Would the Existence of CTCs Allow for Nonlocal Signaling?

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1 Introduction

Quantum information has been a majorly productive research program over the last two decades. It has enabled scientists to make progress theoretically, experimentally, and in terms of the development of technology. However, our understanding of what quantum information is telling us about the world—and what it is telling us about quantum theory itself—remains underdeveloped. Broadly speaking, this paper is an attempt to reconcile the conceptual framework at play in the metaphysical approach to the foundations of quantum mechanics with the framework at play in quantum information science.

In this paper, I focus on a debate about the predicted behavior of quantum systems in the presence of a localized region of spacetime subject to nonlinear laws of evolution. The particular example under consideration is that of a closed timelike curve, or CTC. In this literature, the peculiarities of how time travel could possibly be achieved is not addressed. Rather, CTCs are treated as a resource. In this context, we can formulate the question "what could we do if we had access to a CTC?"

David Deutsch's seminal 1991 paper (Deutsch 1991) set the groundwork for a quantum mechanical analysis of the information-processing capabilities of a quantum system augmented by access to a CTC. Over the last decade, interest has flourished in the particular computational tasks that can be achieved with a CTC-assisted quantum computer circuit. However, a debate has arisen surrounding a particularly strange result: the ability to distinguish non-orthogonal quantum states. This is not allowed in ordinary quantum theory, but the Deutsch's analysis of the behavior of quantum systems in the presence of a CTC seems to predict it.[1] Furthermore, this ability leads to

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[1] In this paper, I will be working exclusively with Deutsch's CTC model for quantum

an even more radical conclusion: quantum CTCs allow for information to be sent between arbitrarily distant parties instantaneously.

However, there has been serious resistance to this conclusion from various parties to the debate. And the attempts to formulate exactly why this admittedly non-quantum-mechanical behavior should be ruled out has been illuminating of the underlying assumptions that are often at play, even in a field such as quantum information, which purports to be formulated entirely in operationalist terms, and neutral with respect to interpretational debates.

A CTC-assisted quantum computational circuit may seem like an exotic example. But analyzing these kinds of systems has proved to be very fruitful. Working with this example has brought to light several common confusions about one of the central concepts of quantum theory: nonlocality. In the foundations literature, quantum nonlocality is often closely connected to the concepts of information and causation. For example, it is often said that the nonlocal correlations quantum mechanics are allowed at spacelike separation because no information is traveling between the two distant systems A and B faster than a light signal could. In cases where nonlocality is exploited as an information channel (as in the quantum teleportation protocol) then quantum information is not present (or useable) at B until after a classical (lightlike) signal is received from A.

I will argue that the exploitation of quantum correlations to send a message—as allowed by the existence of CTCs—should be distinguished from cases where a carrier of information is physically traversing the space between two distant points faster than the speed of light. These two notions are often conflated. A consequence of this argument is that relativity prohibits only the latter, and doesnt rule out the possibility of (what I term) Nonlocal Signaling.

This is contrary to a foundational principle of the quantum information-theoretic approach. The No-Signaling Principle plays a fundamental role in the formulation of quantum information. I argue that this accounts for the resistance to the conclusion that it can be violated under certain conditions. Furthermore, I argue that the No-Signaling Principle's inclusion as a fun-

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systems (D-CTCs). There is an alternative proposal for how to understand the behavior of quantum systems in the presence of CTCs, referred to as P-CTCs. While there are significant differences between the predictions and underlying physics of the two proposals, both allow for the behavior under consideration in this paper, i.e. distinguishing non-orthogonal states, and Nonlocal Signaling. In fact, Gisin showed (Gisin 1990) that a more general nonlinear framework would allow for the same behavior.

damental postulate about the nature of the quantum world, as is the case in the quantum information-theoretic interpretation of quantum theory, as advocated in Bub and Pitowsky's "Two Dogmas About Quantum Mechanics", represents a commitment to a principle-theoretic conception of quantum theory.

Whereas a "constructive" theory is built up from its ontology and dynamics, the fundamental formulation of "principle" theory is "top-down", in terms of inviolable global principles (the paradigm case is special relativity). Principle theories explicitly deny the fundamentality of a theory's ontology, and consider constructive formulations of theories to be secondary, and to have a role only as proofs of the consistency of their principles. I suggest that the framework developed by Deutsch is inconsistent with these commitments, and therefore the justification for including the No-Signaling Principle in the D-CTC framework offered by quantum information theorists is indefensible.

This paper will examine the recent debate surrounding this point in the foundations literature. I will address each argument for the impossibility of Nonlocal Signaling. I will argue that each of these arguments falls short of its goal either for technical reasons, or for reasons of insufficient justification for imposing global constraints on the possible correlations allowed in this context.

I will argue that there is a deeper motivation at play for rule out the possibility of Nonlocal Signaling, which has to do with the fundamental commitments of the quantum information approach. I argue, however, that the framework developed by Deutsch is inconsistent with these commitments, and therefore the justification for including a No Signaling principle in the D-CTC framework offered by quantum information theorists is indefensible.

2 Deutsch's Circuit Model for CTCs

In his well known (1991) paper, Deutsch introduced a model for the analysis of the physical behavior of CTCs. Prior to his work, the standard way of analyzing the physical effects of chronology-violating regions of spacetime was in terms of their underlying geometry. Deutsch considered this approach to be insufficient because it fails to take quantum mechanical effects into account. He proposed an alternative approach which involves analyzing the behavior of CTCs in terms of their information processing capabilities.

He begins his account by defining a notion of equivalence between spacetime-bounded networks containing chronology-violating regions. A network in this context is to be understood as a spacetime geometry which takes as input the initial state of a physical system and outputs the system's final state. Two networks are *denotationally equivalent* if their outputs are the same function of their inputs. That is to say, regardless of whether two networks have differing spacetime geometries if the function that maps their initial states to their final states is the same, they are denotationally equivalent.

Next he introduces the idea that the transformation between any two denotationally equivalent networks is trivial. Insofar as we are interested in analyzing CTCs in terms of their physical effects (that is, their output given a certain input), we are free to use the simplest model available in the denotational equivalence class of a particular network for the purpose of our analysis of the information flow through a CTC.

