Bell's local causality for philosophers

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Abstract

This paper is the philosopher-friendly version of our more technical work (Hofer-Szabó and Vecsernyés, 2014). It aims to give a clear-cut definition of Bell's notion of local causality. Having provided a framework, called local physical theory, which integrates probabilistic and spatiotemporal concepts, we formulate the notion of local causality and relate it to other locality and causality concepts. Then we compare Bell's local causality with Reichenbach's Common Cause Principle and relate both to the Bell inequalities. We find a nice parallelism: both local causality and the Common Cause Principle are more general notions than captured by the Bell inequalities. Namely, Bell inequalities cannot be derived neither from local causality nor from a common cause unless the local physical theory is classical or the common cause is commuting, respectively.

Key words: local causality, Bell inequality, common cause

1 Introduction

Local causality is the principle that causal processes cannot propagate faster than the speed of light. This does not mean that in a physical theory subject to this principle no correlation between spatially separated events can exist; a correlation can well be brought about by a common cause in the past of the events in question. However, since all causal processes propagate within the lightcone, fixing the past of an event in a detailed enough manner, the state of this event will be fixed once and for all, and no other spatially separated event can contribute to it any more.

In a nutshell, this is the idea which becomes primary focus in John Bell's (2004) seminal papers initiating a whole research program in the foundations of quantum theory. In these papers Bell translated the intuitive idea of local causality into a probabilistic language opening the door to treat the principle in a theoretical setting and to test its experimental validity *via* the Bell inequalities derived from the principle. The logical scheme of this translation was the following: if physical events are localized in the spacetime in a certain independent way, then these events are to satisfy certain probabilistic independencies. This manual was highly intuitive, however, to apply it in a formally correct way one had to wait until the advent of a mathematically well-defined and physically well-motivated formalism which is able to integrate spatiotemporal and probabilistic concepts. Without such a framework one could not account for the (otherwise intuitive) inference from relations between spacetime regions to probabilistic independencies between, say, random variables. The most elaborate formalism offering such a general framework is quantum field theory, or its algebraic-axiomatic form, algebraic quantum field theory (AQFT).

Thus, it comes as no surprise that AQFT has soon become an important medium to pursue research on the Bell inequalities (Summers, 1987a,b; Summers and Werner, 1988; Halvorson 2007); relativistic causality (Butterfield 1995, 2007; Earman and Valente, 2014); or the closely related (see below) Common Cause Principle (Rédei 1997; Rédei and Summers 2002; Hofer-Szabó and Vecsernyés 2012a, 2013a). In

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this paper we follow the route pioneered by the algebraists, but we do not go as far as AQFT. Our aim is simply to establish a *minimal framework* which is needed to formulate Bell's notion of local causality in a strict fashion. Thus we will borrow only a part of AQFT to represent something which we will call a *local physical theory.* A local physical theory is a formal structure integrating the two most important components of a general physical theory: spacetime structure and algebraic-probabilistic structure. Our secondary aim in this paper is to clarify the relation of Bell's local causality to such other important notions as local primitive causality, Common Cause Principle and the Bell inequalities.

There is a renewed interest in a deeper conceptual and formal understanding of Bell's notion of local causality. Travis Norsen illuminating paper on local causality (Norsen, 2011) or its relation to Jarrett's completeness criterion (Norsen, 2009); the paper of Seevinck and Uffink (2011) aiming at providing a 'sharp and clean' formulation of local causality; or Henson's (2013) paper on the relation between separability and the Bell inequalities are all examples of this inquiry. Our research runs parallelly in some respect to these investigations and we will comment on the points of contact underway.

In Section 2 we fix our mathematical framework, called local physical theory and list some important relativistic causality principles. In Section 3 we formulate Bell's notion of local causality in a local physical theory. In Section 4 we compare local causality with the Common Cause Principle and relate both to the Bell inequalities. We conclude the paper in Section 5.

This paper is the philosopher-friendly version of our more detailed and more technical work (Hofer-Szabó and Vecsernyés, 2014). Many points (such as local causality in a non-atomic local physical theory; local causality in stochastic dynamics; its complex relation to other locality and causality concepts, etc.) which are treated in a more conceptual way here obtain a more detailed mathematical analysis there. We will not refer to these results point-by-point in the paper.

