I ♥ ♦s

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Abstract

Richard Arthur (2006) and I (Savitt 2009) proposed that the present in (time-oriented) Minkowski spacetime should be thought of as a small causal diamond. That is, given two timelike separated events p and q, with p earlier than q, they suggested that the present (relative to those two events) is the set $I^+(p) \cap I^-(q)$. Mauro Dorato (2011) presents three criticisms of this proposal. I rebut all three and then offer two more plausible criticisms of the Arthur/Savitt proposal. I argue that these criticisms also fail.

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1. Causal Diamonds

At the end of the twentieth century, it looked as if one question at the intersection of physics and metaphysics had been settled. What is the present in Minkowski spacetime, M? The upshot of a series of well-known papers beginning in the 1960s seemed to prove that one had a very limited choice. The present, at or for a spacetime point $e \in M$ could be either the whole spacetime M or just the point e itself. The choice is no wider if one allows the present to be defined relative to a spacetime point $e \in M$ *and* a timelike worldline γ containing e.[1]

It might come as a surprise, then, that I (2009) suggested a third structure for the present (relative to e and γ) in M.[2] I then called these structures *Alexandroff presents*, but now, to conform to the usage that seems to be standard in physics, I will call them *causal diamonds*. The first order of business must be to define them. Even though the discussion below will mostly concern Minkowski spacetime M, it will be useful to define causal diamonds in a larger class of spacetimes that includes M.

Consider relativistic spacetimes $< M,g,\uparrow >$ that are strongly causal and possess a temporal orientation (as indicated by the arrow). Choose two points p,q on a timelike

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[1] The papers from which these ideas emerged were by Howard Stein (1968, 1991) and by Rob Clifton and Mark Hogarth (1995). I will refer to them as *SCH*. These papers were written in response to papers by Cornelis Rietdijk (1966, 1967), Hilary Putnam (1967), and Nicholas Maxwell (1985, 1988). I will discuss the implications of the results in the SCH papers in more detail below.

[2] The same suggestion can be found in Arthur (2006), and a similar idea but to a different purpose in Myrvold (2003, §2). All of us were clearly inspired by the discussion at the end of Stein (1991). One should note also that in the philosophical literature causal diamonds appeared explicitly in Winnie (1997), which in turn was indebted to Robb (1914, 1921, 1936).

worldline ɣ in M with p earlier than q. Then the set $I^+(p) \cap I^-(q)$ is *a causal diamond*.[3] In these spacetimes causal diamonds are guaranteed to exist--for instance, by Theorem 3.27 of Minguzzi and Sánchez (2008). Such spacetimes are free of closed timelike curves, and the topology these sets compose, which is known as the Alexandrov (or Alexandroff) topology, is Hausdorff, giving one what is generally thought to be a physically reasonable spacetime.

Gibbons and Solodukhin (2007a,b) distinguish between small vs. large causal diamonds. Small causal diamonds have a proper time separation between the defining end-points p and q that is small compared to the curvature scale of the ambient spacetime. Larger causal diamonds are those in which the later point q recedes to the future boundary $\mathcal{I}^+$ of an asymptotically de-Sitter spacetime. The cosmologists whose work we will sketch below employ large causal diamonds whereas Arthur and I proposed small causal diamonds (diamonds in which the proper time separation τ between the endpoints p and q is scaled to the human "specious" or psychological present) as (special) relativistic counterparts of the common sense present. But they are all causal diamonds nevertheless.

2. Dorato *contra* Diamonds

Arthur's and my proposal was criticized in Dorato (2011). The aim of this paper is to evaluate these criticisms and then to add a few further thoughts of my own. In the course of this discussion a more detailed understanding of the proposal under fire will emerge.

Dorato crisply sums up his arguments on page 391 of his paper:

[3] The set $I^+(p)$ is the set of all points in M that can be reached from p by an everywhere future-directed, continuous timelike curve. The set $I^-(q)$ is the set of all points in M from which a continuous, everywhere future-directed timelike curve can reach q. The set $J^+(p)$ is the set of all points in M that can be reached from p by an everywhere future-directed, continuous timelike or lightlike curve. Similarly for $J^-(q)$. Some physicists think of sets like $J^+(p) \cap J^-(q)$ as the causal diamonds.

