Prospects for a New Account of Time Reversal

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Abstract

In this paper I draw the distinction between intuitive and theory-relative accounts of the time reversal symmetry and identify problems with each. I then propose an alternative to these two types of accounts that steers a middle course between them and minimizes each account's problems. This new account of time reversal requires that, when dealing with sets of physical theories that satisfy certain constraints, we determine all of the discrete symmetries of the physical laws we are interested in and look for involutions that leave spatial coordinates unaffected and that act consistently across our physical laws. This new account of time reversal has the interesting feature that it makes the nature of the time reversal symmetry an empirical feature of the world without requiring us to assume that any particular physical theory is time reversal invariant from the start. Finally, I provide an analysis of several toy cases that reveals differences between my new account of time reversal and its competitors.

Keywords: time reversal, symmetries, philosophy of time

1. Introduction

The following two questions about time reversal are intimately related to one another:

1. What does the time reversal operator look like (or, equivalently, how does a physical state or a series of physical states change under time reversal)?

2. Which physical theories are time reversal invariant?

In the philosophical literature on time reversal, authors frequently attempt to justify an answer to one of these questions by assuming an answer to the other question. Why they do so is obvious: if one knows how the time reversal operator acts on physical states, it is relatively easy to conjure up a time reversal operator in the context of a particular physical theory and then check to see whether this time reversal operator maps solutions of this physical theory's equations to solutions. Conversely, if one assumes from the beginning that a particular theory is time reversal invariant, one can utilize the mathematical structures of the theory (e.g. the symmetries under which the theory's differential equations are invariant) to determine what properties a time reversal operator should satisfy. I will call accounts of time reversal that assume an answer to 1 and use this response to generate an answer to 2 "intuitive" and accounts of time reversal that assume an answer to 2 and use this response to generate an answer to 1 "theory-relative". This distinction amounts to a difference in how one attempts to *justify* their particular approach to time reversal. These two approaches serve as archetypes that are typically imperfectly instantiated; not every approach to time reversal falls neatly into one of these two categories, and indeed, what I call "intuitive" accounts of time reversal may, in fact, import some facts about the time reversal invariance of particular physical laws into their account. However, the

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new distinction I draw here between intuitive and theory-relative accounts does provide a rough, helpful guide for dividing up the accounts of time reversal that appear in the literature and understanding common concerns about several popular accounts of time reversal. These concerns will motivate the novel account of time reversal I present in the final sections of this paper.

In this paper I examine several intuitive and theory-relative accounts of time reversal. In the first section I consider the work of Horwich <10>, Albert <1>, Malament <18>, and Arntzenius and Greaves <3>, all of whom provide intuitive accounts of time reversal. In the second section I quickly consider the work of Wigner <25> and the "textbook account" considered by Arntzenius and Greaves, both of which are theory-relative accounts of time reversal. I discuss the general shortcomings of both intuitive and theory-relative accounts as a way of motivating a new approach to time reversal which I sketch in section five of this paper. Finally, at the end of the paper, I consider how this new account of time reversal would treat time reversal in toy models governed by select sets of physical laws so that the reader may better understand the implications of my new account of time reversal.

1.1. Mathematical Background

Before proceeding with my analysis, I should quickly provide the reader with the necessary mathematical background to make sense of the analysis to follow. The theories I am concerned with for the purposes of this paper are, roughly, physical theories that at the very least provide us with 1) a set of dependent variables $u = (u_1, u_2, ...)$, 2) a set of independent variables $x = (x_1, x_2, ...)$, 3) a set of algebraic and/or differential equations e which represent the laws of the physical theory and involve all and only the variables found in $x \cup u$ and derivatives of the dependent variables in u with respect to the independent variables in x, and 4) some sort of translation guide that tells us what each of the variables in $x \cup u$ represents in the real world.[3] Call U the space representing the dependent variables in u and X the space representing the independent variables in x.

A theory's algebraic equations can be represented as n-1-dimensional manifolds of the n-dimensional manifold $X \times U$. If the theory is more sophisticated and, like most theories of physical interest, refers to ordinary or partial differential equations additionally or exclusively, then these equations are represented not as submanifolds of $X \times U$ but as submanifolds of the space $X \times U \times U^{(1)} \times ...U^{(n)}$, where $U^{(i)}$ is the ith prolongation or jet space whose coordinates represent the derivatives of the dependent variables in u with respect to the independent variables in x up to order n. So, for instance, if we are dealing with a theory whose variables are the dependent variable p, which represents position, and the independent variable t, which represents time, and whose equations involve velocity($\frac{dp}{dt}$) and acceleration ($\frac{d^2p}{dt^2}$) only, then the theory's equations would be represented by submanifolds of the space $T \times P \times P^{(1)} \times P^{(2)}$. Call $X \times U \times U^{(1)} \times ...U^{(n)}$, the space on which a theory's equations are represented as submanifolds, the theory's variable space.[4]

This framework allows us to define symmetries of our physical theories as maps from points in a theory's variable space to itself that map all and only points that fall within the submanifold representing the theory's equations to points that fall within the submanifold representing the theory's equations. Put more simply, symmetries will always and only take solutions to the equations in e to solutions. This definition of a symmetry has the virtue of serving as a natural extension of the notion of point symmetries found in the mathematical literature on symmetries of differential equations (see, for instance, Olver <20> and Hydon <11>) and providing us with a neutral framework from which we can assess the pronouncements of different accounts of time reversal. We might otherwise worry, for instance, that Horwich's and Albert's accounts of time reversal, which seem to take time reversal to be a map from a phase space to itself, may be, in some sense, incommensurable with Malament's notion of time reversal, which is defined as a transformation of

[3] I am playing fast and loose with the formalities regarding this "translation guide" and how it syncs up the mathematics of our theory with measurements in the real world, but this is because I wish to remain as neutral as possible as to the question of how physical theories represent what they represent as very little of the analysis to come in this paper hinges on the specific features of this "translation guide".

[4] Further details on this part of the framework can be found in Olver <20>.

a geometric object defined on spacetime. Both Albert's notion of time reversal and Malament's, however, induce transformations of the same theory's variable space, and so we can use my framework to compare them straightforwardly.

For the purposes of this paper, I will use the notation $T(x)$ to represent the transformation induced by the symmetry T on the 1-dimensional subspace characterized by the variable x alone. Less technically, the claim that $T(x) = x'$ effectively isolates the effect of the time reversal operator to the variable x and tells us, in effect, how time reversal "acts on" on this variable. In cases where I write $T(\Psi)$, where Ψ is the fully-specified state of a physical system, T should once again be simply understood as a function from variable space to itself.

2. Intuitive accounts of time reversal

I will limit my survey of the literature to accounts of time reversal that deal with one particular physical theory, classical electrodynamics, because this particular theory has generated such fruitful discussion in the literature on time reversal and because it is helpful to see directly how different approaches to time reversal deal with the same physical theory. Intuitive accounts of time reversal may differ from one another, but they all employ the same general strategy for determining whether particular theories are time reversal invariant, which runs basically as follows:

1. Begin with familiar physical properties (such as position and perhaps velocity) of whose behavior under the time reversal operator we have an intuitive grasp.

2. Utilize some formal relations provided for us by the theory in question to determine how the values of other fundamental physical properties of the theory transform under time reversal.

3. Check to see if any solution of the theory is mapped to a solution by the putative time reversal procedure. If so, then the theory is time reversal invariant. If not, then the theory is not time reversal invariant.

A general procedure for laying out a taxonomy of intuitive accounts of time reversal, then, can be given by providing the following information about each account: 1) the intuitions which drive the particular characterization of time reversal needed for the author's account, 2) the consequences of this characterization of time reversal for the transformation of the fundamental properties of a physical theory under time reversal, and 3) the verdict the account delivers concerning the time reversal invariance of particular theories.

Note that this strategy, as it stands, makes room for degrees of purity within the intuitive accounts of time reversal thanks to step 2. Because step 2 explicitly relies on the formal relations provided by the theory in question to relate one physical property to another, and since some of these relations may need to be established by substantive physical laws that are taken to be definitional (or at least constitutive of the property in question) and assumed to be time reversal invariant, these intuitive accounts will be closer to theory-relative accounts than "purer" intuitive accounts, which would take advantage of fewer or no such law-like formal relations. We will see this hybrid approach in the first intuitive account I will consider, Horwich's, and follow it up with a consideration of the "purest" intuitive account I will consider, Albert's. Note, however, that no matter how pure an intuitive account of time reversal may be, the general strategy taken by all such accounts will leave them open to the concerns I raise below in section 4.