2 What is a local physical theory?

First we set the framework, called local physical theory, within which probabilistic and spatiotemporal notions can be treated in an integrated way.

Definition 1. A *$\mathcal{P}_\mathcal{K}$-covariant local physical theory* is a net $\{\mathcal{A}(V), V \in \mathcal{K}\}$ associating algebras of events to spacetime regions which satisfies *isotony*, *microcausality* and *covariance* defined as follows (Haag, 1992):

1. *Isotony.* Let $\mathcal{M}$ be a globally hyperbolic spacetime and let $\mathcal{K}$ be a covering collection of bounded, globally hyperbolic subspacetime regions of $\mathcal{M}$ such that $(\mathcal{K}, \subseteq)$ is a directed poset under inclusion $\subseteq$. The net of local observables is given by the isotone map $\mathcal{K} \ni V \mapsto \mathcal{A}(V)$ to unital C^*-algebras, that is $V_1 \subseteq V_2$ implies that $\mathcal{A}(V_1)$ is a unital C^*-subalgebra of $\mathcal{A}(V_2)$. The *quasilocal algebra* $\mathcal{A}$ is defined to be the inductive limit C^*-algebra of the net $\{\mathcal{A}(V), V \in \mathcal{K}\}$ of local C^*-algebras.

2. *Microcausality* (also called as *Einstein causality*) is the requirement that $\mathcal{A}(V')' \cap \mathcal{A} \supseteq \mathcal{A}(V), V \in \mathcal{K}$, where primes denote spacelike complement and algebra commutant, respectively.

3. *Spacetime covariance.* Let $\mathcal{P}_\mathcal{K}$ be the subgroup of the group $\mathcal{P}$ of geometric symmetries of $\mathcal{M}$ leaving the collection $\mathcal{K}$ invariant. A group homomorphism $\alpha \colon \mathcal{P}_\mathcal{K} \to \mathrm{Aut}\,\mathcal{A}$ is given such that the automorphisms $\alpha_g, g \in \mathcal{P}_\mathcal{K}$ of $\mathcal{A}$ act covariantly on the observable net: $\alpha_g(\mathcal{A}(V)) = \mathcal{A}(g \cdot V), V \in \mathcal{K}$.

If the quasilocal algebra $\mathcal{A}$ of the local physical theory is commutative, we speak about a *local classical theory*; if it is noncommutative, we speak about a *local quantum theory.* For local classical theories microcausality fulfills trivially.

A *state* ϕ in a local physical theory is defined as a normalized positive linear functional on the quasilocal observable algebra $\mathcal{A}$. The corresponding GNS representation $\pi_\phi \colon \mathcal{A} \to \mathcal{B}(\mathcal{H}_\phi)$ converts the net of C^*-algebras into a net of C^*-subalgebras of $\mathcal{B}(\mathcal{H}_\phi)$. Closing these subalgebras in the weak topology one arrives at a net of local von Neumann observable algebras: $\mathcal{N}(V) := \pi_\phi(\mathcal{A}(V))'', V \in \mathcal{K}$. Von Neumann algebras are generated by their projections, which are called *quantum events* since they can be interpreted as 0-1–valued observables. The net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ of local von Neumann algebras also obeys isotony, microcausality, and $\mathcal{P}_{\mathcal{K}}$-covariance, hence one can also refer to a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ of local *von Neumann algebras* as a local physical theory. Although, the local σ-algebras of classical observable events provided by the projections of the local abelian von Neumann algebras are not the most general σ-algebras, still they provide us a rich enough set of examples for classical theories.

One can introduce a number of important locality and causality concepts into the above formalism. Here we only list them in turn and assert their logical relations; for the motivation of these concepts see (Earman and Valente, 2014).

Local primitive causality. For any globally hyperbolic bounded subspacetime region $V \in \mathcal{K}$, $\mathcal{A}(V'') = \mathcal{A}(V)$.

A local physical theory satisfying local primitive causality also satisfies the following two properties:

Local determinism. For any two states ϕ and ϕ' and for any globally hyperbolic spacetime region $V \in \mathcal{K}$, if $\phi|_{\mathcal{A}(V)} = \phi'|_{\mathcal{A}(V)}$ then $\phi|_{\mathcal{A}(V'')} = \phi'|_{\mathcal{A}(V'')}$.

