Notes on the ontology of Bohmian mechanics

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Abstract

It is argued that in Bohmian mechanics the effective wave function of a subsystem of the universe does not encode the influences of other particles on the subsystem. This suggests that the ontology of Bohmian mechanics does not consist only in Bohmian particles and their positions. It is nonetheless pointed out that since the wave function in configuration space may represent the state of ergodic motion of non-Bohmian particles in three-dimensional space, the ontology of Bohmian mechanics may still consist only in particles.

Bohmian mechanics, which is also called the de Broglie-Bohm theory, is a realistic alternative to standard quantum mechanics initially proposed by de Broglie (1928) and later developed by Bohm (1952). According to the theory, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. It has been a hot topic of debate how to interpret the wave function in the theory. Recently Esfeld et al (2013) further developed the nomological interpretation of the wave function, as suggested by Dürr, Goldstein and Zanghì (1997), Goldstein and Teufel (2001) and Goldstein and Zanghì (2012). They argued that the ontology of Bohmian mechanics consists only in Bohmian particles and their positions, and in particular, the effective wave function of a subsystem of the universe encodes the influences of other particles in the universe on the subsystem. In this note, we will argue that this interesting view may be problematic.

The Bohmian law of motion is expressed by two equations, a guiding equation for the configuration of particles in three-dimensional space and the Schrödinger equation, describing the time evolution of the wave function, which enters the guiding equation. The law can be formulated as follows:

$$\frac{dQ(t)}{dt} = v^{\Psi(t)}(Q(t)).$$

(1)
In these equations, \( Q(t) \) denotes the spatial configuration of particles in three-dimensional space, and \( \Psi(t) \) is the wave function of that particle configuration at time \( t \). The status of these equations is different depending on whether one considers the physical description of the universe as a whole or of a subsystem thereof (Esfeld et al. 2013). Bohmian mechanics starts from the concept of a universal wave function, figuring in the fundamental law of motion for all the particles in the universe. That is, \( Q(t) \) describes the configuration of all the particles in the universe at time \( t \), and \( \Psi(t) \) is the wave function at time \( t \), guiding the motion of all particles taken together. To describe subsystems of the universe, the appropriate concept is the effective wave function.

The effective wave function is the Bohmian analogue of the usual wave function in the standard formulation of quantum mechanics. It is not primitive, but derived from the universal wave function and the actual spatial configuration of all the particles ignored in the description of the respective subsystem (see Dürr, Goldstein, Zanghì 1992 for more details). The effective wave function of a subsystem can be defined as follows. Let \( A \) be a subsystem of the universe including \( N \) particles with position variables \( x = (x_1, x_2, ..., x_N) \). Let \( y = (y_1, y_2, ..., y_M) \) be the position variables of all other particles not belonging to \( A \). Then the subsystem \( A \)’s conditional wave function at time \( t \) is defined as the universal wave function \( \Psi_t(x, y) \) evaluated at \( y = Y(t) \):

\[
\psi^A_t(x) = \Psi_t(x, y) \big|_{y=Y(t)}.
\]  

If the universal wave function can be decomposed in the following form:

\[
\Psi_t(x, y) = \phi_t(x)\phi_t(y) + \Theta_t(x, y),
\]  

where \( \phi_t(y) \) and \( \Theta_t(x, y) \) are functions with macroscopically disjoint supports, and \( Y(t) \) lies within the support of \( \phi_t(y) \), then \( \psi^A_t(x) = \varphi_t(x) \) (up to a multiplicative constant) is \( A \)’s effective wave function at \( t \). It can be seen that the temporal evolution of \( A \)’s particles is given in terms of \( A \)’s conditional wave function in the usual Bohmian way, and when the conditional wave function is also \( A \)’s effective wave function, then it also obeys a Schrödinger dynamics of its own. This means that the effective descriptions of subsystems are of the same form of the law of motion as given above.

An important question is the meaning of the effective wave function of a subsystem of the universe. According to Esfeld et al (2013), the effective wave function of a subsystem encodes the non-local influences of other particles on the subsystem via the non-local law of Bohmian mechanics, and the ontology of Bohmian mechanics consists only in the particles and their
positions. For example, in the double slit experiment with one particle at a time, the particle goes through exactly one of the two slits, and that is all there is in the physical world. There is no field or wave that guides the motion of the particle and propagates through both slits and undergoes interference. The development of the position of the particle (its velocity and thus its trajectory) is determined by the positions of other particles in the universe, including the particles composing the experimental setup, and the non-local law of Bohmian mechanics can account for the observed particle position on the screen (Esfeld et al 2013).

In the following, we will argue that this interpretation of the effective wave function may be problematic. First of all, consider the simplest situation in which the universal wave function factorizes so that

$$\Psi_t(x, y) = \phi_t(x) \phi_t(y).$$

(5)

Then $$\psi^A_t(x) = \phi_t(x)$$ is A’s effective wave function at $$t$$. This is the first example considered by Dürr, Goldstein and Zanghì (1992) in explaining the effective wave function. For this situation, it is uncontroversial that sub-system A and its environment (or subsystem B) are independent of each other, and the functions $$\phi_t(x)$$ and $$\phi_t(y)$$ describe subsystem A and its environment, respectively. Thus, the effective wave function of subsystem A is independent of the particles in the environment, and it does not encode the non-local influences of these particles either. Although this situation is extremely unphysical, its existence in logic seems to already contradict the above interpretation of the effective wave function; if the interpretation is valid, it should be also valid for the effective wave function in this situation.