2.1. Horwich

I begin my inquiry into intuitive accounts of time reversal with chapter 3 of Horwich <10>. Horwich, one of the first philosophers to discuss the time reversal invariance of classical electrodynamics in general and to provide an intuitive account of time reversal in classical electrodynamics in particular,[5] begins by pointing out that it is not enough for a time reversal operator to simply reverse the order of physical states; that is, it is not sufficient to characterize $S_n...S_1$, a simple inversion of some sequence of physical states $S_1...S_n$, as the result of every time reversal operation. Though Horwich agrees that intuitively time reversal should at the *very least* reverse the temporal ordering of instantaneous physical states, something more is needed for a full account of time reversal. Thus, some operator (which I shall call T, using my own notation instead of Horwich's) should be employed to transform the individual states into their time-reversed counterparts so that the true time-reversed version of the sequence $S_1...S_n$ is $T(S_n)...T(S_1)$.

Having appealed to the time reversal operator T above, Horwich now owes us an account of how that operator acts on physical states. Horwich begins his short analysis of T by drawing the distinction between what he calls "basic" properties, which are (presumably monadic) properties held by instantaneous physical states, and "non-basic" properties, which are relations between numerous instantaneous states across time. Position and time, for instance, count as basic properties in Horwich's account while velocity is non-basic. Horwich defines the operation of T on $\mathbf{x_n}$, the position of some particle, and t, the temporal coordinate, as follows:

$$T(\mathbf{x_n}) = \mathbf{x_n} \tag{1}$$

$$T(t) = -t \tag{2}$$

These are the only basic properties that Horwich identifies, and so Horwich's account will, of course, be of limited use; however, it does provide enough information for us to understand how the operator T acts on a particle's velocity $\mathbf{v}$. Since the velocity is the time derivative of position, and since T flips the sign of time but not of position, it follows that:

$$T(\mathbf{v}) = -\mathbf{v}. \tag{3}$$

Horwich does not explicitly discuss the ways in which other non-basic properties transform under T, making the extent to which he assumes (or doesn't assume) certain laws in electrodynamics to be time reversal invariant somewhat murky, but he does say that the magnetic field $\mathbf{B}$ should be treated similarly to $\mathbf{v}$ since flipping the direction in which electrons flow in a wire would likewise flip the sign of the magnetic field these electrons induce:

$$T(\mathbf{B}) = -\mathbf{B}. \tag{4}$$

It is not entirely clear why, according to Horwich, we need to flip the sign of the magnetic field under time reversal, but one justification that seems particularly appealing given his discussion of electron flow reversal is to claim that Ampere's circuital law (with Maxwell's correction):

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}; \tag{5}$$

provides a fundamental, definitional constraint on the quantity B and that fundamental definitional relations are time reversal invariant. That is, the reason why flipping the directions of the electrons in a loop of wire

[5]Earman <9> predates Horwich in his discussion of time reversal and, in fact, presents a general account of time reversal that I will make more precise with the account of time reversal I sketch later in this paper. However, as Earman's early work does not deal directly with classical electrodynamics, I begin my discussion here with Horwich instead. In a later work, Earman <8> does deal directly with electrodynamics, and the approach he takes is similar to the one adopted by Malament in my analysis below, so for the sake of brevity, I have omitted a separate examination of this account.

would flip the sign of the magnetic field induced by those electrons is that the above equation is definitional and therefore time reversal invariant. By definitional in this context, I should clarify that I do not mean that Ampere's circuital law is sufficient to tell us what B is, only that this law serves a necessary condition and fundamental constraint on B and thus ought to be treated as time reversal invariant. Regardless of whether or not Horwich would agree with this method of justifying his suggestion, it is clear that Horwich's intuitions about time and velocity alone are not sufficient to justify his account of the action of the magnetic field under a time reversal transformation. He must rely on *some* relations to determine the action of the time reversal operator on other physical properties, and so, in order to better understand how he justifies his account of time reversal, we have to read a bit more into Horwich's account than he actually gives us. It is worth noting Horwich's treatment of $\mathbf{B}$ under T and considering his justification for it here because these will be points of contention between Horwich and Albert, as I shall discuss in the next section.

Finally, given the above descriptions of how the time reversal operator T acts on instantaneous physical states, what can we conclude about the time reversal invariance of particular physical theories? Horwich does not draw any conclusions of his own on this matter in chapter 3 of his book, but his suggestion that the magnetic field flips its sign under time reversal allows us to extrapolate Horwich's views on the time reversal invariance of classical electromagnetism. The fundamental equations in classical electromagnetism are Maxwell's:

$$\nabla \cdot \mathbf{E} = \rho \tag{6}$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j} \tag{7}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{8}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{9}$$

$$\tag{10}$$

and the Lorentz force law:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{11}$$

As it stands, Horwich does not provide us with enough information to determine whether or not these equations are invariant under time reversal because he does not state how T operates on the electric field $\mathbf{E}$, the current $\mathbf{j}$, or the electromagnetic force $\mathbf{F}$. However, it seems like Horwich would likely support the following characterizations of the action of T on these variables:

$$T(\mathbf{E}) = \mathbf{E} \tag{12}$$

$$T(\mathbf{j}) = -\mathbf{j} \tag{13}$$

$$T(\mathbf{F}) = \mathbf{F} \tag{14}$$

The transformation of these properties under the time reversal operator can be justified as follows: first, since $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ by Newton's second law, it seems that $\mathbf{F}$ should not flip signs since T should flip the signs of both $\mathbf{p}$ and t.[6] The electric field $\mathbf{E}$ likewise seems like a property that time reversal should not affect, and

[6] I assume here that $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ is essentially definitional in this context, as is typically assumed in derivations of the Lorentz force law, and not a statement of some sort of substantive law. As stated previously, we can assume that definitions are time reversal invariant and cannot fail to be so while substantive physical laws may fail to be time reversal invariant. So, for instance, even though we may consider forces that obey $\mathbf{F} = \mathbf{v}$ and so seem as if they should flip sign under time reversal since $\mathbf{v}$ does, we can claim that such relations constitute substantive physical laws and so do not contradict our claim, derived from Newton's

so Horwich seems likely to advocate the transformation of $\mathbf{E}$ suggested above. Finally, since the current $\mathbf{j}$ is a quantity that involves a single time derivative, it should intuitively flip signs under time reversal. All three of the above extrapolations, then, seem fairly reasonable given Horwich's view, and as we'll see shortly, Albert agrees with Horwich on these points.

All that remains now is to check and see if T, as I have characterized it, leaves Maxwell's equations and the Lorentz force law invariant. As a matter of fact, the operator T does transform physical states in such a way that if some state Ψ satisfies Maxwell's equations and the Lorentz force law, the time-reversed state $T(\Psi)$ satisfies Maxwell's equations and the Lorentz force law too, and so classical electrodynamics appears to be time reversal invariant.

2.2. Albert

Albert <1>, in the first chapter of his book, provides an account of time reversal that, for the most part, agrees with Horwich's in spirit even as it differs from Horwich's account on details pertaining to classical electrodynamics. In many ways Albert's general project follows Horwich's closely, but his intuitive account is ultimately "purer" than Horwich's (on my reading of Horwich) in that Albert assumes the time reversal invariance of no substantive physical laws. Like Horwich, Albert essentially divides physical properties into basic properties and non-basic properties, though Albert does not use the term "basic" himself. The properties Albert treats as basic (in Horwich's sense) are those which are necessary for characterizing the world fully at some particular time. As such, Albert agrees with Horwich's assessment that the time reversal operator T acts in the following way on the following basic and non-basic physical properties:

$$T(\mathbf{x_n}) = \mathbf{x_n} \tag{15}$$
$$T(t) = -t \tag{16}$$
$$T(\mathbf{v}) = -\mathbf{v} \tag{17}$$

However, Albert's intuitions differ from Horwich's on a few important points. Albert suggests that time reversal ought to simply reverse the temporal ordering of physical states but leave all of the fundamental quantities invariant.[7] That is, according to Albert, T should never flip the sign of a property's value unless that property is either a temporal coordinate or a non-basic property defined as a time-derivative of some other, more basic property whose values do not flip sign under time reversal. Despite Horwich's motivations for flipping the sign of the magnetic field under time reversal, Albert claims that the magnetic field, since it is a basic property and not explicitly the time-derivative of some more fundamental property, should not flip sign under time reversal. Thus, Albert defines the operation of T on the basic and non-basic properties of classical electromagnetism as follows:

$$T(\mathbf{B}) = \mathbf{B} \tag{18}$$
$$T(\mathbf{E}) = \mathbf{E} \tag{19}$$
$$T(\mathbf{j}) = -\mathbf{j} \tag{20}$$
$$T(\mathbf{F}) = \mathbf{F} \tag{21}$$

Albert's version of T yields the same transformations as Horwich's with the sole exception of the magnetic

second law, that $\mathbf{F}$ does not change sign under time reversal. Alternatively, we may claim that, within a particular theoretical context, a substantive law may be treated as definitional and so be assumed to be time reversal invariant even if it is not treated so in other theoretical contexts. So, to those still worried about cases like $\mathbf{F} = \mathbf{v}$ where the $\mathbf{F}$ seems to intuitively flip sign under time reversal, it is sufficient to claim that Newton's second law behaves as a substantive physical law in such contexts even though it is treated as definitional in the context of Lorentz force law derivations.