Stochastic Einstein locality. Let $V_A, V_C \in \mathcal{K}$ such that $V_C \subset J_-(V_A)$ and $V_A \subset V_C''$. If $\phi|_{\mathcal{A}(V_C)} = \phi'|_{\mathcal{A}(V_C)}$ holds for any two states ϕ and ϕ' on $\mathcal{A}$ then $\phi(A) = \phi'(A)$ for any projection $A \in \mathcal{A}(V_A)$.

If a net satisfies Haag duality:

$$\mathcal{A}(V')' \cap \mathcal{A} = \mathcal{A}(V) \tag{1}$$

for all bounded globally hyperbolic subspacetime region V, which is a stronger requirement than microcausality, then it also satisfies local primitive causality. But microcausality alone does not entail local primitive causality.

A global version of local primitive causality (entailed by the local one) is

Primitive causality. Let $\mathcal{K}(\mathcal{C}) \subseteq \mathcal{K}$ be a covering collection of a Cauchy surface $\mathcal{C}$ and let $\mathcal{A}(\mathcal{K}(\mathcal{C}))$ be the corresponding algebra. Then $\mathcal{A}(\mathcal{K}(\mathcal{C})) = \mathcal{A}$.

A local physical theory with primitive causality satisfies

Determinism. If $\phi|_{\mathcal{A}(\mathcal{K}_\mathcal{C})} = \phi'|_{\mathcal{A}(\mathcal{K}_\mathcal{C})}$ for any two states ϕ and ϕ' on $\mathcal{A}$ then $\phi = \phi'$.

In the rest of the paper a local physical theory obeys only isotony, microcausality, and $\mathcal{P}_{\mathcal{K}}$-covariance by definition without any other locality and causality constraints. We turn now to Bell's notion of local causality.

3 Bell's notion of local causality in a local physical theory

Local causality has been playing a central notion in Bell's influential writings on the foundations of quantum theory. To our knowledge it gets an explicit formulation three times: in (Bell, 1975/2004 , p. 54), (Bell, 1986/2004, p. 200), and (Bell, 1990/2004, p. 239-240). In this latter posthumously published paper "La nouvelle cuisine", for example, local causality is formulated as follows:[1]

> "A theory will be said to be locally causal if the probabilities attached to values of local beables in a space-time region V_A are unaltered by specification of values of local beables in a space-like separated region V_B, when what happens in the backward light cone of V_A is already

[1]For the sake of uniformity we slightly changed Bell's denotation and figures.

sufficiently specified, for example by a full specification of local beables in a space-time region V_C. " (Bell, 1990/2004, p. 239-240)

(For a reproduction of the figure Bell is attaching to this formulation see Fig. 1 with Bell's caption.) Bell

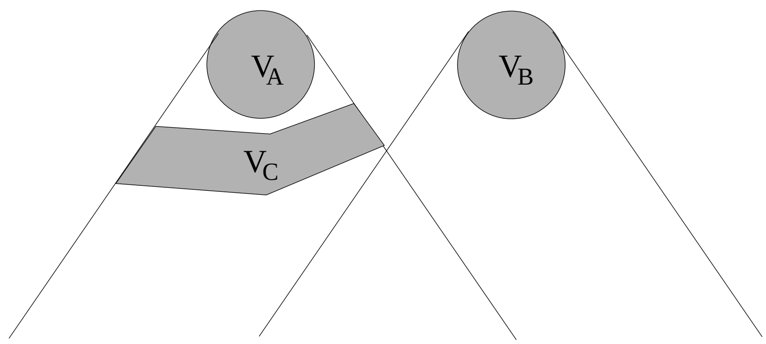

Figure 1: Full specification of what happens in V_C makes events in V_B irrelevant for predictions about V_A in a locally causal theory.

elaborates on his formulation as follows:

> "It is important that region V_C completely shields off from V_A the overlap of the backward light cones of V_A and V_B. And it is important that events in V_C be specified completely. Otherwise the traces in region V_B of causes of events in V_A could well supplement whatever else was being used for calculating probabilities about V_A. The hypothesis is that any such information about V_B becomes redundant when V_C is specified completely." (Bell, 1990/2004, p. 240)

The notions featuring in Bell's formulation has been target of intensive discussion in philosophy of science. Here we would like to give only a brief exposé of them.

