Monadology, Information, and Physics
Part 3: Inertia and Gravity

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Abstract
In Part 3, I will discuss the problems of inertia and gravity in Leibniz, and present three conjectures: (I) If Leibniz were really ready to insist on relativity, he would have to assert the relativity of inertial motion. (II) In Leibniz's theories of dynamics and geometry, there was a struggle between his predilection for straight line and his adherence to an optimality principle. (III) Gravity, as well as inertia, can be considered as a universal feature of the world, so that the foundation of both may have a common root. Further, drawing on the results in Part 1 and Part 2, I will argue for the need of a unified interpretation of Leibniz's metaphysics and dynamics. My three conjectures as regards Leibniz's *possible* treatment of inertia and gravity are proposed along a unified interpretation, in terms of my informational reconstruction of Leibniz's philosophy. Finally, the most important features of the informational interpretation are summarized. The synopsis of the whole paper (Part 1, 2, and 3) is added.

31. Newton's Bucket and Inertia

In this Part 3, we will try a few conjectures, rather than expositions and interpretations of Leibniz's view based on his texts. My first concern is the law of inertia. Despite my assertion in Section 30 (Part 2) of the "new scenario" for establishing the law of inertia (and other laws), there seems to be further important problems to be discussed.

Leibniz repeatedly asserted that the law of inertia is fundamental and based on the nature of things. But I have been unable to find any documents in which Leibniz discussed the *foundation* of the law of inertia. It is true that when he talked about the active and passive forces, he included inertia (a kind of resistance) in the passive force. But this, by itself, does not establish the *law of inertia*, as used in his dynamics. What he has left for us is a few hints and ideas which may become as a clue for our conjectures. But these conjectures may be worthy to try, since they may serve to clarify the problems and potentialities of his dynamics and metaphysics. Moreover, we know, thanks to our hindsight, both inertia and gravity are closely connected with space and time. This may add a more significance to our conjectures, despite my own admission that they are far from "well founded."

In order to begin our conjectures as regards the "Leibnizian foundation" of the law of inertia, we should not forget the problem of Newton's Bucket. Although Clarke alluded to this problem in his 4th Reply (Alexander 1956, 48), by explicitly referring to Newton's Definition 8 (Scholium added to it contains a detailed description of the Bucket experiment), Leibniz disregarded this, simply by saying as follows:

> I find nothing in the Eighth Definition of the Mathematical Principles of Nature, nor in the Scholium belonging to it, that proves or can prove, the reality of space in itself. (5th Paper, sect. 53, Alexander 1956, 74)

This is one of the most disappointing passages in their correspondence. It may be worthwhile to state my reason for this disappointment, because Newton's Bucket has close connections with the law of inertia, as well as with the problems of space and time (for a detailed discussion of the problem of rotation for the relational theory, see Earman 1989, ch. 4.).

It is Huygens (Leibniz's mentor of mathematics in Paris) who first clarified the nature of centrifugal force arising from a rotation. Thus we may assume that Leibniz must have known well about Huygens' clarification. Let me briefly explain. The following Figure 20 shows a (horizontally) rotating disc, where an observer on the disc releases a small ball at B. How does this motion of the ball look to the observer on the disc and to another at rest outside of the disc?

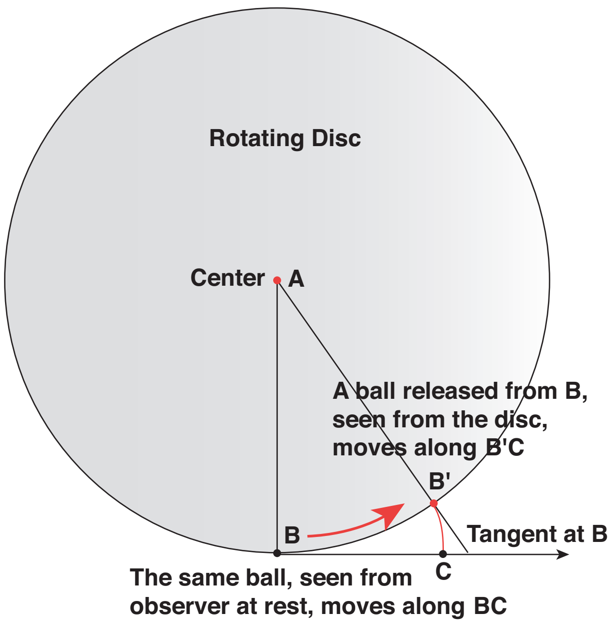

Figure 20: Huygens on Centrifugal Force

Huygens combines two observations from the disc and the observer at rest, a fine relativistic thinking. Although the observer at rest says the ball moved along the tangent at B, the observer on the disc sees a different path, along B'C. Now what does this show?

With a closer look of the same phenomena (Figure 21), within a short time interval, Huygens found the same phenomena as Galileo's free fall! And from this, he deduced the magnitude of the centrifugal force acting on the ball (as regards the nature of trajectories B_1C_1, B_2C_2, etc., see Bertoloni Meli 1993, 46-47).