[7] On this point, Callender <4> agrees with Albert, though Callender's own analysis focuses on the time reversal invariance of non-relativistic quantum mechanics as opposed to electromagnetism.

field; however, this change is sufficient to render classical electromagnetism, which was invariant under Horwich's proposed time reversal operator, non-invariant under Albert's. To see this, consider again Ampere's circuital law, $\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \mathbf{j}$. The right-hand side of this equation flips signs under time reversal while the left-hand side does not. There are no non-zero values of $\mathbf{B}$ which would satisfy both Maxwell's second equation and its time-reversed counterpart, so classical electromagnetism is not, according to Albert, time reversal invariant.

2.3. Malament

Up to this point I have ignored the specific ways in which intuitive accounts typically interpret or understand how the time reversal operator acts on states; however, the issue of interpretation cannot be avoided when discussing the distinctive features of the account given by Malament <18>, so a little bit of backtracking is in order. As North <19> and Arntzenius and Greaves <3> point out, time reversal is typically interpreted or understood in one of two essentially equivalent ways. The first sort of time reversal, active time reversal, involves a temporal flipping of physical states (as characterized by the contents of spacetime and their fundamental physical properties) while leaving the coordinate system and background spacetime of these states unaltered. One can think of such time reversals as making the necessary changes to the progression of physical states such that, if you observed the time-reversed sequence of a movie, for instance, the movie would appear to be running backwards. The second sort of time reversal, passive time reversal, involves simply a reassignment of coordinates without changing any intrinsic features of the physical states in question. One can perhaps think of the passive time reversal of some sequence of events as the way in which someone moving backwards in time would view the original sequence. Essentially, these two physical interpretations of the time reversal operator amount to the same thing: whether one keeps the coordinates fixed while time-reversing the physical states or keeps the physical states fixed while time-reversing the coordinates, the fundamental characterization of the time reversal operator should remain the same.

Malament proposes a new physical interpretation of time reversal. Rather than supposing that time reversal acts on the physical states alone or coordinates alone, Malament takes the time reversal operator to fundamentally invert the temporal orientation of the background spacetime structure and influence properties and states that depend on this structure accordingly. This interpretation, which he calls "geometric", is more general than the passive interpretation since it applies to coordinate-independent models, and Malament claims that, unlike active and passive interpretations of time reversal, his geometric interpretation applies to curved spacetimes. So it seems that of the intuitive accounts considered thus far, Malament's is the best-suited to deliver verdicts on the notion of time reversal in our best currently available physical theories.[8]

So, what verdict does Malament's geometric interpretation deliver in the case of classical electromagnetism? Malament begins by providing us with the entities and properties he takes to be fundamental to the invariant formulation of electromagnetism. These are charged particles and fields, in particular the electromagnetic field F which Malament takes to be a map from the pair $\langle L, q \rangle$ at some point p, where L is the line tangent to some particle's worldline at p and q is the charge of the particle at p, to a four-vector indicating the direction in which the test particle characterized by $\langle L, q \rangle$ would accelerate if placed at p. Malament notes that some information is missing in his characterization of F because, for any timelike worldline, there are two possibilities for the unit tangent vector at a given point: one which points towards the past, and one which points towards the future. Since there is nothing in Malament's characterization so far to suggest that L should be future-directed instead of past-directed, Malament relies on the temporal orientation of the background spacetime to determine the direction in which L points. So F, according to Malament, takes the pair of objects $\langle L, q \rangle$ to some four-vector given a specific background temporal orientation.[9]

Geometric time reversal involves keeping the fundamental quantities the same while flipping the temporal orientation, so the operator T should have no effect on the position or charge of a particle. Thus, to use

[8] This point is also made by North <19>.

[9] It is worth noting that a similar convention for an orientation in spacetime could be picked out by positing something like the local orientation field considered by Pooley <22>.

the notation previously applied to Horwich and Malament for the sake of continuity, Malament would be committed to the following (assuming $\mathbf{x_n}$, in this context, to refer to the spatial coordinates on a spatial surface picked out by a global timeslice):

$$T(\mathbf{x_n}) = \mathbf{x_n} \tag{22}$$
$$T(q) = q \tag{23}$$

However, Malament agrees with both Horwich and Albert that velocities (in this case, understood as tangent vectors to the worldlines of particles) flip sign under time reversal:

$$T(\mathbf{v}) = -\mathbf{v} \tag{24}$$

This follows from the fact that velocities are future-directed four-vectors that lie tangent to worldlines. Thus, the role of temporal orientation in determining velocity requires a velocity sign flip under geometric time reversal. Like velocity, the field F requires a temporal orientation for its mapping to work. Intuitively, L and q do not flip signs under geometric time reversal since these fundamental quantities remain untouched by a geometric time reversal; however, the four-vector that results from the mapping ought to flip its sign under time reversal because of the temporal orientation introduced to F as discussed above. Thus, we should expect the electromagnetic field, represented by the Maxwell-Faraday tensor F^{ab}, to transform as follows:

$$T(F^{ab}) = -F^{ab} \tag{25}$$

This does not, however, tell us how $\mathbf{E}$ and $\mathbf{B}$ transform, so it is hard to compare Malament's account to Horwich's and Albert's accounts without saying a bit more about the relationship between $\mathbf{E}$, $\mathbf{B}$, and F^{ab}. Skimming over the technical details of Malament's account, he defines the electric field $\mathbf{E}$ as the product of the tensor F^{ab} and a "frame", which is a future-directed time-like vector field and thus a physical quantity whose value flips signs under geometric time reversal. Since both F^{ab} and the frame flip signs under time reversal, $\mathbf{E}$ itself is not affected by the time reversal operator. The magnetic field $\mathbf{B}$, however, is determined to be proportional to the product of F^{ab}, the frame, and a volume element representing an antisymmetric tensor field whose values also flip sign under geometric time reversal. Thus, $\mathbf{B}$ itself flips sign under geometric time reversal. So, like Horwich, Malament takes T to operate in the following way on the following quantities of interest in classical electromagnetism:

$$T(\mathbf{B}) = -\mathbf{B} \tag{26}$$
$$T(\mathbf{E}) = \mathbf{E} \tag{27}$$
$$T(\mathbf{j}) = -\mathbf{j} \tag{28}$$
$$T(\mathbf{F}) = \mathbf{F} \tag{29}$$

As previously discussed, Maxwell's equations and the Lorentz force law all still hold under these transformations, and thus classical electromagnetism, according to Malament, is time reversal invariant.

2.4. Arntzenius and Greaves's "Feynman" account

The last intuitive account of time reversal I consider comes from Arntzenius and Greaves <3>. Arntzenius and Greaves agree with Malament's general strategy but provide a new account (which, they claim, is equivalent to Malament's) inspired by Feynman's claim that "antiparticles are just particles moving backwards in time". Essentially, the Arntzenius-Greaves (henceforth AG) program begins by building in an intrinsic temporal orientation to the tangent worldlines L utilized by Malament in his analysis. If worldlines are intrinsically-directed, as Malament claims they are not, then L flips signs under time reversal, which means that the field F is no longer only defined relative to a specific temporal orientation, so:

$$T(F^{ab}) = F^{ab}. \tag{30}$$

If one follows Malament's procedure for deriving the behavior of $\mathbf{E}$ and $\mathbf{B}$ under time reversal from the behavior of F^{ab} under time reversal as described in the previous section, then one finds that $\mathbf{E}$ is defined in terms of the product of a frame which flips signs under time reversal and a F^{ab} which does not, meaning that $\mathbf{E}$ now flips signs under time reversal. Likewise, $\mathbf{B}$ now does not flip its sign under time reversal.

