LOST OPPORTUNITY: WHAT EINSTEIN (PART A OF 1916) MISSED IN §1 OF MINKOWSKI (1908)

Graham Nerlich

Einstein's (1916) first survey of General Relativity is deeply flawed in its informal introductory section, Part A. He presents the salient feature of the new theory as the mere lifting of coordinate restrictions on Special Relativity rather than its being a spacetime theory of gravity. Minkowski (1908) developed a different conception of Special Relativity, independent of light and signalling, with spacetime as its immediate and principal consequence. If Einstein had begun general relativity from that basis he would have avoided the many errors into which Part A fell.

1 Introduction

Einstein (1916) was the first complete survey paper on General Relativity (GR), arguably the most powerful and elegant theory in the history of science, cosmic in scope, new and surprising in its array of concepts, unprecedented in its style of explanation. The paper divides into five Parts. Part A, 'Fundamental Considerations on the Postulate of Relativity', stands to the full theory rather as Newton's Scholium stands to the full theory in *Principia* (Newton 1999). Each expresses what its author saw as the broad meaning of its prime concepts, space and time. Like the Scholium, Part A is of great metaphysical interest. Its immediate influence on philosophical thought as well as thought in physics was very strong and very misleading.

A main aim of this paper is to show that, while Einstein's contributions to science were second to none, he wrote Part A under the spell of a dogmatic empiricist epistemology. It is a cautionary tale: the very brightest among us can be widely misled by bad philosophical convictions.

2 PART A: General remarks

In Part A, and especially in §§1-3, Einstein argued for a conclusion essential to GR: the postulate of relativity of motion must extend beyond its restricted scope in Special Relativity (SR): the theory must be formulated in *generally covariant* style. Einstein took this to show that the theory dispenses with space and time and requires focus just on material point-coincidences.

A glance at the end of §3 and the topic of general covariance suggests which way Einstein thought the new wind should blow through §§1-3. §3 concludes:

> … this requirement of general covariance[1]… *takes away from space and time the last remnant of physical objectivity* [117]. (My emphasis).

That is plausibly the metaphysical goal of these three sections.

A fatal flaw pervades Part A §§1-3. The *only* reason why the "postulate of the special principle of relativity" must be extended is simple: the curvature of spacetime is the heart and soul of GR's theory of gravity. If it is curved in any model, then, obviously, appropriately curved coordinates are needed to cover it in that model. SR's special linear coordinates can't do that job. GR's theory of gravity may not be simple but the move from curved spacetime to curved coordinates is. Since the spacetimes of the theory vary both from model to model and, in any model, may vary from place to place, the range of smooth curvilinear coordinates is virtually unconfined. You *may* use curvilinears in any Riemannian space or spacetime. In variably curved spacetimes you *must* use them.

In these opening sections, Einstein never mentioned the curvature of spacetime: it is nowhere cited as demanding the extension of coordinate systems to the general

[1] For an informal account of general covariance concerned just with specifying the metric of a spacetime, see consecutively numbered paragraphs in §5 below.

group. He argued throughout from examples i.e. thought experiments. Each one introduces massive objects but at so great a distance as to allow a setting in effectively zero gravity, i.e. in an arbitrarily extended inertial frame of reference; i.e. in *flat* spacetime. For instance "Let K be a Galilean system of reference, i.e. a system relative to which … a mass sufficiently distant from other masses, is moving with uniform motion in a straight line." (114)[2]. Thus Einstein pictures GR as if it were SR, save for permitting a wider range of rest bodies or systems, but without the addition of any new gravitational physics. But in such settings there is no *need* for lifting SR restrictions. Every example consistent with the use of Galilean inertial frames and flat spacetime can be adequately, indeed *best*, described and analysed in Lorentz coordinates. They are privileged in SR. Einstein concluded that they can't cope with his examples and that no frames are privileged for them. Both conclusions are false.

He saw GR as an expanded relativity theory, not an advance in understanding gravity.

Why did this pervasive error occur?[3]

3 Part A §1: Observations on the Special Theory of Relativity

Einstein's 2-postulate version of SR

The section begins by asserting that there are two postulates for SR: The first postulate, the special *principle* of relative motion (that motion is a symmetrical relation among inertial frames of reference), is satisfied in both classical mechanics and SR (the 1905 theory of *physics* that respects the principle). In the second paragraph Einstein claims that "the special theory of relativity does not depart from classical mechanics through the postulate of relativity but through the postulate of the constancy of the velocity of light *in vacuo*, from which, in combination with the special theory of relativity, there follow…the relativity of simultaneity, the Lorentzian transformation, and the related laws for the behaviour of moving bodies and clocks" (111).

[2] Numbers in brackets are page numbers in Einstein (1916).
[3] I assume that Part A was written in the light of succeeding Parts.

The quoted claim was first shown to be false in §1 of Minkowski's famous paper (1908) and in his (1915). The light postulate and operations based on it are not needed to gain the results listed above. Two years later than Minkowski, Ignatowski (1910) also deduced the same results just from the special principle of relativity alone. Einstein may have overlooked Ignatowski's paper but we can hardly suppose him ignorant of the claims and arguments advanced by Minkowski. However it may have happened, it is clear that, in Part A, Einstein ignored both spacetime itself and Minkowski's novel and elegant way of arriving at it. Since this aspect of Minkowski's work is, astonishingly, almost totally absent from later literature it seems likely that most students of Einstein (1916) have followed him in ignoring it. Minkowki himself made it a prominent part of his "well-known" paper.