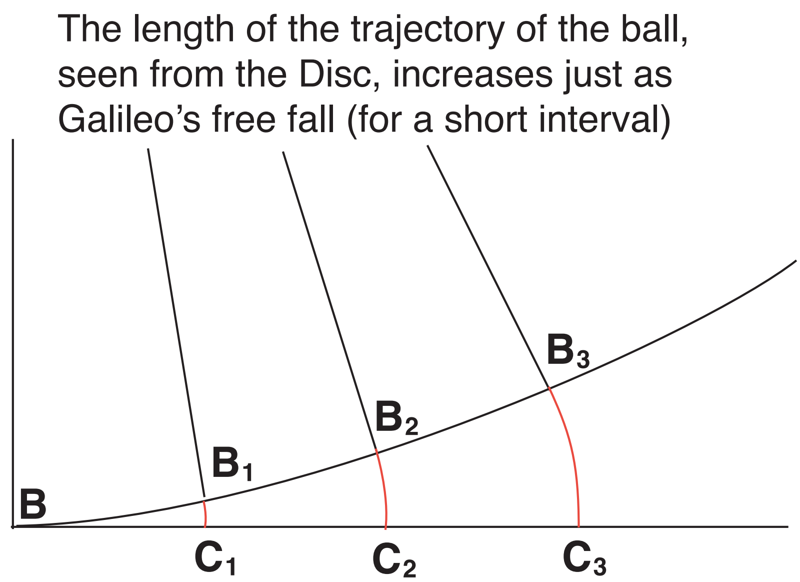

Figure 21: Huygens found a Motion similar to Free Fall

This discovery has been told by many people, and my version owes a great deal to Barbour's (2001, 487-488; Figures are adapted from Uchii 2006, 63). What is clear from this discovery is that the *inertial* motion (for the observer at rest) BC seems as an *accelerated* motion B'C for the observer on the disc (in motion). Here is a sort of relativity, but the crucial point is that without the assumption that BC is an *inertial* motion on a tangential *straight* line, Huygens' inference may lose its validity. Some assumption, such as the law of inertia, or an inertial system as a reference frame, must be *presupposed*. Notice that we are here concerned with laws of *physics*, not geometry alone, and also with *physical space*, not geometrical space alone. The same holds for Huygens' beautiful results as regards collisions (see Barbour 2001, 9.4 and 9.5).

Of course, the need for such presuppositions does not change at all for Newton too. He could claim that "the effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion" (Newton 1962, vol. 1, 10, Cajori's English edition of *Principia*). Newton merely presupposed absolute space and time with a definite metric, and that was the trick for easily presenting the three laws of motion. Moreover, it must be pointed out that the centrifugal force arises *because of the law of inertia*, as well as by the *rotation*.

Thus, if Huygens and Leibniz want to say that Newton's assumption of absolute space is redundant or unnecessary, insisting on the *relativity* of motion in *general*, the burden of proof is on them, and they have to consider not only the *seeming non-relativity of rotation* but also *the law of inertia*. Leibniz, in particular, has to clarify the ground of the law of inertia, together with the nature of space and time, including their metric. That was the topic we discussed in Part 2 (there, my discussion was *restricted to inertial systems*, remember). Leibniz's attempts were incomplete, to say the least. And what I wish to point out here is that, without such preparations, he could not give any satisfactory answer to Clarke and Newton. His allegation of the "wholesale" relativity of motion does not help at all.

Here, my first conjecture comes in.

> (Conjecture I) If he were *really* ready to insist on *relativity*, he would have to assert the *relativity of inertial motion.*

That is to say, he should have insisted: Viewed from the observer on the disc, the "inertial motion for the rest observer" is not inertial, but an *accelerated* motion. In order to see this, just imagine that the observer at B on the disc regards *his own motion* BB′ as inertial, along a *straight* line (notice B is moving with a *uniform* speed). Then, the trajectory BC of the small ball should look *not straight at all but curved and accelerated* vis-à-vis his own motion! See the following Figure 22 (and recall Figure 15 of Part 2, Section 29), which is a sort of coordinate transformation.

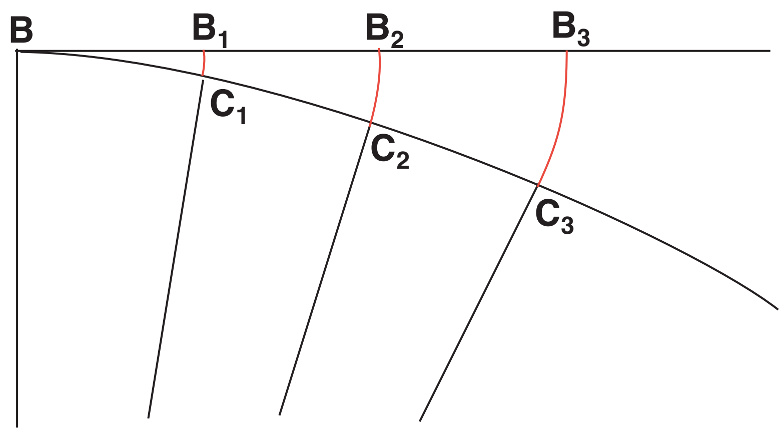

Figure 22: Relativity of Inertial Motion?

I am not saying Figure 21 and Figure 22 are "equivalent," of course; for suggesting anything of this sort, we need differential geometry, and the notion of *general covariance* together with a *coordinate transformation* (briefly, the law must remain the same whatever coordinate system we may adopt for expressing it), which were first used by Einstein's theory of gravity (1915). We can point this out because of our hindsight, of course. No one in Leibniz's days could see this, despite the fact that Galileo's "inertia" was a *circular* motion! But once we see this, we can clearly understand that there is a sort of inconsistency between Leibniz's two conspicuous assertions, the law of inertia and the relativity of motion. The crucial question is: in view of Leibniz' persistent claim that *all* motions are *relative* (at least mathematically), how can he claim also that *inertial motions and straight motions have a special status* in his dynamics? Thus, he definitely has to argue for the *ground* of the law of inertia. That may have opened a new line of research as regards inertia, circular motion, and curvilinear motion in general. The crucial point should be the notion of *invariance*, according to our hindsight. Let me indicate two options:

> (i) If he wishes to insist that inertial motion is *straight*, and straight motions have a special status, "based on the nature of body (or ultimately, of monad)," he has to show why this is so. This option is somehow inconsistent with the results of *Analysis Situs*, because a curvature of space is possible.
>
> (ii) If Leibniz were ready to admit the relativity of inertial motion, he would have to point out *what remains invariant*, in such a coordinate transformation as the one from Figure 21 to Figure 22. (Notice that a *geodesic*, as a surrogate of an inertial path, can be determined even in a curved space-time.)

I am not trying to say that Leibniz was in a position to make a choice of these possibilities. But I imagine that Leibniz might have been aware of the inconsistency contained in (i).

32. Straight or Shortest?

It seems Leibniz's persistent adherence to the law of inertia (the uniform motion on a *straight* line), is closely related with his metaphysical view. At the same time, he often appeals to *optimality*, both in metaphysics and dynamics. The problem here is that it is not clear whether or not he was aware of the gap between *straightness* and *optimality*. Since this seems crucial to me for understanding many problems (mostly unsolved) Leibniz had to face, I may make it my second conjecture:

> (Conjecture II) In Leibniz's theories of dynamics and geometry, there was a struggle between his predilection for straight line and his adherence to an optimality principle.