The notion "beable" is Bell's neologism. (See Norsen 2009, 2011.) "The *be*ables of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real" (Bell, 1990/2004, p. 234). The clarification of the "beables" of a given theory is indispensable in order to define local causality since "there *are* things which do go faster than light. British sovereignty is the classical example. When the Queen dies in London (long may it be delayed) the Prince of Wales, lecturing on modern architecture in Australia, becomes instantaneously King" (p. 236).

Beables are to be local: "*Local* beables are those which are definitely associated with particular space-time regions. The electric and magnetic fields of classical electromagnetism, $\mathbf{E}(t,x)$ and $\mathbf{B}(t,x)$ are again examples." (p. 234). Furthermore, local beables are to "specify completely" region V_C in order to block causal influences arriving at V_A from the common past of V_A and V_B. (For the question of complete *vs.* sufficient specification see (Seevinck and Uffink, 2014).)

One can translate Bell's above terms in the following way. In a classical field theory beables are characterized by sets of field configurations. Taking the equivalence classes of those field configurations which have the same field values on a given spacetime region one can generate local σ-algebras. Translating σ-algebras into the language of abelian von Neumann algebras one can capture Bell's notion of "local beables" in the framework of a local physical theory. More generally, one can use the term "local beables" both for abelian and also for non-abelian local von Neumann algebras, hence treating local classical and quantum theories on an equal footing.

How to translate the term "complete specification"? Complete specification of field configurations in a given spacetime region means that one specifies the field values to a prescribed value in the given spacetime region, that is one specifies the corresponding local equivalence class of a single configuration. In probabilistic language complete specification is translated into a probability measure having support on this local equivalence class of the single specified configuration. In the abelian von Neumann language this corresponds to a change of the original state that results in a *pure state* on the local von Neumann algebra in question with value 1 on the projection corresponding to the local equivalence class of the single specified configuration. We also would like this change of states to be as *local* as possible. Both pureness and locality can be captured in a general local physical theory by some conditions imposed on a completely positive map generating the change of states. If the local algebras of the net are *atomic* (which, by the way, is not the case in a general AQFT), the change of states can be generated by conditioning the original state on an arbitrary *atomic* event (a minimal projection) in the local algebra. In this case "complete specification of beables" will mean a so-called selective measurement by an atomic event in a local algebra (Henson, 2013). With these notions in hand we can formulate Bell's notion of local causality in local physical theories:[2]

Definition 2. A local physical theory represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ of von Neumann algebras is called *(Bell) locally causal*, if for any pair $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ and for every locally normal and faithful state ϕ establishing a correlation, $\phi(AB) \neq \phi(A)\phi(B)$, between A and B, and for any spacetime region V_C such that

(i) $V_C \subset J_-(V_A)$,

(ii) $V_A \subset V_C''$,

(iii) $J_-(V_A) \cap J_-(V_B) \cap \big(J_+(V_C) \setminus V_C\big) = \emptyset$,

(see Fig. 2) and for any atomic event C_k of $\mathcal{A}(V_C)$ $(k \in K)$, the following holds:

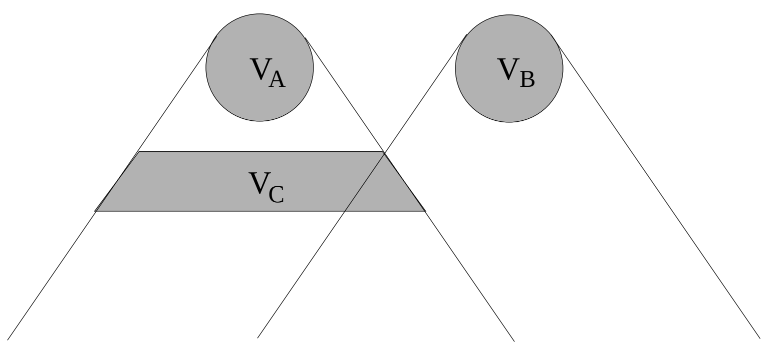

Figure 2: A region V_C satisfying Requirements (i)-(iii).

$$\frac{\phi(C_k ABC_k)}{\phi(C_k)} = \frac{\phi(C_k AC_k)}{\phi(C_k)} \frac{\phi(C_k BC_k)}{\phi(C_k)} \tag{2}$$

Remarks:

1. Again we stress that Definition 2 captures local causality only for local physical theories with *atomic* local von Neumann algebras.