A few more important quantities should also be discussed. If one is to take Feynman at his word when he says that anti-particles are simply particles moving backwards in time, one must saddle Feynman with the view that the time reversal operator flips the charge of the particle instead of just leaving it invariant; after all, how else could electrons and positrons have opposite charges? Perhaps more curiously, velocity and velocity-dependent quantities like j now do not flip signs under time reversal because their temporal directness is treated as basic instead of parasitic on the underlying temporal orientation of spacetime. Thus, the "Feynman proposal" of AG defines the operation of T as follows:

$$T(\mathbf{x_n}) = \mathbf{x_n} \tag{31}$$
$$T(q) = -q \tag{32}$$
$$T(t) = -t \tag{33}$$
$$T(\mathbf{v}) = \mathbf{v} \tag{34}$$
$$T(\mathbf{B}) = \mathbf{B} \tag{35}$$
$$T(\mathbf{E}) = -\mathbf{E} \tag{36}$$
$$T(\mathbf{j}) = \mathbf{j} \tag{37}$$
$$T(\mathbf{F}) = \mathbf{F} \tag{38}$$

If T is defined in this way, it turns out that any set of values $\langle T(\mathbf{x_n}), T(q), T(t), T(\mathbf{v}), T(\mathbf{B}), T(\mathbf{E}), T(\mathbf{j}), T(\mathbf{F})\rangle$ will satisfy Maxwell's equations and the Lorentz force law just in case the set of values $\langle \mathbf{x_n}, q, t, \mathbf{v}, \mathbf{B}, \mathbf{E}, \mathbf{j}, \mathbf{F}\rangle$ does; thus, classical electromagnetism is invariant under the "Feynman" time reversal operator just as it was under Malament's time reversal operator even though these two operators act very differently on the variable space of classical electrodynamics.[10]

3. Theory-relative accounts of time reversal

The alternative to the intuitive accounts of time reversal examined above are what I have called theory-relative accounts of time reversal. These accounts proceed in two seemingly simple but technically complex steps by 1) singling out a particular physical theory which intuitively seems time reversal invariant and 2) determining what features the time reversal operator would need to have in order for the given theory to be time reversal invariant. This approach has been fairly unpopular in the philosophical literature on time reversal despite some popularity in the physics literature, so there are only two truly theory-relative accounts discussed in the literature that I am aware of: one briefly discussed in Arntzenius and Greaves's work, and one that appears in the work of Wigner <25> on the time reversal operator in quantum mechanics.

[10] It is worth noting, having discussed AG's "Feynman" account and Malament's geometric account, that AG believe that the difference between these two accounts is merely conventional. They provide a "structuralist" ontology according to which certain choices of convention allow one to derive the basic features invoked by Malament's account or the basic features invoked in "Feynman's" account. There may, then, be no genuine, non-conventional difference between these two time reversal operators despite appearances to the contrary.

3.1. Arntzenius and Greaves's "textbook" account

Arntzenius and Greaves <3> discuss what they call the "textbook account" of time reversal at the beginning of their paper on the time reversal invariance of electromagnetism as a foil for the accounts that follow. Very quickly, Arntzenius and Greaves claim that most classical electromagnetism textbooks argue first that electromagnetism ought to be time reversal invariant and, from this assumption, derive the following properties of the time reversal operator:

$$T(\mathbf{x_n}) = \mathbf{x_n} \tag{39}$$
$$T(q) = q \tag{40}$$
$$T(t) = -t \tag{41}$$
$$T(\mathbf{v}) = -\mathbf{v} \tag{42}$$
$$T(\mathbf{B}) = -\mathbf{B} \tag{43}$$
$$T(\mathbf{E}) = \mathbf{E} \tag{44}$$
$$T(\mathbf{j}) = -\mathbf{j} \tag{45}$$
$$T(\mathbf{F}) = \mathbf{F} \tag{46}$$

Clearly, the above T operator treats the quantities the same way as Horwich's and Malament's operators treat these quantities. Two points are worth noting here that I will come back to when I provide my general critiques of both the intuitive and theory-relative accounts of time reversal. First, note that the "textbook account" is able to deliver the features of the time reversal operator T without making any assumptions about the fundamental properties of the physical theory or which interpretation of time reversal (active, passive, geometrical) is best. Given this ontological or interpretive agnosticism, the textbook account may be attractive to many philosophers of physics. However, the operator it delivers in this case isn't the only candidate for T that keeps electromagnetism time reversal invariant. As discussed in the last section, the "Feynman" account provides a time reversal operator that acts very differently from the one provided by the textbook account, and yet classical electromagnetism is still time reversal invariant under this strange-looking operator. One may rightly worry, then, that theory-relative accounts of time reversal do not assume enough information about T to pick out a unique form for the time reversal operator to take within the context of a particular physical theory.

3.2. Wigner's account of time reversal in quantum mechanics

Though not directly relevant to our discussion of time reversal in classical mechanics and electromagnetism, it is worth mentioning the theory-relative account provided by Wigner <25> to show that theory-relative accounts are not simply utilized by fictitious interlocutors presented in papers by philosophers. Though the details of Wigner's account will not allow us to directly compare Wigner's theory-relative account to the intuitive accounts previously presented, it is worth briefly discussing Wigner's approach.

Wigner begins his discussion of time reversal on page 325 by assuming that, within the context of quantum mechanics, the time reversal operator T acts as follows on our time variable: $T(t) = -t$. This might lead one to interpret Wigner as if he's giving an intuitive account of time reversal, but he clearly thinks this constraint on T is a minimal one that helps us get a handle on the symmetry he's interested in, not as the starting point for a full intuitive characterization of T. The substantive assumption that quantum mechanics is time reversal invariant comes on the next page when he claims that "The following four operations, carried out in succession on an arbitrary state, will result in the system returning to its original state. The first operation is time inversion, the second time displacement by t, the third again time inversion, and the last one again time displacement by t" (326). This claim is equivalent to the assumption of the time reversal invariance of non-relativistic quantum mechanics since, if we start with a solution to the time-independent Schrödinger equation ψ and its counterpart $\psi(t) = U(t)\psi$, which is a solution to the time-dependent Schrödinger equation, the only way to get the set of transformations $U(t)TU(t)T^*$ to be the unit operation is if we take $T\psi(-t)$ to be a solution to the Schrödinger equation whenever $\psi(t)$ is. The crucial first step in Wigner's derivation of the time reversal operator in non-relativistic quantum mechanics, then, is to assume that the theory in question is actually time reversal invariant and proceed from there.[11]

4. Problems for intuitive and theory-relative accounts

Though both intuitive and theory-relative accounts have their advantages, both seem unappealing for a number of reasons. Let us begin with intuitive accounts. The most popular approach to time reversal in the philosophical literature seems to be the intuitive approach, and sophisticated accounts such as Malament's and AG are certainly interesting and allow for exciting work on time reversal; however, the intuitive approach has problems generally, and many particular intuitive accounts run into idiosyncratic problems. I begin with the specific problems and then work outwards toward more general issues.

One worry about Malament's and Arntzenius and Greaves's projects is that both assume an affirmative answer to the question "Is our spacetime temporally orientable?". One of the uses for a time reversal operator may be to help us answer the questions of 1) what temporal asymmetries in spacetime there may be, and 2) where the temporal asymmetries in spacetime may come from. Horwich himself is motivated by these questions when providing his analysis of time reversal. If one is committed to something like the spacetime symmetry principles advocated by Earman <7>, one of which states that all spacetime symmetries are dynamical symmetries, then perhaps we should conclude that a time reversal asymmetry in one of our dynamical theories should indicate a temporal asymmetry in spacetime. In short, the existing literature gives us a good reason to believe that an analysis of time reversal may help to answer questions about temporal asymmetries in spacetime; however, because Malament and AG assume a temporal orientation in order to formulate their accounts of time reversal, these accounts are incapable of providing independent evidence for a temporal orientation, so those interested in such questions need to look elsewhere for accounts of time reversal that will further their projects.[12]

Perhaps a more difficult challenge to intuitive projects in general is the one that the AG program suggests, namely that all intuitive accounts rely on some basic assumptions about the fundamental ontology of a certain physical theory. The division of properties into basic and non-basic certainly seems a metaphysically weighty endeavor with potentially enormous consequences for one's preferred intuitive account of time reversal. One might prefer, as I suggested in my discussion of the "textbook account", some account of time reversal that does not require us to make distinctions between basic and non-basic properties that may lead us into error; after all, as the recent debate over instantaneous velocities sparked by Albert as well as Arntzenius <2> shows, reasonable people may disagree over the basic or non-basic status of even velocity. If this is so, what are we to make of even stranger properties that appear in our more recent physical theories such as string theory and quantum chromodynamics where even our best physical intuitions do not lead straightforwardly to one particular ontology or another?[13] With such dubious metaphysics, one wonders if the success of our currently available intuitive accounts of time reversal can be repeated in the context of these new theories, and if not, it again seems like there are many cases where our intuitions should not be our guide for determining what the time reversal operator looks like.