Minkowski's absolutist version of SR.

Roughly, Einstein developed SR as a theory of physics; Minkowski drew it more from mathematical invention.

Minkowski set out "from the accepted mechanics of the present day, along a purely mathematical line of thought, to arrive at changed ideas of space and time". The ideas "have sprung from the soil of experimental physics and therein lies its strength." ((1908):75).

In his (1905) Einstein saw that the contemporary neo-Newtonian, Galilean relativity of motion, conjoined with an absolute constant speed for light, foisted bizarre results on electromagnetism. He also saw that the problem lay in time's invariance under the Galilean recipe for change of inertial frame. His solution exploited the invariance of light speed under change of inertial frame, together with new operations for synchronising clocks by light signalling and so on. Minkowski saw the problem and its solution as deeper and more general. Its foundation lay in the geometry of the relation of time to space, not in any particular theory or its operations in physics – e.g. electromagnetism. His motivation in §1 of the revolutionary paper (1908) was that the standard picture of time's relation to space had bizarre features just because it completely separates them. His motivating remarks are terse and scattered and not crucial to his argument. His distinct and more general critique of classical relativity has no overlap with Einstein's.

Briefly, Minkowski invented a metric for a newly conceived unification of space and time that took its departure from the Euclidean spatial metric in Cartesian coordinates. The most obvious (and conservative) approach to this assumes that *pseudo*-Cartesians would coordinate the new 4 dimensional manifold. That entails that the inclusion of dt^2 makes the time coordinate pseudo-orthogonal to the spatial ones. The important difference between time and space is preserved if spatial and temporal differentials have opposite signs in the metric equation. Further, the equation makes sense only when spatial and temporal units are linked, so a constant, *c,* tying units of time to spatial ones, becomes a coefficient of the time differential *dt* (or else of each of the three spatial ones). Then *c* has some such form as metres-per-second or seconds-per-metre or light-years-per-year. It is a speed. This "purely mathematical line of thought" yields the now familiar *form* of the Lorentz metric:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

The value of *c* is remains undetermined although its role is clear. It is not intelligible that it be an infinite speed (1908: 79). Experience suggests that it is very large but measurement is needed to discover it. One need not measure light speed: measurements confirming the addition theorem for velocities will do. It is then a contingent, but epistemologically a highly convenient fact that something – light *in vacuo* – has that speed. But the metric is independent of that: SR would not collapse if light turned out not to travel at *c* - if the photon is massive for example. In the pseudo-geometry thus invented the metric is assumed to be the same at each point of the new manifold just as is assumed for Euclidean space. Thus *c* takes its place as defining a finite, constant, invariant speed. [4]

Minkowski believed that mathematicians should have reached SR along this line of thought before the physicists did. The thoughts are mainly mathematical.[5] He

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[4] For a more detailed account of Minkowski §1 see Nerlich (2013) Ch 3, Cacciatori et. al (2008, Petkov (forthcoming) esp Chapter 8.2. For Ignatowski see Brown (2005) Lange (2013). Cox and Foreshaw (2009): 74-88 reach a "no light" version somewhat differently. Mermin (1984) and Sen (1994) are "no light" papers representative of a series of papers *Am. Jnl of Physics*.

[5] Mathematicians missed the triumphant possibility of arriving first at SR and along a simpler, clearer route.This opinion has distinguished support. See Pauli (1958): 11, who mentions Ignatowski, but, surprisingly, not Minkowski; also Dyson (1972): 640-643 and Cacciotori et al. (2008).

called his method of invention "fancy free" (op. cit.: 79). Of course one needs to look and see whether the world is as the theory says.

It is surprising and regrettable that this part of his lecture is so seldom mentioned. It builds SR on a quite different basis from the more familiar one.

The 2-postulate version of SR held a tenacious grip on Einstein's imagination. His justified, strong satisfaction with its empiricist, operationalist, methods probably obscured the advantages of Minkowski's deeper, more immediate perspective on changed ideas of space and time. One advantage of this perspective is that it puts spacetime at the foundation of the new theory from the start. The basis is no longer a form of relativity but "the postulate of the absolute world" (1908: 83). It points toward a version of GR that also postulates an absolute world and not a relativity theory.

Here's a bare bones sketch of how Einstein might have proceeded.[6] The classical first law describes the trajectories of force-free motion. They project up into straight trajectories of Minkowski spacetime – its geodesics. The "happiest thought of [Einstein's] life"[7] - that to fall freely in a gravitational field is to feel no force - suggests that all purely gravitational trajectories could be the force-free geodesics of a new curved spacetime. He needed to find a law that linked the distribution of mass-energy with the right geometry. That, together with the equivalence of inertial and gravitational mass, gives a reasonably articulate skeleton of GR and how it reduces gravity as a force to the geometric structure of spacetime. Einstein tells us that he was in command of much of this as early as 1908. Seven years were spent in search of a generally covariant formulation of GR. That preoccupation, together with his imperfect grasp of the output of that struggle, may explain why the thoughts of 1908 had become less salient by 1916.