(i) [Causal diamonds have] no important applications in physical theories;

(ii) it does not seem a plausible, strong and non-arbitrary explanation of the extendedness of our subjective present, and

(iii) It does not correctly pick out the events we intend to pick out when we use “now” in ordinary language,

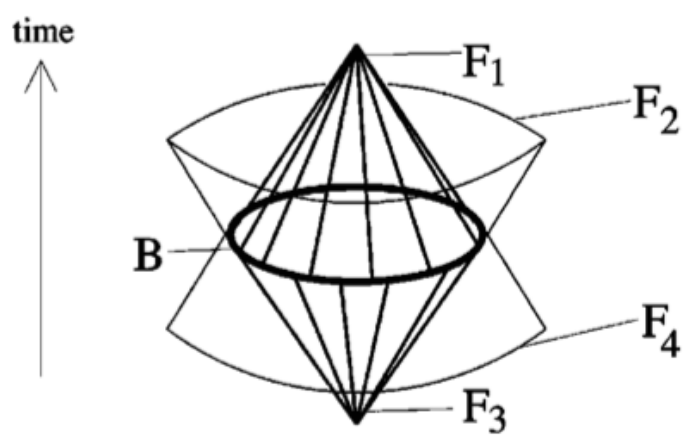
(iv) these seem the only reasons to introduce it.

I conclude that we should drop it.

Let us examine these three criticisms, beginning with the first. My counter-claim is that causal diamonds are well-defined and well-motivated spacetime volumes that have proved, in surprising ways, increasingly handy in recent physics. Let me first advert to authority. Gibbons and Solodukhin (2007a, 2) say that “Causal Diamonds, or Alexandrov open sets, play an increasingly important role in quantum gravity, for example in the approach via casual sets (Sorkin, 2003), in discussions of ‘holography’, and also of the probability of various observations in eternal inflation models (see Bousso et al., 2007, for a recent example and references to earlier work).” Consider, for instance, holography.

Thomas Banks and William Fischler have been working for a decade or so on a generalization of string theory and quantum field theory they call *Holographic Space-Time* (HST). According to Banks in a recent overview of their work (2013, 2), “The basic geometrical object, for which HST provides a quantum avatar, is a causal diamond… A time-like trajectory can be viewed as a nested sequence of causal diamonds.”

To give a simple, related example, let us look at figure 3 of Bousso (2002), a review article on the holographic principle:



The caption of the illustration says this: “The four null hypersurfaces orthogonal to a spherical surface B. The two cones F_1 and F_3 have negative expansion and hence correspond to light sheets. The covariant entropy bound states that the entropy on each light sheet will not exceed the area of B. The other two families of light rays, F_2 and F_4, generate the skirts drawn in thin outline. Their cross-sectional area is increasing, so they are not light sheets. The entropy of the skirts is not related to the area of B.” (Bousso, 2002, 842) The two cones, F_1 and F_3, form a causal diamond. This is only one result of many in the investigation of the holographic principle, but it is one.

The utility of causal diamonds depends on several of their features. First, the volume of a causal diamond is finite, and the area of its boundary is finite. Second, its boundary consists of null or lightlike surfaces. Third, the points in the diamond defined by two points (say p and q) are all those points that can effect some point on a timelike curve extending from p to at q *and* can also be effected by some (other) point on that curve. Bousso imagines an experiment starting at p and ending at q. He claims (Bousso 2000b, especially §2), following Susskind, that physics need take account of only the set of factors that can reciprocally influence the experiment. If so, then physics need consider only events in the causal diamond defined by p and q.

Bousso and Susskind (2011) use causal diamonds for two other purposes. First, they use the boundaries of causal diamonds to define an objective notion of decoherence. When a particle entangled with an apparatus at some event crosses the border of a diamond they define, then (in their view) irreversible decoherence occurs and (in their terms) the event *happens*. Thus they say in §3.3:

> Causal diamonds have definite histories, obtained by tracing over their boundary, which we treat as an observer-independent environment. This gets rid of superpositions of different macroscopic objects, such as bubbles of different vacua, without the need to appeal to actual observers inside the diamond. Each causal diamond history corresponds to a sequence of things that “happen”. And the global picture of the multiverse is just a representation of all the possible diamond histories in a single geometry: the many worlds of causal diamonds!

