett-type of theory allows for possibly non-local influences both of polarizer setting parameters and definite polarization parameters on the outcomes: namely, if **A** and **B** denote respectively two variables that take the value + 1 (−1) according to whether the detectors $\mathbf{D}_1$ and $\mathbf{D}_2$ register (do not register) the arrival of a photon, the value of **A** may depend not only on **a**, **u** and λ but also possibly on **b** and **v** and, similarly, the value of **B** may depend not only on **b**, **v** and λ but also possibly on **a** and **u** (Leggett (2003), pp. 1473-4):

$$\mathbf{A} = \mathbf{A}(\mathbf{a}, \mathbf{u}, \mathbf{b}, \mathbf{v}, \lambda), \qquad \mathbf{B} = \mathbf{B}(\mathbf{b}, \mathbf{v}, \mathbf{a}, \mathbf{u}, \lambda).$$

As we said, the parameters **u** and **v** are introduced in order to ascribe to each member of a photon pair a state of definite polarization. In the Branciard et al. 2008 generalization, the original Leggett states λ, which were originally expressed as pure product states $\lambda = \mathbf{u} \otimes \mathbf{v}$, are expressed as mixed states, namely states that represent photons endowed with a certain *degree* η of polarization. If we denote by α and β the possible binary outcomes, in the original case the local expectation values for such outcomes were

$$< \alpha >_\lambda = \mathbf{u} \cdot \mathbf{a}, \qquad < \beta >_\lambda = \mathbf{v} \cdot \mathbf{b},$$

in the generalized case the corresponding expressions are

$$< \alpha >_\lambda = \eta\, \mathbf{u} \cdot \mathbf{a}, \qquad < \beta >_\lambda = \eta\, \mathbf{v} \cdot \mathbf{b}.$$

Yet, the Leggett-type of theory still preserves a weaker kind of locality assumption, according to which the averages of **A** and **B** – namely, the averages over all values of λ within each subsensemble – depend only on *local* parameters, namely

$$< \mathbf{A} > = \mathrm{A}\,(\mathbf{u}, \mathbf{a}), \qquad < \mathbf{B} > = \mathbf{B}\,(\mathbf{v}, \mathbf{b})$$

It is a locality assumption that makes sense to require, since a Leggett-type of theory satisfying it is not immediately ruled out by the Bell theorem: Leggett himself provides an explicit example of a theory that can satisfy it and at the same time violate the Bell inequalities (Leggett 2003, pp. 1485-1488). If this is the case, then the issue of the compatibility between this class of theories and quantum mechanics is not idle at first sight: as a matter of fact, Leggett derives from his newly introduced theories a new inequality that turns out to be inconsistent with quantum predictions (an inconsistentcy experimentally confirmed by Gröblacher et al. 2007).

3 Replies to the Egg criticisms

In the present section, I will comment on two lines of criticisms raised by Egg that I find plausible: I will then turn to my counter-arguments on the points on which I still disagree with Egg's claims in the following sections.

3.1 Kochen-Specker and individual vs. subensemble realism

In the section 4 of his paper, Egg rightly points out that one of my charges against the foundational significance of the Leggett theories – namely that the Realism assumption in the Gröblacher *et al* 2007 experimental test of the original Leggett inequality is *logically* inconsistent with the Kochen-Specker type of results – overlooks a distinction between the level of the subensembles involved and the level of *individual* measurements performed on the members of the subensembles themselves. The Kochen-Specker type of results blocks Realism only for this latter sort of level, but the Leggett theories, in the Gröblacher *et al* realization, might well be 'realistic' at the subensemble level without conflicting with the Kochen-Specker prohibition. Therefore – Egg argues – in order to assess the foundational plausibility of the Leggett theories, we do not need just logic (as my original charge suggested), but also an experimental test[2].

[2] It should be noted, however, that it would seem highly plausible – in the case of both the abstract setting of the Leggett theories in Leggett 2003 and the experimental test of the Leggett inequality of Gröblacher *et al* 2007 – to motivate the Realism assumption at the subsensemble level with the validity of Realism *also* at the individual level. This point has been raised recently in an exchange between Navascués 2013 and Branciard 2013 that appears to be relevant here. According to Navascués, the Leggett theories in fact assume what he calls the *realistic polarization principle*, on the basis of which individual photons have a definite polarization state. Navascués shows then that, if something like the realistic polarization principle is assumed, then the statistics for the polarization measurements coincide necessarily with the correlations obtained when measuring *separable* states: in turn, this implies that the Leggett theories are in fact local realistic (since any quantum experiment verifying entanglement leads to a refutation of these theories). Branciard objects that assuming a realistic polarization principle is a matter of *interpretation* and in principle is not directly *required* by the strictly mathematical formulation of crypto-nonlocal theories (that there is no

3.2 Bell: theory and experiment

In his section 5, Egg questions my understanding of the relation between the theoretical and the experimental aspect of the Bell theorem. First, Egg quotes the following passage in my paper: “But, as Bell showed, there is little significance in testing against quantum theory a theory (be it local or non-local) that is supposed to satisfy a condition that we already know quantum mechanics cannot possibly and reasonably satisfy” (Laudisa 2008, p. 1123), and then he remarks:

> I am not convinced by this reasoning. Indeed, the best counterexamples to this claim are Bell’s inequalities themselves. The fact that these inequalities are violated by the quantum mechanical predictions shows that quantum mechanics “cannot possibly and reasonably satisfy” the conditions assumed for their derivation. Should we therefore conclude that Aspect’s experiments (to name just the most famous example) are of “little significance”? This would amount to a dubious *a priori* commitment to the truth of quantum mechanical predictions in domains where quantum mechanics has not yet been tested (Egg 2013, p. 877).