Led by his choice of a light postulate and starring inertial frames of reference, clocks, rods and signalling as the principal foundations of the theory, Einstein writes

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In this connection Max Born's (1975:131)[5] cites a letter from Minkowski in which the latter states that Einstein's 1905 came as a great shock to him since he had already reached his own spacetime account of SR before that paper appeared. This does not establish priority and Minkowski never claimed it. It does establish independence. Probably the 'fancy free' mathematician whom Minkowski appears merely to imagine (Minkowski 1908: 78-9.) is in fact his earlier self. Born (op. cit.: 98) mentions an advanced seminar at Göttingen in 1905 given by Hilbert and Minkowski on electromagnetism. Minkowski was working in the area at the time.

[6] For another see Misner et al. (1973): 1-10.

[7] Schilpp (1949): 67, Pais (1982) Part IV §9.

that "the laws of geometry…are to be interpreted directly as laws relating to the possible relative positions of solid bodies at rest…and as laws which describe the relations of measuring bodies and clocks" (112). This fits with his pursuit of stationary systems throughout his examples and the relationalist, operationalist, empiricist tenor of the whole Part. But one does not now need to be Einstein to see that all this sets off in quite the wrong direction.

4 Part A §2: The need for an Extension of the Postulate of Relativity

§2 of Part A aims to establish a need to extend the postulate of relativity beyond the set of inertial frames privileged in both classical mechanics and SR. So it takes on the style of a metaphysical, epistemological *relativity* theory - a thought that springs quite naturally from the operationalist 2-postulate version.

Einstein's strong empiricism is evident throughout. As an empiricist *metaphysical* theory, the relativity of motion, in one or another form, implies two immediately relevant theses. The first is restrictive: statements about motion are intelligible if and only if the motion of an observable thing is referred to an observable object taken as at rest. The second is permissive: any observable object may be taken as at rest. The theses are epistemologically driven. Classical and SR physics reject them both, so, in one plain sense, their conjunction was *a priori*, with no support from physics before 1916. In GR, the holy grail of general covariance doesn't really contain what the permissive thesis claims: that any object can be taken at rest. It permits a free choice among *coordinates* that are consistent with the geometric structure of any model. That does include, for any particle, coordinates that describe it as at rest i.e. its x, y, z coordinates are constant. But that is not the main sense of general covariance: there is no requirement on a coordinate system that some object be at rest in it. Einstein does not spell out either metaphysical thesis yet it was clearly his ardent wish, and his eventual belief, that GR should incorporate them. Unlike Einsteinian SR, GR is not a theory about which frames of reference may be taken as at rest.

Absolute Rotation

Einstein's approach to his first target − the need to extend the SR range of coordinates in GR − begins at a tangent. He argues for the rejection of absolute rotation. He does not directly consider Newton's elegant thought experiment (in the Scholium). Let there be a world containing nothing but a system of two massive balls, joined by a cord, the balls rotating round the centre of the cord in a plane containing it and the centres of the balls. A tension in the cord conserves angular momentum. The tension is observable, and, in classical and SR physics, it is decisive evidence of rotation. Since nothing exists but this system, rotation in absolute space causes the tension in the cord. (See Janssen (2002:)7-8).

Einstein suggests a different thought experiment to oppose this view. It is confusing so I pursue Newton's easier example. It makes a limpid claim to absolute acceleration.

Einstein objects that Newton's example "cannot be admitted as epistemologically satisfactory unless the reason given is an observable fact of experience". Newton gives only a "factitious cause and not a thing that can be observed". He regards this as "a weighty argument from the theory of knowledge". "The cause must lie *outside* the system…the general laws of motion … must be such that the mechanical behaviour…is partly conditioned, in quite essential respects, by distant masses [which are] … the seat of the causes" of the tension in the cord (113).[8]

Clearly Mach's discussion of Newton's bucket experiment was also in Einstein' mind.

However, Newton's example, just as he gave it, is at home in both SR and in GR, which thus inherit any fault, epistemological or other, that the example may bring with it. The GR story is just the SR story in a model set in Minkowski spacetime with negligible perturbation from the small masses in the system − a situation Einstein always preferred in Part A. For SR, as well as for this GR model, the difference between Newton's case and another where there is no rotation lies not only in the different states of the cord but also in the proper times along worldlines of (central particles within) the balls. Without rotation these worldlines

[8] This is confusing in that Einstein has already secured his inertial frame, as always, by putting distant masses so distant as to make gravitation negligible. But how is the behavior of the system conditioned by these masses?

are geodesics; with rotation they are spirals in spacetime. Thus there is a difference in proper-time magnitudes. This would be observable if the balls were replaced by massive clocks. (Dorling (1978)).