Next, consider the general situation in which there is an extra term in the factorization of the universal wave function, which is described by Eq. (4). In this case, the effective wave function of subsystem A is determined by both the universal wave function and the positions of the particles in its environment (via a measurement-like process). If $$Y(t)$$ lies within the support of $$\phi_t(y)$$, A’s effective wave function at $$t$$ will be $$\phi_t(x)$$. If $$Y(t)$$ does not lie within the support of $$\phi_t(y)$$, A’s effective wave function at $$t$$ will be not $$\phi_t(x)$$. For example, suppose $$\Theta_t(x, y) = \sum_n \phi_n(x) \phi_n(y)$$, then if $$Y(t)$$ lies within the support of $$\phi_1(y)$$ (and $$\phi_i(y)$$ and $$\phi_j(y)$$ are functions with macroscopically disjoint supports for any $$i \neq j$$), A’s effective wave function at $$t$$ will be $$\phi_1(x)$$. However, the role played by the particles in the environment is only selecting which function $$\phi_i(x)$$ the effective wave function of subsystem A is, while each selected function $$\phi_i(x)$$ is independent of the particles in the environment and completely determined by the universal wave function. Thus, it seems that in general situations the effective wave function of a subsystem does not encode the influences of other particles in the universe either. When the effective wave function of a subsystem has been selected, the other particles in the universe will have no influence on the
particles of the subsystem. This means that in the double slit experiment with one particle at a time, the development of the position of the particle does not depend on the positions of other particles in the universe (if only the positions of these particles select the same effective wave function of the studied particle during the experiment, e.g. $Y(t)$ has been within the support of $\phi(t,y)$ during the experiment).

This result may have further implications for the ontology of Bohmian mechanics. If the ontology of Bohmian mechanics consists only in particles and their positions, then the effective wave function of a subsystem of the universe, which is not nomological, should be ontologically explained by these particles and their positions. It is uncontroversial that the effective wave function of a subsystem does not supervene on the distribution of the system’s particles’ positions. For instance, for the electron in the hydrogen atom, there are countably many real-valued wave functions corresponding to different energy eigenstates of the electron, but they may all describe a particle that is at rest at all times. Therefore, if the ontology of Bohmian mechanics consists only in particles and their positions, then the effective wave function of a subsystem must encode the influences of the particles which are not part of the subsystem. It seems that this logic was also accepted by Esfeld et al (2013), though they did not explicitly give this argument. As we have argued above, however, the effective wave function of a subsystem of the universe does not encode the influences of other particles on the subsystem. Therefore, it seems that the ontology of Bohmian mechanics cannot consist only in particles and their positions.

Finally, we note that the existence of the above simplest situation in logic might also pose a threat to the nomological interpretation of the universal wave function. First, in this situation, since the universal wave function factorizes and the subsystems A and B are independent of each other, it seems uncontroversial that the effective wave functions of the subsystems have the same nomological or ontological status as the universal wave function. Next, it is generally thought that the wave function being nomological requires that it is time-independent. Even if the universal wave function is time-independent, the effective wave functions of the two subsystems can be both time-dependent. Therefore, it seems that the effective wave functions are not nomological, and thus the universal wave function of the universe is not nomological too. By contrast, both the effective wave functions and the universal wave function can be ontological. In addition, if the universal wave function represents the disposition of motion of all particles in the universe (Esfeld et al 2013), then when the universal wave function factorizes, the effective wave function of each subsystem will also represent the disposition of motion of the particles of the subsystem. This means that Belot’s (2012) objections also apply to the dispositionalist interpretation of Bohmian mechanics suggested by Esfeld et al (2013).

To sum up, we have argued that the effective wave function of a sub-
system of the universe does not encode the influences of other particles in
the universe. This suggests that the ontology of Bohmian mechanics does
not consist only in Bohmian particles and their positions. However, we
note that even if the wave function is admitted as part of the ontology of
Bohmian mechanics, the physical entity described by the wave function is
not necessarily a continuous entity on the high-dimensional configuration
space of the universe (cf. Albert 1996). It has been suggested that the wave
function of an N-body system represents the state of ergodic motion of N
non-Bohmian particles in three-dimensional physical space, and at a deeper
level it represents the dispositional property of the particles that determines
their motion (Gao 2011, 2013, 2014). This interpretation of the wave func-
tion may not only avoid the problems of wave function ontology (cf. Solé
2013), but also overcome Belot’s (2012) objections to the dispositionalist
interpretation of Bohmian mechanics. If it is valid, then a quantum system
will contain two kinds of particles in Bohmian mechanics. The Schrödinger
equation is for the non-Bohmian particles undergoing ergodic motion, and
the guiding equation is for the Bohmian particles undergoing non-ergodic
motion. In this way, the ontology of Bohmian mechanics may consist only
in particles.

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