2. In case of classical theories a locally faithful state ϕ determines a locally nonzero probability measure p by $p(A) := \phi(A) > 0, A \in \mathcal{P}(\mathcal{N}(V))$. By means of this (2) can be written in the following 'symmetric' form:

$$p(AB|C_k) = p(A|C_k)p(B|C_k) \tag{3}$$

[2]For a similar approach to local causality using σ-algebras see (Henson, 2013); for a comparison of the two approaches see our (Hofer-Szabó and Vecsernyés 2014).

or equivalent in the 'asymmetric' form:

$$p(A|BC_k) = p(A|C_k) \tag{4}$$

sometimes used in the literature (for example in (Bell, 1975/2004 , p. 54)).

3. The role of Requirement (iii) in the definition is to ensure that "V_C shields off from V_A the overlap of the backward light cones of V_A and V_B". Namely, a spacetime region *above* V_C in the common past of the correlating events (see Fig. 3) may contain stochastic events which, though completely

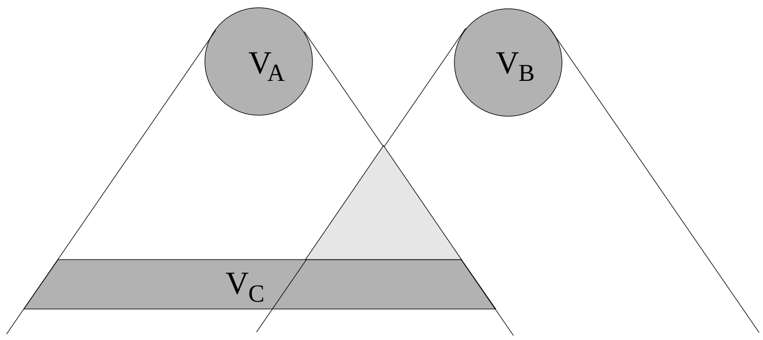

Figure 3: A region V_C for which Requirement (iii) does not hold.

specified by the region V_C, still, being stochastic, could establish a correlation between A and B in a classical stochastic theory (Norsen, 2011; Seevinck and Uffink 2011). If V_C is a piece of a Cauchy surface Requirement (iii) coincides with Requirement (iv):

(iv) $J_-(V_A) \cap J_-(V_B) \cap V_C = \emptyset$

visualized in Fig. 4. However, for algebras corresponding to coverings of Cauchy surfaces Require-

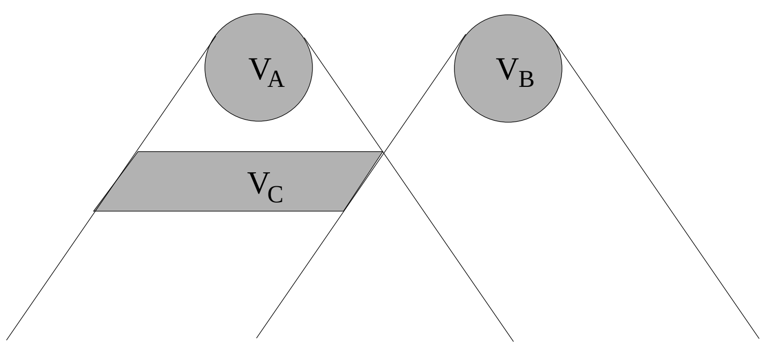

Figure 4: A region V_C for which Requirement (iv) holds.

ment (iii) is weaker than Requirement (iv) since it allows for regions penetrating into the top part of the common past. For local classical theories Requirement (iii) is enough, but for local quantum theories Requirement (iv) should be used.