Dorling (op. cit) and Janssen (2002) argue that this *kinematical*, proper-time, evidence means that the *kinematical* absolute rotation (in spacetime) *is* observable. This would clear the example of epistemological fault. I doubt that Einstein would have agreed. That still falls short of observing the *cause* i.e. the absolute rotation in spacetime, of the tension in the cord.[9] The evidence for absolute rotation remains both decisive and *indirect*. A relative rotation can be seen directly, an absolute one can't. Further, the shorter proper-times along the rotating balls' worldlines are not the tension's cause: their spiralling in spacetime is. That is an absolute rotation. GR fails to endorse the epistemological version of the relativity of motion. It does not forbid "factitious" causes any more than earlier theories did. No doubt much of the charm of the 2-postulate version lay in the hope that it would.

There is a further question that need not be pursued here: does GR admit a satisfactory Machian account of the example? If the solution is required to conform to the epistemological version of the relativity of motion, the answer is no. Boundary conditions on the structure of spacetime at infinity are needed. That transgresses the demands of epistemological relativity (Janssen (2002: 19-22).

So far, none of this is about coordinate systems. Einstein's immediate proposal changes the *physics* of the case not its coordinates. GR's field equation entails that the distant matter on which Einstein insisted will curve spacetime. *That* forbids Lorentz coordinates. Einstein nowhere pursues this theme, obvious though it is from absolutist SR taken with the theory of gravity in Part C.

He writes instead that the distant observable masses "take over the role of the factitious cause R_1 *[i.e. absolute space]*. Of all imaginable spaces, R1, R2 etc., in any kind of motion relative to one another, there is none we may look on as privileged *a priori* without reviving the above-mentioned epistemological objection. *The laws of physics must be of such a nature that they apply to systems of reference in any kind of*

[9] Human observers are continuant spatial beings living in time. Which spacetime structures can they observe? No good answer is obvious to me. If I toss a ball and catch it do I observe that its trajectory is (almost) a spacetime geodesic? I think not, but perhaps my conclusion in this paragraph is too quick.

motion. Along this road we arrive at an extension of the postulate of relativity". (116; original italics).

What is this reasoning? Presumably it envisages changing the reference (rest) frame (thus the space) to admit Mach's striking suggestion: choose a space where the balls are at rest and the outside masses rotate round them. That certainly provides a strong "aha!" moment, but even granted the general laws by which the masses condition the system's behaviour, there is nothing to suggest how the change of system of reference (coordinates) explains anything at all about causation. That may make calculations more intuitive but there is no way that it alters the GR physics. The argument is invalid. It gives no reason to extend the postulate of relativity.

Further, neither Newton's mechanics nor SR privilege any space *a priori*. Inertial frames are certainly privileged but for *good empirical reasons*: the laws of mechanics are invariant with respect to them. The road to that discovery from Aristotle to Galileo, Descartes and Newton was long, hesitant, replete with observations and experiments, false leads, failed theories and confusions. As long as Einstein's examples are set in the context of SR then there are *empirically privileged* frames of reference, adequate within those theories to explain fully the tension in the cord without postulating outside matter. Such examples do not and cannot point to a need to expand the coordinate systems beyond the ones privileged in those empirical, *a posteriori* theories.

Accelerated Frames in SR

Einstein next turns to another familiar thought experiment: "…a well known physical fact…favours an extension of the theory of relativity" (114). Gravity accelerates objects equally whatever their mass and constitution. The example that explores this begins, as before, by setting large masses at large distances thus permitting an inertial reference frame in virtually empty spacetime. Suppose, then, an inertial frame F_1 and an object, O, in uniform motion in it. Consider a second frame F_2 in uniform *acceleration* relative to F_1. Object O is now accelerated relative to F_2 as a rest frame, independently of its mass or makeup. Relative to F_2, the second law of motion requires a field of force in which O and all other objects, fall. Only the postulation of a uniform *gravitational* field can account for this phenomenon in this rest frame.

Einstein's main thesis is that postulating this field gives F_2 an equal right with F_1 to be regarded as a rest system but one in which there is a uniform gravitational field. The frames "may both with equal right be looked on as "stationary"' (114). So an extension of the Relativity Postulate is needed to include such frames. But the frames do *not* have an equal right to be taken as stationary in flat spacetime and no extension of coordinates is *needed*.

Einstein concludes that GR must include a theory of gravitation, 'since we are able to "produce" a gravitational field merely by changing the system of coordinates'. That is one aspect of the crucial link between gravity, acceleration and geometry. It does not include Einstein's "happiest thought" - that fall under gravity is force-free. It gives no reason to extend coordinates. It does not link curvature to mass.

F_1 can be an inertial frame only if SR holds to a good approximation and spacetime is Minkowskian. There is no matter-sourced, real ("tidal-force") gravitational field: it would be represented by spacetime's curvature and thus be non-uniform. Any "gravity" springs wholly from the free choice of coordinates not from the structure of spacetime.

In §2, Einstein provides no sound reason for extending the postulate of the special relativity of motion. Nevertheless that conclusion is needed. It comes quite intuitively through the absolute world version of SR together with later Parts of (1916).

5 Part A §3 The Requirement of General Covariance

Coordinates in space and spacetime

§3 begins by noting that in classical mechanics and SR, coordinates have a *direct* physical meaning: a point on an *x* axis has the coordinate number *n* if it is *n* spatial units[10] along the positive axis of the system. That is a 'direct' physical meaning. The

[10] A background standard of congruence is needed too.

same holds for numbers on the time coordinate axis. This directness must be abandoned in GR. The same point is made earlier in §1 (112).