The final step of his proposal is to introduce a simple standard form into which any spacetime-bounded network can be trivially transformed for the purpose of analysis. The simple standard form involves translating all spacetime-bounded networks into circuits in which each particle traveling in the original network is replaced by sufficiently many carrier particles, each of which have a single 2-state internal degree of freedom (a bit). The regions in which the particles interact are localized (by denotationally trivial transformations) into gates, such that the particles are inert while traveling between them. And finally, all chronology-violating effects of the network are localized to sufficiently many carrier particles on closed loops, which only interact with chronology-respecting particles in gates.

Deutsch points out that chronology violation itself makes no difference to the behavior of a network unless there is a closed loop of information. In the original network, this closed information path could potentially not be confined to the trajectory of any single particle (since the carriers can interact with each other), but for any such network, there is a denotationally trivial transformation which will localize the closed information path on sufficiently many carriers on closed paths.

The real innovation of this approach is that it can very easily accommodate quantum mechanical effects by relaxing the requirement that the carrier particles be in a well-defined classical state after interactions. If viewed classically, networks containing chronology violations can lead to paradoxes that seem to put unnaturally strong constraints on possible initial conditions of physical systems (e.g. you are somehow prohibited from getting in the time machine that would take you back to kill your grandfather). Deutsch uses his model to argue that, when quantum mechanics is taken into account, these unnatural constraints on initial states disappear. Deutsch's fixed point theorem states that CTCs "place no retrospective constraints on the state of a quantum system" (Deutsch 1991, 3203). That is to say, for any possible input state, there will be a paradox-free solution.

This is the result of a consistency condition implied by the quantum mechanical treatment of time-traveling carrier particles interacting with later versions of themselves. If we let $|\psi\rangle$ be the initial state of the "younger" version of the carrier particle, and let $\hat{\rho}$ be the density operator of the "older" version of the carrier particle, then joint density operator of the two particles entering the region of interaction is

$$|\psi\rangle\langle\psi| \otimes \hat{\rho}$$

and the density operator of the two carrier particles after the interaction is

$$U(|\psi\rangle\langle\psi| \otimes \hat{\rho})U^{\dagger}$$

where U is the interaction unitary. The consistency condition requires that the density operator of the younger version of the carrier particle as it leaves the region of interaction is the same as that of the older version as it enters the region of interaction.

$$\hat{\rho} = \mathrm{Tr}[U(|\psi\rangle\langle\psi| \otimes \hat{\rho})U^{\dagger}]$$

This makes intuitive sense, because it is the interaction that causes the earlier version of the carrier particle to become the later version. When translated via a denotationally trivial transformation to a network in which the chronology-violating behavior is localized to a single particle on a CTC, and it interacts with a chronology-respecting (CR) carrier particle, the consistency condition for the CTC system is

$$\rho_{\mathrm{CTC}} = \mathrm{Tr}_{\mathrm{sys}}[U(|\psi\rangle\langle\psi| \otimes \rho_{\mathrm{CTC}})U^{\dagger}].$$

This requirement says that the density operator of the system on the CTC *after* the interaction is the same as it was *before* the interaction. That is to say, after the interaction, the carrier particle on the CTC enters the "future mouth" of the wormhole, and exits the "past mouth" of the wormhole *before* the interaction. The state of the particle that comes out of the past mouth must be the same as the system that enters the future mouth. Furthermore,

ρ_{CTC} depends on $|\psi\rangle$, so the input state on the causality-respecting carrier particle has an effect on the state of the particle it will interact with.

In light of this dependence, Deutsch's claim that CTCs, when properly understood, place no constraints on the possible states of the quantum system may be stronger than is warranted. While it is true that, unlike the classical analysis of time travel paradoxes, his model places no constraints on the input state of the causality-respecting system, it *does* constrain the possible states of the system confined to the CTC.

While this seems more intuitively plausible than the classical time travel paradoxes, it is nonetheless puzzling. In the classical case, it is somehow forbidden that I get in the time machine that will take me back to kill my grandfather. There isn't necessarily any obvious causal mechanism that prevents me. It is simply impossible, to avoid paradox, that I ever actually carry out my mission. This constraint is often described as *superdeterministic*, since it is something above and beyond simple determinism that rules out the possibility of me getting into the time machine. David Lewis's influential formulation of the classical consistency condition from his (1976) alleviates some of this tension by redescribing the time travel narrative as a single, self-consistent history. The drawback of this approach is that it seriously undermines the notion that the time traveler has free will.

In Deutsch's model, this tension is seemingly resolved. Any initial state of the system is allowed—the time traveler could enter the time machine with any intentions whatsoever. Consistency is guaranteed by the state of the system confined to the CTC. This doesn't offend the intuitions as badly as the classical case, because we can imagine the following pseudotime narrative: The causality-respecting qubit begins its journey in some initial state, then encounters and interacts with CTC quibit, precipitating a change of state of both of them. The CTC qubit in its new state then travels back in time to again interact with the causality-respecting qubit (in its initial state), and the interaction again changes the state of the CTC qubit. Over infinite iterations of this process, the CTC qubit converges on some particular state, like a top that's rotation stabilizes after some initial wobbling. The CR qubit *causes* the CTC qubit to be in the right state.

The puzzle arises, though, when we note that the CTC qubit must *always* have been in this stable state. There are no previous interactions with the CR qubit to force it to evolve over time into the right state. So although Deutsch's model has avoided the superdeterminism of the traditional time travel paradoxes, which constrained the initial states of the CR system, it seems to have introduced significant kinematic constraints in another place. Something like Lewis's classical consistency condition must still be at play. That is to say, there must be a deeper metaphysical justification (i.e. the impossibility of a self-contradictory history) which is behind Deutcsh's quantum condition. And as we'll see in Section 5.2, Deutsch seemingly has something like this in mind.

Deutsch's analysis of the physical effects of chronology-violating regions of spacetime in terms of quantum computational circuits and the consistency condition has been very influential in the study of quantum information, and has led to many interesting insights about the nature of the quantum world. One particularly interesting result is due to Brun, Harrington and Wilde. In what follows, I will discuss their work, the debate surrounding their central claim, and further implications of their argument.

3 The BHW Circuit

In (Brun et al. 2009), the authors described a procedure for using CTC-assisted quantum computational circuits to distinguish between non-orthogonal states of a qubit. In this section, I will describe the protocol for distinguishing between the linearly depenent BB84 states $|0\rangle$, $|1\rangle$, $|+\rangle$ and $|-\rangle$, where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$.

