# Classical Black Holes Are Hot[†]

Erik Curiel[‡]

## Contents



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[‡]**Author's address**: Munich Center for Mathematical Philosophy, Ludwig-Maximilians-Universität, Ludwigstraße 31, 80539 München, Deutschland; **email**: `erik@strangebeautiful.com`

## ABSTRACT

In the early 1970s it is was realized that there is a striking formal analogy between the Laws of black-hole mechanics and the Laws of classical thermodynamics. Before the discovery of Hawking radiation, however, it was generally thought that the analogy was only formal, and did not reflect a deep connection between gravitational and thermodynamical phenomena. It is still commonly held that the surface gravity of a stationary black hole can be construed as a true physical temperature and its area as a true entropy only when quantum effects are taken into account; in the context of classical general relativity alone, one cannot cogently construe them so. Does the use of quantum field theory in curved spacetime offer the only hope for taking the analogy seriously? I think the answer is 'no'. To attempt to justify that answer, I shall begin by arguing that the standard argument to the contrary is not physically well founded, and in any event begs the question. Looking at the various ways that the ideas of "temperature" and "entropy" enter classical thermodynamics then will suggest arguments that, I claim, show the analogy between classical black-hole mechanics and classical thermodynamics should be taken more seriously, without the need to rely on or invoke quantum mechanics. In particular, I construct an analogue of a Carnot cycle in which a black hole "couples" with an ordinary thermodynamical system in such a way that its surface gravity plays the role of temperature and its area that of entropy. Thus, the connection between classical general relativity and classical thermodynamics on their own is already deep and physically significant, independent of quantum mechanics.


# 1 Introduction

I aim in this paper to clarify the status of the analogy between black-hole mechanics restricted to general relativity on the one hand (*i.e.*, with no input from quantum field theory on curved spacetime or from any other type of semi-classical calculation) and classical thermodynamics on the other ("classical" in the sense that no quantum and no statistical considerations come into play). Based on the striking formal similarities of the respective mathematical formulæ of the Zeroth, First, Second and Third Laws of classical thermodynamics and of the mechanics of black holes in stationary, axisymmetric, asymptotically flat spacetimes, as I discuss in §2, the best particular analogies seem to be: (1) that between the surface gravity of a black hole as measured on its event horizon and the temperature of a classical system; and (2) that between surface area of the horizon and entropy.[1] When it is also noted that black holes, like ordinary thermodynamical systems, are characterized by a small number of gross parameters independent of any details about underlying microstructure, and that each version of the First Law states a conservation principle for essentially the same quantity as the other, *viz.*, mass-energy, it becomes tempting to surmise that some deep

[1] Both the surface gravity and the surface area in question are defined with respect to the orbits of the Killing fields in virtue of which the spacetime is qualified as 'stationary' and 'axisymmetric'. See Wald (1984, ch. 12) for details.

or fundamental connection between black holes and thermodynamics is being uncovered. But is it of real physical significance in some sense?

The conventional answer to this question is 'no'. Because classical black holes seem to be perfect absorbers, they would seem to have a temperature of absolute zero, even when they have non-zero surface gravity. It is only with the introduction of quantum considerations, the standard account runs, in particular the derivation of Hawking radiation, that one finds grounds for taking the analogy seriously. And yet the startling and suggestive fact remains that one can derive laws for black holes formally identical to those of classical thermodynamical systems from the fundamental principles of general relativity itself with no aid from quantum field theory in curved spacetime. Does the use of quantum field theory in curved spacetime offer the only hope for taking the analogy seriously? I think the answer is 'no'. To attempt to justify that answer, I shall begin by arguing in §3 that the standard argument to the contrary is not physically well founded, and in any event begs the question. I will, therefore, in §4, look at the various ways that the ideas of "temperature" and "entropy" enter classical thermodynamics, which will suggest arguments that show the analogy between classical black-hole mechanics and classical thermodynamics should be taken seriously indeed, without the need to rely on or invoke quantum mechanics. If this is correct, then there may already be a deep connection between general relativity and classical thermodynamics on their own, independent of quantum mechanics.

My arguments in this paper, however, are not only negative. I do think that the connection between gravitational and thermodynamical phenomena intimated by the formal equivalence of their respective Laws is of real physical significance. My strongest argument in favor of this claim is the construction, in §5, of the analogue of a Carnot cycle with the heat sink provided by a stationary black hole. In the process, the black hole's surface gravity and area play, respectively, the physical roles of temperature and entropy of an ordinary heat sink in an ordinary Carnot cycle. The process also grounds the construction of an absolute temperature scale that applies both to black holes and to ordinary classical thermodynamical systems. Finally, there follows from the construction the existence of a universal constant with the physical dimension needed to give surface gravity the physical dimension of temperature and area the physical dimension of entropy. To put the icing on the cake, I also formulate the analogues of the Clausius and Kelvin Postulates—the bases for the introduction of temperature and entropy in classical thermodynamics—in the context of classical black hole thermodynamics, and give arguments for them at least as strong as the arguments for their analogues in classical thermodynamics, based on the laws and properties of black holes.

If surface gravity and area couple to ordinary thermodynamical systems in the same way as temperature and entropy, respectively, do, and if they are introduced into the theory using the same constructions and arguments, then there can be no grounds for denying that they physically *are* a real temperature and entropy. To put it more provocatively, if my claim is correct, then gravity on its own, independent of its relation to the other three known fundamental forces so successfully treated by quantum field theory, already is a fundamentally thermodynamical phenomenon.[2] I want to

[2] If one could show that the sorts of arguments I give here could be translated into the framework of Newtonian gravitational theory, that would provide even stronger support for this last claim.

stress, nonetheless, that I do not consider quantum effects to be irrelevant when considering possible relations between gravitational physics and thermodynamics. I want only to argue for the idea that the analogy between the laws of classical thermodynamics and those of black hole mechanics in classical general relativity is robust and deep in its own right.

I conclude the paper, in §6, with a discussion of possible problems with my arguments and constructions, some remarks on possible lessons my conclusions, if correct, may yield, and some open questions.

Before diving in, I should perhaps say, by way of background, that I am curious about this question in the first place in part because of my curiousity about the larger question of the relation between thermodynamical characteristics of a physical system and the possibility of always being able to or indeed always being required to find an underlying statistical interpretation of those thermodynamical characteristics. That the laws of black hole mechanics follow from the fundamental theory itself (in this case, general relativity), and are not as with classical thermodynamics an independent adjunct connected to the underlying fundamental (Newtonian) theory through the use of statistical devices, could suggest that thermodynamics is itself more of the nature of a fundamental theory than has been thought since the advent of statistical mechanics—or at least that thermodynamical characteristics and quantities of physical systems may be fundamental to them in some way analogous to that of other fundamental characteristics and dynamical quantities, such as the possession of a stress-energy tensor, for example, and its satisfaction of some form of covariant conservation principle. In a similar vein, these sorts of results may also perhaps lend support to the idea that general relativity is an effective field theory, and the Einstein field equation only an equation of state, *à la* Jacobson (1995), and perhaps Bredberg, Keeler, Lysov, and Strominger (2011) and Lysov and Strominger (2011). If that is true, then the entire program of "quantizing gravity" may be misguided from the start. Yet another possibility, contrary to that just mentioned, is that one may take my arguments as showing that the signature of quantum gravity, in particular the traces of whatever statistical quantities it may give us for making traditional sense of the thermodynamical phenomena I discuss here, show up already in purely classical, non-statistical theory.[3] Finally, and I think most importantly, my arguments lend *prima facie* support to projects (especially in cosmology) that want to attribute entropy generically to "gravitational degrees of freedom", as in the work of Clifton, Ellis, and Tavakol (2013), and as required by Penrose's Conformal Curvature Hypothesis (Penrose 1979).

I do not intend to investigate these larger issues here, however. I intend to investigate only the status of the analogy between the laws of classical thermodynamics on the one hand and those of black-hole mechanics in classical general relativity on the other. I mention these larger issues only to give some of my motivation for this work, and to place it in the context of important work being done in many branches of theoretical physics today.

There are other motivations behind this project as well. Although philosophers of physics have recently begun to work on issues arising from proposals for theories of quantum gravity, some

[3] I thank Fay Dowker for elucidating this possibility in a very helpful way in conversation.

of which take as their starting points the seemingly thermodynamical character of gravitational phenomena as exemplified by the laws of black-hole mechanics, almost no philosophical work has been done investigating the nature of this seemingly thermodynamical character as revealed by the structures of general relativity and of quantum field theory formulated on curved, relativistic spacetimes. Because general relativity and quantum field theory are well entrenched, clearly and rigorously articulated physical theories, I believe it behooves philosophers to study them, if not before, at least in conjunction with work done on quantum gravity.

# 2 The Laws of Black-Hole Mechanics and the Laws of Thermodynamics

Within the context of general relativity, one can derive laws describing the behavior of black holes in stationary, asymptotically flat spacetimes bearing a remarkable resemblance to the classical laws of equilibrium thermodynamics. I restrict attention to the asymptotically flat case, because that is the simplest natural analogue of an isolated system for black holes in general relativity.[4] I restrict attention to stationary black holes because those are the simplest natural analogue of an equilibrated system for black holes in general relativity.

Now, for the laws themselves:[5]

### Zeroth Law

[**Thermodynamics**] The temperature $T$ is constant throughout a body in thermal equilibrium.[6]

[**Black Holes**] The surface gravity $\kappa$ is constant over the event horizon of a stationary black hole.