In addition to providing objective decoherence, Bousso and Susskind, then, use causal diamonds as the many worlds out of which they construct the multiverse in their “multiverse interpretation of quantum mechanics”.

These are some (admittedly, speculative) ways in which causal diamonds have entered into physical theory. I must leave it to the reader to decide whether they are "important". What I would like to emphasize is that causal diamonds are a natural structure to fasten on, since they contain all the spacetime events that can interact causally with events on a timelike worldline γ between the two events, p and q, that define the diamond.

Let me tackle next Dorato's third criticism. Suppose I were to say, on some cold, rainy Vancouver morning, "The sun is surely shining now in Rome." What I would have intended by this (as long as I am not explicitly thinking relativistically) is to pick out events in Rome that are happening at the same time as my utterance and to suppose that those events are part of a sunny day there. To be more pedantic, as far as our common sense, pre-relativistic way of conceiving time goes, my utterance occurs in some observer-independent hyperplane of simultaneous events, and it is meant to signify that the part of the hyperplane that includes Rome contains sunny events.

As I point out (352), but as we all knew already, in the special theory of relativity there is no such distinguished set of simultaneous events. So Dorato is surely right when he says that causal diamonds, if proposed as a scientific successor concept to our common sense concept of the present, do "not correctly pick out the events we intend to pick out when we use 'now' in ordinary language." It is true, however, that *nothing* in M does. Let me just repeat the nice quote from Mermin (2005, xii) that I used to make this point: "That no inherent meaning can be assigned to the simultaneity of distant events is the single most important lesson to be learned from relativity."

So one has to make a choice. Perhaps as far as the special theory goes (and the general theory, insofar as it is locally Minkowskian) there just is just nothing like a (common sense) present to be had in those spacetimes.[4] Alternatively, if one wishes to see what elements of our pre-relativistic concept of time one can find in relativistic spacetimes, one can seek some elements of or structures in Minkowski spacetime (or the more general class of spacetimes stipulated earlier) that *more-or-less* play the role that the common sense present did. If one does make such a proposal, one knows in advance that it will *not* encompass precisely the set of points intended when we use "now" in ordinary language. One looks for a "best fit," with the criteria of fitness rather

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[4] I argued this in Savitt (2000).

loosely specified. That is the philosophical task--assuming that there is a philosophical enterprise here at all.

But if that is the game that's afoot, then the suggestion that each event is its own present--no more, no less--certainly has its difficulties. It is not able to assign a truth value to the example above ("The sun is surely shining now in Rome.") spoken by me on the West Coast, although it works well for Dorato in Rome. On the other hand, any reasonably sized causal diamond defined by two events on my world line, one marking the beginning and one the end of my utterance for instance, will include events in Rome and so will afford grounds for assigning a truth value to the example sentence. There will be many, many examples like it. Although the Arthur/Savitt proposal will indeed fail for some other cases (for, say, my musings about what is happening now on Mars), it will do the job in a host of routine situations. I submit that more in the way of correspondence with the common sense present cannot reasonably be asked for in these spacetimes and that therefore Dorato's third criticism is simply beside the point.

Also, if this is the game that's afoot, then Dorato's second criticism above is as wide of the mark as his third. Causal diamonds are not invoked to explain our having experiences of the present that are extended. Rather, our experience of the present as having some duration grounds the requirement (or, more moderately, suggests the possibility) that the relativistic counterpart of the present not be a mere point or an achronal set of points.

In the penultimate paragraph of his paper Dorato says that "violations of achronality are admissible only for the psychological present, but not for the physical present," (393) Viewed one way, this is an eminently sensible view. How could two events that are timelike separated, that are *invariantly* temporally ordered, both be present? But viewed another way, this is the sort of categorical assertion that sometimes comes back to embarrass its author. We live with experienced presents in which a succession of events a second or two long do all seem present, however difficult it may be to articulate this experience coherently. If we are to see what of our commonsense concept of time is afforded to us in relativistic spacetimes, then it is not unreasonable to seek a counterpart of our present that has duration--though, as noted above, it won't be a perfect replica of our commonsense concept. It will be local rather than global, for instance, if Mermin's understanding of Einstein is right.