Why must it be abandoned? The first major job of coordinates is to provide for calculating the magnitude of any very small interval between two *arbitrary but nearby* points in a space or spacetime: i.e. the job of expressing the space's metric structure in a small region. The job can't really be done directly, even given the most amenable spatial structure. However, there is a simplest way.

The exercise in the next five paragraphs is elementary. Surprisingly, it is just the elements that Part A distorts.

To write a whole theory of physics in generally covariant style (with tensors) is a challenging enterprise but in Part A, Einstein is concerned with a much simpler task – merely writing a spacetime metric in that coordinate style. To see how to do this, consider the simplest case: Cartesian coordinates for 2-dimensional Euclidean space.

1 The length of an arbitrary interval is calculated by first projecting each of its end-points orthogonally onto the x and y axes to find its x and y coordinates. Then one finds the x and the y coordinate differences between these coordinates numbers. In general the coordinate differences depend on the orientation of the axes relative to the line in space that joins the points That already goes a little beyond Einstein's account of directness. The squared magnitude of the interval is given via Pythagoras's Theorem from the sum of the squares of the coordinate differences (differentials) thus:

$$ds^2 = dx^2 + dy^2$$

That is the simplest form of a calculation that completes the coordinate job for this space. Cartesians for Euclidean space are beautifully succinct because their square-grid structure directly encodes both the parallels and the Pythagorean orthogonality structure throughout that space itself. Everywhere, the sides of any right-angled triangle bear the Pythagorean relation to the hypotenuse. Cartesians are objectively privileged.