[4]The generalization of the idea of a black hole and of the Four Laws to the non-asymptotically flat case by Hayward (1994), by the use of so-called dynamical trapping horizons, is of great interest, but to treat them would take us beyond the scope of this paper. Also, I will not discuss the so-called Minus-First Law of Brown and Uffink (2001); much work has been done to prove, or at least argue for, its correlate in black-hole mechanics (though not referred to as such in that literature), that perturbed black holes tend to settle down to equilibrium, and, in particular, that the sorts of perturbations I consider here do not destroy the event horizon. There are now strong plausibility arguments in favor of it (Hollands and Wald 2012), but its status in black-hole mechanics is still, to my mind, very much up for grabs, though, as a betting man, my money is on there being arguments for it at least as strong as for the Third Law (which, perhaps, is not to say very much).

[5]For proofs of the laws for black holes, see Wald (1984, ch. 12), Israel (1986), Wald (1994), and Wald and Gao (2001).

[6]This is not the standard formulation of the thermodynamical Zeroth Law, which is "If two systems are in thermal equilibrium with a third, then each is in thermal equilibrium with the other". Because the formulation I use and the standard formulation are essentially equivalent when the systems at issue are assumed to be thermally homogeneous, as is the case for all the types of system my constructions rely on, and because I think this is a reasonable restriction when treating the Zeroth Law in any case, this is not a problem for my arguments. Indeed, standard statements of the meaning of "thermal equilibrium" usually include the qualification that the system be thermally homogeneous, in the sense that the system contain no boundary with a permeability to heat flow different than that of the rest of the system.

### First Law

**[Thermodynamics]**

$$\mathrm{d}E = T\mathrm{d}S + p\mathrm{d}V + \Omega\mathrm{d}J$$

where $E$ is the total energy of the system, $T$ the temperature, $S$ the entropy, $p$ the pressure, $V$ the volume, $\Omega$ the rotational velocity and $J$ the angular momentum.[7]

**[Black Holes]**

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega_{\mathrm{BH}}\delta J_{\mathrm{BH}}$$

where $M$ is the total black hole mass, $A$ the surface area of its horizon, $\Omega_{\mathrm{BH}}$ the "rotational velocity" of its horizon,[8] $J_{\mathrm{BH}}$ its total angular momentum, and '$\delta$' denotes the result of a first-order, linear perturbation of the spacetime.[9]

### Second Law

**[Thermodynamics]** $\delta S \geq 0$ for any process in an isolated system.[10]

**[Black Holes]** $\delta A \geq 0$ in any process.[11]

[7] Strictly speaking, this is not the First Law, but rather the Gibbs Relation, which is equivalent to the First Law for thermodynamical systems in equilibrium and for systems that deviate from equilibrium only "quasi-statically". Since all my arguments involve only systems in equilibrium, and, as is standard in thermodynamical arguments, systems that deviate from equilibrium only by quasi-stationary effects, this is not a problem.

[8] See Wald (1984, ch. 12, §3, pp. 319–320).

[9] For an exact definition and thorough discussion of the perturbations used, see Wald and Gao (2001). There is an oddity about this formulation of the law, however, that I have not seen addressed in the literature but is surely worth puzzling over. While the $\delta$ acting on $M$ is the same as that acting on $J_{\mathrm{BH}}$, it is not the same as that acting on $A$. The $\delta$ acting on $M$ and $J_{\mathrm{BH}}$ represents a perturbation of a quantity taken asymptotically at spatial infinity; the other represents perturbations taken "at the event horizon". I know of no other physically significant equation where different differential operators act on different mathematical spaces in such a way that, as in this case, there's no natural mapping between them. What's going on here?

[10] Again, this is not the usual formulation of the Second Law in classical thermodynamics, which is standardly given as the Clausius or Kelvin Postulate (*e.g.*, Fermi 1956, §7). Because the principle of entropy increase follows from either Postulate (Fermi 1956, §§11–13), and because the appropriate analogues for those Postulates hold for black holes (§5.3 below), this again is not a problem for my arguments.

[11] Note that, because we are considering by fiat only asymptotically flat black holes, the appropriate analogue of an isolated classical thermodynamical system, it would be redundant to stipulate in the statement of the Law that the process takes place in an isolated system. Indeed, the Area Theorem (as the Second Law for black holes is often called) is a a result in pure differential geometry, the only input with a physical interpretation required being the so-called null energy condition. That condition essentially rules out only macroscopic fluxes of negative energy, so the scope of the quantifier in "any process" in the statement of the Law should be taken very broadly indeed. In particular, one need not even assume the process is quasi-static, nor even that the processes are restricted to the sorts of first-order, linear perturbations used in the formulation and proof of the First Law. (See Curiel 2014c for a discussion of the physical content of the null energy condition and its role in the proofs of the Laws for black holes.)

**Third Law**

> [**Thermodynamics**] $T = 0$ is not achievable by any process.[12]
>
> [**Black Holes**] $\kappa = 0$ is not achievable by any process.

The most striking architectonic similarity between the characterization of ordinary thermodynamical systems (in equilibrium) by the laws of thermodynamics and the characterization of black holes (in equilibrium, *i.e.*, stationary) is that in each case the behavior of the system, irrespective of any idiosyncracies in the system's constitution or dynamical history, is entirely captured by the values of a small number of physical quantities, 6 for ordinary thermodynamical systems, 4 for black holes: in the former case, they are temperature, entropy, pressure, volume, angular velocity and angular momentum;[13] in the latter, they are surface gravity, area, angular velocity and angular momentum. The Zeroth and Third Laws suggest that we take the surface gravity of a black hole as the analogue of temperature. The Second Laws suggest that we take area as the analogue of entropy. This is consistent with the First Law, if we treat $\frac{1}{8\pi}\kappa\delta A$ as the Gibbsian "heat" term for a system in thermal equilibrium. Indeed, if we do so then the analogy for the First Law becomes exact: relativistically, energy just is mass, so the lefthand side terms of the First Law for ordinary systems and for black holes are not just analogous, they are physically identical; likewise, $\Omega_{\text{BH}}\delta J_{\text{BH}}$ as a work term in the law for black holes is physically identical to the corresponding term in the law for ordinary systems.

Now the force of the question motivating this paper should be clear: the mathematical analogy is perfect, and there are already some indications that the analogy may reach down to the level of physics, not just mathematics. But how far should we take the analogy? What can it mean to take seriously the idea that the surface gravity of a black hole is a physical temperature, and its area a physical entropy?

## 3 The Standard Argument Does Not Work

There are well-known difficulties with taking the surface gravity of a classical black hole to represent a physical temperature. One important method for defining the thermodynamic temperature of an object derives from the theory of thermal radiation from black bodies. If a normal black body immersed in a bath of thermal radiation settles down to thermal equilibrium, it will itself emit thermal radiation with a power spectrum characteristic of its equilibrium temperature as measured using a gas thermometer. This power spectrum can then be used to define a temperature scale. It is this definition of thermodynamic temperature that is almost always (at times implicitly) invoked

[12] I actually think this is a defective statement of the Third Law of thermodynamics. (See, *e.g.*, Schrödinger 1960, Aizenman and Lieb 1981 and Wald 1997 for a discussion of some of its problems.) Schrödinger (1960) provides a far more satisfactory statement of the Third Law, which I think carries over well into black-hole thermodynamics. I do not have room to go into the matter here, though.

[13] Of course, the First Law guarantees that not all these quantities will be independent, and, if one is considering a particular species of thermodynamical system, then one may have available an equation of state that will further reduce the number of independent quantities, but all that is beside the point for my purposes.

when the claim is made that if one considers classical general relativity alone then black holes, being perfect absorbers and perfect non-emitters, have an effective temperature of absolute zero.[14]

To try to be a little more precise, I will offer a reconstruction of the standard argument. It is not given in exactly this form by anyone else in the literature, but I think it captures both the spirit and the letter of the orthodox view. Put a Kerr black hole in a box with perfectly reflective sides, which are far from the event horizon (in the sense that they are many times farther away from the event horizon "in natural spacelike directions" than its own "natural" diameter). Pervade the box with thermal radiation. According to classical general relativity, the black hole will absorb all incident thermal radiation, and emit none, until eventually all thermal radiation in the box (outside the event horizon) has vanished, so the black hole must have a temperature of absolute zero. Thus, the surface gravity $\kappa$, which is never zero for a non-extremal Kerr black hole, cannot represent a physical temperature of the black hole in classical general relativity. Conventional wisdom holds, as a result, that if the formal similarities mentioned above were all there were to the matter then they would most likely represent a merely accidental resemblance or perhaps would indicate at best a superficial relationship between thermodynamics and black holes, but in any event would not represent the laws of classical thermodynamics as extended into the realm of black holes.[15]

In 1974, using semi-classical approximation techniques Hawking discovered that stationary, axisymmetric black holes appear to radiate as though they were perfect black-body emitters in thermal equilibrium with temperature $\frac{\hbar}{2\pi}\kappa$, when quantum particle-creation effects near the black hole horizon are taken into account (Hawking 1974; Hawking 1975). It is this result that is generally taken to justify the view that the resemblances between the laws of black hole mechanics and the laws of classical thermodynamics point to a fundamental and deep connection among general relativity, quantum field theory and thermodynamics, and in particular that $\kappa$ *does* in fact represent the physical temperature of a black hole, and therefore $A$ its entropy.[16]

I have two problems with this orthodoxy. First, I find the physical content of the standard argument not to stand up to scrutiny. While it is true that the Kerr black hole in the box, according to classical general relativity, will emit no blackbody radiation while it absorbs any incident on it, that is not the end of the story. Classical general relativity does tell us that the Kerr black hole will emit some radiation, *viz.*, *gravitational* radiation, while it is perturbed by the infalling thermal radiation, and that gravitational radiation will in fact couple with the thermal radiation still outside the black hole. If we are trying to figure out whether purely gravitational objects, such as black holes, have thermodynamical properties, we should surely allow for the possibility that gravitational radiation, or, indeed, the exchange of "gravitational energy" in any form, may count as a medium for

[14] See for example the remarks in Bardeen, Carter, and Hawking (1973), Carter (1973) and Wald (1999). There is another form of argument for attributing the temperature absolute zero to all classical black holes, that it seems to be possible to use them to convert heat into work with 100% efficiency. I address this type of argument in §6.