As regards Leibniz's predilection for a motion on a straight line, we have already seen in Section 7, Part 1 of this paper. There I said that this feature is "indispensable" for understanding his theory of motion, without any criticism. The most important textual evidence (from *Specimen Dynamicum*) may be repeated (this time with an abbreviation) here:

> since only force and *nisus* [effort] arising from it exist at any moment …, and since every *nisus* tends to in a straight line, it follows that *all motion is either rectilinear or composed of rectilinear motions*. (Ariew and Garber 1989, 135)

Since we have clarified in the preceding Section 31 the problem Leibniz should have considered, in order to harmonize the law of inertia with relativity, we are now in a much better position to discuss this. First, we have to point out that this view appeared after Leibniz's work (*Tentamen*, 1689) on planetary motion was published. In this work, Leibniz actually tried to reconstruct a planetary motion in terms of the "polygonal representation" composed of rectilinear motions. This work, and other related manuscripts were thoroughly examined by Bertoloni Meli (1993). And his judgement is certainly worth quoting. He asks, whether the polygonal representation corresponds to the real trajectory of the body.

> The answer to this question can only be a resounding no, and the reason is straightforward. The choice of the specific polygon entails a degree of arbitrariness depending on the progression of the variables. (Bertoloni Meli 1993, 83)

However, Bertoloni Meli distinguishes mathematical grounds and physical grounds. And his judgment is qualified as follows:

> Leibniz's mathematical representations of curvilinear motion are fictitious. The Polygonal curve, however, corresponds in principle to physical laws involved, rather than the infinitesimal details of the trajectory of the body. Moreover, …, for Leibniz change takes place not instantaneously, but gradually and in accordance with the law of continuity. Thus each vertex ought to represent a smooth rather than a sudden transition. (*Ibid.*)

I think this is a fair judgement, in pointing out that, despite the mathematical arbitrariness, the Leibnizian reconstruction can give, physically, a smooth motion of a planet. But all the same, the *straightness* of the action of a force seems to be assumed *a priori*, presumably in conformity with the *law of inertia*. Recall that, for Leibniz, a force [*nisus*] is acting even in inertial motions (Section 4, Part 1). Leibniz may have intended that the straight inertial motion can result (with no other intervening actions) from this straightness of *nisus*, but this would be merely question begging. That may well be the reason why he still tried, in his last days, to define a straight line based on his *Analysis Situs*.

In this paper, we have referred to De Risi's book (2007) several times. In Chapter 2 of his book, he begins to examine Leibniz's treatment of a straight line (226-264). By going through De Risi's detailed examination, we may be surprised and "riddled by Leibniz's numberless, exhausting sprays of definitions and corrections" (226). This can be ascertained even by examining a much simpler description in *Initia Rerum*, to which I referred in Section 30, Part 2. There, in relation to straight line, "minimal path," "simplest path," "maximal determination," etc. are mentioned. For the sake of simplicity, let us take only two characterizations of a straight line, (a) straight as *no curvature*, and (b) straight as a *minimal path* between two given points. We can easily see that (a) and (b) are different, in general, and that they can coincide only if we are talking about Euclidean geometry. And this gap between *straightness* and *minimal distance* should reappear as regards *inertial motion*! This is indeed one of the major reasons for my conjecture (2). On the one hand, *straightness* of inertial motion, on the other hand, any inertial motion between two given points traverses a *minimal path,* and the difference between these two can disappear only if space is Euclidean. And this is exactly the reason why inertial motion is given a special status in Leibniz's dynamics; and because of this special status, inertial motion resisted relativization.

However, De Risi's work also revealed that Leibniz had a possible escape route from this difficulty. If he had chosen "minimal path" instead of "straight line" as an essential feature of inertial motion, he might have obtained a new prospect! For, Leibniz has left a few papers in which he tried to determine the *shortest line* that passes from a point to another, on surfaces with constant and variable *curvature* (De Risi 2007, 236). In fact, Leibniz's two papers from 1690's are added in Appendix of De Risi's book (592-595). Such attempts were no doubt premature (because Gaussian coordinates and differential geometry are needed), but they clearly show that Leibniz did certainly considered the possibility of Non-Euclidean space, as an arena for his dynamics. Thus, inertial motion taken as a motion along a "minimal path" may have opened a new possibility for his dynamics.

The reader may wonder that this is merely a "wishful thinking." But just recall what I have pointed out in Section 30, Part 1. In *Initia Rerum* Leibniz introduced the notion of the *path of a motion*. And I have pointed out that this notion can be extended to the *situation*, since a motion is nothing but a change of situation. And if we can remove Leibniz's predilection for straightness of any elementary motion, a possible path of situation (which is equivalent with a possible state-transition of the phenomenal world) can be turned into a trajectory in all possible situations (similar to Barbour's configuration space, utilizing our hindsight, of course). Applying the consideration of optimality, we should be able to determine a *geodesic* in this arena. Thus God can choose the best geodesic among all possible paths! I am not trying to ascribe this fancy project to Leibniz; I am merely suggesting that such potentialities can be entertained in view of the ideas and works Leibniz has in fact provided.

And I may add that in Leibniz's metaphysics, according to my informational interpretation, I do not see anything which requires the *straightness* of elementary motion or action (in phenomena). As I have clarified in Part 1, there is certainly a correspondence between metaphysics and dynamics, but since neither spatial nor temporal concepts are applicable to the monads, these concepts must be constructed on the *basis* of certain features of monads and their forces. And I have repeatedly emphasized that, in order to produce the phenomenal world with space and time, God's coding is indispensable; indeed there must be at least two different encodings, one for representation, the other for phenomena. Leibniz has not provided the foundation of the law of inertia, and in his foundational considerations on geometry, he has failed to provide a satisfactory definition of straight line. However, there were various possibilities for utilizing the idea of *optimality*, for constructing dynamics, because this idea is applicable both to monads and to phenomena which result from the former. My conjecture (2) is intended to call the reader's attention to this.

