

Choosing the Analytic Component of Theories

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Abstract

I provide a compact reformulation of Carnap's conditions of adequacy for the analytic and the synthetic component of a theory and show that, contrary to arguments by Winnie and Demopoulos, the conditions need not be supplemented. Specifically, the axiomatization and the observational vocabulary of a theory determine its synthetic component uniquely but leave some freedom in the choice of its analytic component. This throws a new light on the process of the rational reconstruction of theories and renders adequate a suggestion by Bedard for the interpretation of theoretical terms (when expressed within the confines of standard predicate logic). I suggest a possible analytic component for theories that is stronger than the Carnap sentence and thus closer to Bedard's suggestion.

Keywords: analyticity; analytic-synthetic distinction; Carnap sentence; Ramsey sentence; reduction sentence; Przełęcki reduction pair; relativization sentence

1 Introduction

The notion of analyticity is a difficult beast in natural language, too unclear to do serious philosophical work according to some (Quine 1951). In spite of critics (Quine 1951, again) this problem may be solved by explicating the notion (Mates 1951, 532–33). Such an explication is unlikely to succeed for all domains of application at once (Martin 1952, 42–45), and it is the restriction to theories that can be phrased in higher order predicate logic that has led to the probably most satisfying and influential explication. This explication, given by Carnap (1963, 24.D), has been used for a variety of analyses, from metaphysics (Lewis 2009, §3) and philosophy of science (Przełęcki 1980) to philosophical methodology (Papineau 2009, §III).

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Besides his explication, Carnap also suggests three conditions of adequacy that his explication meets and which I will simplify in the following (§2). Winnie proposes (and Demopoulos endorses) an additional condition of adequacy that singles out Carnap’s explication as the uniquely adequate one for all possible theories (§3). I will show that it leads to implausible restrictions on analytic sentences and theoretical terms. So Winnie’s condition should be given up, also because it lacks a convincing argument in its support: Winnie’s argument rests on a false analogy (§4), and Demopoulos’s argument for the need for an additional condition of adequacy rests on an unjustified assumption about the rational reconstruction of theories (§5). Without an additional condition of adequacy, there is a certain leeway in determining the analytic component of a theory, of which I take advantage when developing a generalization of Carnap’s explication that is more constructive in the case of empirically inadequate theories and that comes closer to an informal suggestion by Bedard (§6).

2 Conditions of Adequacy for an Analytic-Synthetic Distinction

The starting point of my discussion is not that of a typical explication. Instead of beginning with a vague explicandum given in normal language and searching for a precise explicans that should be used instead, I will analyze the properties of an extant explicans, namely Carnap’s. (My discussion is not meant as a historical interpretation of Carnap’s writings, although I will at times comment on the relation between Carnap’s positions and my results.)

Since Carnap’s explicans assumes higher order predicate logic, which is not a formalism in which theories are typically phrased, most theories will have to be axiomatized before the explicans can be applied. Carnap also assumes a bipartition of the language’s vocabulary into a set \mathcal{O} of observational terms and a set \mathcal{T} of theoretical terms.¹ With this bipartition comes a tripartition of the set of sentences of the language into observational sentences, whose only terms are observational, theoretical sentences containing only theoretical terms, and mixed sentences containing both observational and theoretical terms. As Suppe (1971, §1) has argued, the bipartition of the vocabulary crucially rests on the assumption of an artificial language, since it cannot be found in natural languages (see also Demopoulos 2008, 365). The exact nature of the bipartition is contentious, and I will restrict my comments to the implications of Carnap’s explicans; historically, Carnap assumed the bipartition to be a result of language choice (Oberdan 1990).

With these assumptions, Carnap aims at finding a way of separating the factual, synthetic content of a theory ϑ from its non-factual, analytic content, which he takes to be conventional (cf. Demopoulos 2008, 364–65). Carnap (1963,

1. As is usual in this debate, ‘term’ is used as a synonym of ‘non-logical symbol’ in the following.

24.D) suggests a general solution for cases in which the theory can be expressed in a single sentence, so that there are only finitely many observational terms $\mathcal{O} = \{O_1, \dots, O_m\}$ and theoretical terms $\mathcal{T} = \{T_1, \dots, T_n\}$, and ϑ can be written as $\vartheta(O_1, \dots, O_m, T_1, \dots, T_n)$.² Then, according to Carnap, an adequate synthetic component of ϑ is its Ramsey sentence

$$R_{\mathcal{O}}(\vartheta) := \exists X_1 \dots X_n \vartheta(O_1, \dots, O_m, X_1, \dots, X_n),^3 \quad (1)$$

which results from ϑ by existentially generalizing on all theoretical terms in ϑ . As an adequate analytic component of ϑ , he suggests its Carnap sentence

$$C_{\mathcal{O}}(\vartheta) := R_{\mathcal{O}}(\vartheta) \rightarrow \vartheta. \quad (2)$$

Because the Ramsey sentence does not contain any theoretical terms, it cannot determine their meaning. And since the Carnap sentence contains the Ramsey sentence as its antecedent, the theoretical terms are only given meaning if the Ramsey sentence is true. If the Ramsey sentence is false, the Carnap sentence is always true, no matter the interpretation of the theoretical terms. (Since $\neg R_{\mathcal{O}}(\vartheta)$ is an observation sentence, this means that on Carnap's account analytic sentences can be empirically verified.)

According to Carnap, $R_{\mathcal{O}}(\vartheta)$ and $C_{\mathcal{O}}(\vartheta)$ provide an adequate synthetic and, respectively, analytic component of ϑ . To develop explicit conditions of adequacy, Carnap (1963, 963) defines the observational content of any sentence S as follows:

Definition 1. “The *observational content* or O-content of $S =_{\text{Df}}$ the class of all non-L-true [not logically true] sentences in $L'_{\mathcal{O}}$ which are implied by S .”

$L'_{\mathcal{O}}$ refers to the “logically extended observation language”, whose sentences contain only observational terms, logical symbols, and variables of any order (959). In my terminology, the sentences of $L'_{\mathcal{O}}$ are the observational sentences. On the basis of definition 1, Carnap (1963, 963) suggests

Definition 2. “ S' is *O-equivalent* (observationally equivalent) to $S =_{\text{Df}}$ S' is a sentence in $L'_{\mathcal{O}}$ and S' has the same O-content as S .”⁴

Finally, Carnap's discussion (965) suggests the following conditions of adequacy:⁵

2. To be more precise, one could write $\vartheta^*(O_1, \dots, O_m, T_1, \dots, T_n) = \vartheta$, so that ϑ^* is an $m + n$ -place formula of higher order and ϑ one of its instantiations. The notation used here has become standard, however, and is less cumbersome.