A similar problem rears its head when considering issues of an intuitive account's purity. If my reading of Horwich is correct, he and Albert will disagree about the time reversal invariance of classical electrodynamics, and their disagreement will ultimately boil down to which law-like, time reversal invariant formal relations they are willing to posit when determining the time reversal invariance of the theory in general. What support can either provide for their position aside from intuitions about what constitutes a substantive physical law and what constitutes a definitional constraint? So intuitive accounts may lead to unproductive

[11] For a more detailed discussion of Wigner's treatment of time reversal, please consult Roberts <23>. My thanks to Bryan Roberts for a helpful conversation on Wigner and for bringing the theory-relativity of Wigner's treatment of time reversal to my attention.

[12] It's worth pointing out that these accounts could take advantage of something like the consistency constraint I rely on in my hybrid account below in order to account for time reversal in temporally non-orientable spacetimes; however, as Malament's and Arntzenius and Greaves's accounts currently stand, they don't permit such a broad, general definition of time reversal.

[13] Take, for instance, the disagreement over the particle and field ontologies of quantum field theory.

disagreements not only about which metaphysically basic properties are most intuitive but also about how pure an intuitive account of time reversal ought to be.

Another challenge to these accounts along the same lines is that while our intuitions may be good guides to determining how properties like classical position and momentum transform when time-reversed, our intuitions may mislead or abandon us when we're faced with strange new physical theories and odd physical properties. We may know intuitively how velocity should transform under time reversal, for instance, but what about lepton number, isospin, quark color or flavor, or any of the other numerous properties that we find in our best available physical theories? With few, weak, or unreliable intuitions to guide us, it seems likely that we will miss something when providing an intuitive account of time reversal, so we have good reason to turn to some other approach in these cases[14].

Finally, when compared to the theory-relative approach, the intuitive accounts seem to rely on the problematic assumption that we know, *a priori*, what the time reversal operator looks like in the context of a given physical theory. As my previous criticisms have suggested, our intuitions do not seem to be excellent guides for what we should believe to be true in our fundamental physical theories, so why should we think our assumptions about the time reversal operator hold fixed in the context of new physical theories? What mechanism is in place in an account like Albert's, Horwich's, or Malament's to allow us to revise our conception of time reversal given enough empirical evidence? One advantage of the theory-relative approach is that it makes the question of what the time reversal operator looks like an empirical one: since our theories are open to revision, the discrete symmetries of these theories may be too, and so our conception of what operator represents time reversal in a particular physical theory may evolve as our theory does. Thus, those swayed by empiricist or positivist considerations may think that the *a priori* character of time reversal as portrayed by intuitive accounts gives us a good reason to abandon such accounts in favor of their competitors, the theory-relative accounts.

The upshot of these criticisms is simply that even the best intuitive accounts are only as good as our intuitions, and there are unfortunately many cases in contemporary physics where our intuitions should only guide us so far. So while intuitive accounts may work well in cases like classical mechanics or electromagnetism, such approaches are not suitably general to deliver the best results in cases like quantum mechanics or quantum field theory. For this reason, then, theory-relative accounts may seem the superior alternative, especially for the empiricist philosopher of physics.

However, theory-relative accounts have problems of their own. First, as pointed out by Arntzenius and Greaves, physical theories have many symmetries, and if one is willing to devote the time and effort, one can determine a class of symmetries under which any particular physical theory is invariant. However, after doing all of this work, Arntzenius and Greaves ask, why do we have any reason to call one of these operations "time reversal" in particular? If "time reversal" simply becomes synonymous with "that symmetry under which our theory is invariant", it seems like all theories are tautologously time reversal invariant and questions about time reversal are no longer of any physical interest. But let us grant that the proponent of the theory-relative account wants to provide a better definition and restricts his claims about the time reversal operator to one theory, as Wigner does with quantum mechanics. Even then, one could argue, it seems that we have no good reason to think that quantum mechanics is time reversal invariant. Why should it be? If the theory-relative account advocate accuses the intuitive account advocate of extending her intuitions of how quantities transform under time reversal beyond their reasonable bounds, the intuitive account advocate can just as easily accuse the theory-relative account advocate of extending his intuitions of which theories are time reversal invariant beyond their reasonable bounds. The time reversal invariance of classical mechanics does not justify the presumption that quantum mechanics is time reversal invariant, so what reason do we have to think quantum mechanics or any other physical theory is time reversal invariant without some basic notion of what the time reversal operator is to work from?

But the problems for theory-relative accounts run deeper, for in many cases, theory-relative accounts underdetermine the time reversal operator. Arntzenius and Greaves's textbook account, for instance, seems like it *should* allow for an operator that acts as Arntzenius and Greaves's "Feynman" account suggests.

[14]My thanks to Bryan Roberts for bringing this criticism to my attention.

This underdetermination would not be so problematic if, as with the "Feynman" and Malament accounts, according to the AG project, all such disagreements can be directly attributed to specific disagreements about fundamental ontology or convention, but we have no reason to believe that this is generally the case, and even if it is, these differences, while metaphysically only conventionally, may be epistemically weighty. Alternatively, theory-relative accounts may make substantive assumptions about how T acts on properties like t, as Wigner does, to help determine which of several available candidates we ought to take to "really" be time reversal, but by making these assumptions, theory-relative accounts may find themselves committing the same crimes they accuse intuitive accounts of committing. One may thus legitimately worry that the procedure utilized in the creation of theory-relative accounts of time reversal will not yield true time reversal operators but rather a class of similar-looking symmetries, some of which may be time reversal operators and some of which may not be.

5. Prospects for a third way

If both the intuitive and theory-relative accounts run into the kinds of problems I have suggested, one may be tempted to look for a new way to answer questions about time reversal. In any particular theoretical context, is there some way to avoid assuming too substantive an answer to the question "what does a time reversal operator look like?" while still providing an adequate answer to the question "which theories are time reversal invariant?"and vice-versa? I believe there is, though the details of the proposal will depend on the theories in question. Though I cannot articulate a general approach to time reversal that delivers answers to these questions in every case of physical interest, in what follows I will provide an intuitive/theory-relative hybrid account of time reversal for physical theories that satisfy certain constraints that steers a course between some of the more problematic aspects of both intuitive and theory-relative accounts. This account, as shall become clear, owes a great deal to the spirit of many of the accounts I have discussed previously as well as the discussion of time reversal in Earman <9>, but the mathematical framework it utilizes and the way it utilizes this framework, while not novel to mathematicians, do represent a novel contribution to the philosophical literature on time reversal.

My hybrid account centers on theories that satisfy the following criteria: 1) the theories in question each provide us with a set a physically interpretable independent variables, a set of physically interpretable dependent variables, and a set of physical laws represented by differential equations that relate some of these independent and dependent variables to one another, and 2) it must be clear when two different theories refer to the same physical property. These constraints can be taken as additions to or clarifications of the mathematical framework outlined in Section 1.1. Here is the basic procedure I suggest we rely on to tell us what the time reversal operator looks like in the context of a particular set of physical theories: take the set TR_P to be composed of the symmetries, in the sense introduced in Section 1.1, of some set of physical theories P. The set TR_P must also possess the following properties:[15]

1. Each symmetry in TR_P must have some basic intuitive properties, such as being a discrete symmetry of a physical theory and being an involution.

2. Every theory in P must be invariant under a symmetry consistent with every physical symmetry in TR_P.

If TR_P is the null set, then the theories in P are not invariant under time reversal. If TR_P is non-empty, then we can rely on the consistency constraint to help us identify elements of TR_P with one another. Consistent symmetries, in this sense, treat variables corresponding to the same physical properties the same way, modulo variables that appear in one theory that don't correspond to the same physical property as any variables in the other theory. For instance, the symmetry that takes a point (x, y) to $(x, -y)$ and the symmetry that

[15]Here, I follow Peterson <21>, and I refer readers interested in a further explication and defense of the consistency condition to this work.

takes a point (x, y, z) to $(x, -y, z)$ are consistent while the symmetry that takes a point (x, y) to $(x, -y)$ and the symmetry that takes a point (x, y, z) to $(-x, -y, z)$ are not, assuming the x- and y-coordinates in each theory represent the same physical quantity. If we can use the consistency criterion to identify every element of TR_P with every other element of TR_P, then the elements of TR_P are physical symmetries representing the same overarching time reversal symmetry in different physical theories. If there is no such way to unify all of the elements of TR_P, then the set TR_P picks out a number of different overarching time reversal symmetries, no one of which is *the* time reversal symmetry. Either way, so long as TR_P is non-empty, we can say that the set of laws P is invariant under time reversal.