3.1 Details of the BHW Circuit

The authors begin by detailing a protocol for distinguishing between two non-orthogonal states. The setup involves two qubits: system A in the unknown initial state $|\psi\rangle$ (either $|0\rangle$ or $|-\rangle$), and system B, a qubit in some state $|\phi\rangle$ on a CTC. The procedure is simple: (1) perform a SWAP of systems A and B, (2) perform a controlled-Hadamard transformation with system A as the control and system B as the target, and (3) measure system A in the computational basis. A measurement of system A that yields the output $|0\rangle$ means that the input state $|\psi\rangle = |0\rangle$. A measurement of system A that yields the output $|1\rangle$ means that the input state $|\psi\rangle = |-\rangle$.

This result obtains because of Deutsch's consistency condition. Whatever state system B is in when it enters the future mouth of the wormhole must be the same state that comes out of the past mouth of the wormhole. That is, steps (1) and (2) must have no net effect on system B. The density matrix of the system on the CTC (system B) depends on the input state of system A:

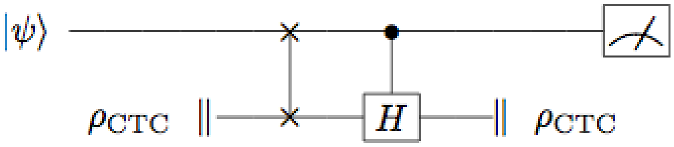
$$\rho_{\mathrm{CTC}} = \mathrm{Tr}_{\mathrm{sys}}[U(|\psi\rangle\langle\psi| \otimes \rho_{\mathrm{CTC}})U^{\dagger}]$$

and the output of the circuit (i.e. the final state of the CR quibit) depends on the input of system A and ρ_{CTC}:

$$\rho_{\mathrm{output}} = \mathrm{Tr}_{\mathrm{CTC}}[U(|\psi\rangle\langle\psi| \otimes \rho_{\mathrm{CTC}})U^{\dagger}].$$

Since the only two possible input states are $|\psi\rangle = |0\rangle$ and $|\psi\rangle = |-\rangle$, the consistency condition requires that the only possible initial states of system B are $|\phi\rangle = |0\rangle$ and $|\phi\rangle = |1\rangle$.

Figure 1: BHW circuit for distinguishing between $|\psi\rangle = |0\rangle$ and $|\psi\rangle = |-\rangle$ (from Brun et al. 2009).



Consider the situation where the input state of system A is $|\psi\rangle = |0\rangle$. If the initial state of system B is $|\phi\rangle = |1\rangle$, then the effect of the first gate (SWAP) would be to transform system A into the state state $|1\rangle$ and system B into the state $|0\rangle$. Since system A is in the state $|1\rangle$, the action of the second gate (controlled-Hadamard with A as the control and B as the target) would transform system B into the state $|+\rangle$. Since the consistency condition requires that the state of B after the action of the two gates is the same as the state of B before the action of the two gates, it is clear that $|\phi\rangle = |1\rangle$ is not an allowed initial state of system B when $|\psi\rangle = |0\rangle$.

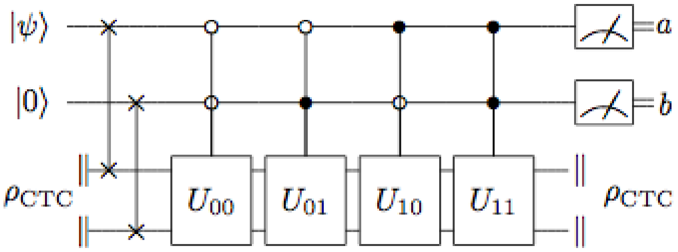
However, if the initial state of system B were $|\phi\rangle = |0\rangle$, then after the action of the first gate (SWAP), system A would be in state $|0\rangle$ and system B would be in state $|0\rangle$. The second gate (controlled-Hadamard) would not be activated since the control qubit is in state $|0\rangle$, so the consistency condition for system B holds. The measurement of system A would yield a result of $|0\rangle$, which indicates that the initial input state was $|\psi\rangle = |0\rangle$.

Now consider the case where system A is initially in the state $|\psi\rangle = |-\rangle$. If the initial state of system B is $|\phi\rangle = |0\rangle$, then after the action of the first gate (SWAP), system A would be in the state $|0\rangle$ and system B would be in the state $|-\rangle$. Since A is the control qubit for the second gate (controlled-Hadamard), it would not be activated and system B would pass through unchanged. It would therefore enter the future mouth of the wormhole in the state $|-\rangle$, violating the consistency condition.

However, if system B had initially been in the state $|1\rangle$, after the first gate, system A would be in the state $|1\rangle$ and system B would be in the state $|-\rangle$. The control qubit would activate the controlled-Hadamard gate, and system B would be transformed into the state $|1\rangle$, which is consistent with its original state. The measurement on system A will yield a result of $|1\rangle$, which indicates that the input was initially $|\psi\rangle = |-\rangle$.

Brun and his collaborators were able to scale this protocol up to allow for the discrimination between the four non-orthogonal BB84 states $|0\rangle$, $|1\rangle$, $|+\rangle$ and $|-\rangle$. They achieve this by adding an ancillary chronology-respecting quibit in the state $|0\rangle$, using two CTC-bound qubits, performing two SWAPs and four controlled unitary transformations, and making two measurements.

Figure 2: BHW circuit for distinguishing the four BB84 states (from Brun et al. 2009).



The unitary transformations are as follows:

$$U_{00} \equiv SWAP$$

$$U_{01} \equiv X \otimes X$$

$$U_{10} \equiv (X \otimes I) \circ (H \otimes I)$$

$$U_{11} \equiv (X \otimes H) \circ (\mathrm{SWAP})$$

The circuit performs the following map ($|\psi 0\rangle \to |ab\rangle$):

$$|00\rangle \to |00\rangle$$

$$|10\rangle \to |01\rangle$$
$$|{+}0\rangle \to |10\rangle$$
$$|{-}0\rangle \to |11\rangle$$

3.2 Using BHW to Signal

In Cavalcanti et al.'s (2012), the authors point out that the evolution of the quantum state through the BHW circuit which allows for the possibility of distinguishing the BB84 states is of the right kind to fit into a protocol for instantaneous signaling proposed by Gisin (1990).