33. Informational Interpretation as an Attempt at Unification

Before proceeding to the hard topic of gravity, let me add a remark on my way for reconstructing, and for conjecturing Leibniz's ideas and thinking. I know there are some influential interpreters of Leibniz who tend to deny the "picture of a unified Leibnizian system" (Garber 1985, 73). For instance, Daniel Garber, by criticizing Russell's and Couturat's way of interpreting Leibniz, says:

> I think that is wrong to see Leibniz's thought as deriving from his logic, either as a historical or a philosophical claim. This is not because logic wasn't important to Leibniz; it was, and was a source of many arguments and philosophical positions. Rather, I would claim, Leibniz's philosophy doesn't derive from his logic because it doesn't *derive*, strictly speaking, from any one source at all. (Grabber 1985, 73)

Garber argues that Leibniz's philosophy is "a complex of interrelated and mutually reflecting positions, principles, and arguments"; and he proposes, "rather one ought to see how these domains are interconnected in Leibniz's thought, which is, to use Michel Serres' apt image, more like a net than a chain" (*ibid*.). But such phrases as "interconnected" and "like a net than a chain" are a mere *catch phrase*, rather than a clarification, unless you get into details of such interconnections. Garber's long and impressive paper is rich in quotations and history of philosophy, but I have to confess I have learned little, as regards the *interconnections* between Leibniz's dynamics and metaphysics, in particular, from this paper (Garber 1995 is much better).

Let us see another example. Bertoloni Meli also says something similar about Leibniz:

> his system is based on an extraordinary complex interplay of themes and disciplines with no fixed centre. Neither mathematics, nor logic, nor metaphysics, nor theology nor any other field, can be taken to be at the foundations of the whole system. (Bertoloni Meli 1993, 78)

Despite this statement, however, Bertoloni Meli has examined the *details* of the "complex interplay of themes and disciplines," and that's one of the most valuable contributions to Leibniz scholarship. And I have gained many hints from his work, for my own attempt at looking for a sort of unified interpretation of Leibniz's physics and metaphysics.

Garber, in his new book (2009, 384) repeats a similar assertion as regards Leibniz's project of *Monadology*:

> Certain metaphysical argument convinced Leibniz that, at root, simple substances, monads, had to be at the bottom of everything. What he hadn't fully figured out, though, is how exactly bodies are to be grounded in the world of monads. This, I would claim was the project of the letters with de Volder, the letters with Des Bosses, and other texts of this period. And, I would claim, there is no single doctrine that one can say is *the* Leibnizian solution to that problem. There are different stands that recur throughout the texts, but I don't think that he ever arrived at an answer that fully satisfied him.

Well, as regards the *factual* statements contained in this quotation, I am not going to complain. But from these alleged facts, does it follow that we *should not* try a unified interpretation of Leibniz's attempts and texts? I do not think so. Garber (and, to some extent, Bertoloni Meli also) is merely saying that he *did not find* any unified view in Leibniz's texts; and this does not imply it is *impossible* to find any, or we *should not* try a unified interpretation. Unification is a "never ending story" in science and philosophy as well, and if you do not try to attribute such a story to Leibniz, to whom else? *Monadology* is in fact a manifestation of this dream, at least.

Moreover, Leibniz's metaphysics does seem to contain the *core* for possible unification. In Section 13, Part 1 of this paper, I *assumed* that the *information of the monads is conserved*, in order to discuss Leibniz's distinction between action and passion. There, I assumed so simply for the sake of avoiding a digression from the main topic there. But, here, we can take up this "assumption" again, since, as I understand, this assumption is the key for any possible unification of Leibniz's dynamics with metaphysics.

The most important reason for Leibniz's introducing substances or monads is that there must be the *same reality*, behind the ever changing world of phenomena. Recall that, although the state of each monad changes in one sense, the whole sequence of the states is given in *timeless* reality, and in this sense, reality is unchanging and remains the same. Thus, God created the timeless world of monads all at once, including all the sequences (transition functions, in my words) of their states. This *totality of reality* must be always the *same*, despite the fact that the phenomenal world is always changing. On this basis, I have argued that the conservation of living force is closely connected with the conservation of the primitive force, which in turn stems from the *transition function* of a monad (Section

13). I have also pointed out that the *recursion* of elastic collision (as the basic interaction among bodies) at every level of the infinite divisibility of body nicely corresponds to the *recursive structure* of the program of monads (Section 15).

In this way, the *information of reality is conserved, individually as regards each monad, and hence as regards the whole reality* (Section 13). Thus, as long as we are talking about Leibniz's philosophy, this assumption must be accepted. And this is nothing but the *basis* of any possible unification of Leibniz's philosophy.

Then, this assumption imposes an *ideal* of unification on Leibniz. I say "ideal" because his dynamics is *not* unified; it is left incomplete! But Leibniz sketched a scenario for unification, that (a) dynamics must be founded on metaphysics, that (b) space and time must be constructed or explained in terms of a relational theory, and (c) geometry (of space) or *Analysis Situs* must be founded on metaphysics. And I myself sketched (d) how metric time can be obtained from the succession of states in the monads (Sections 21-24); and I also argued that (e) even *relativistic metric* can be obtained from the same basis (Sections 27-28). According to my informational interpretation, I have thus tried to fill in some details (insufficient though, no doubt) by supplying connections between informational (metaphysical, for Leibniz) concepts and dynamical concepts.

And these should be a good enough reason for *trying*, rather than *denouncing*, a unified interpretation of Leibniz's philosophy. My informational interpretation is, in fact, inspired by Leibniz's *vision* manifested in *Monadology*. However, in order to pursue a unified view of Leibniz's philosophy, I had to make *my own choice* among possible alternatives; without such a choice, it should be quite hard to obtain any consistent interpretation. But whenever I have made such a choice, I have indicated which option I have taken from the specified alternatives. And I will continue to do so. With this confession, let us proceed to the problem of gravity.