The intuition that my approach relies on is that time reversal operators in one theory can act as a constraint on the proper time reversal operator of another theory. We can rely on this intuition to constrain what time reversal should generally look like, but we must also rely on some minimal intuitions about what it means to be time reversal (as opposed to, say, parity reversal) as well. Because my account relies on such intuitions, it does not avoid all of the problems that the limited nature of our intuitions poses for an account of time reversal, but I believe it minimizes them since the intuitions appealed to by my account ascribe features to the time reversal operator that the intuitive accounts I have discussed (as well as the theory-relative accounts, for that matter) agree upon. I will not, then, be picking fights with these theories over arcane issues of metaphysics or purity. What is more, my account allows the question of what the time reversal operator looks like as well as which theories are invariant under time reversal to both be empirical questions that are typically discovered together, which is a result that no other account of time reversal can boast. There is something for everyone to love (and hate) about this third way; intuitive account proponents will like that their intuitions come into play when comparing the constraints placed on the time reversal operator across numerous physical theories, and theory-relative account proponents will like that we determine the time reversal operator by paying attention to which symmetries a particular physical theory is invariant under and using these symmetries as constraints in our analysis.

Most importantly, my approach need not assume anything about which particular theories are time reversal invariant, precisely how the time reversal operator acts on each property, which properties are basic or non-basic, or what temporal features spacetime does or doesn't have in order to derive its results. The hope, then, is that this holistic and mathematically involved project will be able to deliver non-question-begging answers to the kinds of questions we'd like an account of time reversal to answer within a particular theoretical context. In what follows, I'll attempt to put some flesh on these bare bones and provide a more detailed methodology for exactly how one following my account should determine both which theories are time reversal invariant and what the time reversal operator looks like in cases where the theories in question satisfy the two constraints laid out above.

5.1. A few desiderata for a time reversal operator

Though I do not agree with purely intuitive accounts of time reversal, the basic point that one must start with some minimal notion of time reversal in order to answer either of our questions of interest is certainly true. In what follows I present some minimal constraints that must be satisfied in order for a time reversal operator to be a *time reversal* operator (as opposed to, say, a parity reversal operator, or a spacetime reversal operator) for the kinds of theories my account is capable of dealing with. First of all, a time reversal operator should be a discrete point symmetry of time reversal invariant systems, meaning that it should be a function from solutions (represented as sets of points on some variable space) to solutions which, unlike continuous symmetries like translations and rotations, shouldn't depend on some variable parameter. This follows intuitively from the fact that we never talk about time reversing some state of affairs to some extent or by some value in the way that we talk about, say, translating some figure to the left three meters or rotating some figure by an angle of ninety degrees. Time reversal also ought to be an involution, meaning that two time reversals leave the system where it started. Stated formally, this constraint requires that $T(T(p)) = p$ for every point p in a theory's variable space. This is another obvious constraint on a time reversal operator since two applications of the time reversal operator should leave us with the same state we started with. This constraint also sticks close to the criteria on a time reversal operator laid out by Earman <9>.

But requiring that T be an involution is not enough, for there are other physical symmetries that satisfy this condition as well. The most notable of these, parity reversal, I will represent by the operator P. The parity reversal operator is essentially the spatial analogue of time reversal. To distinguish the two, then, I simply impose the following restrictions: a physical state Ψ must inhabit the same spatial location as its time-reversed counterpart and the same "temporal location" as its parity-reversed counterpart.[16] That is, $T(\mathbf{x_n}) = \mathbf{x_n}$ and $P(t) = t$. Any candidate operator that satisfies both conditions in a case where Ψ is a function of time and position only will be called the "trivial" or "identity" operator I since it maps each state to itself. What is more, any candidate for T or P that does not satisfy either of these criteria may still be what I will call a "spacetime reversal operator", which I'll represent as A, which satisfies the condition $A(\mathbf{x_n}, t) = (u_1(\mathbf{x_n}, t), u_2(t, \mathbf{x_n}))$ for some function u_1 which satisfies $u_1(u_1(\mathbf{x_n}, t), u_2(t, \mathbf{x_n})) = x$ and for some function u_2 which satisfies $u_2(u_2(t, \mathbf{x_n}), u_1(\mathbf{x_n}, t)) = t$ (since we require that A must still be an involution). In looking for time reversal operators, then, I restrict my attention to only those operators that satisfy the condition that $T(\mathbf{x_n}) = \mathbf{x_n}$. This is not to say that parity reversal and spacetime reversal are not philosophically interesting or deeply connected with the project of constructing a proper account of T; rather, it's just that A and P are different operators from T and are thus not the subject of this particular inquiry. I do believe that, should no candidate T operators be forthcoming, candidate A operators may provide the closest thing to a time reversal operator we may find, but since the condition for T fits better with our intuitions of what separates time reversal from other involutions, we should look for T-candidates before we go searching for A-candidates to play the role of the time reversal operator in our theory.

This is all the analysis of what we're looking for in a time reversal operator I will provide. The only other constraint, consistency, will be applied only after we have determined which of our physical theories are invariant under which discrete symmetries, so I will turn next to how to determine which discrete symmetries our theories are invariant under.

5.2. Hydon's method

We now know some basics about the kind of symmetries we're looking for when we look for time reversal symmetries, but in order to determine whether a given theory is time reversal invariant, we first need to know what discrete symmetries the theory is invariant under. These symmetries provide us with the candidate time reversal operators in the first place and will allow us to utilize consistency as a constraint on time reversal operators. Hydon <11, 12, 13, 14, 15, 16> provides a clear and (relatively) simple method for calculating all of the discrete symmetries of a given differential equation.[17] Hydon's methodology works as follows: first, find the Lie algebra of point symmetry transformation generators for the differential equation in question[18]. These symmetry generators will give us all of the symmetries continuously connected to the identity component but will not directly give us the discrete symmetries. However, the Lie algebra of the generators of the point symmetry transformations constitutes a vector space with the following property: the adjoint action of any symmetry transformation of the original differential equation on the Lie algebra of generators of the point symmetry transformations will simply act as a change of basis. Relying on the fact that the commutation relations between basis elements of the Lie algebra of generators of the point symmetry transformations are unchanged under a change of basis, we can derive a series of coupled equations whose solutions will give us all of the symmetries of the differential equation in question. All that remains, then, is to factor out the continuous symmetries of the differential equation, and we're left with all of the discrete symmetries of our differential equation. Though computationally difficult to implement, this procedure

[16] Clearly this constraint only applies when states Ψ are associated with particular regions in space and time. For a theory whose physical states are not functions of space and/or time, the conditions imposed by this paragraph on T, P, and A can be ignored; what will guide us to distinguishing time reversal from parity reversal in such situations will be how well certain discrete symmetries line up with parity reversal operators and time reversal operators in other physical theories.

[17] This method is fairly complicated to implement, and I would direct those interested in implementing Hydon's method to the YaLie Mathematica package, detailed in Díaz <6>. I will briefly outline Hydon's method here, but those interested in the technical details should consult Hydon's work. Those interested only in the results of Hydon's method can skip to the next section.

[18] For details on how to do this, consult Olver <20>.

is guaranteed to give us all of the discrete symmetries of any differential equation which a) is invariant under some continuous symmetries and b) has a non-abelian Lie algebra of point symmetry transformation generators. Both of these conditions are satisfied by most differential equations of interest.

6. The Equations

The previous sections introduced a new hybrid approach to time reversal that requires one to examine the differential equations utilized by a number of different physical theories and the discrete symmetries under which these differential equations are invariant in order to determine both what form the a time reversal transformation should take and what physical theories are invariant under that transformation. Hydon and others such as Levi and Rodríguez <17> have produced all of the discrete symmetries for number of differential equations. I will draw on this work to analyze the discrete symmetries of eleven differential equations and, more importantly, the involutions under which these differential equations are invariant. I will take this catalogue of involutions and discuss what my approach, when applied to this catalogue of involutions, tells us about the time reversal operator in worlds governed solely by select subsets of the equations I will analyze. Finally, I will conclude by drawing comparisons between my analysis and the analysis suggested by both intuitive and theory-relative time reversal theorists. The equations I have to work with are, in many cases, of limited physical interest, in part because, while Hydon's work has led to productive analysis of differential equations and their symmetries by mathematicians, few physicists have extended this work to cases of physical interest.