Of course the main question is how to ensure that a local physical theory is locally causal. Generally the question is difficult to answer; here we simply mention a sufficient condition in case of *atomic* local algebras:

1. A local *classical* theory is locally causal if the local von Neumann algebras are *atomic* and satisfy *local primitive causality.*

   Proof. Due to isotony and local primitive causality $\mathcal{N}(V_A) \subset \mathcal{N}(V_C'') = \mathcal{N}(V_C)$ and hence for any atom C_k of $\mathcal{N}(V_C)$: either (i) $AC_k = 0$ or (ii) $AC_k = C_k$. In case of (i) both sides of (2) is zero, in case of (ii) (2) holds as follows:

$$\frac{\phi(ABC_k)}{\phi(C_k)} = \frac{\phi(BC_k)}{\phi(C_k)} = \frac{\phi(AC_k)}{\phi(C_k)}\frac{\phi(BC_k)}{\phi(C_k)}. \tag{5}$$

2. A local *quantum* theory is locally causal if the local von Neumann algebras are *atomic* and satisfy *local primitive causality*, and if Requirement (iii) in the definition of local causality is replaced by Requirement (iv).

   Proof. Since region V_C is spatially separated from region V_B, $B \in \mathcal{N}(V_B)$ and an atomic event $C_k \in \mathcal{N}(V_C)$ will commute due to microcausality. Using $C_k A C_k = r\, C_k$ (where $r \in [0,1]$ depends on both A and C_k) we obtain:

$$\frac{\phi(C_kABC_k)}{\phi(C_k)} = \frac{\phi(C_kAC_kB)}{\phi(C_k)} = r\frac{\phi(C_kB)}{\phi(C_k)} = \frac{\phi(C_kAC_k)}{\phi(C_k)}\frac{\phi(BC_k)}{\phi(C_k)}. \tag{6}$$

Looking at Point 2 the reader may justly ask: how can a local *quantum* theory be locally causal if local causality implies various Bell inequalities, which are known to be violated for certain set of quantum correlations. Does Definition 2 correctly grasp Bell's intuition of local causality? We answer these questions in the next section.

4 Local causality, Common Cause Principle and the Bell inequalities

Local causality is closely related to Reichenbach's (1956) Common Cause Principle. The *Common Cause Principle* (CCP) states that if there is a correlation between two events A and B and there is no direct causal (or logical) connection between the correlating events, then there always exists a common cause C of the correlation. Reichenbach's original classical probabilistic definition of the common cause can readily be generalized to the local physical theory framework. (See (Rédei 1997, 1998), (Rédei and Summers 2002, 2007), (Hofer-Szabó and Vecsernyés 2012, 2013) and (Hofer-Szabó, Rédei and Szabó 2013).)

Let $\{\mathcal{N}(V), V \in \mathcal{K}\}$ be a net representing a local physical theory. Let $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ be two events (projections) supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ which correlate in a locally normal and faithful state ϕ. The common cause of the correlation is an event screening off the correlating events from one another and localized in the past of A and B. But in which past? Here one has (at least) three options. One can localize C either (i) in the *union* $J_-(V_A) \cup J_-(V_B)$ or (ii) in the *intersection* $J_-(V_A) \cap J_-(V_B)$ of the causal past of the regions V_A and V_B; or (iii) more restrictively in $\cap_{x \in V_A \cup V_B} J_-(x)$, that is in the spacetime region which lies in the intersection of causal pasts of *every* point of $V_A \cup V_B$. We will refer to the above three pasts in turn as the *weak past*, *common past*, and *strong past* of A and B, respectively (Rédei, Summers, 2007).

Depending on the choice of the past we can define various CCPs in a local physical theory:

Definition 3. A local physical theory represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$ is said to satisfy the *(Weak/Strong) CCP*, if for any pair $A \in \mathcal{N}(V_A)$ and $B \in \mathcal{N}(V_B)$ of projections supported in spacelike separated regions $V_A, V_B \in \mathcal{K}$ and for every locally faithful state ϕ establishing a correlation between A and B, there exists a nontrivial common cause system, that is a set of mutually orthogonal projections $\{C_k\}_{k\in K} \subset \mathcal{N}(V_C), V_C \in \mathcal{K}$ summing up to the unit of the algebra, satisfying

$$\frac{\phi(C_kABC_k)}{\phi(C_k)} = \frac{\phi(C_kAC_k)}{\phi(C_k)}\frac{\phi(C_kBC_k)}{\phi(C_k)}, \qquad \text{for all } k \in K \tag{7}$$

such that the localization region of V_C is in the (weak/strong) common past of V_A and V_B.