34. Gravity

It seems that Leibniz's treatment of gravity is one of the weakest spots of his dynamics. The root of the problem may be his mechanistic philosophy as regards the phenomenal world. According to its general strategy, gravity as well as inertia must be reduced to action-reaction by contact, an accumulation of a multitude of dead or living forces.

In *Specimen Dynamicum*, after stating that all motions can be reduced to rectilinear motions (quoted in Section 7, Part 1), Leibniz outlines how such forces as gravity or centripetal force can be explained.

> From this it not only follows that what moves in a curved path always tries [*conari*] to proceed in a straight line tangent to it, but also—something utterly unexpected—that the *true notion of solidity* derives from this. … For if we assume something we call solid is rotating around its center, its parts all try [*conabuntur*] to fly off on the tangent; indeed, they will actually begin to fly off. But since this mutual separation disturbs the motion of the surrounding bodies, they are repelled back, that is, thrust back together again, as if the center contained a magnetic force for attracting them, or as if the parts themselves contained a centripetal force. Thus, the rotation arises from the composition of the rectilinear *nisus* for receding on the tangent and the centripetal *conatus* among the parts. (Ariew and Garber 1989, 135-136)

This qualitative description may look insufficient, but Leibniz actually applied this senario, in mathematical and quantitative terms, to planetary motion in his *Tentamen* (1689), as was already mentioned. And although he could reproduce not all of Newton's major results, he succeeded in giving some credibility to this scenario, depending on the tradition of vortex theory, and without assuming any "action-at-a-distance" like Newton's attractive force. His view on solidity, however, is problematic (for, once we get into *quantitative* calculation of the strength of gravity, we will immediately see the invalidity of Leibniz's view), although it is understandable in view of the importance he gave to the notion of elasticity (a principle of interaction among bodies).

Now, Leibniz's scenario seems credible as far as it can show that some counter force has to work in order to keep a circular motion or a curvilinear motion. However, since it needs an additional assumption of *vortex* which carries a planet, and moreover the assumption of *harmonic circulation* (the velocity of rotation is inversely proportional to the radius), the question of their foundation remains. Leibniz's ingenious trick for producing an elliptic orbit of a planet is that he introduced *two* *opposite tendencies* along the radius of the harmonic circulation: one is centrifugal force, and another is gravity (see Bertoloni Meli 1993, 117). Gravity is explained in terms of a multitude of impacts from ether, pushing the planet towards the center of circulation. Depending on the balance of these two forces, the orbit can deviate from the circle of harmonic circulation and become *elliptic*.

Despite this ingenuity, an additional difficulty remains, for explaining orbits of comets which moves *across* the plane of orbits of the planets. Moreover, it seems quite hard to adapt his theory to the *universal* character of gravity, by relying on the vortex theory. Like inertial motions, gravity seems to work irrespective of *directions* in space. Presumably because of such difficulties, Leibniz tried other ideas in his "Zweite Bearbeitung" (written probably 1689-1690; see Bertoloni Meli 1993, 155, 306), a revised version of *Tentamen*.

In this revised version, he *changed* his model for explaining gravity. Instead of the fluid carrying a planet, he now considers a fluid *emitted from* the center of revolution, and he uses an analogy with *light*, for suggesting the inverse square law. And this time, it is assumed that gross bodies (planets, e.g.) have many pores, and the assumed fluid penetrates these pores. Now, depending on the distance from the center, any such bodies are penetrated by the same fluid according to the *inverse square law* (density of the fluid in the body); thus this fluid *can be* the cause of gravity acting according to that proportion. It may seem strange, but this fluid is a *medium* producing an *attractive force* between the center and the body. Although this idea is fanciful, it has two important features: (a) the inverse square law by analogy of *light*, and (b) the action of this fluid *takes time*, in order to reach another body. Thus it may suite the taste of the modern reader! And I myself am inclined to consider this change as another signal of Leibniz's informational turn. The mechanism of production of attractive force is *widely* left open, indeed, but at least the *route and medium of gravitational interaction* between two bodies are indicated.

Leibniz repeated the same ideas several times in his later years. For instance, in a letter to Johann Bernoulli (18 Nov. 1698), he writes:

> It was my opinion long before Newton's work, that gravity is inversely proportional to the square of the distance, a theory at which I arrived not merely by a posteriori processes but also by an a priori reason which I am surprised that he did not notice. Leaving out of consideration the physical basis of gravity namely, and remaining within mathematical concepts, I consider gravity as an attraction caused by certain radii or attractive lines going out from an attracting center; so like the density of illumination in

rays of light, the density of radiation in gravitational attraction will be inversely proportional to the square of the distance from the radiant point …. (Loemker 1969, 513)

The qualification “remaining within mathematical concepts” may be understood as suggesting a *possible* “phenomenal expression” of an informational interaction between two groups of monads (underlying gravity), as I have suggested in Part 1.

Again, in *Illustratio Tentaminis de Motuum Coelestium Causis* (c. 1705, henceforth *Illustratio*) , the same model and analogy are repeated (pt. 1 sect. 2; both *Tentamen* and *Illustratio* have been translated into Japanese, in vol. 9, Shimomura *et al*. eds., 1999). And a mathematical proof of the inverse square law, as a general feature of any radiation, is provided with a figure (*ibid.*). This figure is adapted with an explanation, as the following Figure 23.