3. The subscript stands for the vocabulary that remains after the existential generalization.

4. Note that Carnap's definition is asymmetric: S' but not S has to be in $L'_{\mathcal{O}}$.

5. Contrary to definition 3 and the assumption of Winnie (1970) and Demopoulos (2008), Carnap only intended the conditions in definition 3 to be sufficient (see appendix B). This does not change the results of my discussion, however, since I want to elucidate the implications of treating the conditions as also necessary.

Definition 3. σ is an *adequate synthetic component* and α an *adequate analytic component* of ϑ if and only if

- “(a) The two components together are L-equivalent [logically equivalent] to $TC[:= \vartheta]$.
- (b) The first component is O-equivalent to TC .
- (c) The second component contains theoretical terms; but its O-content is null, since its Ramsey-sentence is L-true [logically true] in L'_O .”

Like Ramsey and Carnap sentences, these conditions of adequacy are defined for single theories. Of course, they can be applied to multiple theories and additional statements by conjoining the theories and statements. Since ‘O-content’ is defined relative to the extended observation language, the observational content of a theory is equivalent to its Ramsey sentence (appendix A, corollary 14), which leads immediately to

Claim 1. σ is an *adequate synthetic component* of ϑ and α is an *adequate analytic component* of ϑ if and only if

1. $\sigma \wedge \alpha \vDash \vartheta$,
2. $R_\vartheta(\sigma) \vDash R_\vartheta(\vartheta)$,
3. σ contains no theoretical terms, and
4. $\vDash R_\vartheta(\alpha)$.

Claim 1 straightforwardly leads to

Corollary 2. $R_\vartheta(\vartheta)$ is an *adequate synthetic component* of ϑ and $C_\vartheta(\vartheta)$ is an *adequate analytic component*.

The conditions of adequacy are themselves adequate only under a number of presumptions. For one, there can be no analytic observational sentence besides logical truths. For if there were such a sentence ω , $R_\vartheta(\omega) \vDash \omega$ would be analytic without being a logical truth, contrary to condition 4. Therefore every observational sentence must be synthetic, which in turn suggests that the observational terms of the language must be completely precise. Otherwise, a theory’s Ramsey sentence may be not logically true but still analytic, because it might only make some vague concepts more precise. For instance, an observational term O may be vague so that an object named by the observational term c may or may not be O . If $\omega \vDash Oc$, then $R_\vartheta(\omega) \vDash Oc$, but by assumption, Oc is an analytic stipulation about the extension of O . Thus observational terms are empirically and precisely interpreted, and it is a matter of empirical fact whether observational sentences are true (excepting logical truths, of course).

The conditions of adequacy further presume higher order predicate logic. Thus, although Carnap (1934, §17) considered logic conventional, its conventionality is not captured in the conditions of adequacy. In other words, the meaning of the logical symbols is treated as having been chosen beforehand. The meaning of mathematical terms, on the other hand, is not presumed to have been fixed. For even though higher order logic arguably suffices for defining or at least axiomatizing all concepts of mathematics, the definitions or axioms introduce new terms (namely the mathematical ones), and these have to be either observational or theoretical.⁶ If mathematical terms were observational, however, mathematical sentences would not be analytic, and thus mathematical terms are theoretical terms.

Finally, the conditions of adequacy presume that an empirical theory plays a significant role in determining the meaning of its theoretical terms, since it has to entail the analytic sentences which give them meaning. Thus the proponent of an empirical theory ϑ develops ϑ to account for observation sentences, but to do so, has to engage in language choice (cf. Carnap 1966, 188). The Carnap sentence hence expresses the idea that our concepts are not somehow *a priori* given to us, but rather chosen as conventions in response to empirical information, and indeed useless if that information turns out false: If the Ramsey sentence is false, the theoretical terms are completely unrestricted in their interpretation.

Williamson (2007, 54) states that if analytic truths are true by convention (“stipulation”), then the “distinction between analytic truth and synthetic truth [...] distinguish[es] different *senses* of ‘true’”. He then argues that this is problematic because analytic and synthetic truths are not truth-functional, so that from “the perspective of compositional semantics, the analytic-synthetic distinction is no distinction between different ways of being true” (58). One response to his argument is that according to the Carnap sentence analyticity is not truth-functional (appendix A, claim 11), but can still be consistently understood as conventional. A more direct response is that while it is not particularly clear how truth by convention is a “different way of being true”, it is not more suspicious than, say, a truth of physics: If a true physical theory ϑ entails sentence τ but not true sentence ϱ , one can say that unlike ϱ , τ is true because of ϑ . And if it is specifically $C_{\vartheta}(\vartheta)$ that entails τ , one can say that τ is true because of the analytic component of ϑ . In this sense, and compared to the truth of ϱ , being analytically true *is* a different way of being true, in the same way that being true because of the laws of physics is.

Thus it seems that one of the central requirements of Carnap’s philosophy of science, and indeed of logical empiricism in general, has been fulfilled: For each theory, there is a way of distinguishing precisely between its analytic and its synthetic component. Even better, there is an effective procedure that does so

6. Carnap (1956, 43) states that acceptance of higher order logic with a denumerably infinite domain seems to be “sufficient to make sure that [the language] includes all of mathematics that is needed in science”, but it is clear from his further expositions that he considers the acceptance sufficient to *define* all of mathematics.

automatically for all theories, simply based on the choice of the axiomatization and the choice of the observational terms.

3 Winnie and Demopoulos on the Conditions of Adequacy

For a consistent theory ϑ in a first order language without identity, Winnie (1970, theorem 5) shows that beyond $C_\vartheta(\vartheta)$, $C_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta)$ also fulfills the conditions for the analytic component of ϑ (cf. Williams 1973, 404–8).⁷ The lack of identity is necessary so that ϑ does not restrict the cardinality of its domain, but as Demopoulos (2008, 376–77) points out, this can also be made an explicit condition. Then Winnie’s result is easy to recover in higher order logic:

Claim 3. *If ϑ does not restrict the cardinality of its domain, $C_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta)$ is an adequate analytic component and $R_\vartheta(\vartheta)$ is an adequate synthetic component of ϑ .*

Proof. That $R_\vartheta(\vartheta)$ fulfills conditions 2 and 3 of claim 1 follows from corollary 2. Clearly, $R_\vartheta(\vartheta) \wedge C_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta) \vDash \vartheta$. Furthermore, if ϑ does not restrict the cardinality of its domain, $\vDash R_\vartheta(\vartheta) \vDash [\neg R_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta)] \vee [R_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta)] \vDash [\neg R_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta)] \vee R_\vartheta(\vartheta) \vDash R_\vartheta([\neg R_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta)] \vee \vartheta) \vDash R_\vartheta([R_\vartheta(\vartheta) \rightarrow \vartheta] \wedge R_{\mathcal{T}}(\vartheta)) \vDash R_\vartheta(C_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta))$. \square

Winnie (1970, 294–96) and Demopoulos (2007, §v) consider this non-uniqueness result something of a confirmation of the Quinean charge that the distinction between analytic and synthetic sentences is arbitrary (cf. Quine 1951). As a defense of Carnap’s approach, Winnie (1970, 296–97) and Demopoulos (2007, §v) suggest an additional condition of adequacy for analytic components that is based on

Definition 4. ϱ is *observationally vacuous* in ϑ if and only if $\vartheta \vDash \varrho$ and for any sentence τ with $\vartheta \vDash \tau$ and observational sentence ω , $\tau \wedge \varrho \vDash \omega$ only if $\tau \vDash \omega$.