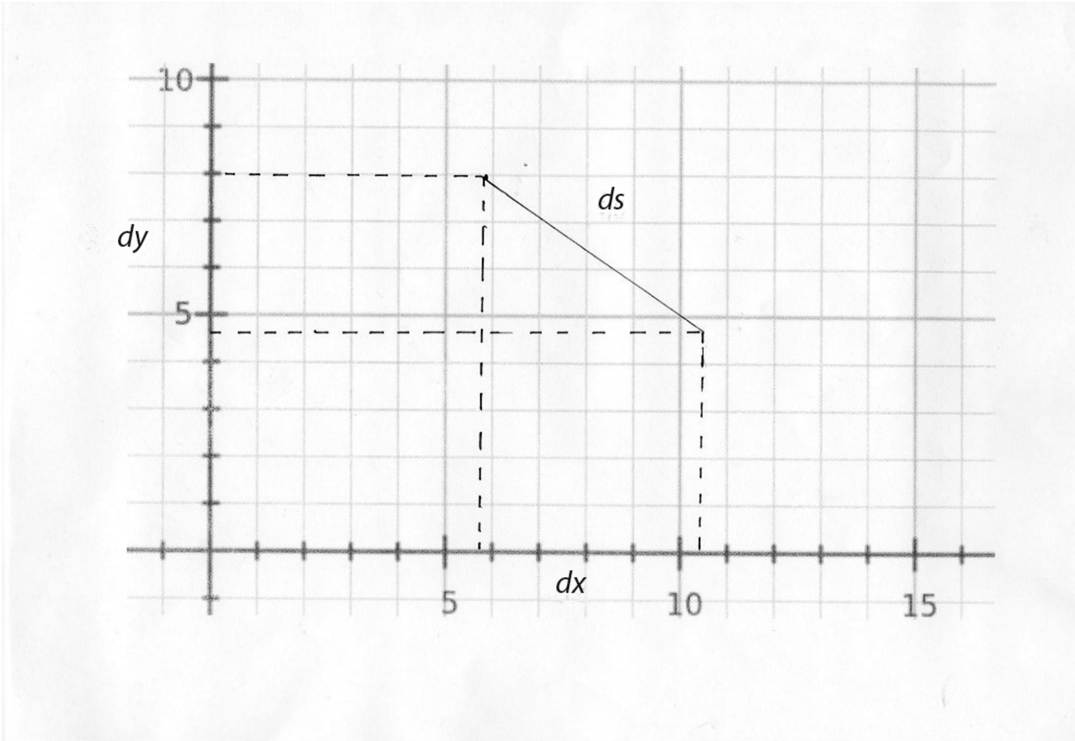

Figure 1

2 In Euclidean space, a simple departure from Cartesians is to use skew coordinates.

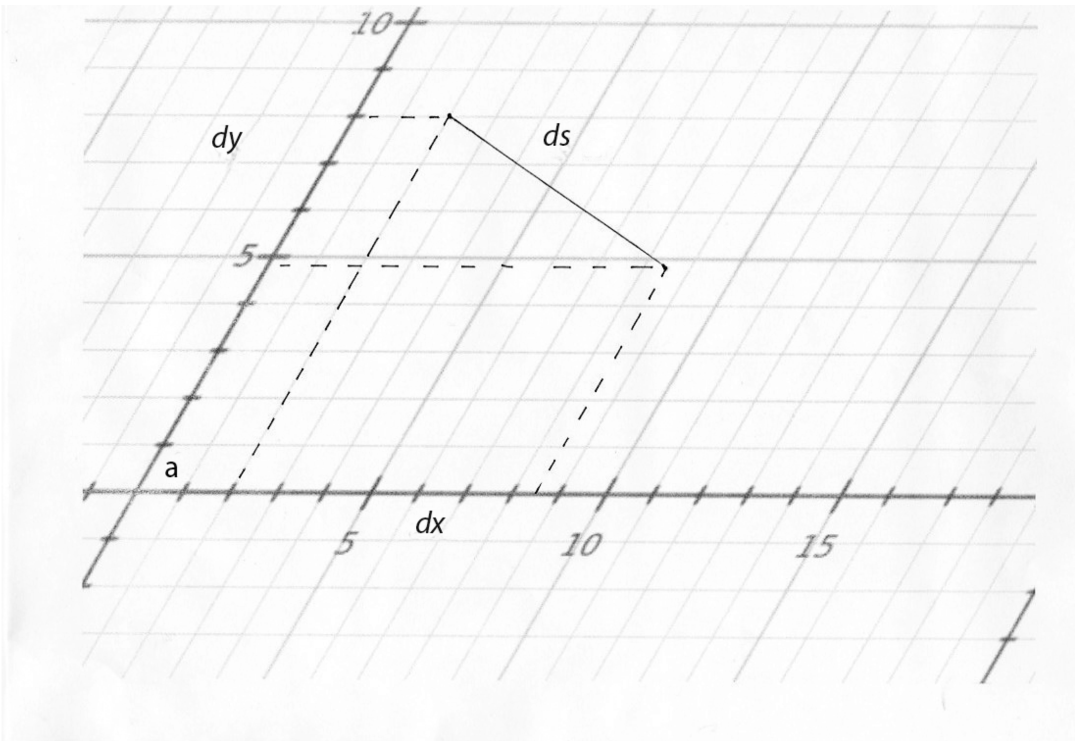

Figure 2

The preceding simplest coordinate representation of the metrical relation between spatially separated points immediately fails. The metric calculation must now take account of the angle, a, between skew coordinate axes as a coefficient of the cross product of the differentials.

$$ds^2 = dx^2 + 2\cos a\, dx.dy + dy^2$$

The spatial distance itself between the points is unaffected. This takes a first easy step towards general coordinates. The metrics in skew and in general coordinates differ from the succinct Cartesian expression only in complexity. The space remains Euclidean. But their greater complexity hints at a deeper similarity in the structure of *all* metrics among *all* coordinates. An *underlying* general form for all Riemannian spaces is:

$$ds.ds = g_{11}dx_1.dx_1 + g_{12}\, dx_1.dx_2 + g_{21}dx_2.dx_1 + g_{22}dx_2.dx_2$$

A metric is often referred to just as g_{ik} where i and k are index variables describing the $n\times n$ (for n dimensional space) array of coefficients. This is a massive abbreviation in characterising metrics. All Riemannian geometries (of 2 dimensions) have metrics in which the coordinate differences (differentials) are thus pairwise multiplied together in every way. Different coefficients (g_{ik} etc.) are attached to each product in the order indicated. A product vanishes if the coefficient is 0 and is unchanged, and omitted, if it is 1. General coordinates are freely chosen within that loose constraint. The equation is called the *metric* tensor.

3 That free choice allows us to choose coordinate curves instead of straight lines and arbitrary angles of intersection of x with y coordinates. The correspondence between points in the space and coordinate number pairs (triples, quadruples etc. for higher dimensions) is required only to be 1-1, continuous and differentiable (smooth). Any direct relation between coordinate number differences and spatial length is abandoned. These coordinates are general. They form the largest group of coordinates that correspond 1-1 with ordered number pairs such that the topology – the continuous smooth relations between points or numbers – is

invariant. The transformation equations that take us from one of these general systems to another are smooth and continuous.

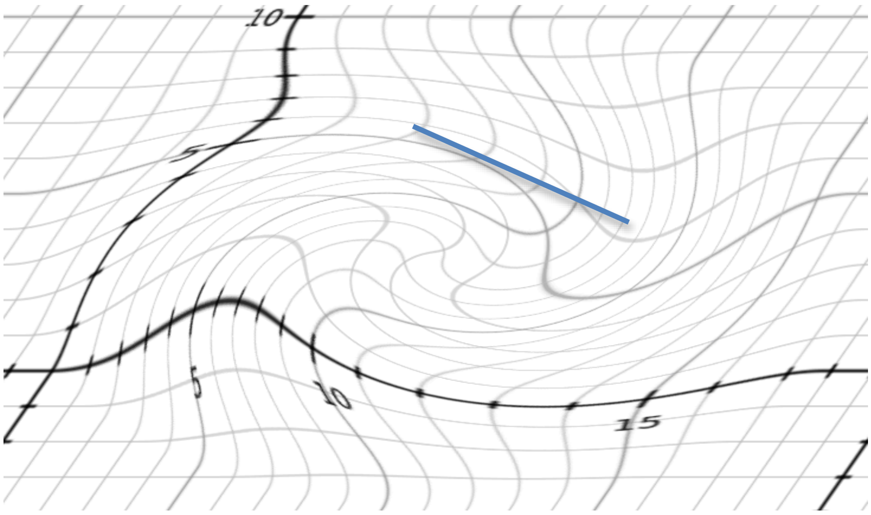

Figure 3

4 Cartesian coordinates encode a great deal of space's structure just in the conventions by which we set them up. That is how the metric can be expressed so simply. In skew and in general coordinates this encoding is deliberately erased and the information replaced explicitly in the coefficients of the differential products. So what is contained in the metric equation is the same but conveyed by different means.