A common cause is called *nontrivial* if $C_k \not\leq X$ with $X = A, A^\perp, B$ or $B^\perp$ for some $k \in K$. If $\{C_k\}_{k \in K}$ commutes with both A and B, then we call it a *commuting* common cause system, otherwise a *noncommuting* one, and the appropriate CCP a *Commutative/Noncommutative CCP.*

The status of these six different notions of the CCP has been thoroughly scrutinized in a special local quantum theory, namely algebraic quantum field theory (AQFT). Here we only give a brief overview.

The question whether the Commutative CCPs are valid in a Poincaré covariant local quantum theory was first raised by Rédei (1997, 1998). As an answer, Rédei and Summers (2002, 2007) have shown that the Commutative Weak CCP is valid in Poincaré covariant AQFT. Since local algebras in a Poincaré covariant AQFT are atomless (type III) von Neumann algebras, the question has been raised whether Commutative Weak CCP is valid in local quantum theories with locally finite dimensional, hence atomic local von Neumann algebras. Deciding the question, Hofer-Szabó and Vecsernyés (2012a) have given an example in the local quantum Ising model where the Commutative Weak CCP is *not* valid. A natural reaction to these facts was to ask what role commutativity plays in these propositions. Addressing this question, Hofer-Szabó and Vecsernyés (2013) have shown that allowing common causes *not* to commute with the correlating events, the Noncommutative Weak CCP can be proven in local (UHF-type) quantum theories with finite dimensional local von Neumann algebras.

Concerning the Commutative (Strong) CCP less is known. If one also admits projections localized only in *un*bounded regions, then the Strong CCP is known to be false: von Neumann algebras pertaining to complementary wedges contain correlated projections but the strong past of such wedges is empty (see (Summers and Werner, 1988) and (Summers, 1990)). In spacetimes having horizons, e.g. those with Robertson–Walker metric, the common past of spacelike separated bounded regions can be empty, although there are states which provide correlations among local algebras corresponding to these regions (Wald 1992). Hence, CCP is not valid there. Restricting ourselves to projections in *local* algebras on *Minkowski* spacetimes the situation is not clear. We are of the opinion that one cannot decide on the validity of the (Strong) CCP in this case without an explicit reference to the dynamics.

Now, what is the relationship between the various CCPs and Bell's local causality? The following list of *prima facie* similarities and differences may help to explicate this relationship:

Similarities:

1. Both local causality and the CCPs are *properties of a local physical theory* represented by a net $\{\mathcal{N}(V), V \in \mathcal{K}\}$.

2. The core mathematical requirement of both principles is the *screening-off condition* (2) or equivalently (7).

3. The *Bell inequalities* can be derived from both principles. (But see below.)

Differences:

1. In case of local causality the screening-off condition (2) is required for *every* atomic event (satisfying certain localization conditions). In case of the CCP for every correlation *only a single* subset of events is postulated satisfying the screening-off condition (7).

2. In case of local causality the screening-off condition is required only for *atomic* events. In case of the CCPs these atomic screener-offs of the algebra $\mathcal{A}(V_C)$ are called trivial, since they screen any correlation off. What one is typically looking for are *nontrivial* common causes.

3. In case of local causality screener-offs are localized *'asymmetrically'* in the past of V_A; in case of the CCP they are localized *'symmetrically'* in either the weak, common or strong past of V_A and V_B.

Let us come back to Point 1 of the Similarities, that is to the relation of local causality and the CCPs to the Bell inequalities. In (Hofer-Szabó and Vecsernyés, 2013b, Proposition 2) we have proven a proposition which clarifies the relation between the CCPs and the Bell inequalities. It asserts that the Bell inequalities can be derived from the existence of a (local, non-conspiratorial joint) common cause system for a set of correlations *if* common causes are understood as *commuting* common causes. However, if we also allow for *non*commuting common causes, the Bell inequalities can be derived only for *another* state which is *not* identical to the original one. And indeed in (Hofer-Szabó and Vecsernyés, 2013a,b) a noncommuting common cause was constructed for a set of correlations violating the Clauser–Horne inequality. Moreover, this common cause was localized in the *strong* past of the correlating events.