The information on all the non-trivial involutive candidate time reversal operators for six differential equations is summarized on the following two tables. For each of these equations, I will use the shorthand u_x to stand for $\frac{\partial u}{\partial x}$, u_{xx} as shorthand for $\frac{\partial^2 u}{\partial x^2}$, and so on. In cases with one dependent variable and one independent variable, I will call the dependent variable x and the independent variable t, while in cases where there are one dependent and two independent variables, x and t will refer to the independent variables while u will refer to the dependent variable. My notation is not shared by all of the authors whose work I cite, but it makes it easier to see similarities across the discrete symmetries I analyze.[19]

Table 1: Equations all of whose discrete symmetries are known

Eq Abbrev	**Equation**	**Solved in**
Chazy	$x_{ttt} = 2xx_{tt} - 3y_t^2 + \lambda(6x_t - x^2)^2$	Hydon <11, 12>
EPD	$u_{tt} - u_{xx} = \frac{p(p+1)}{t^2}u$	Hydon <16>
Harry-Dym	$u_t = u^3 u_{xxx}$	Hydon <11, 13, 16>
K	$x_{tt}^2 = (\frac{x_t}{t} - e^x)^2$	Hydon <15>
L	$x_{tt} = \frac{x_t}{t} + \frac{4x^2}{t^3}$	Hydon <12, 16>
Toda	$u_{tt} = e^{u_+ - u} + e^{u - u_-}$	Levi and Rodríguez <17>

[19] The Toda equation, which appears on the list, is somewhat unique and so requires some further discussion. It is a discrete differential equation and contains the term $u_\pm = u(x \pm h, t)$, where h is the lattice step which separates two adjacent values of x. Further details may be found in Levi and Rodríguez <17>.

Table 2: Involutive Candidate T Operators

Name	**Equation**	**Coordinate Transformation**
T_{C1}	Chazy	$T_{C1}(t,x) = (-t,-x)$
T_{C2}	Chazy	$T_{C2}(t,x) = (-\frac{1}{t}, t^2x+6t)$
T_{C3}	Chazy	$T_{C3}(t,x) = (\frac{1}{t}, -(t^2x+6t))$
T_{E1}	EPD	$T_{E1}(x,t,u) = (-x,t,u)$
T_{E2}	EPD	$T_{E2}(x,t,u) = (x,-t,u)$
T_{E3}	EPD	$T_{E3}(x,t,u) = (x,t,-u)$
T_{E4}	EPD	$T_{E4}(x,t,u) = (-x,-t,u)$
T_{E5}	EPD	$T_{E5}(x,t,u) = (x,-t,-u)$
T_{E6}	EPD	$T_{E6}(x,t,u) = (-x,t,-u)$
T_{E7}	EPD	$T_{E7}(x,t,u) = (-x,-t,-u)$
T_{E8}	EPD	$T_{E8}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{t}{t^2-x^2}, u)$
T_{E9}	EPD	$T_{E9}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{t}{t^2-x^2}, u)$
T_{E10}	EPD	$T_{E10}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{-t}{t^2-x^2}, u)$
T_{E11}	EPD	$T_{E11}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{t}{t^2-x^2}, -u)$
T_{E12}	EPD	$T_{E12}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{-t}{t^2-x^2}, u)$
T_{E13}	EPD	$T_{E13}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{-t}{t^2-x^2}, -u)$
T_{E14}	EPD	$T_{E14}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{t}{t^2-x^2}, -u)$
T_{E15}	EPD	$T_{E15}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{-t}{t^2-x^2}, -u)$
T_{HD1}	Harry-Dym	$T_{HD1}(x,t,u) = (x,-t,-u)$
T_{HD2}	Harry-Dym	$T_{HD2}(x,t,u) = (-x,t,-u)$
T_{HD3}	Harry-Dym	$T_{HD3}(x,t,u) = (-x,-t,u)$
T_{HD4}	Harry-Dym	$T_{HD4}(x,t,u) = (-\frac{1}{x}, t, \frac{u}{x^2})$
T_{HD5}	Harry-Dym	$T_{HD5}(x,t,u) = (\frac{1}{x}, t, \frac{-u}{x^2})$
T_{HD6}	Harry-Dym	$T_{HD6}(x,t,u) = (-\frac{1}{x}, -t, \frac{-u}{x^2})$
T_{HD7}	Harry-Dym	$T_{HD7}(x,t,u) = (\frac{1}{x}, -t, \frac{u}{x^2})$
T_{K1}	K	$T_{K1}(t,x) = (-t,x)$
T_{K2}	K	$T_{K2}(t,x) = (\frac{1}{t}, x+4\ln\|t\|)$
T_{L1}	L	$T_{L1}(t,x) = (-t,-x)$
T_{L2}	L	$T_{L2}(t,x) = (\frac{1}{t}, \frac{x}{t^2})$
T_{L3}	L	$T_{L3}(t,x) = (-\frac{1}{t}, -\frac{x}{t^2})$
T_{T1}	Toda	$T_{T1}(x,t,u) = (x,-t,u)$
T_{T2}	Toda	$T_{T2}(x,t,u) = (-x,t,-u)$
T_{T3}	Toda	$T_{T3}(x,t,u) = (-x,-t,-u)$

7. Analysis

Before using the information on these tables to determine what time reversal would look like if some subset of differential equations examined above served as the fundamental laws of our world, here's a quick review of how to apply my account to determine what the time reversal operator is:

Step 1: Find a set of differential equations that seem the best available candidates for the fundamental physical laws governing the world.

Step 2: Determine all of the discrete symmetries under which these differential equations are invariant.

Step 3: Apply the intuitive criteria for a time reversal operator to rule out those discrete symmetries that cannot be time reversal operators. This means:
A: Rule out all non-involutions.
B: Determine which of the variables represent spatial and temporal coordinates, and rule out all symmetries that do not leave the spatial coordinates unchanged while changing the temporal coordinates.
C: Rule out all symmetries that do not satisfy the consistency condition.

Step 4: Any remaining discrete symmetries can be properly called "time reversal".

So, having investigated a number of differential equations and determined all of the involutions under which these equations are invariant, there are essentially three more important pieces of information necessary to determine what the time reversal operator should look like: first, we need to determine which of these differential equations provide the best available candidates for the fundamental physical laws governing the world; secondly, we need to determine which variables appealed to by these fundamental equations correspond to the same physical properties; and finally, we need to know which of these variables is time and which is position. Unfortunately, on the first count, none of these differential equations are the sort of things that contemporary physicists would call our best candidates for physical laws governing the world since none appear in the most successful current fundamental physical theories (e.g. quantum field theory, general relativity, statistical mechanics, etc.). The Chazy equation, for instance, is of interest because of certain mathematical properties it has but as of yet is of no physical interest. The differential equations listed here as equations K and L are likewise of interest mathematically but, to my knowledge, carry no physical significance, so to call them the best available candidates for the fundamental physical laws governing our world would be laughable.

In the absence of any differential equations in my analysis that seem the best available candidates for the fundamental physical laws governing *our* world, then, we cannot, given the tools above, answer the crucial question of what time reversal *in our own world* must be; however, we can answer the interesting question of what the time reversal operator is in worlds fundamentally governed by the above differential equations (that is, worlds in which the set or some subset of the above differential equations provides the set of best available candidates for the fundamental physical laws governing the world). By turning our focus from the actual world to these other possible worlds, we have an added three degrees of flexibility that allow us to posit the information necessary for applying my method: 1) we can stipulate which of the above differential equations we take to be fundamental at the world in question, 2) we can stipulate which of the independent and dependent variables in each differential equation correspond to the same physical properties, and 3) we can stipulate which of the variables corresponds to time and which of the variables corresponds to position.

I will proceed to analyze a number of these possible worlds and adopt the following naming convention for these worlds: I will label worlds as $W = \langle L; I; T; X \rangle$, where L is the set of differential equations that fundamentally govern the world, I is the set of claims about which variables correspond to the same physical properties, and T and X are the sets of variables appearing in L which representing temporal and spatial coordinates respectively. I will append to each law in L a numerical subscript so that it is clear which variables correspond to the same physical properties as other variables, and I will translate $A = B$ as "A and B correspond to the same physical property".

With these preliminaries out of the way, let's turn to a few worlds of interest. I will limit my attention in the remainder of this section to two cases of interest: first, a world in which time reversal works as one might intuitively expect, which provides a first glimpse of how my account is practically applied, and second, a world where time reversal may transform the time coordinate in an unusual way, thus suggesting a way in which the method I propose may be used to discover novel potential time reversal operators.