Gisin's proposal involves two players, Alice and Bob, each sharing one half of a singlet pair. Alice measures her particle either in the X direction (yielding $|1\rangle$ or $|0\rangle$) or the Z direction (yielding $|+\rangle$ or $|-\rangle$), forcing Bob's particle into the same state. Bob then subjects his particle to a nonlinear evolution of a certain type that allows him to determine its state. Gisin proposed a particular nonlinear Hamiltonian that would do the job, but Cavalcanti and Menicucci point out that the BHW circuit has the right features to fit into this framework.

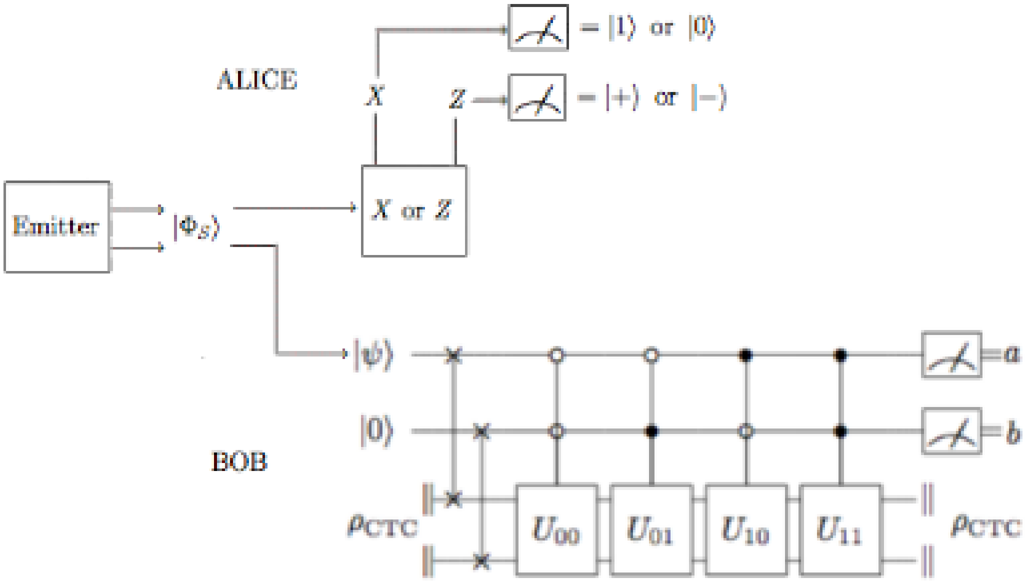
The BHW circuit will allow Bob to perfectly distinguish between all four states. Therefore, if Alice wants to send a 1-bit message, she can choose either to measure in the X direction (for "yes") or the Z direction (for "no"). Bob, using the BHW circuit, can recover Alice's message, which is transmitted instantaneously. That is, if the output of Bob's device is $|10\rangle$ or $|11\rangle$, he knows Alice measured her half of the singlet pair in the X direction (intending the message to be "yes"), and if his results are either $|00\rangle$ or $|01\rangle$, he knows she measured in the Z direction (meaning "no").

3.3 The Bub-Stairs Consistency Condition

In a recent paper (2014) Jeffrey Bub and Allen Stairs propose a consistency condition to solve one of the outstanding conceptual problems with Nonlocal Signaling. Their condition solves some potential ambiguity associated with the possibility of signaling.

The issue that the consistency condition is designed to solve arises because of the fact that the nature of Nonlocal Signaling allows for cause/effect to happen at spacelike separation. Alice's choice of measurement causes Bob to get the result he does faster than a light signal would have been able to

Figure 3: Cavalcanti and and collaborators' proposal for using the BHW circuit in Gisin's instantaneous signaling device. Alice measures first measures her particle along the X or Z axis. Bob then uses the BHW circuit to determine what state his particle is in.



traverse the distance. Since the event of Alice's input and the event of Bob's output are at spacelike separation, observers in different frames will disagree about which event comes before the other.That is, for some observers, Bob will measure his particle before Alice measures hers. In those frames, Alice and Bob's shared Bell State will not have been disentangled by Alice's measurement, and therefore Bob will input a particle in the state $I/2$ into the BHW circuit, which will yield any of the four possible outcomes with equal probability, meaning that he has a 1/2 probability, in that frame, of getting an output that corresponds to the wrong input for Alice. In frames where Alice measures first, her choice determines Bob's output by disentangling their shared Bell State, leading to Bob measuring a particle in a definite state with the BHW circuit. In frames where Bob measures first, he inputs his still-entangled particle into the BHW circuit, yielding each of the four possible outputs with equal probability, regardless of the input Alice later chooses.

To protect against problem, they introduce a simple and elegant new consistency condition. It consists of the conjunction of the following two claims:

(C1) Observers in differently moving reference frames agree on which events occur, even if they disagree about the order of events.

(C2) If an event has zero probability in any frame of reference, it does not occur.

C1 ensures the two observers would agree about the outcomes of the two measurements (namely that the output Bob gets corresponds to the input Alice makes, regardless of who goes first). C2 ensures that the contradiction will never arise, since according to one observer, the probability of Bob getting the outcome that is inconsistent with Alice's input is 0.

While the consistency condition seems unobjectionable, I'll argue in Section 5.3 that the conclusions Bub and Stairs attempt to draw from it are more problematic.

4 Arguments Against Nonlocal Signaling

4.1 The Linearity Trap

In a reply, Bennett et al. argue that the BHW circuit could not in fact be used to distinguish between non-orthogonal states. This is because the adversarial nature of such a task would preclude a player from having the right kind of knowledge of the input state to be able to use the circuit. They argue that, in the context of CTC-assisted computation, it is not generally true that the evolution of a mixture is equal to the corresponding mixture of the evolutions of individual states. They call failing to see this distinction "the linearity trap", and claim that Brun and his collaborators' argument falls into it. they argue that while the BHW circuit could (for example) map the definite input state $|-0\rangle$ to the output $|11\rangle$, it fails when faced with a mixed input state.

In general in quantum mechanics, a state density operator can be used to represent two different kinds of "impure" states—a probability distribution over a classical mixture, and a system which is entangled with some other system not included in the representation.