[15] The remarks of Wald (1984, p. 337), for example, are exemplary in this regard.

[16] See again, for example, the remarks of Wald (1984, p. 337). Indeed, some of the most important researchers in the field make even stronger claims. Unruh and Wald (1982, p. 944), for example, claim that "the existence of acceleration radiation [outside the event horizon, a fundamentally quantum phenomenon,] is vital for the self-consistency of black-hole thermodynamics."

thermodynamical coupling.[17] Indeed, just as electromagnetic radiation turned out to be a medium capable of supporting a physically significant coupling of electromagnetic systems with classical thermodynamical systems, it seems *prima facie* plausible that gravitational radiation may play the same role for gravitational systems. Just as "heat" for an electromagnetic system may be measured by electromagnetic radiation, at least when transfer processes are at issue, so it may be that "heat" for a gravitational system may be measured by gravitational radiation, or any form of exchange of gravitational energy, again at least when transfer processes are at issue. Electromagnetic energy is just not the relevant quantity to track when analyzing the thermodynamic character of purely gravitational systems.

Second, I do not think this definition of temperature is the appropriate one to use in the context of a purely classical description of black holes, for the electromagnetically radiative thermal equilibrium of systems immersed in a bath of thermal radiation is essentially a *quantum* and *statistical* phenomenon, by which I mean one that can be correctly modeled only by using the hypothesis that radiative thermal energy is exchanged in discrete quanta and then computed correctly only with the use of statistical methods. To use that characterization of temperature to argue that we must use quantum mechanics in order to take surface gravity seriously as a physical temperature, therefore, is to beg the question. If my qualm is well founded, it follows that the standard argument does not bear on the strength of the analogy as indicating a real physical connection between classical general relativity and thermodynamics. After all, if one is trying to determine the status of the analogy between *classical* gravitational theory and *classical* thermodynamics independently of any quantum considerations, then the most appropriate characterizations of temperature to use are those grounded strictly in classical thermodynamics itself. (I make the idea of this qualm precise in §6, in discussing possible problems with my arguments.)

There is yet another *prima facie* problem, however, with trying to interpret surface gravity as a true temperature and area as a true entropy, which my arguments so far do not address: neither has the proper physical dimension. In geometrized units, the physical dimension of temperature is mass (energy), and entropy is a pure scalar. The physical dimension of surface gravity, however, is mass$^{-1}$, and that of area mass$^2$. There are no purely classical universal constants, moreover, available to fix the dimensions by multiplication or division.[18] The only available universal constant to do the job seems to be $\hbar$, which has the dimension mass$^2$.[19] I cannot address this problem at this stage of my arguments. Remarkably, however, it will turn out as a natural sequela to my construction of the appropriate analogue of a Carnot cycle for black holes, in §5.2, that the existence of a universal constant in the classical regime with the proper dimension is guaranteed.

[17] I use scare-quotes for 'gravitational energy' because that is an infamously vexed notion in classical general relativity, with no cogent way known to localize it, and indeed strong reasons to think there can be no localization of it in general. (See, *e.g.*, Curiel 2014b.) I will discuss this issue, and the potential problems it may raise for my arguments, in §6.

[18] All the classical universal constants, such as the speed of light and Newton's gravitational constant, are dimensionless. This is actually a puzzling state of affairs, that surely deserves investigation.

[19] I am grateful to Ted Jacobson and Carlo Rovelli for pushing me on the issue of the physical dimensions of the quantities, and on the seeming need to introduce $\hbar$ to make things work out properly.

# 4 Temperature and Entropy in Classical Thermodynamics

I think there are grounds for taking the analogy very seriously even when one restricts oneself to the classical theories, without input from or reliance on quantum theories. To make the case more poignant, imagine that we are physicists who know only classical general relativity and classical thermodynamics, but have no knowledge of quantum theory. How could we determine whether or not to take black holes as thermodynamical objects in a substantive, physical sense, given that we know the deep formal analogy between the two sets of laws? In such a case, we ought to look to the way that temperature and entropy are introduced in classical thermodynamics and the various physical roles they play there. If the surface gravity and area of black holes can be introduced in the analogous ways and play the analogous physical roles, I contend that the global analogy is already on strong ground. In other words, the surface gravity and area must play the same role in the new theory *vis-à-vis* other theoretical quantities as temperature and entropy do in the original theory *vis-à-vis* the analogous theoretical quantities there. If, moreover, it can be shown that surface gravity couples to ordinary classical thermodynamical systems in the same formal way as ordinary temperature does, then there are no grounds for denying that it is a true physical temperature.[20] And if area for black holes is related to surface gravity and to the proper analogue of heat in the same way as entropy is to ordinary temperature and heat, and if it is required for formulating an appropriately generalized Second Law, then there are no grounds for denying that it is a true physical entropy. Indeed, it was exactly on grounds such as these that physicists in the 19th century concluded that the power spectrum of blackbody radiation itself encoded a *physical* temperature and entropy, not merely that there was an analogy between thermodynamics and the theory of blackbody radiation. Planck (1926) himself had doubts about the thermodynamical character of blackbody radiation until he had satisfied himself on these points.

There are three fundamental, related ways that temperature is introduced in classical thermodynamics, which themselves ground the various physical roles temperature can play in the theory (how it serves as the mediator of particular forms of coupling between different types of physical system, *e.g.*). The first derives from perhaps the most basic of the thermodynamic characteristics of temperature and is perhaps most definitive of the cluster of ideas surrounding the concepts of "temperature" and "heat": it is that when two bodies are brought into contact, heat will spontaneously flow from the one of higher temperature to the one of lower temperature.[21] The second arises from the fact that increase in temperature is positively correlated with increases in the capacity of a system to do work.[22] This fact allows one to define an empirical scale of temperature through,

[20] Since entropy directly mediates no coupling between thermodynamical systems, the same argument is not available for it. This is one of the properties of entropy that makes it a truly puzzling physical quantity: there is no such thing, not even in principle, as an entropometer.

[21] It is important for some of my later arguments to note that this characterization of comparative temperature does not preclude processes in which heat at the same time flows from the colder body to the hotter. It says only that it is always the case that heat flows from hotter to colder, irrespective of what may or may not happen in the reverse direction.

[22] See, *e.g.*, the exemplary remarks of Sommerfeld (1964, p. 36): "Thermodynamics investigates the conditions that govern the transformation of heat into work. It teaches us to recognize temperature as the measure of the work-value

*e.g.*, the use of a gas thermometer: the temperature reading of the thermometer is made directly proportional to the volume of the thermometric gas used, which is itself directly proportional to the work the gas does on its surrounding container as it expands or contracts in response to its coupling with the temperature of the system being measured. The utility of such a scale is underwritten by the empirical verification that such empirical scales defined using a multitude of different gases under a multitude of different conditions are consistent among one another.[23] The third arises from an investigation of the efficiency of reversible, cyclic engines, *viz.*, Carnot engines, which yields a definition of the so-called absolute temperature scale associated with the name of Kelvin.[24] It is the possibility of physically identifying the formally derived absolute scale with the empirically derived scale based on capacity to do work (increase in volumes, *e.g.*) that warrants the assertion that they both measure the same physical quantity.[25]

Likewise, there are (at least) three ways that entropy enters classical thermodynamics. The first historically, and perhaps the most physically basic and intuitive, is as a measure of how much energy it takes to transform the heat of a thermal system into work: generally speaking, the free energy of a thermodynamical system is inversely proportional to its entropy.[26] The second is as that perfect differential $\mathrm{d}S$ into which temperature, as integrating factor, transforms exchanges of heat $\mathrm{d}Q$ over the course of quasi-stationary processes (Fermi 1956, ch. IV): the integral of $\mathrm{d}Q$ along a quasi-stationary path between two equilibrium states in the space of states of a thermodynamical system is not independent of the path chosen, whereas the integral of $\frac{\mathrm{d}Q}{T}$ is. (Indeed, Sommerfeld 1964 uses this fact to conclude that entropy is a true physical property of a thermodynamical system, whereas heat content is not.) The third also arises from the analysis of the efficiency of Carnot cycles (Fermi 1956, ch. IV).

Now, the following fundamental theorem of classical thermodynamics provides the basis both for the definition of the absolute temperature scale and for the introduction of entropy as the perfect differential derived from exchanges of heat when that temperature is used as an integrating factor.

**Theorem 4.1** *Any two reversible, cyclic engines operating between temperatures $T_2$ and $T_1$ (as measured using gas thermometry) have the same efficiency. The efficiency of any non-reversible engine operating between $T_2$ and $T_1$ is always less than this.*

This theorem is a direct consequence of either the Clausius or the Kelvin postulate, which can be argued on physical grounds both to be equivalent to each other and to directly imply the principle of entropy increase (for the proofs of which statements see, *e.g.*, Fermi 1956):

of heat. Heat of higher temperature is richer, is capable of doing more work."