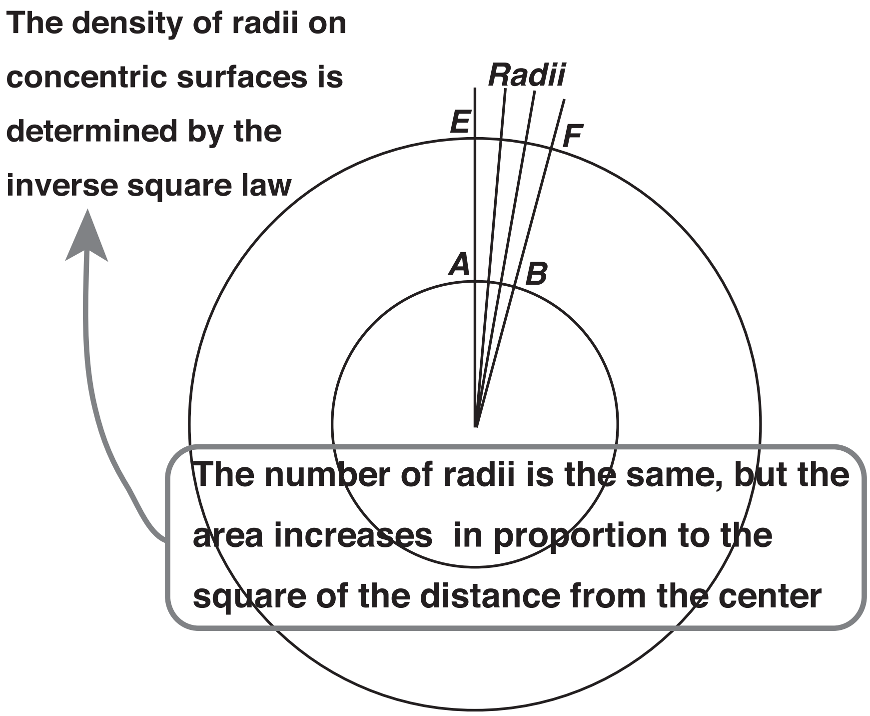

Figure 23: Radiation and the Inverse Square Law

Although several radii in Figure 23 are added by myself, this addition is quite in conformity with Leibniz’s own idea. And the reader may notice a striking affinity of these radii with Faraday’s line of electric force, or of magnetic force. In fact, in the introduction of “Zweite Bearbeitung,” there is a reference to Gilbert’s theory of magnetism (see Bertoloni Meli 1993, 156). And also recall that penetration of light and magnetism through the “vacuum” was one of Leibniz’s reasons against the *emptiness* in the alleged "vacuum" region (after all, "ether" persisted and coexisted until the beginning of the 20th century, together with electro-magnetic field).

Further, in Leibniz's 5th paper to Clarke, he refers to the same ideas, and generalizes this model to all phenomena of gravity.

> For, both quicksilver and water, are masses of heavy matter, full of pores, through which there passes a great deal of matter void of heaviness …; such as is probably that of the rays of light, and other insensible fluids; and especially that which is itself the cause of gravity of gross bodies, by receding from the center towards which it drives those bodies. … the gravity of sensible bodies towards the center of earth, ought to be produced by the motion of some fluid. (sect. 35, Alexander 1956, 66)

The phrase "matter void of heaviness" (graviton?) would strike the modern reader! However, it must be pointed out that, if gravity works *generally*, between the sun and the earth, between planets, between a stone and the earth, etc., the flow of such insensible fluids (kinds of ether) must be *mutual*! If two bodies attract each other, this *action-reaction is mutual* and depends on the mass of *each* body; which means that the flow of fluids from one is accompanied by another reversed flow from the latter to the former. I have been unable to find Leibniz's explicit statement on this point, but I presume that this is the case. Otherwise, gravity cannot be a principle of motion comparable with the law of inertia. But on this presumption, Leibniz's simple derivation of the inverse square law may become problematic (as regards gravity); at least it must be appropriately modified.

Anyway, it should be clear that such a model may disturb his own explanation in terms of the vortex theory, although it may be in conformity with the assumption of ether. One of the main reasons why he adheres to the vortex theory is that all planets moves on the *same plane*, and Newton cannot provide any reason for this. But, whether or not Leibniz had less confidence in the vortex theory (but adhering to the assumption of ether) in his later years, Leibniz must have to go back to his metaphysics, for grounding any such theories, in addition to empirical confirmation. Thus we are led to the *metaphysical* foundation of gravity. In Leibniz's days no one could imagine theories of field, presumably because of the preconception of the vortex theory and the rudimentary knowledge of electro-magnetic phenomena.

In sum then, Leibniz's treatment of gravity seems to have two aspects: (i) One, that is firmly rooted in the tradition of the vortex theory and mechanistic philosophy, which tries to *reduce* gravity to *local* action by contact, and (ii) the other, that seeks a more general feature, such as elasticity or the law governing gravity. As I understand, although Leibniz's strategy may look ambivalent between (i) and (ii), these have coexisted, and in later years, (ii) became more prevalent. On the assumption of this reading, then, we can suggest a similar move for Leibniz as that in our previous case of the foundation of inertia. Thus the following is my third conjecture:

> (Conjecture III) Gravity, as well as inertia, can be considered as a universal feature of the world, so that the foundation of both may be suspected to have a common root. Just as the foundation of inertia may be obtained by optimality of the path of the situation (in the whole world), so may be the foundation of gravity.

The reader may criticize this by saying "it is too much contaminated by our hindsight." I will not deny that there is indeed *some* contamination, from our knowledge of Einstein's theory of gravity. But we can find some ground for this conjecture in Leibniz's own text. As Bertoloni Meli (1993) points out,

> In the *Hypothesis Physica Nova* of 1671 Leibniz explained gravity and elasticity in terms of interaction between the aether and matter. (53)

That is to say, gravity and elasticity are somehow similarly treated; and since elasticity is given, as we have seen in Part 1, the status of basic interaction between bodies (repeated at all levels of infinite divisibility of matter), so is gravity, by analogy. And this is not an isolated evidence. See another remark by Bertoloni Meli:

> In *Propositiones Quaedam Physicae* of 1672 Leibniz assumed that the whole universe is elastic and that elasticity and gravity differ only in name. A body lifted in the air would be heavy because the elasticity of the universe would tend to restore the original position of equilibrium through a series of impacts. (*ibid.*)

"Elasticity of the universe" is again a striking expression. The universe has many spots where "elastic force" becomes uneven (stress is there, and this is nothing but *internal energy*; see Sections 8, 28), and this in turn dictates a body there, "how it should move." This reminds me of J. A. Wheeler's famous dictum: "Space tells matter how to move, and matter tells space how to curve" (Misner, Thorne, and Wheeler 1973, 5).