That is to say, if the player Alice were trying to determine the state of a system prepared for her by an adversary Rob, she would be unable to.

This is because (the argument goes) the output of the BHW circuit when acting on a mixed input state will itself be a mixture, and will not yield the desired information. That is to say, Alice creates a density matrix encoding her epistemic relationship to the input (she assigns probabilities to the four possible inputs based on her knowledge of Rob's proclivities, say). When the circuit acts on *this* object, it will not yield a definite result telling Alice the actual input. As Bennett et al. say:

> Much of the apparent power of CTCs and nonlinear quantum mechanics comes from analyzing the evolution of pure states, and extending the results linearly to find the evolution of mixed states. However, because mixed states do not have a unique decomposition into pure states this does not give an unambiguous rule for evolution. Indeed the very nature and meaning of mixed states may be ill defined in such theories." (Bennett et al. 2009, 4)

They go on to claim that a theory with the resources for identifying the "correct" decomposition of mixed states into pure states would return the power to CTCs, but at the cost of being a theory that fails to reduce to standard quantum mechanics in regions distant from the CTC.

4.2 Cavalcanti and Menicucci Reply

In a response to this argument, Cavalcanti and Menicucci point out that this clearly isn't what Brun and collaborators had in mind. They argue that Bennett et al. are confusing the density matrix which Alice may create to encode her partial knowledge of the input with an actual physical mixed input state. Rob has (by construction) prepared and presented to Alice a pure state, and Alice therefore inputs a pure state into the device. If it were true that the output depended on the density matrix Alice constructs to encode her partial knowledge of the *actual state*, then the device would not work deterministically *even for Rob.* Therefore the nonlinear action of the device could not be verified, and we would not even be justified in claiming to have knowledge of how it would act on a pure state. As they say:

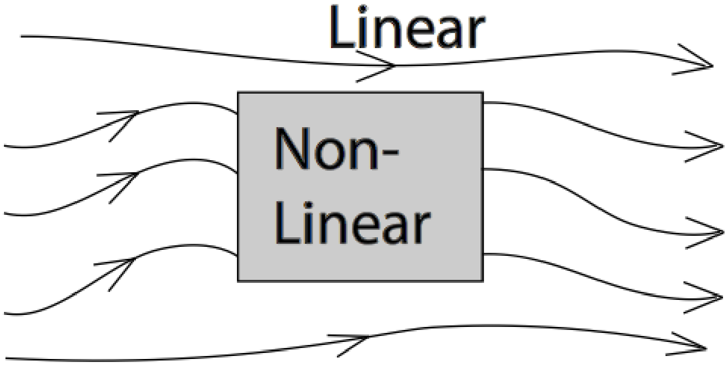
> In summary, empirical verifiability of the deterministic, nonlinear action of some physical device for some set of pure-state inputs generally precludes using density matrices to represent *proper mixtures* of such inputs (Cavalcanti and Menicucci 2010).

That is to say, in the context of CTC-assisted quantum computation, density matrices cannot be used to assign probabilities over possible pure-state inputs to a device. Alice will need to find another way to encode her epistemic state. However, Bennett et al. have identified a potential problem for this protocol. We have, by construction, been considering cases where Rob can unproblematically produce pure quantum states. However, if he were unable to do so, and entanglement persisted between the preparation apparatus and the system being prepared, the system presented to Alice would genuinely be in a mixed state, and the argument made by Bennett et al. would go through. The implications of this fact are beyond the scope of the present paper.

4.3 The Preparation Problem

Cavalcanti and his collaborators argue against the possibility of signaling in this way. They claim that superluminal signaling is ruled out by special relativity, and we should therefore conclude that something has gone wrong in the argument that led us to predict this effect. They begin by defining a notion of a nonlinear extension of quantum mechanics that reduces to ordinary quantum mechanics everywhere outside of a particular spacetime region. That is, they describe an ordinary quantum world, with one localized extraordinary region, where the output states are a nonlinear function of the input states. They refer to this region as the "nonlinear box".

Figure 4: A localized region of nonlinear evolution, from (Cavalcanti et al. 2012).



According to Deutsch's model, the effects of the existence of a CTC can be localized to a particular region by a denotationally trivial transformation, so locating all of the effects of a CTC in a nonlinear box is allowed. In this model, standard linear quantum mechanics is true everywhere outside the box, and the only nonlinear evolutions happen to systems inside the (or traveling through) the box.

In order to block the failure of the No Signaling principle, they argue that different preparation procedures that unproblematically yield the same pure state according to linear quantum mechanics, will in fact yield different states, which will have different effects in the context of this nonlinear extension of quantum theory. Specifically, even though we consider the following two preparation procedures to yield the same pure state in linear quantum mechanics, the equivalence fails in the theory including the nonlinear box:

> **Procedure 1** Measure the an ensemble of qubits in the computational basis, and post-select those that are in the state $|1\rangle$.
>
> **Procedure 2** Take an ensemble of pairs of maximally entangled qubits in the Bell state $|\Phi^+\rangle$, and measure the states of the A quibits in the computational basis, then post-select the B qubits that were entangled with the A qubits for which the measurement result was $|1\rangle$.

As Cavalcanti and his collaborators say:

> Signaling can be avoided *only if* the remote preparations in [Procedure 2] are not in the same equivalence class as the corresponding preparations in [Procedure 1] when nonlinear transformations are considered. (Cavalcanti et al. 2012, 3)

But this prescription leads to what they call the "preparation problem", which has two parts. Firstly, if it is the case that a pure quantum state does not uniquely determine a system's evolution through a nonlinear box (because it may be the result of either preparation procedure), then the formalism of quantum states seems to be insufficient to account for the physical situation. Secondly, the model is incomplete without a specification of which preparation procedures fall into which equivalence classes in the nonlinear box.

4.4 Reply to Cavalcanti et al.

Cavalcanti et al. argue that, since instantaneous signaling is impossible, something must have gone wrong with the analysis of the BHW circuit that led to the prediction that Alice could send a signal to Bob.

> Under these assumptions, Alice may send information to Bob instantaneously. Since consistency with relativity forbids this, we must rethink our assumptions. (Cavalcanti et al. 2012, 4)

In this section I will argue that the constraint on superluminal travel from special relativity is distinct from—and does not entail—a prohibition against signaling via the exploitation of quantum nonlocality.