The problem with this criticism is that I was referring not to any general condition, but to that form of realism that I commented upon when, in the previous lines, I have drawn a difference between two different ways of understanding what it means to go *beyond* quantum mechanics. In the same spirit, Egg points then to an alleged imprecision of mine when I have provided a logical reconstruction of the Bell-Clauser-Horne argument concerning local stochastic hidden variable theories. He focuses on the following three lines (Laudisa 2008, p. 1127)

2. $QM \rightarrow \neg BI$ [Experimental fact]
3. QM [Assumption]
4. $\neg BI$ [2, 3 *Modus ponens*]

by noting

(i) that the implication $QM \rightarrow \neg BI$ can be shown to hold on a purely theoretical basis, so that the justification ‘Experimental fact’ is out of place,

(ii) that the QM Assumption is misleading, in that it suggests that we need to hold the *truth* of quantum mechanics to conclude for the violation of the Bell inequalities.

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necessity, anyway, is something that is already acknowledged by Navascués in the first lines of the above quotation). However, it is Branciard himself (Branciard 2013, p. 3) who stresses that the Navascués analysis shows how *physically unreasonable* crypto-nonlocality turns out to be when not supplemented with a physical interpretation along the lines highlighted by Navascués (and consistent with my 2008 paper).

Now, I acknowledge that I stripped into brackets both the theoretical fact (quantum mechanical *theoretical* probabilities violate the Bell inequalities) and the subsequent experimental confirmation of such theoretical predictions. But I thought that it was clear enough that what is necessary to assume is *both* the validity of at least the portion of quantum theory that is needed to make sense of the theoretical setting involved in the derivation of the Bell inequalities *and* the reliability of at least the portion of quantum theory that is needed to make sense of the experimental setting involved in the test (something that Egg himself points out few lines after his criticism)[3]. Once both points are made clear, the aim of the logical reconstruction was to show that the Realism assumption played no role (just like the case with strict anticorrelation).

Now, it is true that both I and Egg, when referring to 'the Bell theorem', we refer to the ideal case, namely the case in which perfect correlation is assumed. It is also true, however, that the inequality that has been experimentally tested is not the perfect correlation-Bell inequality but rather some form of the CHSH inequality, in which no perfect correlation is required at the outset. Do my arguments in my 2008 paper concerning the irrelevance of "Realism" (that Egg criticizes while still referring to the Bell inequality and not to the CHSH inequality) still hold when referred to the CHSH inequality? The answer is yes, as I had tried to show in my 2008 paper (section 4) with an argument that was related to the Bell-CH inequality but that holds in essentially the same way when referred to the CHSH inequality.

The point is that also the CHSH inequality can well be derived from the only assumption of locality. Even if their original paper the authors made believe that the 'realistic' assumption (be it in terms of hidden variables, pre-existing properties and the like) concerning the complete states λ was crucial (Clauser *et al.* 1969, p. 881), the only assumption on λ that was relevant for the derivation was locality, so that the empirically observed violation of the CHSH inequality should be traced back to a failure of locality in nature (see also Norsen 2007). In my 2008 paper I had drawn and motivated the same conclusion with reference to the Bell-CH framework, as explained in the classic 1981 Bell paper. We can introduce an EPR-Bohm set-up in very general terms, in which we are interested in the joint probability distribution

$$P(A, B \mid a, b),$$

where each A and B may be a 'yes' or a 'no' and a and b stand respectively for two possible adjustable parameters (with the obvious interpretation). Since A and B are supposed to be so far away from each other that it is not imaginable at the outset that there is some direct influence at work, a sound scientific attitude would lead us first — Bell claims — to make the hypothesis that there are some factors that contribute *locally* to the distribution. The attitude toward such

[3] This seems to be a standard practice: see for instance Ghirardi, Grassi 1994, Goldstein et al. 2011.

justification of the locality condition (in the derivation of the Bell inequality for stochastic hidden variables models this justification is in terms of a 'factorizability' condition) is essentially the same: "let us suppose that the correlations in the EPR experiment are likewise « locally explicable » (Bell 1981, in Bell [2004, p. 152]). Namely, the core of the argument lies in stating what preventing any action-at-a-distance amounts to, *whatever the factors at* A *and* B *might be*. The above assumption need not be grounded on the additional assumption that there are some *pre-existing properties* in the common past of the relevant events at A and B that enhance the correlation. Such assumption would be certainly sufficient for the assumption of existence of local factors, but *not necessary*.