Winnie (1970, 296–97) points out that definition 4 is similar to but stronger than the notion of observational conservativeness relative to an empty set in first order logic (appendix A, definition 6, cf. Mates 1972, 200). He and Demopoulos (2007, 259) further point out that an observationally vacuous sentence can never contribute to the inference of an observational sentence. This, of course, is shorthand for the claim that an observationally vacuous sentence can never contribute to the inference of an observational sentence *from a sentence entailed by ϑ* .⁸ In fact, ϱ is observationally vacuous in ϑ if and only if, first, it is entailed by ϑ , and

7. $R_{\mathcal{T}}(\vartheta) = \exists X_1 \dots X_m \vartheta(X_1, \dots, X_m, T_1, \dots, T_n)$, see n. 3. To be precise, Winnie shows that any theoretical sentence entailed by ϑ can be conjoined with $C_\vartheta(\vartheta)$.

8. As Ayer (1946, 11–12) realized the hard way, almost any sentence ϱ can contribute to the inference of almost any observational sentence ω , namely in conjunction with $\varrho \rightarrow \omega$ when $\varrho \rightarrow \omega \not\vDash \omega$ (Lewis 1988).

second, it is observationally conservative relative to *every sentence* entailed by ϑ . This is a much stronger condition than, for example, observational conservativeness relative to ϑ , which would not strengthen Carnap’s conditions. For every sentence entailed by ϑ is observationally conservative relative to ϑ . Hence it is misleading when Demopoulos (2007, n. 12) calls observational vacuity a “special case” of observational conservativeness. Indeed, observational vacuity is so strong that one might suspect that no consequence of ϑ at all is observationally vacuous in ϑ , since for any sentence ϱ and observational sentence ω entailed by ϑ , $\vartheta \models \varrho \rightarrow \omega$. Since $\varrho \wedge (\varrho \rightarrow \omega) \models \omega$, ϱ therefore fails to be observationally vacuous unless $\varrho \rightarrow \omega \models \omega$, that is, $\neg\omega \models \varrho$. In other words, a sentence ϱ is observationally vacuous only if it is entailed by ϑ and by the negation of every observational sentence ω entailed by ϑ . Since $R_\vartheta(\vartheta)$ is one such observational sentence, ϱ is observationally vacuous only if $\neg R_\vartheta(\vartheta) \models \varrho$ and $\vartheta \models \varrho$, that is, $\neg R_\vartheta(\vartheta) \vee \vartheta \models \varrho$, or simply $C_\vartheta(\vartheta) \models \varrho$. Winnie (1970, corollary 12) also proves the converse, so that the following holds:

Claim 4 (Winnie). ϱ is observationally vacuous in ϑ if and only if $C_\vartheta(\vartheta) \models \varrho$.

Since Winnie demands as an additional condition of adequacy that the analytic component of ϑ be observationally vacuous in ϑ , he thereby shows that only the Carnap sentence is an adequate explication of the analytic component of ϑ and the Quinean charge of arbitrariness is met.

4 Against the Carnap Sentence

As already noted, the Carnap sentence of ϑ does not restrict the interpretation of the observational terms at all if the Ramsey sentence is false and thus, by claim 13 (appendix A), if ϑ asserts a single false observational sentence. Therefore the Carnap sentence formalizes a very weak notion of the meaning given to theoretical terms, and much stronger notions have been suggested. Lewis (1970, 432) suggests that a nearly realized theory interprets its theoretical terms by their near-realizers, and he suggests that the notion of near-realizers can be explicated as the interpretations of the theoretical terms of a true, slight weakening of the theory. Unfortunately, Lewis leaves the notion of slight weakening unexplicated. Bedard (1993, 508–9) avoids this problem by suggesting that if a theory is not realized, its terms “partially denote” the things that realize its strongest true subtheories (whether slight weakenings or not), where subtheories of ϑ are entailed by ϑ (503–5).

Like Lewis, Bedard relies on a semantics that is alien to the standard formalism of higher order logic in that not all interpretations in which a theory ϑ is true are possible realizations. Instead, the possible realizations are restricted to those interpretations that are “natural” in some sense (cf. Schurz 2014, §5.8.3). This restriction does not underlie the Ramsey and Carnap sentences: There, once the observation terms are interpreted, the denotations of the theoretical terms

are determined *solely* through their formal relations to the denotations of the observational terms as given by the theory ϑ . Therefore ϑ is realized if and only if $R_\vartheta(\vartheta)$ is true (appendix A, lemma 12). Bedard’s suggestion can be adapted to this assumption so that the logically strongest sentence τ that is entailed by ϑ and has a true Ramsey sentence provides the interpretation of the theoretical terms. Of course, $R_\vartheta(\tau)$ will typically not be logically true, while the Ramsey sentence of ϑ ’s analytic component has to be. Thus, in the adaptation of Bedard’s suggestion, the analytic component of ϑ must be the strongest sentence entailed by ϑ that has a logically true Ramsey sentence. Unfortunately, no general method is known that would always produce such a sentence for any ϑ , but it is clear that $C_\vartheta(\vartheta) \wedge R_{\mathcal{T}}(\vartheta)$ is closer to Bedard’s suggestion than $C_\vartheta(\vartheta)$, because the former is logically stronger than the latter.

Bedard’s and Carnap’s suggestions for the analytic component are as different as suggestions with different assumptions about the basic formalism can be. The question is which suggestion one should choose. There are a number of reasons why in many situations $C_\vartheta(\vartheta)$ is the wrong choice. One is the following: Take any theory ϑ with theoretical content that is not logically true and does not restrict the cardinality of its domain. Then, if the theory is extended in any way so that its observational content increases, some of the theory’s analytic implications will become non-analytic (see appendix A):

Claim 5. *Let ϑ be such that $R_{\mathcal{T}}(\vartheta)$ is not logically true and has models of any cardinality, and let τ be any sentence. Then $C_\vartheta(\vartheta \wedge \tau) \vDash C_\vartheta(\vartheta)$ if and only if the observational content of $\vartheta \wedge \tau$ is equivalent to that of ϑ .*