This question is given a more thorough treatment in (Dunlap 2014b). But here it should be sufficient to note the following points. Firstly, Tim Maudlin argues for a similar conclusion in his book *Quantum Non-locality and Relativity*, saying

> And we have further found that none of the restrictions can be derived, in any strictly formal sense, from the Lorentz transformations, or from the fundamental relativistic space-time structure. On the contrary, explicitly relativistic theories of tachyons and of superluminal signals have been constructed. The fundamental feature of the Lorentz transformations is that they leave the speed of light invariant, not that they render it an insuperable boundary. (Maudlin 2011, 173)

Furthermore, Bub and Pitowsky's influential paper on the information-theoretic interpretation of quantum theory explicitly states that the two notions are conceptually distinct, saying:

> Note that 'no signaling' is not specifically a relativistic constraint on superluminal signaling. It is simply a condition imposed on the marginal probabilities of events for separated systems [...] and this might well be considered partly constitutive of what one means by separated systems. (**?**, 443)

Christopher Timpson's analysis of the information-theoretic features of nonlocal quantum effects makes the case that there is no carrier of information being transmitted between the two distant locations. Timpson's analysis takes place in the context of a discussion of quantum teleportation, but the argument applies equally well to the case of quantum signaling.

Timpson argues by way of conceptual (and linguistic) analysis that "information" in the technical sense—as it appears in Information Theory, which he denotes as "information$_t$ so as to distinguish it from the non-technical notion—is an abstract noun, which does not refer to "a spatio-temporal particular, to a concrete entity, or to a physical substance" (Timpson 2013, 74). He argues that we shouldn't think of information as being an entity that is somehow transported from one location to another.

Rather, "information transmission" needs to be understood as a physical process in which a new token of information$_t$ is created at B that is subsequent on the prior existence of another token of that same type having existed at A. There is no *thing* traversing the space between A and B. Rather, there is a physical process that results in another token of the information$_t$ being created. Since no physical entity of any kind need pass between A and B for this to be the case, there is nothing that need travel faster than the speed of light.

In the teleportation protocol, information$_t$ (which, in this case, is the unknown quantum state to be teleported) is tokened at location A, and subsequently tokened at location B, having been caused by a physical process.

He believes that we are led astray when we take the phrase "the information$_t$" to denote a particular.

> The assumption [...] is that we need to provide a story of how some located *thing* denoted by 'the information$_t$' travels from Alice to Bob. Moreover, it is assumed that this supposed thing should be shown to take a spatio-temporally continuous path. (Timpson 2013, 82)

By recognizing "information$_t$" as an abstract noun, we solve this problem. Furthermore, this recognition provides us with the only legitimate reading of the question of how the information$_t$ "got to Bob". It is a question that is answered by reference to the physical processes that produce at B another token of the information$_t$ that was tokened at A. In the case of quantum information protocols, this answer will be quantum mechanical.

> I am not claiming that there is no such thing as the transmission of information$_t$, but simply that one should not understand the transmission of information$_t$ on the model of transporting potatoes, or butter, say, or piping water. [...] The transmission of a piece of information$_t$ from A to B will consist in the production at B of another token of the type produced at A, where the production at B is consequent on the token's being produced at A. (Timpson 2013, 83)

Timpson then goes on to say that the unique feature of the teleportation protocol is that the information$_t$ can be tokened at B subsequent to its having been tokened at A, but cannot be tokened at any point in between A and B in the meantime. However, this feature is clearly shared by the signaling protocol under consideration.

Timpson addresses the possibility the impossibility of signaling and the relativistic constraint in this section.

> The constraint is that superluminal signalling is ruled out on pain of temporal loop paradoxes. What this means is that no *physical process* is permissible that would allow a signal to be sent superluminally and thus allow information to be transmitted superluminally. What are ruled out are certain types of physical processes, not, save a metaphor, certain types of motion of information. (Timpson 2013, 96)

Here I disagree with him. Certain types of motion of information are ruled out. Namely, a single token of information can't travel through the space between two different locations faster than a light signal could.

He says that superluminal signaling is ruled out "on pain of temporal paradoxes". But this assumes that there is no possible solution to the temporal paradoxes, and that the possibility of sending a message into the past necessarily opens the door to paradox.

Here I'll simply note that in the context of the present debate, this issue has been solved by Deutsch's consistency condition, the express purpose of which which was to offer a solution to the threat of paradox generated by the existence of a CTC. More will be said on this topic in Section 5.3.

But one thing is clear: Given that the Cavalcanti et al. paper is participating in this very debate, and given that relativity's only claim against the possibility of signaling is that it would allow for messages to the past, we can take Timpson's analysis of the transmission of information from A to B to undermine Cavalcanti's stated justification for ruling out the possibility of signaling.

Finally, in (Dunlap 2014b) I argue that we need to distinguish between superluminal transmission of information (FTLIT), in which a carrier of information traverses the space between the two distant regions faster than a light signal could, and Nonlocal Signaling (NS), which relies on nonlocal quantum effects, and in which no information carrying system traverses the intermediate space.

It is conceptually possible for one to exist without the other. After all, we could discover that c is not the maximum speed for material particles, without having to invoke quantum effects. And it was an open question as to whether the quantum correlations precisified in Bell's Theorem would allow for the transmission of messages. The quantum formalism itself does not require that there be carrier particles responsible for the nonlocal effects. The existence of the ordinary non-signaling quantum correlations seems to already be enough to be in conflict with relativity, if this were not the case. If we assume that the correlations between the distant measurements must be explained by the transmission of a causal influence through spacetime, then the correlations allowed by ordinary quantum theory would already be in conflict with relativity. It is often argued that this is not the case, because the superluminal causal influence traveling between the two distant regions does not result in any epistemically relevant experimental outcome. That is to say, Bob cannot discover Alice's input based on his output, even though a superluminal causal influence has traveled to him from Alice's experimental setup, causing him to get the particular outcome he does. However, if this is the reason that ordinary quantum correlations are not in conflict with relativity, then information is playing an important role here. The prohibition seems to be weakened to the point where it is no longer that nothing can travel faster than light (in fact, causal influence can, so long as it has no noticeable effect). What is prohibited from traveling faster than light is interpretable information (see e.g. Norton 2013). Maudlin (2011) argues against this point as well.