Further, Bertoloni Meli quotes the following from Leibniz's letter to Jacob Bernoulli (3 Dec. 1703, Bretoloni Meli's own translation from Latin, Gerhardt M, vol. 3, 81)

> I hold all the bodies of the universe to be elastic, not through in themselves, but because of the fluids flowing between them, which on the other hand consist of elastic parts, and this state of affairs proceeds *in infinitum*. (55)

Together with other quotations in this Section, these statements by Leibniz clearly show that the view expressed in these is a *consistent trend* as regards gravity, in Leibniz's thinking from 1670's to his later years. My conjecture (III) is based on this fact, and I have only added the connection with *Analysis Situs* and optimality, which are also one of the prominent tenets in Leibniz, and closely connected with his saying that "everything is connected" in our world.

A more radical conjecture would be that the phenomenal world is a huge *cellular automaton* composed of all bodies floating in the sea of ether, including the sea itself, and this cellular automaton results from the whole monadic cellular automaton composed of all smaller cellular automata (organized groups of monads), as was suggested in Section 12, Part 1. However, at present, I cannot supply any specific description as regards the genesis (program) of gravitational phenomena in this automaton. Thus, this conjecture is too general, and maybe too far away from Leibniz's own ideas. All we can say is that the flow of information in this whole cellular automaton is the basis of of the phenomena of inertia and gravity. However, recall that this is *another* way ("as if"-mode) to express the operation of a *single* monad, as I have pointed out in Section 13 in relation to Leibniz's Demon; each single monad can represent the whole world. Anyway, it is clear that the details are widely left open.

35. The Flexibility and the Unity of Leibniz's Philosophy

If we may rely on our hindsight, however, it is easy to continue our conjecture as follows: Both inertia and gravity are closely connected with the structure of space and time. Thus, when Leibniz said "motion is a change of situation" and "the analysis of situation provides the foundation of geometry (of the actual world, in particular)," he certainly hit the mark. Moreover, as we have already seen in this paper, Leibniz's theory of dynamics and metaphysics, because of its two-layer structure, is amazingly flexible. His whole theory (including metaphysics and dynamics as its core) can adapt itself to various possibilities of space, time, and dynamics. The reason for this flexibility can be explained, more specifically, and by way of a summary of the whole paper, as follows.

(1) Leibniz's world of monads can be interpreted as the source of *invariant information*, and this produces various *invariant structures*. "Invariant" in the sense that, in whatever manner these structures may be expressed or represented, they remain the *same*. As I have repeatedly asserted, the world of monads exists *without space* and *without time*. This means that *various* theories of space and time can be constructed on the same structures. Further, all the *changes* of monadic states are given at once, and the *transition functions* (containing all the information) which determine these changes are the source of invariant structures. Thus the possibilities of dynamics (which we humans can construct) are also widely open.

(2) On the basis of such invariant structures, Leibniz can introduce various means for producing various representations and appearances (phenomena). When Leibniz says that "a monad represents the whole world in its own way," and when he distinguishes "well-founded phenomena" from other phenomena, he clearly implies this. Thus, in this paper, I have adopted formal expressions such as *R(W)* or *Ph(R(W))*, in order to make this point explicit. The reason why we have to distinguish *R* and *Ph* has been explained (Section 21). And, although Leibniz seldom says explicitly, both representations and phenomena need (I would emphasize, logically) *coding*, respectively. Described in this way, it becomes quite clear that Leibniz's metaphysics can be rightly called *informational*. This is really an amazing innovation!

(3) As a consequence of these two points, it should be clear that the *program* of the monads and the *coding* for representation and phenomena are the key for whatever happens in the phenomenal world. How can we, finite humans, know these? Leibniz assumed certain (qualitative) *correspondences* between the world of monads and the phenomena. These are homomorphisms (partial isomorphism). And since

Leibniz's mechanistic philosophy dictates we should explain phenomena in terms of motions, dynamics is given the status as the *primary means* for knowing not only the laws of phenomena but also the invariant structures of the world of monads.

(4) However, it may seem hard to point out, in Leibniz's texts, relevant statements as regards *decoding*, i.e., the method for knowing the original programs of monads from phenomena. It may be that Leibniz had little means for working out his metaphysics, as an ambitious theory of information. Aside from isomorphism and homomorphism, what he has explicitly mentioned is a "divine machine," various bodies resulting from an *organized group* of monads. I have pointed out that this corresponds to von Neumann's *cellular automaton*. A recent trend in Leibniz scholarship may be termed "biological" (see Smith 2011, Smith and Nachtomy 2011), and proponents of this trend also emphasize the importance of "divine machine." But unfortunately, there are very few references to the theory of information, or to cellular automaton, which, according to my own view, is indispensable for clarifying this "divine machine." And what I have emphasized in Section 12 is that God's *organization* of monads into many groups (each has its own entelechy) is crucially important for understanding Leibniz's *dynamics*, rather than biology!

(5) Because, bodies are *situated* in the world, and this discloses important information of the world of monads, where monads are also organized. Thus *geometry* of space is at least one essential key for our *decoding*, for inferring God's message through phenomena. Then, by an obvious analogy, *motion* and *time* are also another key for decoding, since motion is a *change of situation*, and time is nothing but encoded expression of the *order of succession* of monadic states. Thus, although Leibniz himself did not have any such concepts as encoding or decoding, he is in effect saying that *dynamics* is an *essential tool for decoding*, for knowing the invariant structures of the world of monads.

(6) In this way, Leibniz has at least indicated how we should try to decode. But, of course, details are widely left open. His most optimistic statement is given in terms of "Leibniz's Demon" (Section 13), and I have shown two possible ways to make this idea meaningful in his dynamics (Sections 26 and 27). And in this process, I have also shown how we can construct *different systems* of space and time, starting from the same invariant structures in the monadic world. This implies that the same can be said with respect to dynamics; since Leibniz's (metaphysical) invariant structures are *informational* (not physical) at bottom, this flexibility is easily obtained.