So why keeping to hold that some form of realism is presupposed in the non-locality issue?

3.3 "Local Realism" and beyond

According to the first Egg's objection in his section 3 («Bell, Leggett and "Local Realism"»), he claims that, even if we reject the 'local-realistic' (LR) grounding of the Leggett theories, we can defend the motivation behind such theories:

> Once we recognize that Bell did not assume realism for the derivation of his theorem, we see that the LR view is mistaken in suggesting that the violations of Bell's inequalities leave us with a choice to give up *either* locality *or* realism. Instead, they simply force us to give up locality. But this leaves open the question whether there is a sense of realism which has to be given up as well. (Egg 2013, p. 875)

We can reply to this objection by drawing a *meta-theoretical* comparison between the foundational strategies that underlie the Bell result on one side and the Leggett result on the other. This comparison will help us to show what is really at the heart of the matter, namely the fact that there is no unique way to state what it takes to go *beyond* quantum mechanics, so that it may turn out to be highly disputable what we really learn by *this or that* way of going beyond quantum mechanics.

In the area of investigations opened nearly half a century ago by John S. Bell, the question naturally arose of what would have been the implications of *extending* quantum mechanics, in view of the emergence of phenomena that were not easy to accommodate within a familiar view of the physical world, non-locality being the most urgent case. Due to the unavoidable existence of entangled states – something that makes quantum mechanics a non-local theory in a fundamental sense (due to the linearity of the theory, of which entanglement is a consequence) – it has seemed plausible to put things in the following way: let us ask whether quantum mechanics might be seen as a 'fragment' of a more general theory which – at a 'higher' level – may recover that locality that turns out not to hold at the strictly quantum level. One of the *strong points* of the original Bell strategy that led to the Bell-named theorem was exactly that this hypothetical extension was confined to the locality/non-locality issue and needed not say *anything* on further details concerning 'realistic' or 'non-realistic' properties, states or whatever: in addition to being useful for the economy of the theorem, this point was absolutely plausible since it makes sense to require from the extension the only condition that we are interested to add to the new hypothetical super-theory, namely locality.

In the Leggett case, things are different. A minimum of locality is preserved in the extension, but *in addition* a condition concerning the definiteness (Leggett 2003) or the partial definiteness (Branciard et al. 2008) is required: standard quantum mechanics, however, has nothing to say on these conditions precisely because its way of describing the states of investigated systems is in strong tension with this sort of definiteness, be it total or partial. If this is the case, the sort of plausibility that was inherent in requiring locality from the theory that was supposed to extend quantum mechanics is absent, since it is unclear what insight would we gain in having a super-theory that is slightly more non-local than any theory ruled out by the Bell theorem but that requires definite (or semi-definite) properties that standard quantum mechanics sees no reason to require anyway. It follows that any implication descending from a theory that assumes such 'definiteness' or 'semi-definiteness' condition can hardly be really significant for the issue of what it means to *extend* quantum mechanics and for the issue of what we have to give up in constructing this or that sort of extension.

3.4 Leggett, Bohm and all that

Finally, Egg raises two objections concerning my understanding of the relation between the Leggett theories and Bohmian mechanics.

First, he reads my claim that Bohmian mechanics satisfies Realism as if this satisfaction were valid for *all* properties. Should this be the case, Egg would be perfectly right in his objection but it is clear that Realism holds in a Bohmian world just for position: this circumstance forces Bohmian mechanics to provide a robust justification for the measurement independence and determinism concerning position (that follow from position having a privileged status in Bohmian mechanics), a justification that is provided on the background of the overall theory and not simply put on top of it, like the case of the Leggett theories.

Second, Egg claims that a supporter of Bohmian mechanics, far from being suspicious about the unwarranted assumptions of the Leggett theories, should welcome their main claim, namely that no degree of reality can be attributed to the polarization of individual photons: this latter result should be interpreted as an experimental confirmation of one of the features of Bohmian mechanics, namely that no property other than position is ‘real’ in a deep sense.

The Bohmian should therefore not join Laudisa in denouncing Leggett’s research program as irrelevant, but should rather welcome it as significantly supporting his own position, by showing that non-realism about the polarization of individual photons is not just a theoretical postulate, but an experimental fact (Egg 2013, p. 879).

This claim is totally controversial, however, in view both of the interpretation of the significance of the experimental refutation of the Leggett inequality and of the peculiar form of realism that Bohmian mechanics can be seen to satisfy. The privileged status of the position in Bohmian mechanics, with the contextual character of all other physical properties, is justified by providing a robust and global image of the physical world, in which particles move around along well-defined trajectories but in which our access to the totality of the information concerning the position of particles is limited. On the other hand, the kind of Realism assumed by the Leggett theories is simply put on top of a theoretical structure whose only function is to derive an inequality that we expect to be violated by quantum probabilities, and whose only aim is to show that such ‘Realism’ is incompatible in any form with ordinary quantum mechanics.

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