[23] Planck (1926, §1, p. 1) remarks that quantitative exactness is introduced into thermodynamics through this observation, for changes of volume admit of exact measurements, whereas sensations of heat and cold do not, nor even comparative judgments of hotter and cooler on their own.

[24] See, *e.g.*, Fermi (1956, §§8–10).

[25] Maxwell (1888, chs. VIII, XIII) gives a wonderfully illuminating discussion of the physical basis of the equivalence of the absolute temperature scale with the one based on gas thermometry.

[26] Again, the discussion of Maxwell (1888, ch. XII) about this idea is a masterpiece of physical clarification and insight.

**Postulate 4.2 (Lord Kelvin)** *A transformation whose only final result is to transform into work heat extracted from a source that is at the same temperature throughout is impossible.*

**Postulate 4.3 (Clausius)** *A transformation whose only final result is to transfer heat from a body at a given temperature to a body of a higher temperature is impossible.*

I claim that these last two postulates, and the fact that they provide grounds for the proof of the efficiency theorem, for the introduction of temperature and entropy as physical quantities, and for proof of the principle of entropy increase, encode essentially all that is of physical significance in the ways I sketched that both temperature and entropy enter into classical thermodynamics.

The Clausius Postulate captures the idea that when two bodies are brought into thermal contact, heat flows from the body of higher temperature to the other. The Kelvin Postulate captures the idea that the capacity of a body to do work on its environment tends to increase as its temperature increases. If one could show that appropriately formulated analogues to these two propositions about classical black holes hold in general relativity, with surface gravity playing the role of temperature and area that of entropy, one would have gone a long way towards showing that surface gravity *is* a true thermodynamical temperature and area a true entropy. If one could further show that ordinary thermodynamical systems equilibrate with black holes in a way properly mediated by their ordinary temperature and by the black hole's surface gravity, so as to allow for the construction of a Carnot-like cycle and the definition of an absolute temperature scale, the analogy would have been shown to be far more than analogy: it would be physical equivalence in the strongest possible sense. I prove all these propositions in §5 below.

# 5 Taking Black Holes Seriously as Thermodynamical Objects

What is needed, first, is a way to characterize "thermal coupling" between black holes and ordinary thermodynamical systems: granted that "heat" in the gravitational context is gravitational energy of a particular form, such as that carried in the form of gravitational radiation or that responsible for red-shift effects in monopole solutions, then it follows that black holes *are not perfect absorbers*. When there is an ambient electromagnetic field, the black hole will radiate gravitationally as it absorbs energy and grows from the infalling electromagnetic radiation. So to conclude that surface gravity is a physical temperature, one need show only that the gravitational energy exchanged between a black hole and other thermodynamical systems in transfer processes depends in the appropriate way on the surface gravity of the event horizon.[27] This approach has *prima facie* physical plausibility: to take the energy in gravitational radiation, *e.g.*, to be the gravitational equivalent of heat is the same as to take the energy in electromagnetic radiation to be the electromagnetic equivalent of heat—it is what couples in the appropriate way to the average kinetic energy of molecules in ordinary thermodynamical systems, *viz.*, what makes it increase and decrease, and that with respect to which equilibrium is defined.

[27] I will discuss in §6 below the fact that there is no well defined notion of localized gravitational energy in general relativity, and how that may bear on my arguments.

Just as the concept of "thermal coupling" had to be emended in the extension of classical thermodynamics to include phenomena associated with radiating black bodies, so we should expect it to be in this case. In classical thermodynamics before the inclusion of black-body phenomena, thermal coupling meant immediate spatial contiguity: heat was known to flow among solids, liquids and gases only when they had surfaces touching each other.[28] In order to extend classical thermodynamics to include black-body phenomena, the idea of thermal coupling had to be extended as well: two black bodies thermally couple when and only when the ambient electromagnetic field each is immersed in includes direct contributions from the electromagnetic radiation emitted by the other. They do not need to have surfaces touching each other.

In order to characterize the correct notion of thermal coupling among systems including black holes (or more generalized purely gravitational systems, such as cosmological horizons), we first need to characterize an appropriate notion of "heat" for black holes, and the concomitant notion of free energy. That will put us in a position to construct the appropriate generalization of Carnot cycles for them, and so to formulate the appropriate generalizations of the Clausius and Kelvin Postulates for such systems.

## 5.1 Irreducible Mass, Free Energy and "Heat" of Black Holes

In analyzing the ideas of reversibility and irreversibility for processes involving black holes, Christodoulou (1970) introduced the *irreducible mass* $M_{\text{irr}}$ of a black hole of mass $M$ and angular momentum $J$:[29]

$$M_{\text{irr}}^2 := \frac{1}{2}[M^2 + (M^4 - J^2)^{\frac{1}{2}}]$$

(From hereon, I shall drop the subscripted 'BH' on terms denoting quantities associated with black holes, except in cases where ambiguity may arise.) Inverting the definition yields

$$M^2 = M_{\text{irr}}^2 + \frac{1}{4}\frac{J^2}{M_{\text{irr}}^2}$$

and so, for a Kerr black hole,

$$M > M_{\text{irr}}$$

(Clearly, $M_{\text{irr}} = M$ for a Schwarzschild black hole.) Thus, the initial total mass of a black hole cannot be reduced below the initial value of $M_{\text{irr}}$ by any physical process. A simple calculation for a Kerr black hole, moreover, shows that,

$$A = 16\pi M_{\text{irr}}^2 \tag{5.1.1}$$

Thus, it follows from the Second Law that $M_{\text{irr}}$ itself cannot be reduced by any physical process, and so any process in which the irreducible mass increases is a physically irreversible process. In

[28] This fact, perhaps, contributed to the historical idea that heat was a fluxional, perhaps even fluid, substance, such as phlogiston or caloric.

[29] I will discuss only Kerr black holes, not Kerr-Newman black holes that also have electric charge, as the ensuing technical complications would not be compensated by any gain in physical comprehension.

principle, therefore, the free energy of a black hole is just $M - M_{\text{irr}}$, in so far as its total mass $M$ represents the sum total of all forms of its energies, and $M_{\text{irr}}$ represents the minimum total energy the black hole can be reduced to.[30]

In classical thermodynamics, it makes no sense to inquire after the absolute value of the quantity of heat a given system possesses. In general, that is not a well defined property accruing to a system. One rather can ask only about the amount of heat tranferred between bodies during a given process.[31] Consider, then, a classical thermodynamical system with total energy $E$ and free energy $E_{\text{f}}$. $E - E_{\text{f}}$ is the amount of energy unavailable for extraction, what Kelvin called its dissipated energy, $E_{\text{d}}$. Say that through some quasi-stationary process, we know not what, both $E$ and $E_{\text{d}}$ change so that the system now has less free energy than it did before; therefore, the entropy of the system must have increased, which can happen only when it absorbs heat, which will in general be the difference between the total change in energy and the change in free energy. If they both change so that the system has more free energy, the same reasoning applies, and it must have given up a quantity of heat equal to that difference.

These remarks suggest defining the "quantity of heat transferred" to or from a black hole during any quasi-stationary thermodynamical process to be the change in its free energy, which is to say the change in total black hole mass minus the change in its irreducible mass, $\Delta M - \Delta M_{\text{irr}}$.[32] If, for instance, the irreducible mass of a black hole does not change, while the total mass decreases, then it would have given up a quantity of heat. As a consistency check, it is easy to see that, according to this definition, when an ordinary thermodynamical system in equilibrium is dumped into a Kerr black hole, the black hole absorbs the quantity of heat the ordinary matter contained as characterized by the Gibbs relation, *viz.*, its temperature times its entropy, as only that energy contributes to its total mass without directly changing its angular momentum. Based on this characterization of "quantity of heat transferred", I claim that the appropriate notion of thermal coupling for systems involving black holes is any interaction where there is a change in the black hole's free energy. For purely gravitational interactions, this includes emission and absorption of that part of the energy of gravitational radiation not due to angular momentum, energy exchange due to simple monopole- or multipole-moment couplings in the near-stationary case, and so on.

Some care must be taken in applying this definition to Schwarzschild black holes, however. Because $M = M_{\text{irr}}$ for a Schwarzschild black hole, one can never give up heat while remaining Schwarzschildian. Schwarzschild black holes, essentially, have achieved heat death—one cannot extract energy from them without perturbing them in an appropriate way. Similarly, they cannot absorb heat in s straightforward sense: if one absorbs ordinary heat from a classical thermodynamical

[30] Some—*e.g.*, Wald (1984, ch. 12, §4)—interpret $M - M_{\text{irr}}$ as the rotational energy of a Kerr black hole, in so far as extracting that much energy from a black hole would necessarily reduce its angular momentum to zero. Based on the arguments I will give in this section, I prefer to think of it as a thermodynamical free energy, which cannot necessarily be decomposed in a canonical way into different "forms", *e.g.*, that much heat and that much rotational energy, *etc.*

[31] See Maxwell (1888, chs. I, III, IV, VIII, XII).