I conclude that Dorato's three arguments fail. I should stress, however, that even if this claim right, the discussion so far does not show that the Arthur/Savitt proposal is correct. It shows only that certain purported objections are not really impediments to the proposal. There may be other objections to be considered.

3. Region-Relative Becoming

I spoke at the beginning of this paper of theorems that seem to show that the present for a given event in Minkowski spacetime could only be either the event itself or the whole of the spacetime. If that claim is correct, isn't the Arthur/Savitt proposal straightforwardly ruled out?[5] My answer will be: no, I don't think so. How could that be? Well, theorems have conditions, and it may be possible to introduce causal diamond presents by (plausibly) denying one of the conditions of a key theorem. Although the SCH theorems are sufficiently complicated that a full discussion of them is not possible within the available space constraints, it is fortunate that a complete discussion of them is not required. A corollary that contains the material essential for my purpose here was extracted from the SCH results by Craig Callender (2000), and I will restrict my discussion to this corollary.

Let me first just state Callender's "No Go" result. At issue is the definition of a binary relation R, which is intended to represent the relation of "having become". That is, the goal is to define a specific binary relation B such that Bxy holds if and only if y has become with respect to x, where x and y are spacetime points. Stein had proposed (and the proposal seems eminently reasonable) that for such a relation at least all events y in or on the past light cone of an event x should have become as of or for x. Hence condition iii) in Callender's No Go result:

> For any binary relation R on time-oriented Minkowski spacetime, if R is i) implicitly definable from time-oriented metrical relations, ii) transitive, iii) such that, if $y \in J^{-}(x)$, then Rxy, and iv) satisfies non-uniqueness, then R is the universal relation U. (S592-S593)

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[5] Neither Dorato (2011) nor I (2009) discuss this objection.

Condition iv), non-uniqueness, is this:

$$\text{(NU)} \quad (\exists x)(\exists y)(Bxy \ \& \ Byx \ \& \ {\sim}(x{=}y))$$

NU, according to Callender (S592), "merely says that at least one event in the universe shares its present with another event's present." Any reasonable representation of becoming should, on this understanding, satisfy condition iii. If two distinct points share a present, as they would in a causal diamond, then it seems that condition iv will be satisfied, and the becoming relation is forced to be the universal relation.[6] This looks to be a disastrous result for any account of the present other than Stein's view that each point event is its own present.

Notice, however, that Callender's gloss on NU contains a metaphysical assumption that, it seems to me, can be reasonably denied. Suppose, for example, that one wished to find an analog for the psychological present in a relativistic spacetime and proposed that some small stretch of a timelike world line γ were the appropriate structure. Then it would turn out that--even given the standard Stein requirement on becoming that we find in condition (iii) of the No Go result and even given the existence of pairs of distinct timelike separated events in that small segment of γ--there would not be two distinct points in that "thick" present that satisfied NU. Having mutually become (which is what NU postulates) is *not* the same relation as "sharing a present."

Similarly a causal diamond will contain (in addition to pairs of timelike separated events) pairs of spacelike separated events x and y such that *neither* Bxy *nor* Byx, but it will not contain events such that *both* Bxy *and* Byx, given the standard Stein condition above. The supposition that the present in a suitable class of relativistic spacetimes can be represented by a causal diamond does not, it seem to me, run afoul of the SCH theorems--unless one requires that events in (or "sharing") a present have become with respect to each other. One need not suppose this, however, if one thinks of the present as a locus of becoming rather than as the "cutting edge" of what has become. Inside that present, events can be partially ordered with respect to becoming in the usual way.