In the context of the current debate, however, this worry seems misplaced, since the nonlinear evolution that originated this line of inquiry was that caused by the presence of a CTC, which explicitly allows information to travel into the past.[2]

4.5 Signaling and Relativity

I have argued in this section that two attempts to rule out the possibility of signaling in the presence of a CTC fail. The "linearity trap" objection of Bennet et al. was handled by Cavalcanti and Menicucci's analysis of verifiability. However, Cavalcanti and his collaborators suggested their own justification for ruling out signaling based on special relativity.

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[2] This point will be explored in more detail in Section 5.3.

I've argued that the relativistic constraint has no direct bearing on the possibility of signaling, especially when we take Timpson's analysis of information$_t$ transmission into account.

To summarize, if the concern with the BHW signaling protocol is that something (a causal influence, say) is traveling faster than the speed of light, then the teleportation protocol should already have given Cavalcanti et al. pause. Both the signaling and the teleportation protocol share the feature that the information appears at B instantaneously after being sent from A.[3]

I contend that the problem lies elsewhere. This can be illustrated by the implausibility of the following scenario assuaging any of their concern: Alice uses the BHW signaling setup to send a message to Bob, but Bob waits to make his measurement (and therefore receive the message) until after a light signal could have potentially traversed the space between them, even though no such signal was sent.

This hypothetical scenario is actually similar to the teleportation protocol in one important respect: even though Bob's system is instantaneously affected by Alice's choice of measurement, the information isn't present at B until after enough time has elapsed to ensure that no information was transmitted faster than the speed of light. Of course, in our imagined scenario, this feature is purely accidental, whereas in the teleportation protocol, this feature is necessary.

The important differences between teleportation and signaling are two. Firstly, what is sent to Bob in the teleportation protocol is an unknown quantum state, and therefore not information he can *use*. Secondly, the token of the information isn't actually produced at Bob's location until after two classical bits of information are sent through a classical (subluminal) channel. (One in four times, however, that information tells Bob that he already has the correct state in hand. In those cases, even though the target state has already been reproduced, he doesn't have verification of this fact until the classical bits arrive. This is a good example to draw out the fact that the exploitability of the information is what is being prevented in teleportation.) These two features prevent the teleportation protocol from being used to send signals.

With respect to Cavalcanti et al.'s argument, I'll simply say that the nonlinear evolution from which they were generalizing in their paper (and which

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[3] And, as Maudlin showed, there are consistent relativistic models containing superluminal causal influences.

is the only one to have been explicitly shown to allow for signaling) is that generated by the presence of a CTC, which explicitly allows for information to be sent to the past, and which has a feature (namely, the consistency condition) that protects against temporal paradoxes.

However, even in the absence of the relativistic justification for ruling out the impossibility of instantaneous signaling, Cavalcanti et al. would likely still want to rule it out on other grounds. If they can establish that instantaneous signaling is in principle impossible, then their argument that "we must rethink our assumptions" in our analysis of the BHW circuit would go through.

In the following section, I'll examine why this may be. I argue that the impossibility of Nonlocal Signaling is a fundamental principle of quantum information, and the project of an information-theoretic interpretation of quantum theory relies on the exceptionless truth of such principles. The reason for this is a deep difference in the conception of physical theories between the proponents of the quantum information perspective, and those who consider the ontology of a theory to be among its most fundamental elements.

5 Quantum Information-Theoretic Motivations

5.1 Why Maintain No Signaling?

I've argued that the relativistic justification for the No-Signaling Principle doesn't stand up to scrutiny. I've presented evidence that nonlocal signaling is not itself inconsistent with relativity. Why do people want to maintain it?

I will argue that the reason for the reticence to give up the No-Signaling Principle has to do with its status in the field of quantum information. In particular, it is one of the most promising principles in the reconstructions of quantum theory literature. The principle of information causality, which would fail if No Signaling failed, is the current contender for a principle that can differentiate quantum correlations from superquantum correlations.

Information Causality says that the most classical information that can yield from the transmission of one classical bit is one classical bit. Classical information needs to follow an explicit continuous causal model. Signaling can yield classical information at B without anything traversing between A and B. So that would violate information causality.

Information Causality is the latest in a list of potential physical principles that is taken by proponents of the quantum information approach to be part of the fundamental formulation of quantum theory.

This is a statement of the principle-based conception of physical theories. This is a conception in which the fundamental formulation of a physical theory is in terms of principled restrictions on the kinematical level. These principles never need to be justified by ontological or dynamical considerations. An empirically equivalent theory theory formulated in terms of dynamics and ontology is taken on this approach to represent a less fundamental formulation, which serves merely as a consistency proof for the set of principles that constitute the definitive version of the theory.

The special theory of relativity is taken to be the paradigm example of this kind of theory. Just as the Principle of Relativity and the Light Postulate pick out Minkowski spacetime as the space of events in SR, and constrain the structure of events in spacetime, the information-theoretic interpretation of quantum theory take there to be principles that define a space of events for quantum theory, and that space to constrain the structure of those events.

> In the case of quantum mechanics, these principles are information-theoretic and include a 'no signaling' principle and a 'no cloning' principle. The structure of Hilbert space imposes kinematic (i.e. pre-dymanic) objective probabilistic constraints on events to which a quantum dynamics of matter and fields is required to conform, through its symmetries [...]. (Bub and Pitowsky 2010, 439)

And, as with relativity, they hold that there is no deeper explanation of the structure of events than that they are subject to the constraints embodied in the principles.

> There is no deeper explanation for the quantum phenomena of interference and entanglement than that provided by the structure of Hilbert space, just as there is no deeper explanation for the relativistic phenomena of Lorentz contraction and time dilation than that provided by the structure of Minkowski spacetime. (Bub and Pitowsky 2010, 439)

In most cases these two ways of formulating a theory (principle and constructive) don't come into any kind of conflict. But in the case of the nonlinear extensions of quantum theory, the principle version of the theory makes different predictions than the constructive version. Following the dynamics of the systems under consideration leads us to conclude that signaling is effected in the BHW circuit. But this is in explicit conflict with the No-Signaling Principle.