[32] I thank Harvey Brown for drawing to my attention the fact that Carathéodory (1909), in his ground-breaking axiomatization of classical thermodynamics, introduced the notion of heat in a way very similar to this, not as a primitive quantity as is usually done, but as the difference between the internal and the free energies of a system.

system, say, being thrown into it, then after it settles down again to staticity it will once again have its total mass equal to its irreducible mass (unless it acquires angular momentum in the process, and so becomes a Kerr black hole). In this case, I think it still makes sense to say the black hole has absorbed heat, in so far as, between the time the system is thrown in and the time the black hole equilibrates again, its irreducible mass will not be equal to its total mass. The maximum of this difference, during the equilibration process, will presumably equal the energy of the system the black hole absorbed. There are many challenges one could reasonably pose to the approximations involved in attempting to carry out such a calculation with anything approaching rigor (which I have not done), but they are all the same sort of challenge one could pose to the analogous problem in classical thermodynamics, so there is no problem here peculiar to black-hole thermodynamics.

### 5.2 Carnot-Geroch Cycles for Schwarzschild Black Holes

As I remarked at the end of §4, the strongest evidence that the formal equivalence of the laws of black holes and those of ordinary thermodynamical systems in fact constitutes a true physical equivalence, and that surface gravity is a physical temperature and area a physical entropy, would consist in a demonstration that black holes thermally couple with ordinary thermodynamical systems in such a way that $\kappa$ plays the same role in that coupling as ordinary temperature would if the system at issue were coupling with another ordinary thermodynamical system and not with a black hole, and the same for area. My proposed construction of the appropriate analogue for a Carnot cycle including black holes, which I give in this subsection, will kill three birds with one stone: not only will it show that $\kappa$ can be characterized as the absolute temperature of the black hole using the same arguments as classical thermodynamics uses to introduce the absolute temperature scale; it will do so by showing that in the coupling of black holes with ordinary thermodynamical systems, $\kappa$ does in fact play the physical role of temperature and area that of entropy; and it will have as a natural corollary the existence of a universal constant that renders the proper physical dimensions to surface gravity as a measure of temperature and area as a measure of entropy.[33]

I call the constructed process a "Carnot-Geroch cycle" both to mark its difference from standard Carnot cycles, and because it relies essentially on the mechanism at the heart of the most infamous example in this entire field of study, Geroch's thought experiment of slowly lowering towards a black hole a box filled with thermal matter, with the argued consequence being that classical black holes must have temperature absolute zero.[34] (I discuss Geroch's original example and argue that it does not in fact support the conclusions he wanted to draw from it in §6 below.) I will first sketch the

[33]I am grateful to Ted Jacobson for bringing to my attention after I wrote this paper the insightful analysis of Sciama (1976), in some ways quite similar to mine. (See Jacobson 2003 for a *prècis* of Sciama's analysis.) Sciama, however uses quantum systems all the way through and assumes that the analogy between black holes and ordinary thermodynamical systems is merely formal when one does not take quantum effects into account.

[34]According to Jakob Bekenstein (private correspondence) and Robert Wald (conversation), Geroch first proposed the example during a colloquium he gave at Princeton in 1970. (Bekenstein tells me that he considers it the first attempt to attribute a temperature to a black hole.) I cannot resist pointing out that my construction is essentially a jiu jitsu move against Geroch's original intent, turning the force of the example against itself, using Geroch's proposed mechanism to show that surface gravity really is a temp.

steps of the proposed cycle informally, then work through the calculations.

**Reversible Carnot-Geroch Cycle Using a Schwarzschild Black Hole as a Heat Sink**

1. start with a small, empty, essentially massless, perfectly insulating box "at infinity", one side of which is the outer face of a piston; in particular, the box is "small" in the sense that it will experience negligible tidal forces as it is lowered toward the black hole; very slowly ("quasi-statically", so that the process is well approximated as an isentropic process) draw the piston back through the inside of the box, so filling the box with fluid from a large heat bath consisting of a large quantity of the fluid at fixed temperature $T_0$, so the fluid does work against the piston as it moves; when the piston has withdrawn part but not all of the way to the opposite side of the box, quickly seal the box, leaving the space opened by the piston filled with a mass of the fluid $M_0$ in thermal equilibrium at temperature $T_0$, and with entropy $S_0$; assume the entire energy of the box is negligible compared to the mass of the black hole
2. very slowly, lower the box towards the black hole using an essentially massless rope; during this process, an observer inside the box would see nothing relevant change; in particular, as measured by an observer co-moving with the box, the temperature, volume and entropy of the fluid remain constant[35]
3. at a predetermined fixed proper radial distance from the black hole, stop lowering the box and hold it stationary
4. very slowly, draw the piston back even further, so lowering the temperature of the fluid to a fixed, pre-determined value $T_1$ while keeping its entropy the same; the value of the temperature is to be fixed by the requirement that the change in total entropy vanishes during the next step (*i.e.*, entropy of black hole plus entropy of everything outside black hole does not change after the fluid is dumped into the black hole)
5. open the box and eject the fluid out of it by using the piston to push it out, so the fluid falls into the black hole delivering positive mass-energy and positive entropy to it, and the piston returns to its initial state; by the way the temperature of the fluid was fixed in the previous step, this is an isentropic process
6. pull the box back up to infinity (which takes no work, as the box now has zero mass-energy, and so zero weight), so it returns to its initial state

Because the total entropy remains constant during every step in the process, these cycles are reversible in the sense of classical thermodynamics. Because the irreducible mass of the black hole increases, however, it is not an irreversible process in the sense of black-hole mechanics.[36]

[35] The mass-density distribution of the fluid would change, increasing towards the side facing the black hole; this, however, does not affect the analysis, since this is what one expects for a system in thermal equilibrium in a quasi-static "gravitational field". In any event, given our assumption about the size of the box, this effect would be negligible.

[36] In Curiel (2014a), I propose another form of Carnot-Geroch cycle for a Kerr black hole, one that exploits its angular momentum in such a way as to make the process both reversible in the sense of classical thermodynamics and physically reversible according to black-hole mechanics.

Now, let us make the following assumptions: first, that it makes sense to attribute a physical temperature $T_{\text{BH}}$ and entropy $S_{\text{BH}}$ to a black hole (though we do not yet know what they are); second, that the entropy of ordinary thermodynamical systems and the entropy of the black hole are jointly additive; and third, that the appropriate temperature at which to eject the fluid into the black hole for the entire cycle to be isentropic ($T_1$ in step 5) is that one would expect for a thermally equilibrated body in thermal contact with another at temperature $T_{\text{BH}}$ sitting the given distance away in a nearly-static gravitational field. It will then follow that the physical temperature must be $8\pi\alpha\kappa$ and the physical entropy $\frac{A}{\alpha}$, where $\kappa$ is the black hole's surface gravity, $A$ its area, and $\alpha$ is a universal constant, the analogue of Boltzmann's constant for black holes (to be derived below).

Let the static Killing field in the spacetime be $\xi^a$ (timelike outside the event horizon, null on it). Let $\chi = (\xi^n \xi_n)^{\frac{1}{2}}$, and $a^a = (\xi^n \nabla_n \xi^a)/\chi^2$ be the acceleration of an orbit of $\xi^a$. Then a standard calculation[37] shows that

$$\kappa = \lim(\chi a)$$

where the limit is taken as one approaches the event horizon in the radial direction, *i.e.*, near the black hole $\chi a$ is essentially the force that needs to be exerted "at infinity" to hold an object so that it follows an orbit of $\chi^a$, which is to say, to hold it so that it is locally stationary. Thus $\chi$ is essentially the "redshift factor" in a Schwarzschild spacetime.

Let the total energy content of the box when it is initially filled at infinity be $E_0$ (as measured with respect to the static Killing field). In particular, $E_0$ includes contributions from the rest mass of the fluid $M_0$, and from its temperature $T_0$ and entropy $S_{\text{b}}$; let $W_0$ be the work done by the fluid as it pushes against the piston in filling the box. By the Gibbs relation and by the First Law of thermodynamics, therefore, we can compute the quantity of heat $Q_{\text{b}}$ initially in the box:

$$Q_{\text{b}} = T_0 S_{\text{b}} = E_0 + W_0$$

As the box is quasi-statically lowered to a proper distance $\ell$ from the event horizon, its energy as measured at infinity becomes $\chi E_0$, where $\chi$ is the value of the redshift factor at $\ell$. Thus, the amount of work done at infinity in lowering the box is

$$W_\ell = (1 - \chi) E_0$$

(Recall that we assumed the box to be so small that $\chi$ does not differ appreciably from top to bottom.) This is not standard thermodynamical work, as the volume of the fluid, as measured by a co-moving observer, has not changed. It is rather work done by "the gravity of the black hole".

Now, when the box is held at the proper distance $\ell$ from the black hole and the piston slowly pushes or pulls so as to change the temperature of the fluid from $T_0$ to $T_1$ (as measured locally), the piston does work (as measured at infinity)

$$W_1 = \chi(E_0 - E_1)$$

[37] See, *e.g.*, Wald (1984).

where $E_1$ is the locally measured total energy of the fluid after the fluid's (locally measured) volume has been changed by the piston. When the fluid has reached the desired temperature $T_1$, the box is opened and the piston pushes the fluid quasi-statically out of the box, so it will fall into the black hole; in the process, the piston does work $W_2$ (as measured at infinity).[38] Now, by the First Law, the total amount of energy the fluid has as it leaves the box is

$$E_1 - \frac{W_2}{\chi} = T_1 S_{\mathrm{b}} \tag{5.2.2}$$

as measured locally.