5 Why do this? Cartesians are possible only in Euclidean space (Lorentzians only in Minkowski spacetime) where there are parallels and a Pythagorean structure. Their simplicity may be lost in two ways: first by arbitrarily choosing new coordinate styles, as we did, simply to show that the actual spatial magnitudes may be detached from the arbitrarily varied coordinate numbering conventionally chosen; or by being *forced* to adopt more complex coordinates *by the structural properties of the space or spacetime itself*. Spacetime has intrinsic curvature if and only if it has no parallels. If curved, the metric, in whatever possible coordinates, can't be Cartesian ("direct" in Einstein's usage). However, in Riemannian spaces, it still takes the basic quadratic form: the coefficients of the cross-product terms generally contain non-zero and non-unit coefficients. (See e.g. Norton (1993), Rynasciewicz (1999)).

Armed with this sketch of basics let's return to Einstein Part A §3.

The Minowskian spacetime version of SR makes it obvious that a generally covariant style of coordinates, even in flat spaces or spacetimes, won't give a metric structure simply encoded in coordinate numbering alone: you also need explicit reference to the metric tensor in the chosen coordinates. It is easy - at least in thought - to separate purely coordinate complexities from those that spring from variable curvatures in spacetime structure in GR proper. Einstein omits that distinction in Part A.

Even in the simple departure of skew coordinates, it is obvious that coordinate numbers and differences seldom directly tell us the distance between points in space itself. The metric tensor is needed too. Substitute the appropriate coordinate-derived numbers in the metric tensor equation, solve it, and the result is *exactly* what one got 'directly' through Cartesian (Lorentzian) coordinates. In those coordinates the tensor is simplified by its coefficients being either 0 or 1. While the calculation in skew (or in more general) coordinates is more articulated and more complex the information delivered is always the same.

In randomly curved general coordinates, even in flat spacetimes, the geometry within spacelike hypersurfaces picked out by t-constant points may curve and vary wildly. Their geometry will not be Euclidean. Further, since numbering on any spacelike coordinate curve need encode only topological information, coordinate differences say nothing about lengths along the curve between points on it. Similarly on x,y,z-constant timelike coordinate curves, the magnitude of coordinate number differences between points on the curve makes no attempt to match information about the magnitude of the proper-time intervals on a clock whose trajectory tracks the coordinate curve between the points. They do not convey the *wrong* measures of things in space. They convey *no* measures. The natural ordering relations between the number quadruples that identify the points in the general system have to mimic only the smooth, continuous spatial (spatiotemporal) ordering of the relations of separation and connection among points in the space (spacetime) itself. So number mismatches between coordinate intervals and spatial (spatiotemporal) metric intervals are inevitable. Arbitrarily singling out some wobbly spacelike hypersurface changes nothing physical in spacetime. No new definitions of space and time are needed and no new theory, unless a theory of coordinates.

As for rods and clocks, whether or not spacetime is curved, good clocks (by definition of ‘good’) measure proper-time intervals and good rods measure spatial ones. New coordinate styles don’t change that. Variations in the coordinate times between ticks of a clock tell us nothing about whether the clock runs fast or slow.

Until we enter the territory of curvature or gravitation none of this goes beyond Minkowski spacetime or undermines the *privilege* of inertial frames. That new territory does need a new understanding of spatiotemporal concepts. When there are neither parallels nor a Pythagorean orthogonality structure[11] no coordinates can be direct in Einstein’s sense or simplest in the sense described above i.e. when the cross terms in the metric tensor vanish. A generally coordinate formulation is usually inescapable in GR. This is clear in the later Parts of (1916) but appears nowhere in Part A except, indirectly, in §4.

Another 2-postulate misdirection

The 2-postulate version, together with the fatal attraction of the epistemological theory of the relativity of motion, led Einstein to address specific examples, rather than the simple, general feature just mentioned - that matter curves spacetime. Examples dominate, since he wants always to pick out some new stationary system. Consider Einstein’s Part A §3 example (115-6) of a disk rotating relative to an inertial system F_1 with its centre at rest in F_1. There are clocks fixed to the disk’s circumference and rotating with it relative to F_1. There is also a clock at its centre at rest relative both to the disk and to F_1. There are also standard measuring rods attached to the disk and used to survey it. Relative to F_1, the circumference clocks “run slow” but the central one does not. The rods on the disk will be “contracted” in length by their circular motion when measuring the circumference of the disk, but not when used to measure its radius.

If the disk is now taken as the rest frame i.e. as not rotating, then clocks fixed to its circumference will “run slow” despite being at rest. So the rate of a clock seems to depend on where it is. Rods along the circumference will be “contracted” compared with those lying along the radius. So, in this (non-inertial) frame the ratio of radius to circumference of the disk will not be π. The geometry on the

[11] Except in very small regions.