5.2 Deutsch's Metaphysics

Deutsch would reject the claim that a principle version of quantum theory is more fundamental than a constructive version. There is an extended argument to this effect in (Dunlap 2014a). Here I will simply note two points.

Firstly, Deutsch is an Everettian. By virtue of this, he has serious metaphysical commitments about the way the world is fundamentally structured. He that the right way to understand quantum theory is primarily as a theory with a definite ontology—the wavefunction—and dynamical laws. This is in explicit contradistinction to the principle-theoretic conception of quantum theory.

Furthermore, as I argue in (Dunlap 2014a), the D-CTC model relies for its formulation on a strong ontic realism about the many worlds of the Everett Interpretation. The definite existencet of counterpart systems in parallel worlds is necessary for solving the quantum CTC paradoxes that motivate his work. As he says, interpretations that insist on a single outcome of an experiment do not have the resources to solve the paradoxes. The Everett Interpretation's extra structure (i.e. the uncollapsed branches of the wavefunction) is what allows it to solve the paradoxes.

> In the Everett Interpretation, it is only the state, which describes, roughly speaking, a collection of values taken as a whole, which must be unchanged after passage round a closed timelike line. (Deutsch 1991, 3206)

For example, his analysis of a grandfather-like paradox runs as follows:

> In all universes the observer approaches the chronology-violating region on a trajectory which would go back in time. But only in half of them does the observer remain on that trajectory, because in half the universes there is an encounter with an older version of the observer after which the younger version changes course and does not go back in time. After that, both versions live on into the unambiguous future. (Deutsch 1991, 3206–3207)

There is an interesting question about whether the details of his consistency condition are actually consistent with the standard Everett Interpretation. His condition requires that ρ, a mixed state, be consistent when traveling around the CTC. He interprets ρ as a collection of worlds, with counterpart systems present in all of them. However, these worlds do not arise in the normal Everettian way (as branches of a wavefunction). This question is addressed in detail in (Dunlap 2014a).

Since Deutsch's metaphysics plays such a central role in the formulation of his CTC model, it cannot be ignored. It is therefore problematic for quantum information theorists to adopt this model for analysis in a principle-theoretic context. Despite their claim to be neutral to questions of interpretation, the metaphysics is playing a fundamental role in the example.

As a consequence, the principle-theoretic prohibition against Nonlocal Signaling cannot be taken seriously in this debate. The D-CTC itself cannot be formulated in the principle-theory context. So the imposition of the No-Signaling Principle is inappropriate.

5.3 Bub-Stairs Consistency

Another conflict with Deutsch's approach that is present in this debate is present in the Bub and Stairs paper, and is related to the point about nolocality and relativity from above. Bub and Stairs argue that their consistency condition allows for a "radio to the past", or a protocol for sending classical information back in time. They contend that the existence of this protocol opens the door to temporal paradox. As mentioned in 4.4, I believe that Deutsch's consistency condition wold apply to this classical information channel as well.

The evidence for this claim is comes from the fact that Deutsch, as an Everettian, would deny that there was any principled distinction between classical information and quantum information. Ultimately, classical information supervenes on quantum systems. In order to be consistent with his broader view on the interpretation of quantum theory, he must treat the classical domain and the quantum domain as subject to the same laws, particularly one as fundamental as a consistency condition.

In fact, in his paper, he explicitly states that he conceiving of computation for the purposes of this argument as

> a representative physical process—representing the behavior of

> general physical systems under the unfamiliar circumstances of chronology violation. (Deutsch 1991, 3197)

He develops a standard form for a CTC-assisted quantum circuit for the purposes of defining his consistency condition in a simple way. But he says that any spacetime bounded network, which he uses to represent general physical systems, can be trivially transformed into a denotationally equivalent standard form, which localize any closed loop of information onto a CTC.

> ...[T]he transformed version would be intuitively very different from the original one which might represent a time traveler, whereas the transformed version appears to represent an ordinary space traveler meeting a time traveler who spontaneously comes into existence as an identical twin of the space traveler, exists for a finite period of time on an "eternal" loop, and then ceases to exist. (Deutsch 1991, 3199)

It is clear that Deutsch takes these quantum-circuit representations to be completely general. Therefore, his consistency condition should apply to all physical systems.

Bub and Stairs consider the radio to the past protocol to be potentially paradoxical because they insist on a strict distinction between the classical domain and the quantum domain.

They say that they see their consistency condition as allowing for a 'radio to the past', which opens the door for the reemergence of the time travel paradoxes in the classical domain. This comes from the fact that they are implicitly taking on a Heisenberg (or operationalist) picture, which is characteristic of quantum information, but is rejected by the realist approaches to the interpretation of quantum theory. This is the same problem we saw above: the tenets of the quantum information-theoretic interpretation of quantum theory are doing work behind the scenes to justify the approach to the problem.

And finally, it should be noted that even in a purely classical context, there are analogues of Deutsch's consistency condition that are taken equally scientifically seriously (e.g. Lewis 1976 and Novikov 2002). So even if they argue that Deutsch's consistency condition only applies to quantum information, there are consistency conditions in reserve ready to step in.

6 Conclusion

The considerable recent interest among the quantum information community in the D-CTC model has produced genuinely interesting results. The BHW circuit and its use in the Nonlocal Signaling protocol are considerable contributions to our understanding of how quantum systems behave in nonlinear extensions of quantum theory.

However, we must be sensitive to the fact that there are significant constraints on the generality of the D-CTC model. Its formulation presupposes significant metaphysical commitments, and is therefore applicable only in contexts where those metaphysical commitments are shared. Failing to recognize this feature of the model threatens to undermine its application. I argue that this problem is present in the debate in the quantum information literature, in particular in the attempts to impose the No-Signaling Principle on the framework in which the system is being analyzed.

Because of these underlying commitments, the D-CTC model serves as an important example for the divergence between the principle-theoretic approaches to quantum theory, and the more metaphysically robust constructive approaches. Deutsch himself is unambiguously an advocate of the latter, and the model is arguably incoherent on the former approach.

While this discrepancy does not necessarily have any bearing on the question of which is the better approach to the foundations of quantum theory, it does help bring into contrast their differences. My own view, as advocated in (Dunlap 2014c), is that foundational progress in a physical theory will only come with a fully realized account of the fundamental ontology of the physical world.

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