In order to compute the total amount of energy and the total amount of heat dumped into the black hole as measured at infinity, we must compute the temperature of the box as measured from there. It is a standard result (Tolman 1934, p. 318) that the condition for a body at locally measured temperature $T$ to be in thermal equilibrium in a strong, nearly static gravitational field is that the temperature measured "at infinity" be $\chi T$. Thus the temperature of the box as measured from infinity will be $\chi T_1$. It follows from equation (5.2.2), therefore, that the total amount of heat dumped into the black hole is

$$\chi T_1 S_{\mathrm{b}} = \chi E_1 - W_2$$

But $\chi E_1 = \chi E_0 - W_1$ and $\chi E_0 = E_0 - W_\ell$, so

$$\chi T_1 S_{\mathrm{b}} = E_0 - W_\ell - W_1 - W_2$$

The expression on the righthand side of the last equation, however, is just the total amount of energy in the box as measured at infinity, and so $\chi T_1 S_{\mathrm{b}}$ is the total amount of energy the black hole absorbs, as measured from infinity, which is entirely in the form of heat.

Now, because we have assumed that the entropy for the fluid and for the black hole is additive, the total change in entropy is

$$\Delta S = -S_{\mathrm{b}} + \frac{\chi T_1 S_{\mathrm{b}}}{T_{\mathrm{BH}}}$$

For the process to be isentropic,

$$\Delta S = 0$$

and so

$$\frac{\chi T_1 S_{\mathrm{b}}}{T_{\mathrm{BH}}} = S_{\mathrm{b}} \tag{5.2.3}$$

Thus, $T_1 = \dfrac{T_{\mathrm{BH}}}{\chi}$, precisely the temperature one would expect for a thermally equilibrated body in thermal contact with another body at temperature $T_{\mathrm{BH}}$ a redshift distance $\chi$ away. Write $Q_{\mathrm{BH}}$ for

[38] One may worry that this process cannot be quasi-static, not even in principle, in so far as the phase-space volume available to the fluid as it is expelled from the box and before it is absorbed by the black hole is, in principle, unbounded, *i.e.*, the entropy of the fluid increases by an arbitrary amount. A superficial, but I think still adequate, answer to this problem is that one can arrange a telescopically extending mechanism from the box to the black hole to ensure that the volume available to the fluid never changes. A deeper and I think more satisfying answer is that, when the fluid passes the event horizon, as all of it must do, its available phase-space volume only decreases, and arbitrarily so. I thank Tim Maudlin for pushing me on this point.

the amount of heat the black hole absorbs ($= \chi T_1 S_\mathrm{b}$), so equation (5.2.3) becomes

$$\frac{Q_\mathrm{BH}}{T_\mathrm{BH}} = S_\mathrm{b}$$

Now, in the limit as the box, and so the heat and entropy it contains, becomes very small (while the temperature remains constant), we may think of this as an equation of differentials,

$$\frac{\mathrm{d}Q_\mathrm{BH}}{T_\mathrm{BH}} = \mathrm{d}S_\mathrm{b} \tag{5.2.4}$$

This expresses the well known fact that temperature plays the role of an integrating factor for heat. Since $\mathrm{d}Q_\mathrm{BH}$ is the change in mass of the black hole, $\mathrm{d}M_\mathrm{BH}$, due to its being the entirety of the energy absorbed, there follows from the First Law of black-hole mechanics[39]

$$\frac{8\pi \mathrm{d}Q_\mathrm{BH}}{\kappa} = \mathrm{d}A \tag{5.2.5}$$

Thus, $\kappa$ is also an integrating factor for heat. It is a well known theorem that if two quantities are both integrating factors of the same third quantity, the ratio of the two must be a function of the quantity in the total differential, and so in this case

$$\frac{T_\mathrm{BH}}{\kappa} = \psi(A) \tag{5.2.6}$$

for some $\psi$. (It is also the case that $\frac{T_\mathrm{BH}}{\kappa} = \phi(S_\mathrm{b})$ for some $\phi$, but we will not need to use that.) It follows from equations (5.2.4) and (5.2.5) that

$$\frac{1}{8\pi}\psi(A)\mathrm{d}A = \mathrm{d}S_\mathrm{b} \tag{5.2.7}$$

and so integrating this equation yields the change in the black hole's area, $\Delta A$ as a function of $S_\mathrm{b}$, say $\Delta A = \theta(S_\mathrm{b})$. (From hereon, we fix some arbitrary standard value for $A$, and so drop the '$\Delta$'.)

In order to complete the argument, and make explicit the relation between $A$ and $S_\mathrm{b}$, and at the same time fix the relation between $\kappa$ and $T_\mathrm{BH}$, consider two black holes very far apart, and otherwise isolated, so there is essentially no interaction between them. Perform the Geroch-Carnot cycle on each separately. Let $A_1$ and $A_2$ be their respective areas, $\theta_1$ and $\theta_2$ the respective functions for those areas expressed using $S_{\mathrm{b}1}$ and $S_{\mathrm{b}2}$, the respective entropies dumped into the black holes by the cycles, and let $\theta_{12}(S_{\mathrm{b}12})$ be the function for the total area of the black holes considered as a single system, expressed using the total entropy $S_{\mathrm{b}12}$ dumped into the system. Both the total area

[39] At least two conceptually distinct formulations of the First Law of black-hole mechanics appear in the literature, what (following Wald 1994, ch. 6, §2) I will call the physical-process version and the equilibrium version. The former fixes the relations among the changes in an initially stationary black hole's mass, surface gravity, area, angular velocity, angular momentum, electric potential and electric charge when the black hole is perturbed by throwing in an "infinitesimally small" bit of matter, after the black hole settles back down to stationarity. The latter considers the relation among all those quantities for two black holes in "infinitesimally close" stationary states, or, more precisely, for two "infinitesimally close" black-hole spacetimes. Clearly, I am relying on the physical-process version, for the most thorough and physically sound discussion and proof of which see Wald and Gao (2001).

of the black holes and the total entropy dumped in are additive (since the black holes, and so the elements of the Carnot-Geroch cycles, have negligible interaction), *i.e.*,

$$\theta_1(S_{\mathrm{b}1}) + \theta_2(S_{\mathrm{b}2}) = \theta_{12}(S_{\mathrm{b}12}) = \theta_{12}(S_{\mathrm{b}1} + S_{\mathrm{b}2})$$

Differentiate each side, first with respect to $S_{\mathrm{b}1}$ and then with respect to $S_{\mathrm{b}2}$; because $\theta_{12}$ is symmetric in $S_{\mathrm{b}1}$ and $S_{\mathrm{b}2}$,

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}S_{\mathrm{b}1}} = \frac{\mathrm{d}\theta_2}{\mathrm{d}S_{\mathrm{b}2}}$$

Since the parameters of the two black holes and the two cycles are arbitrary, it follows that there is a universal constant $\alpha$ such that

$$\frac{\mathrm{d}\theta}{\mathrm{d}S_b} = \frac{\mathrm{d}A}{\mathrm{d}S_b} = \alpha$$

for all Schwarzschild black holes. It now follows directly from equations (5.2.6) and (5.2.7) that

$$T_{\mathrm{BH}} = 8\pi\alpha\kappa \tag{5.2.8}$$

and from equation (5.2.3) that

$$S_{\mathrm{BH}} = \frac{A}{\alpha} \tag{5.2.9}$$

up to an additive constant we may as well set equal to zero.[40] $\alpha$ is guaranteed by construction to have the proper dimensions to give $T_{\mathrm{BH}}$ the physical dimension of temperature (mass, in geometrized units), and $S_{\mathrm{BH}}$ the physical dimension of entropy (dimensionless, in geometrized units).

As a consistency check, it is easy to compute that the total work performed in the process,

$$W_T = W_0 + W_\ell + W_1 + W_2$$

equals the total change in heat of the box during the process, $Q_{\mathrm{b}} - \chi T_1 S_{\mathrm{b}}$, exactly as one should expect for a Carnot cycle. One can use the total work, then, to define the efficiency of the process in the standard way,

$$\eta := \frac{W_T}{Q_{\mathrm{b}}} = 1 - \frac{\chi T_1 S_{\mathrm{b}}}{Q_{\mathrm{b}}}$$

from which it follows that

$$\eta = 1 - \frac{8\pi\alpha\kappa}{T_0}$$

Thus, one can use the standard procedure for defining an absolute temperature scale based on the efficiency of Carnot cycles, and one concludes that the absolute temperature of the black hole is indeed $8\pi\alpha\kappa$.

Unfortunately, one cannot use similar arguments as in the classical case to prove the analogue of theorem 4.1, as the Carnot-Geroch Cycle for Schwarzschild black holes is not reversible in the physical sense. Under restricted conditions, however, the Carnot-Geroch cycle for Kerr black holes *is* physically reversible, and so in that case one can use the classical arguments to prove the analogue of theorem 4.1, as I plan to discuss in future work (Curiel 2014a).

[40] In contradistinction to classical thermodynamical systems, geometrized units for the entropy of black holes can be naturally constructed: let a natural unit for mass be, say, that of a proton; then one unit of entropy is that of a Schwarzschild black hole of unit mass. Why does classical black-hole thermodynamics allow for the construction of a natural unit for entropy when purely classical, non-gravitational thermodynamics does not?

## 5.3 The Generalized Clausius and Kelvin Postulates for Black Holes

Although I consider the construction of the Carnot-Geroch Cycle and the arguments based on it to be the most decisive in favor of conceiving of classical black holes as truly thermodynamical objects, I think it is still worthwhile to show that the appropriately translated analogues of the Clausius and Kelvin Postulates hold for black holes as well. Because those Postulates provide the ground for all ways of introducing temperature and entropy in classical thermodynamics, to show that they hold of black holes as well will show that the physical behavior of black holes conforms as closely as possible to that of classical thermodynamical in all fundamental respects.