The introduction of any sentence into a theory that increases the theory’s observational content would thus render some previously analytic sentence non-analytic. For instance, Carnap (1966, 238) states that as long as they avoid inconsistency, physicists “are free to add new correspondence rules”, which not necessarily (or even typically) lead to an observationally equivalent theory.⁹ More spectacularly, if ϑ is a theory about quarks and leptons, some of its analytic sentences must be non-analytic in a theory $\vartheta' \vDash \vartheta \wedge \varrho$, where ϱ is any sentence with empirical content, for instance a claim about the number of letters in this sentence. Or ϑ could be Newtonian mechanics and ϱ the claim that there is a planet that is closer to the sun than Mercury. In this sense, then, the Carnap sentence is too holistic, since a minuscule change of a seemingly unrelated and possibly completely observational aspect of a theory changes the meaning of its theoretical terms. That specifically some previously analytic sentences can become non-analytic also goes against Carnap’s intentions, since he thought that by adding new correspondence rules, physicists are “increasing the amount of interpretation specified for the theoretical terms” (238). As claim 5 shows, however,

9. How important such introductions of new correspondence rules are for Carnap’s account of scientific theories can be seen from his discussion of the interplay of empirical and conventional content in sequences of reduction sentences (Carnap 1936, 445–46).

any correspondence rule that increases the observational content of a theory also decreases the amount of interpretation specified for the theoretical terms.

The preceding considerations can also be phrased without reference to any change of a theory: If one theory entails another and has additional empirical content, then some of the analytic sentences of the logically stronger theory are, counterintuitively, not analytic sentences of the weaker one. And it is not just a small number of theoretical term that are affected. Two theories that differ in their empirical content differ in almost every analytic sentence. This follows from the basic logical form that every sentence entailed by the Carnap sentence must have:

Claim 6. $C_\theta(\vartheta) \models \alpha$ if and only if $\vartheta \models \alpha$ and $\alpha \models R_\theta(\vartheta) \rightarrow \alpha$.¹⁰

Proof. ‘ \Rightarrow ’: Since $\vartheta \models C_\theta(\vartheta) \models \alpha$, the first conjunct holds. Trivially, $\alpha \models R_\theta(\vartheta) \rightarrow \alpha$. Now assume $C_\theta(\vartheta) \models \alpha$. Then $\neg R_\theta(\vartheta) \vee \vartheta \models \alpha$ and thus $\neg R_\theta(\vartheta) \models \alpha$, so that $R_\theta(\vartheta) \rightarrow \alpha \models \neg R_\theta(\vartheta) \vee \alpha \models \alpha$.

‘ \Leftarrow ’: Since $\vartheta \models \alpha$ and $R_\theta(\vartheta) \rightarrow \alpha \models \alpha$, $C_\theta(\vartheta) \models R_\theta(\vartheta) \rightarrow \vartheta \models R_\theta(\vartheta) \rightarrow \alpha \models \alpha$. \square

Thus every analytic sentence of a theory is a conditional with that theory’s Ramsey sentence as its antecedent. (Note that the Carnap sentence does not even allow sufficient conditions $\forall x[\varphi(x) \rightarrow Tx]$ for theoretical terms T and proper observational formulas φ , let alone explicit definitions.¹¹) Hence, adding empirical content to a theory weakens every of its analytic sentences, with the exception of those sentences that already happen to have the added empirical content as an antecedent.

Two theories ϑ and τ with different empirical content thus share only those analytic sentences α that are conditional on *both* $R_\theta(\vartheta)$ and $R_\theta(\tau)$. Specifically, the only analytic sentences shared by two theories with incompatible empirical content are logical truths:

Corollary 7. Let $R_\theta(\vartheta) \models \neg R_\theta(\tau)$, $C_\theta(\vartheta) \models \alpha$ and $C_\theta(\tau) \models \alpha$. Then $\models \alpha$.

Proof. Assume $C_\theta(\vartheta) \models \alpha$. By claim 6, $\alpha \models R_\theta(\vartheta) \rightarrow \alpha$, and hence $\neg \alpha \models R_\theta(\vartheta) \wedge \neg \alpha \models R_\theta(\vartheta)$. Analogously, $\neg \alpha \models R_\theta(\tau)$ so that $\neg \alpha \models R_\theta(\tau) \wedge R_\theta(\tau) \models \perp$. Thus $\models \neg \perp \models \alpha$. \square

Thus, if ϑ is the conjunction of Newtonian mechanics and the claim that there is a planet closer to the sun than mercury, while τ is the conjunction of Newtonian mechanics and the claim that there is no such planet, then the two theories have different concepts of mass, assuming ‘mass’ is a theoretical term. In practice, corollary 7 states that even if the scientist wants to use the same concept in competing theories, the Carnap sentence makes this impossible.

10. The result is consistent because $R_\theta(\vartheta) \rightarrow [R_\theta(\vartheta) \rightarrow \alpha] \models R_\theta(\vartheta) \rightarrow \alpha$.

11. Caulton (2012) was the first to show that the Carnap sentence does not allow explicit definitions and argue that this is a severe problem.

Another problem for the Carnap sentence stems from the use of mathematics in scientific theories. As already noted, higher order logic itself does not contain mathematical symbols, and so any axiom of mathematics must be explicitly included in a theory (be it in the form of a definition of a mathematical symbol from logical symbols or an axiom about the relation of not further defined mathematical symbols). Furthermore, mathematical symbols must be treated as theoretical, and so mathematical axioms must be theoretical sentences. The problem is that then some mathematical axioms are not analytic according to the Carnap sentence. The reason is a theorem by Winnie (1970, theorem 4), who shows that in a first order language without identity, only tautological theoretical sentences follow from the Carnap sentence of a consistent theory with observational content (cf. Williams 1973, theorem 5). In higher order logic, the result is a corollary of claim 6 if the theoretical sentences do not restrict the cardinality of their domain (see appendix A):

Corollary 8. *Let α be a theoretical sentence with models of any cardinality, and let ϑ have observational content. Then $C_{\vartheta}(\vartheta) \models \alpha$ only if $\models \alpha$.*

Hence if only the Carnap sentence were an adequate analytic component, mathematical axioms and theorems would be either cardinality claims or logical truths. The axioms of group theory, for instance, do not restrict the cardinality of their domain and would thus be logical truths, which they are not. In practice, corollary 8 means that even if the scientist decides that, say, $\forall x(T_1x \leftrightarrow T_2x)$ should hold analytically in her theory ϑ (that is, T_1 and T_2 should be synonyms), the Carnap sentence does not allow it. By claim 6, the strongest analytic claim she can make is $R_{\vartheta}(\vartheta) \rightarrow \forall x(T_1x \leftrightarrow T_2x)$.