Now, an analogous proposition holds for the relation between local causality and the Bell inequalities. We assert here only the proposition without the proof since the proof is step-by-step the same as that of the proposition mentioned above.

Proposition 1. Let $\{\mathcal{N}(V), V \in \mathcal{K}\}$ be a locally causal local physical theory with atomic (type I) local von Neumann algebras. Let $A_1, A_2 \in \mathcal{A}(V_A)$ and $B_1, B_2 \in \mathcal{A}(V_B)$ be four projections localized in spacelike separated spacetime regions V_A and V_B, respectively, which pairwise correlate in the locally faithful state ϕ that is

$$\phi(A_m B_n) \quad \neq \quad \phi(A_m)\,\phi(B_n) \tag{8}$$

for any $m, n = 1, 2$. Let furthermore $\{C_k\}_{k\in K} \subset \mathcal{N}(V_C), V_C \in \mathcal{K}$ be a maximal partition of the unit, where the set $\{C_k\}_{k\in K}$ contains mutually orthogonal *atomic* projections satisfying Requirements (i)-(iii) in Definition 2 of local causality. Then the Clauser–Horne inequality

$$-1 \leqslant \phi_{\{C_k\}}(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0. \tag{9}$$

holds for the state $\phi_{\{C_k\}}(X) := \sum_k \phi(C_k X C_k)$. If $\{C_k\}$ commutes with A_1, A_2, B_1 and B_2, then the Clauser–Horne inequality holds for the original state ϕ:

$$-1 \leqslant \phi(A_1B_1 + A_1B_2 + A_2B_1 - A_2B_2 - A_1 - B_1) \leqslant 0. \tag{10}$$

The moral is the same as in the case of the CCPs: the Bell inequalities can be derived in a locally causal local physical theory only for a modified state $\phi_{\{C_k\}}$; it can be derived for the original state ϕ *if* the set of atomic projections $\{C_k\}$ localized in V_C commutes with A_1, A_2, B_1 and B_2. What is needed for this to be the case?

In local *classical* theories any element taken from any local algebra will commute, therefore the Bell inequalities will hold in local classical theories. In locally causal local *quantum* theories, commutativity of $\{C_k\}$ and the correlating events is not guaranteed. If V_C is spatially separated from V_B (due to Requirement (iv) in Definition 2), then $\{C_k\}$ will commute with B_1 and B_2 and hence (2) will be satisfied. However, for noncommuting A_1 and A_2 one cannot pick a maximal partition $\{C_k\}$ commuting with both projections, and therefore the theorem of total probability, $\sum_k \phi(C_k A_m C_k) = \phi(A_m)$, will not hold for the original state ϕ at least for one of the projections A_1 and A_2 (it will hold only for the state $\phi_{\{C_k\}}$). This fact blocks the derivation of Bell inequalities for the original state ϕ. (For the details see (Hofer-Szabó and Vecsernyés, 2013b, p. 410) In short, the Bell inequalities can be derived in a locally causal local quantum theory only if all the projections commute.

Coming back to the question posed at the end of the previous Section, namely how a local *quantum* theory can be locally causal in the face of the Bell inequalities, we already know the answer: the Bell inequalities can be derived from local causality if it is required that the 'beables' of the local theory are represented by *commutative* local algebras. This fact is completely analogous to the relation shown in (Hofer-Szabó and Vecsernyés, 2013b), namely that the Bell inequalities can be derived from a (local, non-conpiratorial, joint) common cause system if it is a *commuting* common cause system. Thus, the violation of the Bell inequalities for certain quantum correlations is compatible with locally causal local quantum theories but not with locally causal local classical theories. Local causality is a more general notion than captured by the Bell inequalities.

5 Conclusions

In this paper we have shown the following:

(i) Bell's notion of local causality presupposes a clear-cut framework in which probabilistic and spatiotemporal entities can be related. This aim can be reached by introducing the notion of a *local physical theory* represented by an isotone net of algebras.

(ii) Within this general framework we have defined Bell's notion of *local causality* and shown sufficient conditions on which local physical theories will be locally causal.

(iii) Finally, we pointed out some important similarities and differences between local causality and the CCPs and showed that in a locally causal local quantum theory one cannot derive the Bell inequalities from local causality just as one cannot derive them from noncommuting common causes.

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