The standard arguments in favor of the Clausius and Kelvin postulates (as given, *e.g.*, in Fermi 1956, ch. 3), which rely on the impossibility of constructing a *perpetuum mobile* of the second kind, do not translate straightforwardly into the context of general relativity, where there is no general principle of the conservation of energy. Remarkably enough, however, one can still give arguments for them at least as strong as those given for their analogues in classical thermodynamics.

**Postulate 5.3.1 (Generalized Clausius Postulate for Black Holes)** *For any two systems, at least one of which is a stationary black hole, a transformation whose only final result is that a "quantity of heat" (as defined in §5.1) is transferred from the system with lower temperature (surface gravity) to the one of higher temperature (surface gravity) is impossible.*

Assume that initially the black hole is at the lower temperature, and that such a transformation as described in the antecedent of the theorem were possible. Then the change in irreducible mass of the black hole would have to be strictly greater than the change in its total mass during the interaction, with no other change in the spacetime than that another system absorbed heat. In particular, its irreducible mass must increase. However, it follows from equation (5.1.1) that an increase in irreducible mass must yield an increase in the black hole's area, and so its entropy, violating the assumption that nothing else thermodynamically relevant in the spacetime changed. Analogous reasoning in the case where the black hole is initially at a higher temperature shows that the irreducible mass would also have to change in such a process.

**Postulate 5.3.2 (Generalized Kelvin Postulate for Black Holes)** *A transformation whose only final result is that a "quantity of heat" (as defined in §5.1) is extracted from a stationary black hole and transformed entirely into work is impossible.*

The argument is essentially the same as for the Clausius Postulate for black hole. Again, for such a process to occur, the irreducible mass of the black hole would have to increase, but that would necessitate a change in the area of the black hole, violating the conditions of the theorem.

# 6 Problems, Possible Resolutions, Possible Insights, and Questions

I conclude the paper with a brief discussion of some *prima facie* problems with my arguments, suggestions for their resolutions, an examination of what insights my conclusions, if correct, may offer, and some general questions that I think need to be addressed, possibly with the help of my arguments and conclusions.

An obvious complaint against the argument based on the construction of the Carnot-Geroch Cycle is that it is circular: why assume a classical black hole has an entropy in the first place? The best answer to this is implicit in the questions Wheeler initially posed in the late 1960s that inspired the entire field of black-hole thermodynamics in the first place: if we don't assume black holes have entropy, then we would, with effortless virtuosity, be able to achieve arbitrarily large violations of the Second Law of thermodynamics. The world external to a black hole is isolated from the interior of the black hole. So, take your favorite highly entropic system and throw it into a black hole: the entropy of that system vanishes from the external world, so lowering the total entropy of an isolated system. The only escape from this possibility is to assign the black hole itself an entropy in such a way that, when an ordinary entropic system passes into a black hole, then the black hole's entropy increases at least as much as the entropy of the system entering it. This postulate is generally referred to as the Generalized Second Law: the total entropy of the world, *viz.*, the entropy of everything outside black holes plus the entropy of black holes, never decreases (Bekenstein 1973; Bekenstein 1974).

This attempt to answer the first problem leads naturally to the next, possibly the most serious potential problem: the derivation of the relation between black-hole entropy and area based on the Carnot-Geroch cycle does not by itself guarantee that there is no process that violates the Generalized Second Law. In particular, though in footnote 34 I claimed to turn Geroch's infamous thought-experiment on its head, nothing seems to preclude Geroch's original use of it to argue that, were classical black holes to have physical temperature, it would have to be absolute zero independently of what value its surface gravity had. If one arranges matters just so, the weight lifted by the lowering of the box will have extracted *all* the energy content of the box when it reaches the event horizon; one can then dump into the black hole the stuff in the box, which still has its original entropy but zero mass-energy; thus, one will have converted thermal energy into work with 100% efficiency, implying the black hole must have temperature absolute zero. Because the matter dumped into the black hole has no mass-energy, the area of the black hole does not increase; because the matter still has its original entropy, however, the total entropy of the world outside the event horizon has decreased, thus violating the Generalized Second Law.

There are (at least) two possible responses.[41] First, one can note that the procedure requires measurements of arbitrarily fine precision: the violation of the Generalized Second Law occurs *only*

[41] Perhaps the most influential response in the physics literature to this problem is given by Unruh and Wald (1982). I will not consider their response, as it inextricably relies on quantum effects.

if the matter has *exactly* zero stress-energy when it is released *precisely* when the box is contiguous with the event horizon. Otherwise, the area of the black hole will increase, and will always do so in way so as to preserve the Generalized Second Law. If one holds that classical thermodynamics is only an effective theory in the first place, as seems reasonable, then the notion of arbitrarily precise measurements never gets off the ground.[42]

The second possible response accepts the possibility of arbitrarily precise measurements in the context of classical thermodynamics. If one allows the possibility of such measurements, however, then it is not a justified idealization to ignore the stress-energy contained in the rope holding the box above the black hole. One may be justified in treating the rope as having *initially* zero stress-energy, as an idealization, but once the box approaches the black hole, the internal tension in the rope will become a non-trivial momentum flux (as different parts of the rope, at different distances from the horizon, pull on each other with different force), and so one has to take account of that stress-energy.[43] One will, therefore, never be able to get the internal energy of the box exactly to zero before one dumps the entropic stuff in it into the black hole.

There is, however, another possible mechanism for producing arbitrarily large violations of the Generalized Second Law if one treats classical black holes as truly thermodynamical objects. Put a Kerr black hole in a reflecting box and pervade the box with thermal electromagnetic radiation at a lower (Planck) temperature than the classical Bekenstein-Hawking temperature of the black hole. The black hole will eventually absorb the thermal radiation: heat would spontaneously transfer from a system at a lower temperature to one at a higher temperature, a seeming violation of the Generalized Second Law.[44] First, one should note that this is *not* a violation of the Generalized Clausius Postulate, as the irreducible mass and so the area of the black hole increase after absorption. If one takes the Generalized Clausius Postulate as the appropriate formulation of the Generalized Second Law in the context of classical black-hole mechanics and thermodynamics, as the ordinary Clausius Postulate is in classical thermodynamics alone, then there is no violation of the Generalized Second Law.

Although I think this response is correct, it may still seem unsatisfying in so far as it still looks as though there may be violations of the generalized principle of entropy increase (*i.e.*, what is standardly called the Generalized Second Law in the literature). Now, in order to justify the claim that this constitutes a violation of the generalized principle of entropy increase, in the sense that the sum of the external entropy and black-hole area is less after absorption than it was before, one has to verify that in fact the increase in the black hole's entropy after it absorbs all the radiation (and settles

[42] This is not the same issue as arises with arguments over the possibility of a Maxwell demon, though the demon may have to make arbitrarily fine measurements in order to function. The Maxwell demon in classical thermodynamics will eventually thermalize, and so one will have to continually produce a new demon from a low-entropy source in order to produce arbitrarily large deviations from the Second Law, whereas nothing in the black-hole case thermalizes, so if one could make arbitrarily precise measurements then one should be able to systematically produce arbitrarily large deviations from the Generalized Second Law.

[43] See Thorne, Price, and MacDonald (1986) for detailed calculations taking account of the rope's stress-energy during such a lowering process.

[44] I thank Robert Wald for proposing this example to me.

back down to equilibrium) will be less than the entropy originally contained in the radiation. The energy content of the radiation is proportional to its (Planck) temperature raised to the fourth power, $T_P^4$, and its entropy to $T_P^3$. For simplicitly, assume that those powers of $T_P$ just are the radiation's temperature and entropy respectively. The increase in the entropy of the black hole then is its increase in area. Because $A = E^2$ (ignoring constant factors), $\Delta A = (E + T_P^4)^2 - E^2 = T_P^8 - 2ET_P^4$, which may be greater or less than $T_P^3$ depending on the values of $E$ and $T_p$ (and the ignored constants).[45]

Now, one may want to say that this shows that classical black holes cannot be conceived consistently as thermodynamical objects, in so far as we may have here a case of heat spontaneously flowing from a colder to a hotter system, irrespective of whether or how total entropy changes. In the event, however, the violation turns out to depend crucially on the fact that in this case one models the radiation as a *quantum* system, while treating the black hole as a purely classical system—for one will get *exactly the same behavior* for any classical thermodynamical system put in place of the classical black hole. Put an ordinary classical thermodynamical system (*e.g.*, a classical fluid) in a reflecting box and pervade the box with thermal radiation at a lower (Planck) temperature, $T_P$, than that of the classical fluid, $T$. Because the radiation is modeled using quantum mechanics and the fluid using classical thermodynamics, it is ambiguous how to model their interaction and joint evolution. There are two possibilities. First, one may assume that the fluid will absorb the radiation. Second, one may assume it does not. In either case, because the fluid is modeled using classical thermodynamics, it will not *emit* any radiation, black body or otherwise. It is only the first possibility that interests us here, because that is the case analogous to the black hole's behavior.