If the Carnap sentence is the wrong analytic component for some theories, then observational vacuity, the condition of adequacy that (up to equivalence) uniquely picks out the Carnap sentence as the only adequate analytic component of a theory, must itself be inadequate. Therefore its motivation must be flawed; and it is. Winnie and Demopoulos justify observational vacuity as a condition of adequacy by pointing out that observationally vacuous sentences cannot contribute to the inference of an observational sentence. But this shows at best that the condition is not too inclusive. Demopoulos does not argue that all observationally non-vacuous sentences do so contribute, and Winnie's argument to the effect (Winnie 1970, 296) fails. He argues that observational vacuity is to observational consequences what logical vacuity is to consequences in any vocabulary, and he points out that logical truths can be removed from any set of premises without invalidating the inference of a sentence. Thus one would expect that observationally vacuous sentences can be removed from any set of premises without invalidating the inference of an observational sentence. But this is not what definition 4 of observational vacuity states. It states that observational vacuous sentences *are entailed by* ϑ and can be removed from any set of premises *that are entailed by* ϑ without invalidating the inference of an observational sentence. Thus observational vacuity is far from being directly analogous to logical truth.

A more analogous concept would be for instance that of observational conservativeness with respect to ϑ (definition 6, appendix A), which is fulfilled by a sentence if it can be removed from any set of premises *that contains ϑ* without invalidating the inference of an observational sentence. And since Carnap’s conditions of adequacy already demand that the conjunction of the analytic and the synthetic components be equivalent to ϑ , the analytic component is automatically observationally conservative with respect to ϑ .

Thus observational vacuity has to be given up as a condition of adequacy for the analytic component of theories. It might therefore seem that the Carnap sentence has to be given up as well, and a new condition of adequacy must be found that leads to a new method for determining the analytic component of theories. But this is not the case. What has to be given up is the assumption that there must be an automatic method of determining, for every ϑ , the analytic component of ϑ . Specifically, Carnap’s conditions of adequacy suffice, and the freedom they leave in the choice of the analytic component of a theory is justified. Or so I argue next.

5 Against a Unique Adequate Analytic Component

Winnie and Demopoulos claim that without the demand for observational vacuity of the analytic component of any theory ϑ , the analytic-synthetic dichotomy is arbitrary. But this is a tendentious formulation, for one because at most the analytic component of ϑ can be arbitrary, as its synthetic component is uniquely determined up to equivalence by Carnap’s conditions of adequacy:

Claim 9. *A sentence σ is an adequate synthetic component of ϑ if and only if $\sigma \vDash R_\vartheta(\vartheta)$.*

Proof. Assume that σ is an adequate synthetic component of ϑ . Then it is an observational sentence, and hence $\sigma \vDash R_\vartheta(\sigma)$. By claim 1, it further holds that $R_\vartheta(\sigma) \vDash R_\vartheta(\vartheta)$. Hence $\sigma \vDash R_\vartheta(\vartheta)$. Given claim 1, the converse is immediate. \square

Winnie and Demopoulos’s claim of arbitrariness is also too strong because the analytic component is far from being completely unrestricted.

Corollary 10. *α is an adequate analytic component of ϑ if and only if $\vDash R_\vartheta(\alpha)$ and $\vartheta \vDash \alpha \vDash C_\vartheta(\vartheta)$.*

Proof. By claims 1 and 9, α is adequate if and only if $\vDash R_\vartheta(\alpha)$ and $R_\vartheta(\vartheta) \wedge \alpha \vDash \vartheta$. If the latter condition holds, then $\vartheta \vDash \alpha$ and $R_\vartheta(\vartheta) \wedge \alpha \vDash \vartheta$, that is, $\alpha \vDash R_\vartheta(\vartheta) \rightarrow \vartheta$. Conversely, if $\vartheta \vDash \alpha$, then $\vartheta \vDash R_\vartheta(\vartheta) \wedge \alpha$ and if $\alpha \vDash C_\vartheta(\vartheta)$, then $R_\vartheta(\vartheta) \wedge \alpha \vDash R_\vartheta(\vartheta) \wedge C_\vartheta(\vartheta) \vDash \vartheta$. \square

Corollary 10 specifically entails that $C_\vartheta(\vartheta)$ is the weakest possible analytic

component of ϑ ,¹² and that an analytic component α can be logically at most as strong as ϑ and must be weak enough so that $\models R_{\vartheta}(\alpha)$.

To see how benign the non-uniqueness of the analytic component of ϑ is, consider what goes into a philosopher’s reconstruction of a scientist’s theory T according to Carnap’s formalism. For one, the philosopher has to infer the scientist’s intentions from her pronouncements about T and her use of T , and he has to develop an axiomatization ϑ in higher order logic. If this is not possible, neither is the reconstruction in Carnap’s formalism. When ϑ has been developed or while developing ϑ , the philosopher also has to find out which part of the vocabulary the scientist intends to be observational. ϑ alone contains no information about \mathcal{O} at all. That the analytic component of ϑ is not unique given only ϑ and \mathcal{O} just means that to a certain extent the philosopher also has to rely on the scientist’s intentions when determining the analytic component of ϑ . In other words, the philosopher has to determine the analytic component in the same way he has to determine the axiomatization and the observational terms: as part of the process of the rational reconstruction of T . The conditions of adequacy only provide a check of the rational reconstruction as a whole: If one has determined ϑ , \mathcal{O} , and the analytic component of ϑ , the conditions of adequacy have to be met. Thus it could be the case that, for instance, a sentence clearly intended to be analytic by the scientist forces the philosopher to treat some term as theoretical rather than observational, or to axiomatize the theory in one way rather than another.

In his defense of Winnie’s additional condition of adequacy, Demopoulos (2008, 255–56, emphases removed) draws a bright line between the delineation of \mathcal{O} , which he calls the “first phase of Carnap’s reconstruction”, and the split of ϑ into $R_{\vartheta}(\vartheta)$ and $C_{\vartheta}(\vartheta)$, which he calls the “second phase”. For Demopoulos, the rational reconstruction of the theory T is complete with the axiomatization ϑ and the delineation of \mathcal{O} . The distinction between the analytic and the synthetic component has to *follow* from the result of the rational reconstruction: “An initially plausible response (Maxwell 1963) holds that the arbitrariness is harmless if it attaches only to the unreconstructed sentences of a science. But [. . .] the objection applies even to the second phase of Carnap’s proposed reconstruction, and this appears to be a complete vindication of Quine” (Demopoulos 2008, v). But Demopoulos’ bright line is itself arbitrary. The delineation of \mathcal{O} is as dependent on the intentions of the scientist as the delineation of the analytic component of ϑ ; both have to be rationally reconstructed. Indeed, this is what Maxwell (1963, 403–4) holds, for his description of the process of rational reconstruction is very much like the one above:

For any reformation [i. e., rational reconstruction], it will be necessary to presuppose—perhaps to stipulate—that [certain sentences] are synthetic. [. . .] There will be a rule to the effect that the selection of sentences which are to be taken as A-true [analytically true]

12. This is shown for first order logic and sets of sentences by Williams (1973, 404).

must be such that no conjunction of A-true sentences L-implies a synthetic sentence. From this point on, as far as I can see, one must proceed to a large extent by trial and error. As Carnap has pointed out there is (certainly!) no decision procedure for A-truth. One will try to select a set of sentences from $[T]$ which, if taken as A-true, will “fix” to a satisfactory extent the meanings of the relevant expressions.