"stationary" disk will not be Euclidean. A gravitational field must be imposed within the frame of reference despite the absence of spacetime curvature.

That is misleading.

General covariance permits coordinates that describe the disk as stationary but they are neither neat nor necessary since F_1 describes the same world more simply. The 2-postulate version's fixation on rest states, material clocks, rods and light signals easily leads to confusion of coordinate space and time with proper length and time, to a groundless suspicion that time and space are being redefined, that clocks run at different rates at different places that the speed of light is somehow inconstant and so on. It tells us nothing about GR.

Final remarks

Einstein writes: "…in the general theory of relativity space and time cannot be defined in such a way that differences in the spatial coordinates can be directly measured by the unit measuring rod, or differences in the time coordinates by a standard clock." (117) True, but not simply because of new choices of coordinates. In the presence of masses, the matter tensor cannot be Euclidean since spacetime curves and may curve variably. Then you can't use Cartesian or Lorentz coordinates. *That* is why "*The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)".* (117 Einstein's italics). That, too, is true but no Part A argument entails or explains it. Last, Einstein draws his main metaphysical conclusion: "…this requirement of general covariance…takes away from space and time the last remnant of physical objectivity" (loc. cit.). It does not follow.

Presumably he intended this as an intuitive thesis that joins hands with what's known as the Point Coincidence Argument: "…the results of our measurings are nothing but verifications of such meetings as the material points of our measuring instruments with other material points…and observed point-events happening at the same place and at the same time." (loc. cit.)

General covariance takes nothing away from the reality of space and time. It is merely about changes in coordinates. In Part A it seems clear that Einstein confused the arbitrariness of coordinate choice with a lack of structure in spacetime itself.

Minkowski's version of SR, where the fundamental entity is spacetime, readily accommodates the path-breaking concepts forged in later Parts − spacetime's curvature in the presence of mass, the reduction of tidal gravitation force to curvature, the need for general coordinates.

That Einstein's bold ontological claim was right nevertheless has found distinguished support (Norton (2011), Earman and Norton (1987)). It rests mainly on the contentious hole argument. This is not the place to pursue that theme in detail. For an extensive critique of it along these lines see Nerlich (2013 ch. 10).

Briefly, the hole argument fails because the ideas underlying Leibniz Equivalence as a metaphysical thesis are indefensible. In spaces that are non-Euclidean, the kinematic and dynamic shifts are never symmetries: there are no parallel trajectories for them to exploit. The Principle of the Identity of Indiscernibles, metaphysical, certainly, but always dubious, can't then convert symmetries into identities because the differences resulting from the shifts reveal that space does have a real observable structure. They also reveal something of what it is. It is clear, further, that which spatial relations can be postulated in some region of space or spacetime is not independent of the geometry of the region. It is no longer assumed as a philosophical thesis that space is Euclidean so the price of substantivalism does not break the metaphysical bank despite the suggestions of the hole argument (Nerlich (1991 and 2013), ch. 1).

Leibniz Equivalence does no better when one turns to the differentiable manifold but I shall not pursue the matter further than to say that active diffeomorphisms of the manifold do not have point coincidences as direct invariants. The commanding requirement on diffeomorphisms is that they preserve relations of separation and connection among manifold points; i.e. the topology is invariant and the transformations reveal it as a real spatiotemporal structure. Point coincidences and tensors in general are indeed "dragged along' in diffeomorphisms but the transformations are not directly aimed at that.

Leibniz Equivalence can be invoked as good pragmatic advice: ignore differences that your theory itself tells you are inconsequential.

The Point Coincidence thesis is false. If we are to have any coherent and synoptic physics at all then we must necessarily observe that some point-coincidences occur elsewhere or elsewhen from others, being smoothly separated and connected by spacetime intervals. In an aphorism "Time is nature's way of keeping everything from happening at once; space is what prevents everything from happening to me."[12] These are observable truths.

Kretschmann (1917) pointed out that using generally covariant coordinates does not affect the geometric content of a theory. Einstein acknowledged the error but seems never to have revised the 2-postulate approach to GR or fully embraced the *reduction* of gravity to geometry; i.e. identifying spacetime geodesics as free-fall trajectories and thus necessarily purely *kinematic*. Geodesics have zero acceleration vectors at every point and therefore no force vectors at any point. In the Leyden lecture (Einstein (1983)) and the Princeton lectures (Einstein (1953)) gravity is treated as an action - as a dynamical, not a kinematical, motion. (See Petkov (2012). Spacetime is tentatively regarded as an ether in order to escape action at a distance. The spacetime metric field is seen as coincident with, but not identical to, the gravitational field. In the later editions of his popular exposition (Einstein (1954)), GR is introduced just as it was in 1916 Part A.

In Part A, Einstein's dogmatic empiricism blinded him to structures in later Parts that lay in plain sight and were the shining jewels of his unsurpassed inventive genius. The unfortunate influence of this, especially on positivism in succeeding decades, is a topic for another paper. Minkowski's tragically early death (1864-1909) robbed us of a constructive, imaginative and rather different perspective on GR.

Graham Nerlich

University of Adelaide

graham.nerlich@adelaide.edu.au

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[12] Attributed to John Wheeler in the dedication page of Hagar (2014).

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