One might wish, however, in addition to the standard Stein definition, to define a notion of becoming relative to that present. More generally, one might wish to define

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[6] Of course, I also assuming that the first two conditions are met.

becoming relative to some portion or region of a spacetime, like a causal diamond D. The idea is that what has become relative to D would be all events that have become relative to any event in D, minus D itself. If we call those events B(D), then

$$B(D) \stackrel{\text{def}}{=} \{y: (\exists x)\ (x \in D\ \&\ y \notin D\ \&\ y \in I^{-}(x))\}.$$ [7]

One might call this *region-relative becoming.*

If the above defence of small causal diamonds as presents in relativistic spacetimes is successful, it might be argued that I have proved too much. Consider just Minkowski spacetime for the moment. Malament (1977) has shown that, given an inertial world line γ and an event $e \in \gamma$, one can also define the unique hyperplane Σ orthogonal to γ at e. That hyperplane looks very much like the pre-relativistic present, at least as far as the "observer" represented by γ is concerned. B(Σ) would then be the part of spacetime that has become relative to Σ, the past, while the rest of spacetime that is neither Σ nor B(Σ) is the future relative to Σ. Given the naturalness of these ideas, should one not say that Σ, rather than D, is the (counterpart of the) present for γ at e in M?

Given the title of this paper, the reader should not be surprised to discover that I think not, but I do not have a knock-down argument for my view. What I can do is offer three considerations that I hope will incline the reader in its favor.

Suppose that two "observers" represented by inertial world lines γ and γ' intersect at some spacetime point e. Both agree as to what has become at e in Stein's sense, $I^{-}(x)$ (or perhaps $J^{-}(x)$). This is a natural and desirable feature of a relation of having become. When it comes to region-relative becoming, however, neither D nor Σ will have this feature. Under reasonable assumptions, however, D will come *very* close.

If the specious presents along γ and γ' centered on an event e are roughly the same temporal length, then their two causal diamond presents (call them D and D') nearly coincide. For each diamond there is a small finite volume of spacetime which will have become relative to one but not the other.[8] The temporal difference of two points in such

[7] Cf. Myrvold (2003, §2).

[8] See the estimate in Savitt (2009, 357-358).

regions (in proper time) will be at most of the order of the proper times of the two specious presents along γ and γ'.

For a pair of hyperplanes Σ and Σ' orthogonal at e to γ and γ' respectively, the case is quite different. There is an infinite volume of spacetime that will have become with respect to each one but not the other, and there is no upper bound on the proper time difference between two points in these regions. D, then, comes *much* closer than Σ to satisfying one desideratum on a notion of the present in the way that it meshes with region-relative becoming.

Secondly, the overlap of D and D' can be used to explain our common sense intuition that at any given time we share a present. The hyperplanes Σ and Σ' have no such large overlap. In the standard presentations of relativity in 1 + 1-dimensional spacetimes, in fact, the only event they have in common is just the point of intersection e.

Thirdly, if one focuses on Σ rather than D in thinking about time in M, then it seems to me that one is willfully ignoring the lesson that one should learn from relativity. Let me quote Mermin again: "That no inherent meaning can be assigned to the simultaneity of distant events is the single most important lesson to be learned from relativity." There is no reason to choose this one hyperplane as opposed to the infinity of others.

The events in a causal diamond do have an inherent meaning, as thinkers from Alexandrov to Dorato have pointed out. Given an inertial world line containing the events p and q, the causal diamond defined by p and q contains all the events that are "both a possible effect and a possible cause of events on the segment of the worldline [from p to q]." (Dorato 2011, 382) When it comes to understanding time, it might seem odd that the diamonds are local. But our experience is confined to our local region of spacetime, and relativity robs us of justification for extrapolating that experience along an arbitrary hyperplane.

I think these last insights capture at least some of the thought behind my slogan: "Philosophy of time should aim at an integrated picture of the experiencing subject with its felt time in an experienced universe with its spatiotemporal structure." (351) Dorato protested that the causal diamonds I proposed could not fulfill the expectations raised by this slogan, and in this he is surely correct. But I did not think that the mere suggestion that one might usefully think of causal diamonds as successor concepts for the present in relativistic spacetimes would complete this program in one go. At best, and if successful, it would be a small first step. It would locate the bits of spatiotemporal structure to be coordinated with the experiencing subjects and with their experiences as one small part of a complex whole that we wish to understand.[9]

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