Thus for Maxwell and (according to Maxwell) also for Carnap the rational reconstruction of T does not end with the axiomatization ϑ and the delineation of θ , but continues with the delineation of the analytic component of ϑ . The demand that a conjunction of analytically true sentences must not logically entail a synthetic sentence is a condition of adequacy on the relations between ϑ , its synthetic sentences, and its analytic sentences. Carnap’s conditions of adequacy in fact entail this condition: Since all analytic sentences are entailed by ϑ ’s analytic component, their conjunction is at most as strong as the analytic component itself and therefore, by Carnap’s conditions of adequacy, cannot entail a synthetic sentence.

That Carnap’s conditions do not determine a unique analytic component of a theory solely based on the delineation of the set of observational terms does not mean that the analytic components of some theories have to be non-unique, of course. For each theory, one can still choose exactly one analytic component. It is only that one could also choose a different analytic component that would also be adequate.

6 Przełęcki Reduction Pairs and Relativization Sentences

If the Carnap sentence is not a theory’s only possible analytic component, what other options are there? Bedard’s suggestion does not lead to a general solution: While every theory has its Carnap sentence as unique weakest adequate analytic component, not every theory has a unique strongest adequate analytic component. For assume a theory

$$\vartheta \models \forall x[\varphi(x) \rightarrow Tx] \wedge \forall x[\psi(x) \rightarrow \neg Tx] \quad (3)$$

consisting of two reduction sentences (Carnap 1936, 442), where φ and ψ are observational formulas and T is a theoretical term. Then

$$\forall x[\varphi(x) \rightarrow Tx] \wedge \forall x[\psi(x) \wedge \neg\varphi(x) \rightarrow Tx] \quad (4a)$$

and

$$\forall x[\varphi(x) \wedge \neg\psi(x) \rightarrow Tx] \wedge \forall x[\psi(x) \rightarrow Tx] \quad (4b)$$

are both adequate analytic components, and neither entails the other.

ϑ 's Carnap sentence

$$C_{\vartheta}(\vartheta) \equiv \forall x[\varphi(x) \rightarrow \neg\psi(x)] \rightarrow \forall x[\varphi(x) \rightarrow Tx] \wedge \forall x[\psi(x) \rightarrow \neg Tx] \quad (5)$$

(cf. Carnap 1963, 964–66) is the weakest adequate analytic component of ϑ and thus clearly does not meet Bedard's suggestion. Przełęczki (1969, §7.III) suggests the logically stronger sentence

$$\forall x[\varphi(x) \wedge \neg\psi(x) \rightarrow Tx] \wedge \forall x[\psi(x) \wedge \neg\varphi(x) \rightarrow \neg Tx]. \quad (6)$$

I will call the two conjuncts (6) the 'Przełęczki reduction pair for ϑ '. One advantage of the Przełęczki reduction pair is that it retains the symmetry of the original reduction sentences. More importantly, it is entailed by both of the stronger adequate analytic components (4). Thus, having chosen Bedard's over Carnap's suggestion, one cannot decide between the two stronger analytic components without deferring to the intentions of the scientist, but one *can* rely on ϑ 's Przełęczki reduction pair rather than ϑ 's Carnap sentence.

In a particularly intuitive way of arriving at the Przełęczki reduction pair, one can consider it the relativization of the concepts of ϑ to the domain in which $R_{\vartheta}(\vartheta)$ is true. Even if $R_{\vartheta}(\vartheta) = \forall x\neg[\varphi(x) \wedge \psi(x)]$ is false, there may be some objects a in the domain for which $\neg[\varphi(a) \wedge \psi(a)]$ is true. The relativization to these objects, that is, the relativization $\vartheta^{(\xi)}$ of ϑ to $\xi := \lambda x\neg[\varphi(x) \wedge \psi(x)]$ (cf. Hodges 1993, 203)¹³ results in

$$\begin{aligned} \vartheta^{(\xi)} \equiv \forall x([\neg\varphi(x) \vee \neg\psi(x)] \rightarrow [\varphi(x) \rightarrow Tx]) \\ \wedge \forall x([\neg\varphi(x) \vee \neg\psi(x)] \rightarrow [\psi(x) \rightarrow \neg Tx]), \quad (7) \end{aligned}$$

which is equivalent to the Przełęczki reduction pair. This consideration makes it especially transparent that in contradistinction to $C_{\vartheta}(\vartheta)$, ϑ 's Przełęczki reduction pair allows for the relation T to have meaning even if $R_{\vartheta}(\vartheta)$ turns out false.

The reasoning that led to the Przełęczki reduction pair suggests the following generalization:

Definition 5. A *relativization sentence* for ϑ is any sentence $\exists x\xi(x) \rightarrow \vartheta^{(\xi)}$ such that $\forall x\xi(x) \equiv R_{\vartheta}(\vartheta)$.

The antecedent $\exists x\xi(x)$ of the relativization sentence ensures observational conservativeness (see the proof of claim 15). Relativization sentences are in a way analogous to Carnap sentences. For those structures in which a theory's Ramsey sentence is true, the Carnap sentence stipulates that the whole theory is true, and for those structures in which the Ramsey sentence is false, the Carnap sentence stipulates nothing. Analogously, for sets of objects to which a theory's Ramsey sentence applies, relativization sentences stipulate that the theory

13. A relativization of a sentence to some one-place formula ξ restricts all quantifiers occurring in the sentence to ξ .

applies to them as well, and for sets of objects to which the Ramsey sentence does not apply, they stipulate nothing. Unlike the Carnap sentence, however, relativization sentences are not uniquely determined by a theory and its observational terms, since the only requirement for the relativizing formula ξ is that its universal closure must be equivalent to the theory's Ramsey sentence. And this can be achieved in different ways, for even Carnap sentences are relativization sentences. This can be shown by choosing $R_{\mathcal{O}}(\vartheta)$ as the relativizing formula; since it contains no free variable, $\forall x R_{\mathcal{O}}(\vartheta) \equiv R_{\mathcal{O}}(\vartheta)$ and, as is easily shown, $\exists x R_{\mathcal{O}}(\vartheta) \rightarrow \vartheta^{(R_{\mathcal{O}}(\vartheta))} \equiv R_{\mathcal{O}}(\vartheta) \rightarrow \vartheta$.

Relativization sentences are adequate analytic components of relational first order theories (appendix A, claim 15), so that they can often be used instead of Carnap sentences. Of course, relativization sentences are not mandatory analytic components either; they rather provide choices besides $C_{\mathcal{O}}(\vartheta)$ and $C_{\mathcal{O}}(\vartheta) \wedge R_{\mathcal{O}}(\vartheta)$.