To determine whether or not there is a violation of the generalized principle of entropy increase, note first that the change in the fluid's temperature, after it absorbs the radiation, will be $C_V T_P^4$, where $C_V$ is the fluid's specific heat. Because the fluid's initial entropy is $E/T$ (from the Gibbs relation, $E$ being the inital total energy of the system), the change in the fluid's entropy will be

$$\frac{E + T_P^4}{T + C_V T_P^4} - \frac{E}{T}$$

This may be greater than or less than $T_P^3$ depending on the values of $E$, $T$, $C_V$ and $T_P$. (This may be easily seen from the fact that the first term in the sum diverges as $T_p$ goes to zero, whereas $T_p^3$ does not, and the second term diverges as $T_p$ goes to infinity more slowly than does $T_p^3$ while the first term approaches a constant.) This, however, is exactly the situation as for the classical black hole. Thus, in fact, the classical black hole behaves *exactly like a classical thermodynamical system*, which is the only conclusion I am arguing for.

One may want to conclude that this is not a satisfactory result: *any* systematic way to violate

[45] This simple calculation ignores the fact that the black hole will emit gravitational radiation as it is perturbed by the in-falling electromagnetic radiation. It is possible that, though the resulting gravitational radiation will carry very little, essentially negligible, energy, it may still contain non-negligible entropy, if purely gravitational entropy in such a form has the same outlandishly high relative values as it does for black holes, as the arguments of Clifton, Ellis, and Tavakol (2013) suggest. If that is correct, then it may be that, when the entropy of the gravitational radiation is taken into account, it would not be possible to violate even the principle of entropy increase by this mechanism.

the generalized principle of entropy increase should be considered illegitimate, irrespective of the constraints one has to impose on the systems one models, and how one models them, to arrange it.[46] I think, however, that these arguments rather show that it is simply inconsistent, in the context of thermodynamics, to model one system using quantum mechanics and another using classical thermodynamics, when one treats the systems as interacting, as I suggested in §3. Another possible lesson one may want to draw from these arguments, one I am sympathetic to, is that the appropriate form of the Generalized Second Law should not be the generalized principle of entropy increase, but rather the Generalized Clausius Postulate.

Before turning to what we may learn from my conclusions, if they are correct, I consider a few more possible problems, none of which I consider severe. Indeed, the resolution of all of them lies in showing that, again, the proposed problem really is a problem for treating classical black holes as thermodynamical systems if and only if it is also a problem for ordinary classical thermodynamical systems—again, classical black holes behave in every way like ordinary thermodynamical systems.

As is well known, the surface gravity $\kappa$ is well defined only for stationary black holes; does this mean that my analysis cannot apply to non-stationary black holes? Yes, it is the case that my analysis cannot apply to non-stationary black holes, but that is no problem. Non-stationary black holes are ones out of equilibrium, and so this presents the same situation as obtains in classical equilibrium thermodynamics. I think we often forget that, strictly speaking, temperature in ordinary thermodynamics is well defined only for bodies in (or quite close to) thermal equilibrium. One way to see this is to note that, for systems far from equilibrium, different kinds of thermometric device will return very different readings, as fine details of their different couplings to the system which are negligible for equilibrium systems become non-trivial, in particular due to phenomena manifesting themselves at temporal and spatial scales below the hydrodynamic scale.[47]

Another problem is that it seems as though we can attribute heat to a Schwarzschild black hole only when it is being perturbed. Again, the situation is in fact much the same as in classical thermodynamics, wherein it never makes sense to attribute a definite quantity of heat to an isolated system in equilibrium. The only definite claims we can make, as Maxwell himself so insightfully and eloquently discussed (footnote 31), are about the quantification of heat *transfer*. In any event, one *can* extract both "heat" and work from a Schwarzschild black hole by perturbing it; indeed, this is in excellent analogy with ordinary thermodynamical systems that have reached heat death, from which heat and work can be extracted only if one perturbs them properly. In fact, the analogy is even better than that brief remark suggests: stationary classical black holes do not "radiate heat", but neither do ordinary classical thermodynamical systems in equilibrium; classical systems exchange heat only when they are in direct contact (contiguous) with another system at a different temperature, but the same holds for stationary classical black holes, in so far as their immediately contiguous environment is "at the same temperature", *viz.*, has essentially the same effective surface

[46] I believe this is Wald's reaction to the situation.

[47] See, *e.g.*, Benedict (1969, §§4.1–4.4, pp. 24–9). This reference is not the most up-to-date with regard to the international agreement on defining the standard, practical methods for the determination of temperature, but I have found no better reference for the nuts and bolts of thermometry. See Curiel (2010, §3.4) for a discussion of the details.

gravity as measured at infinity, as the black hole does. Still, one may protest, in the construction of the Carnot-Geroch Cycle, I ignored perturbations to the black hole from the lowering of the box, so how can one say, given my definitions and arguments, that energy was extracted from it? Given the assumption that the total energy of the box is negligible compared to the mass of the black hole, I claim it is a good approximation to ignore any perturbations to the black hole while still accounting for the (relatively negligible) amount of energy the box gains by being lowered through the black hole's "gravitational field".

Another potential problem: it is clear that black holes have, by the standard definition, negative specific heat, since their surface gravity decreases as their mass-energy increases. Standard arguments, however, conclude that two bodies with negative specific heat cannot thermally equilibrate. There is, though, a hidden assumption in the standard arguments, to wit, "conservation of heat"—it is always assumed, that is to say, that for two bodies in thermal contact one can gain heat only if the other loses it, and that in the same amount. Heat, however, is not a substance, as everyone from Maxwell (1888) to Planck (1926) to Sommerfeld (1964) is at pains to emphasize, and so obeys no conservation law. There is no reason why two bodies with different temperatures in thermal contact cannot both "gain or lose heat from or to each other" at the same time. When two black holes in quasi-stationary orbit[48] about each other equilibrate, the temperatures of both bodies simultaneously decrease as they both gain heat from the other, the one of higher temperature decreasing more quickly than the other, so they will eventually reach the same temperature.

A potentially more serious problem with my analysis is that it is difficult to see what sense can be made of "exchange" between a global energetic quantity (in the case of stationary, asymptotically flat black holes, ADM mass) on the one hand, and localized stress-energy of ordinary systems on the other. A more poignant way of posing the problem is to note that gravitational energy is strictly non-local in the precise sense that there is no such thing as a gravitational stress-energy tensor (Curiel 2014b), and so it satisfies no general conservation law. How, then, can one talk about exchange for such a *recherché* quantity?[49] There are, I think, two responses to this problem, one stronger than the other. The first, weaker, response is that one always has in place a quasi-local notion of mass-energy in stationary and axisymmetric spacetimes, which suffices for the purposes of my arguments, just as it does in Newtonian gravitational theory (*à la* the "Poynting integral" of Bondi 1962). The stronger response, which is more to the point, is that neither is heat a localized form of energy in classical thermodynamics—it is not a perfect differential (as the discussion of Sommerfeld 1964 makes particularly clear), and so it also has no corresponding conservation law—just like gravitational energy—and yet we feel no inconsistency in talking there about exchange of energy for a quantity that can be represented only as a total magnitude, with no corresponding localized density. Sauce for the goose is surely sauce for the gander.

My arguments, I think, have not only residual possible problems; they also open the possibility for real insight into existing questions about black-hole mechanics and thermodynamics. Although

[48] There are no solutions to the Einstein field equation representing two Kerr black holes in stable orbit about each other (Manko and Ruiz 2001).

[49] I thank Jim Weatherall for pushing me on this point.

the following is not a problem peculiar to my analysis, it is a general one in the field I believe my analysis can give some insight into. Black holes have enormous entropy, far more than any reasonably conceivable material system that could form them on collapse (Penrose 1979). There must, therefore, be a correspondingly enormous and discontinuous jump in entropy when a collapsing body passes the point at which an event horizon forms. How can one explain that? It is here that I believe my old-fashioned approach to entropy bears some of its sweetest fruit. More modern characterizations of entropy, whether of a Boltzmannian, Gibbsian, von-Neumann-like, or Shannon-like form, have no explanation for this jump. If, however, one conceives of entropy as a measure of how much work it takes to extract energy from a system, how much free energy a system has, what forms its internal energy (as opposed to free energy) are in, then black holes have enormous energy, only a very small amount of which is extractible, and there is a clear physical discontinuity in extractability of energy when an event horizon forms.

I leave the reader with a question concerning this entire field that, though not peculiar to my arguments here, I feel strongly needs to be investigated further by both philosophers and physicists. The Laws of thermodynamics are empirical generalizations, indeed, the paradigm of such. I know of no other fundamental propositions in physics whose support comes *entirely* from experimental evidence, with not even the suggestion of the possibility of a formal derivation from "deeper" physical principles. Also, I know of no other propositions, with the possible exception of the Newtonian inverse-squared distance dependence of gravitational attraction between two bits of matter, that are more deeply entrenched empirically than the Laws of thermodynamics. But, entirely to the contrary, and with the exception only of the Third Law (which is also the most weakly supported by experimental evidence in classical thermodynamics), all the Laws of black-hole mechanics are theorems of differential geometry. They require no input from physical theory at all. One will sometimes see the claim that one or the other of the Laws requires the assumption of the Einstein field equation, but this is not true: all the Laws are independent of the Einstein field equation in the strong sense that one can assume its negation and still derive the Laws; the Einstein field equation enters only when one wants to give a physical interpretation of the quantities involved by way of its asserted relation between the Ricci tensor and the stress-energy tensor of matter.[50] So how can laws that, in one context, are nothing but empirical generalizations, magically transform into mathematical theorems when extended into a new context?

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[50]See Curiel (2014c) for a thorough discussion.

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