(7) Then, the next question is: Given this flexibility of Leibniz's philosophy, what instruction can he give us, as regards our choice of a *best* theory of dynamics? The answer seems obvious to me. He would surely recommend the use of the *optimality* principle, but only *in the light of our experience*! The reader may wonder: "why experience?" Leibniz does not deny the importance of experience at all, because phenomena are, to him, mostly *appearances* of the activities of monads in one way or another. We have to distinguish well-founded phenomena from others, but Leibniz has the distinction of *qualitative* and *quantitative* features of phenomena. Well-founded phenomena are the products of the monads via God's coding, and quantities come from this coding. Thus, since well-founded phenomena are all *coded messages* of God, we have to base any of our theories on them. We *humans* may imagine any hypotheses we like, but in order to *decode* the phenomena and to know God's message, we have to essentially rely on *experience*. I think this is Leibniz's *metaphysical empiricism*, and the optimality principle must be used on this assumption.

(8) Needless to say, the optimality principle in various forms, such as the principle of least action, or the variational principle, is mostly advocated, refined, and used by later people, especially after analytical mechanics was established. But Leibniz was by far the most prominent figure, among his contemporaries, advocating the basic *vision*, and several ideas in mathematics, dynamics, and especially in metaphysics. Thus my reference to this principle is not anachronism. Despite the fact Leibniz's version is vague or ambiguous, it is one of his major tenets.

Although my research on the informational interpretation of Leibniz still continues, I may stop here. My last statement at this stage is, in view of (1)-(8), my informational interpretation is qualified as a *unified interpretation* of Leibniz's philosophy. Of course, it is *not* derived from any *single* discipline, but I think I have shown how various elements of Leibniz's philosophy can be tightly *unified* around the core of his theory of information, i.e., *Monadology*.

For the sake of the reader's convenience, I will add the Synopsis of the paper so far (Part 1, 2, and 3), as an Appendix.

Appendix: The Synopsis of the Paper

Part 1: Metaphysics and Dynamics

1. Preliminaries
   Leibniz's informational turn/Whole series of change given at once/Phenomena, space, and time/Need for coding
2. Leibniz on Forces
   Impenetrability and inertia/Active force, passive force
3. *Specimen Dynamicum*
   Motion does not exist/What is active force?/"Act on" and "being acted on"
4. How does Dynamics correspond to Metaphysics? (Figure 1: Inertial Motions change Situation)
   Significance of *analysis situs*/Motion as a change of situation/Force even in inertial motion
5. Living Force and Dead Force (Figure 2: Leibniz's reconstruction of Acceleration)
   How Leibniz reconstructs acceleration
6. Collisions and Relativity of Motion (Figure 3: Collision of Two Bodies, Figure 4: Collision and the Center of Gravity)
   Infinite divisibility/Collision and elasticity/Collision and relativity/Conservation of living force due to elasticity
7. All Motion is Rectilinear or composed of Rectilinear Motions
   Effort gives rise to force, and tends to in a straight line
8. Living Force: Total and Partial
   Partial living force may imply internal energy
9. Active vs. Passive in *Monadology*
   Active and passive/How should we interpret this distinction?
10. The Structure of a Program: Turing Machine (Figure 5: Turing Machine)
    Nested recursion
11. A Program has a Structure with many Layers
    Active and passive in the hierarchy of programs/Leibniz was aware of recursion
12. Divine Machines and Cellular Automata (Figure 6: Von Neumann's Cellular Space, Figure 7: Leibniz's Cellular Space)
    Flow of information/Organization of bodies and of monads/Leibniz and cellular automata/Program and its subprograms
13. Relativity of Action and Passion, based on Recursion
    Active and passive are relative/Recursion is crucial/Leibniz's Demon
14. Recursion in the Phenomena
    Elastic collision recurs/Program version of the law of inertia

27. Relativistic Metric in Leibniz (Figure 14: Bi-metric Relativity for Humans and Angels)

Relativistic metric is also possible with no change in Leibniz metaphysics/ Moreover, even a bi-metric system is possible on the same basis/Leibniz's demon is possible even with relativistic metric

28. Mass and Energy

The relation of mass and energy can be incorporated in Leibniz's dynamics, in conformity with his notion of partial living force

29. What is the Problem with Inertia and Relativity? (Figure 15: Can Relativity be Generalized?)

Leibniz did not discuss the foundation of the law of inertia/Law of inertia inconsistent with wholesale relativity

30. How can Leibniz establish the Law of Inertia?

Leibniz's last research on *Analysis Situs* shows he is still ambivalent as regards his definition of straight line/But the notion of path can be extended to the whole situation/Why not apply optimality to paths?

Appendix to Section 30: Barbour's *Platonia* and *Shape Space* (Figure 16: A Configuration Space, Figure 17: *Platonia* for Possible Triangles, Figure 18: Two Representations of Planetary Motion, Figure 19: *Shape Space* and the Planetary Motion)

Barbour's relational dynamics has many things in common with Leibniz's dynamics

Part 3: Inertia and Gravity

31. Newton's Bucket and Inertia (Figure 20: Huygens on Centrifugal Force, Figure 21: Huygens found a Motion similar to Free Fall, Figure 22: Relativity of Inertial Motion?)

In order to consider Leibniz's position as regards the law of inertia, Newton's bucket is a good starting point/Huygens' work on centrifuge force is illuminating in many ways/Conjecture I: Leibniz could have entertained the relativity of inertial motion

32. Straight or Shortest?

Conjecture II: there was a struggle in Leibniz between his predilection for straight line and his adherence to optimality principle

33. Informational Interpretation as an Attempt at Unification

The reasons for trying unified interpretation of Leibniz's philosophy/ *Monadology* implies the conservation of information, which can be the core of unification

34. Gravity (Figure 23: Radiation and the Inverse Square Law)

Leibniz's treatment of gravity is not impressive / But after *Tentamen*, he changed his model for gravity / Analogy between light and gravity suggests the inverse square law and a flow of gravitational force / Because of his insistence on the connection of elasticity and gravity, gravity may be regarded as another basic interaction / Conjecture III:gravity as well as inertia may be a universal feature of the phenomenal world, and they may have a common root.

35. The Flexibility and the Unity of Leibniz's Philosophy

The unity of Leibniz's metaphysics and dynamics stems from *Monadology* as a theory of information. The monads and their transition functions are the source of *invariant structures*, and the world of monads is always the same. On this basis, coding of representation and coding of phenomena are added, and these are the source of amazing flexibility.

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