7 Conclusion

Starting from an analysis of Carnap's conditions of adequacy for the analytic and the synthetic component of a theory, it has become clear that there is and should be no automatic method for determining a theory's analytic component even when the theory is axiomatized and the observational vocabulary is delineated. The delineation of the analytic sentences of a theory is as much part of its rational reconstruction as the delineation of the theory's observational vocabulary. Therefore it is no problem that, unlike a theory's synthetic component, a theory's analytic component is not uniquely determined by Carnap's conditions of adequacy. Specifically, a rational reconstruction does not have to identify a theory's Carnap sentence as its analytic component. Reduction sentences provide one particularly vivid illustration of the possibility of choosing a theory's analytic component differently, since Przełęcki reduction pairs and, more generally, relativization sentences turn out to be well-motivated, applicable, and hence anything but inadequate.

A Additional Definition and Proofs

Definition 6. A sentence ϱ is *first order observationally conservative* with respect to a sentence δ if and only if for any observational sentence ω it holds that $\varrho \wedge \delta \models \omega$ only if $\delta \models \omega$.

Claim 5. Let ϑ be such that $R_{\mathcal{T}}(\vartheta)$ is not logically true and has models of any cardinality, and let τ be any sentence. Then $C_{\mathcal{O}}(\vartheta \wedge \tau) \models C_{\mathcal{O}}(\vartheta)$ if and only if the observational content of $\vartheta \wedge \tau$ is equivalent to that of ϑ .

Proof. ‘ \Leftarrow ’: By corollary 14 (appendix A), the observational content of $\tau \wedge \vartheta$ is equivalent to that of ϑ if and only if $R_{\mathcal{O}}(\vartheta) \models R_{\mathcal{O}}(\vartheta \wedge \tau)$. It is to be shown that if $C_{\mathcal{O}}(\vartheta \wedge \tau)$ holds, $R_{\mathcal{O}}(\vartheta)$ entails ϑ . Since $R_{\mathcal{O}}(\vartheta \wedge \tau) \rightarrow \vartheta \wedge \tau$ holds and $R_{\mathcal{O}}(\vartheta)$ entails by assumption $R_{\mathcal{O}}(\vartheta \wedge \tau)$, it also entails $\vartheta \wedge \tau$ and hence ϑ .

‘ \Rightarrow ’: If $C_{\mathcal{O}}(\vartheta \wedge \tau) \models C_{\mathcal{O}}(\vartheta)$, then $\models C_{\mathcal{O}}(\vartheta \wedge \tau) \rightarrow C_{\mathcal{O}}(\vartheta)$ and, by propositional logic, $\models R_{\mathcal{O}}(\vartheta) \rightarrow R_{\mathcal{O}}(\vartheta \wedge \tau) \vee \vartheta$. Hence $R_{\mathcal{O}}(\vartheta) \models R_{\mathcal{O}}(\vartheta \wedge \tau) \vee \vartheta$ and thus $R_{\mathcal{O}}(\vartheta) \models R_{\mathcal{O}}(\vartheta \wedge \tau) \vee R_{\mathcal{T}}(\vartheta)$. Since $R_{\mathcal{T}}(\vartheta)$ is not a tautology and does not restrict the cardinality of its domain, any model for \mathcal{O} of $R_{\mathcal{O}}(\vartheta)$ can be expanded such that $R_{\mathcal{T}}(\vartheta)$ is false. Hence $R_{\mathcal{O}}(\vartheta \wedge \tau)$ must be true in every model for \mathcal{O} of $R_{\mathcal{O}}(\vartheta)$, so that $R_{\mathcal{O}}(\vartheta) \models R_{\mathcal{O}}(\vartheta \wedge \tau)$ and thus $R_{\mathcal{O}}(\vartheta) \models R_{\mathcal{O}}(\vartheta \wedge \tau)$ \square

Corollary 8. Let α be a theoretical sentence with models of any cardinality, and let ϑ have observational content. Then $C_{\mathcal{O}}(\vartheta) \models \alpha$ only if $\models \alpha$.

Proof. By claim 6, $C_{\mathcal{O}}(\vartheta) \models \alpha$ only if $R_{\mathcal{O}}(\vartheta) \rightarrow \alpha \models \alpha$ and thus only if $\neg R_{\mathcal{O}}(\vartheta) \vee \alpha \models \alpha$. Thus $\neg R_{\mathcal{O}}(\vartheta) \models \alpha$. Since $\neg R_{\mathcal{O}}(\vartheta)$ and α do not share any vocabulary and higher order logic has the Craig-interpolation property (Shapiro 1991, §6.6.2), $\neg R_{\mathcal{O}}(\vartheta) \models \alpha$ only if there is an interpolation sentence without non-logical symbols that is entailed by $\neg R_{\mathcal{O}}(\vartheta)$ and that entails α . Since ϑ has observational content, $\neg R_{\mathcal{O}}(\vartheta)$ is not logically false, and thus the interpolation sentence cannot be logically false, so that it can at best restrict the cardinality of its domain. But since α does not restrict the cardinality of its domain, $\models \alpha$. \square

Claim 11. According to the Carnap sentence, analyticity is not compositional.

Proof. It is to be shown that for some Carnap sentence, there are a sentence schema Φ and two ways of completing the schema with sentences of the same truth values and the same statuses regarding analyticity such that only in one case the completion is analytically true. Let ϑ be $O \wedge T_1$, where O is an observation sentence and T_1 a theoretical sentence whose Ramsey sentence is logically true. Then $C_{\mathcal{O}}(\vartheta) \models O \rightarrow (O \wedge T_1) \models O \rightarrow T_1$. Let Φ be $\lceil X \rightarrow Y \rceil$ and the sentences T_1 and T_2 have the same extensional truth value. None of the sentences O , T_1 , and T_2 is entailed by $C_{\mathcal{O}}(\vartheta)$ and hence none is analytically true (nor is any analytically false). Thus O , T_1 , and T_2 agree on their extensional truth values and their analytic truth values. But while $O \rightarrow T_1$ is analytically true, $O \rightarrow T_2$ is not. \square

Lemma 12. *Structure \mathfrak{A} for \mathcal{O} can be expanded to a model of sentence ϑ if and only if $\mathfrak{A} \models R_{\mathcal{O}}(\vartheta)$.*

Proof. A sentence ϑ is Ramseyfied by substituting every theoretical term T_i , $1 \leq i \leq n$ in ϑ by a variable X_i and existentially quantifying over each X_i , leading to $\exists X_1 \dots X_n \vartheta[T_1/X_1, \dots, T_n/X_n]$. Define $g : \{T_i\}_{1 \leq i \leq n} \rightarrow \{X_i\}_{1 \leq i \leq n}$, $T_i \mapsto X_i$.

‘ \Leftarrow ’: Assume that \mathfrak{A} is a structure for \mathcal{O} and $\mathfrak{A} \models R_{\mathcal{O}}(\vartheta)$. Then there is a satisfaction function ν mapping each variable X_i , $1 \leq i \leq n$ to an extension of the same type over $\text{dom}(\mathfrak{A})$ such that $\mathfrak{A}, \nu \models \vartheta[T_1/X_1, \dots, T_n/X_n]$. Induction on the complexity of formulas then shows that any extension of $\nu|_{\{X_1, \dots, X_n\}} \circ g$ to all theoretical terms can be used to expand \mathfrak{A} to a model of ϑ .

‘ \Rightarrow ’: Similar. □

Claim 13. *For any sentence ϑ , $R_{\mathcal{O}}(\vartheta)$ entails the same observational sentences as ϑ .*

Proof. Since $\vartheta \models R_{\mathcal{O}}(\vartheta)$, ϑ entails all observational sentences entailed by $R_{\mathcal{O}}(\vartheta)$. Conversely, assume that $\vartheta \models \omega$, where ω is an observational sentence. If \mathfrak{A} is a structure for \mathcal{O} and $\mathfrak{A} \models R_{\mathcal{O}}(\vartheta)$, then there is by lemma 12 an expansion \mathfrak{B} of \mathfrak{A} such that $\mathfrak{B} \models \vartheta$. By assumption, $\mathfrak{B} \models \omega$. Since ω is an observation sentence, the reduct of \mathfrak{B} to \mathcal{O} , \mathfrak{A} , is a model of ω . Thus $R_{\mathcal{O}}(\vartheta) \models \omega$. □

Corollary 14. *The observational content of ϑ is equivalent to $R_{\mathcal{O}}(\vartheta)$.*

Proof. ϑ ’s O-content is the set of observational sentences that it entails. Since $R_{\mathcal{O}}(\vartheta)$ is an observational sentence that entails all observational sentences entailed by ϑ (claim 13), $R_{\mathcal{O}}(\vartheta)$ is equivalent to the observational content of ϑ . □

Claim 15. *A relativization sentence for a relational first order sentence ϑ is an adequate analytic component of ϑ .*

Proof. By claims 1 and 9, it suffices to show that $R_{\mathcal{O}}(\vartheta) \wedge [\exists x \xi(x) \rightarrow \vartheta^{(\xi)}] \models \vartheta$ and $\models R_{\mathcal{O}}[\exists x \xi(x) \rightarrow \vartheta^{(\xi)}]$. The former is straightforward: $R_{\mathcal{O}}(\vartheta) \wedge [\exists x \xi(x) \rightarrow \vartheta^{(\xi)}] \models \forall x \xi(x) \wedge [\exists x \xi(x) \rightarrow \vartheta^{(\xi)}] \models \forall x \xi(x) \wedge \vartheta^{(\xi)} \models \vartheta$. For the latter, it has to be shown that every structure \mathfrak{A} for \mathcal{O} can be expanded to a model of $\exists x \xi(x) \rightarrow \vartheta^{(\xi)}$ (lemma 12).

If $\mathfrak{A} \not\models \exists x \xi(x)$, then any expansion \mathfrak{B} of \mathfrak{A} is such that $\mathfrak{B} \not\models \exists x \xi(x)$, and hence $\mathfrak{B} \models \exists x \xi(x) \rightarrow \vartheta^{(\xi)}$. Thus assume that $\mathfrak{A} \models \exists x \xi(x)$ and let \mathfrak{C} be the restriction $\mathfrak{A}|_{\xi}$ of \mathfrak{A} to ξ .¹⁴ Since ϑ is relational, \mathfrak{A} can be assumed to be relational without loss of generality, and hence such a \mathfrak{C} exists. Trivially, $\mathfrak{A} \models \forall x [\xi(x) \rightarrow \xi(x)] \models [\forall x \xi(x)]^{(\xi)}$, so that $\mathfrak{C} \models \forall x \xi(x) \models R_{\mathcal{O}}(\vartheta)$ by the relativization theorem (Hodges 1993, 203). Hence there is an expansion \mathfrak{B}^* of \mathfrak{C} such that $\mathfrak{B}^* \models \vartheta$ (lemma 12). Since $\mathfrak{C} \subseteq \mathfrak{A}$, there is an expansion \mathfrak{B} of \mathfrak{A} such that $\mathfrak{B}^* \subseteq \mathfrak{B}$. Since $\mathfrak{B}^* \models \vartheta$ and $\mathfrak{B}^* = \mathfrak{B}|_{\xi}$, $\mathfrak{B} \models \vartheta^{(\xi)}$, again by the relativization theorem. □

¹⁴ $\mathfrak{C} = \mathfrak{A}|_{\xi}$ if and only if $\mathfrak{C} \subseteq \mathfrak{A}$ and $\text{dom}(\mathfrak{C}) = \{a : a \text{ satisfies } \xi \text{ in } \mathfrak{A}\}$ (cf. Bell and Slomson 1974, 73).

B Carnap on the Non-uniqueness of the Analytic Component

Winnie (1970, 293) states that Carnap was aware that the Carnap sentence is not the only possible analytic component of a theory given his conditions of adequacy (definition 3). His evidence is the following remark by Carnap (1963, 965) on the Carnap sentence: “It may be that we wish to establish still further sentences as A_T -postulates [i. e., postulates for theoretical terms] in addition to those formed from a theory TC in the way described. But we shall admit as A_T -postulates only sentences whose conjunction satisfies the condition [(c) of definition 3].” But Carnap here does not explicitly claim that the further sentences can be seen as A_T -postulates stemming from TC . The A_T -postulates could also be additional sentences that do not follow from TC .

Demopoulos (2007, 258, n. 11) claims that Carnap was not aware that there are other adequate analytic components of a theory besides its Carnap sentence. He states that this is “evident from his [Carnap’s] remarks” on the relation of truth and analytic truth (Carnap 1963, 915), but there Carnap only states that one can define analytic truth independently of truth, and thereafter prove that every analytic truth is also true. This seems to be independent of the question whether only the Carnap sentence fulfills Carnap’s conditions of adequacy.

According to Maxwell (1963, 404), “Carnap pointed out [that] there is (certainly!) no decision procedure for A-truth” (see the quote at the end of §5), which speaks in favor of Winnie’s position. But the question whether Carnap thought that the conditions in definition 3 determined a unique analytic component for every theory is moot, because he did not consider the conditions necessary. For Carnap (1963, 963–65) allowed analytic observational sentences that are not logically true (called A_O), and thus there can be sentences whose Ramsey sentence is not logically true, but that are nonetheless analytic (for instance any ω with $A_O \models \omega$ and $\not\models \omega$). Carnap (1963, 965) rather takes the conditions to be *sufficient* for showing that the Ramsey and Carnap sentences are adequate. After pointing out that the two sentences fulfill the conditions, he continues: “These results show, in my opinion, that this method supplies an adequate explication for the distinction between those postulates which represent factual relations between completely given meaning, and those which merely represent meaning relations.” And this does not mean there cannot be other results that would also show that the Ramsey and the Carnap sentence are adequate.

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