7.1. An Analysis of W_1

Take $W_1 = \langle \text{EPD}_1, \text{Toda}_2; x_1 = x_2, t_1 = t_2; t_1, t_2; x_1, x_2 \rangle$. This is the world fundamentally governed by the Euler-Poisson-Darboux equation and the Toda equation where the x coordinates refer to spatial location and the t coordinates refer to time. We assume that the two fields generated by these different differential equations, u_1 and u_2, refer to different physical features of the world so that the two laws are not inconsistent with one another. According to Tables 2 and 3, since the two fundamental differential equations in W_1 are the EPD and Toda equations, the candidate non-trivial involutive time reversal operators are the following transformations:

$$T_{E1}(x,t,u) = (-x,t,u) \tag{47}$$
$$T_{E2}(x,t,u) = (x,-t,u) \tag{48}$$
$$T_{E3}(x,t,u) = (x,t,-u) \tag{49}$$
$$T_{E4}(x,t,u) = (-x,-t,u) \tag{50}$$
$$T_{E5}(x,t,u) = (x,-t,-u) \tag{51}$$
$$T_{E6}(x,t,u) = (-x,t,-u) \tag{52}$$
$$T_{E7}(x,t,u) = (-x,-t,-u) \tag{53}$$
$$T_{E8}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{t}{t^2-x^2}, u) \tag{54}$$
$$T_{E9}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{t}{t^2-x^2}, u) \tag{55}$$
$$T_{E10}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{-t}{t^2-x^2}, u) \tag{56}$$
$$T_{E11}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{t}{t^2-x^2}, -u) \tag{57}$$
$$T_{E12}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{-t}{t^2-x^2}, u) \tag{58}$$
$$T_{E13}(x,t,u) = (\frac{x}{t^2-x^2}, \frac{-t}{t^2-x^2}, -u) \tag{59}$$
$$T_{E14}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{t}{t^2-x^2}, -u) \tag{60}$$
$$T_{E15}(x,t,u) = (\frac{-x}{t^2-x^2}, \frac{-t}{t^2-x^2}, -u) \tag{61}$$
$$T_{T1}(x,t,u) = (x,-t,u) \tag{62}$$
$$T_{T2}(x,t,u) = (-x,t,-u) \tag{63}$$
$$T_{T3}(x,t,u) = (-x,-t,-u) \tag{64}$$

By considering only these symmetries as time reversal operator candidates, we have applied step 3A of my procedure. For step 3B, we rule out all symmetries that do not leave the spatial coordinates unchanged. This means that we can ignore all of the above symmetries that non-trivially transform the x-coordinates, leaving us with the following time reversal operator candidates:

$$T_{E2}(x_1, t_1, u_1) = (x_1, -t_1, u_1) \tag{65}$$
$$T_{E3}(x_1, t_1, u_1) = (x_1, t_1, -u_1) \tag{66}$$
$$T_{E5}(x_1, t_1, u_1) = (x_1, -t_1, -u_1) \tag{67}$$
$$T_{T1}(x_2, t_2, u_2) = (x_2, -t_2, u_2) \tag{68}$$

The only time reversal candidate under which the Toda equation is invariant, we see, is T_{T1}. We can now apply step 3C of my method by using the consistency constraint to rule out T_{E3} as a suitable time reversal operator since it transforms the time coordinate t_1 in a way that T_{T1} does not transform t_2. This leaves us with three candidates for the time reversal operator:

$$T_{E2}(x_1, t_1, u_1) = (x_1, -t_1, u_1) \tag{69}$$
$$T_{E5}(x_1, t_1, u_1) = (x_1, -t_1, -u_1) \tag{70}$$
$$T_{T1}(x_2, t_2, u_2) = (x_2, -t_2, u_2) \tag{71}$$

This tells us that time reversal operator in W_1 works as follows on the following coordinates:

$$T(x) = x \tag{72}$$
$$T(t) = -t \tag{73}$$
$$T(u_2) = u_2 \tag{74}$$

But what about $T(u_1)$? Sine both T_{E2} and T_{E5} are time reversal operator candidates and since these two transformations only differ with respect to their action on u_1, the answer is underdetermined, and there is no fact of the matter (at the moment, at least) as to how time reversal acts on u_1. One of the advantages of my approach to time reversal, however, is that it is holistic: what constitutes a time reversal operator in a given world depends on all of the fundamental laws that govern the world in question. So let's consider two different worlds that share a great deal in common with W_1 that will give us different fully determined time reversal operators.

The first world to consider is $W_1' = \langle \text{EPD}_1, \text{Toda}_2, \text{Chazy}_3; x_1 = x_2, t_1 = t_2 = t_3; t_1, t_2, t_3; x_1, x_2 \rangle$. We will make one further stipulation: the coordinate x_3 in the Chazy equation corresponds to the coordinate u_1 in EPD. Adding our candidate time reversal operators from the Chazy equation to our set of remaining time reversal operators for W_1, we get the following set of candidate time reversal operators for W_1':

$$T_{E2}(x_1, t_1, u_1) = (x_1, -t_1, u_1) \tag{75}$$
$$T_{E5}(x_1, t_1, u_1) = (x_1, -t_1, -u_1) \tag{76}$$
$$T_{T1}(x_2, t_2, u_2) = (x_2, -t_2, u_2) \tag{77}$$
$$T_{C1}(t_3, x_3) = (-t_3, -x_3) \tag{78}$$

We can now apply Step 3C from my analysis again by ruling out symmetries that aren't consistent with one another. Since T_{C1} transforms t_3 in a way that the others don't transform t_1 or t_2, we can rule this symmetry out. But what's more, our stipulation that u_1 and x_3 are the same variable tells us that we can rule out T_{E2} from contention since it transforms u_1 in a way that T_{C1} does not transform x_3 while T_{E5} does. This leaves us with the following set of time reversal operators:

$$T_{E5}(x_1,t_1,u_1) = (x_1,-t_1,-u_1) \tag{79}$$
$$T_{T1}(x_2,t_2,u_2) = (x_2,-t_2,u_2) \tag{80}$$
$$T_{C1}(t_3,x_3) = (-t_3,-x_3) \tag{81}$$

and we know that our time reversal operator works as follows:

$$T'(x) = x \tag{82}$$
$$T'(t) = -t \tag{83}$$
$$T'(u_1) = -u_1 \tag{84}$$
$$T'(u_2) = u_2 \tag{85}$$

So, by adding the Chazy equation to the set of fundamental laws at W_1, we can determine how time reversal acts on every property of physical interest.

Now consider the world $W_1'' = \langle \text{EPD}_1, \text{Toda}_2, \text{K}_3; x_1 = x_2, t_1 = t_2 = t_3; t_1, t_2, t_3; x_1, x_2 \rangle$. Again, we will make one further stipulation: the coordinate x_3 in K corresponds to the coordinate u_1 in EPD. Adding our candidate time reversal operators from K to our set of remaining time reversal operators for W_1, we get the following set of candidate time reversal operators for W_1'':

$$T_{E2}(x_1,t_1,u_1) = (x_1,-t_1,u_1) \tag{86}$$
$$T_{E5}(x_1,t_1,u_1) = (x_1,-t_1,-u_1) \tag{87}$$
$$T_{T1}(x_2,t_2,u_2) = (x_2,-t_2,u_2) \tag{88}$$
$$T_{K1}(t_3,x_3) = (-t_3,x_3) \tag{89}$$
$$T_{K2}(t_3,x_3) = (\frac{1}{t_3}, x_3 + 4\ln|t_3|) \tag{90}$$

We can now apply Step 3C from my analysis again by ruling out symmetries that aren't consistent with one another. Since T_{K2} transforms t_3 in a way that the others don't transform t_1 or t_2, we can rule this symmetry out. But as with W_1', our stipulation that u_1 and x_3 are the same variable tells us that we can rule out another symmetry from contention, this time T_{E5}, since it transforms u_1 in a way that T_{K1} does not transform x_3 while T_{E2} does. This leaves us with the following set of time reversal operators:

$$T_{E2}(x_1,t_1,u_1) = (x_1,-t_1,u_1) \tag{91}$$
$$T_{T1}(x_2,t_2,u_2) = (x_2,-t_2,u_2) \tag{92}$$
$$T_{K1}(t_3,x_3) = (-t_3,x_3) \tag{93}$$
$$\tag{94}$$

and we know that our time reversal operator works as follows:

$$T''(x) = x \tag{95}$$
$$T''(t) = -t \tag{96}$$
$$T''(u_1) = u_1 \tag{97}$$
$$T''(u_2) = u_2 \tag{98}$$

That is, by adding K to the set of fundamental laws at W_1, we can determine how time reversal acts on every property of physical interest, and, perhaps most interestingly, obtain a different time reversal operator from T'.

The time reversal operator in W_1 is underdetermined by the evidence available. It could act as either T' or T'', which seems to be a problem for my account since it is now open to many of the criticisms I lobbed at theory-relative accounts. But this underdetermination is a consequence of the holistic virtues of my account. If we take either the Chazy equation or equation K to be fundamental in W_1, we break the underdetermination and are presented with a time reversal operator whose action on each variable is fully determined; however, since the Chazy equation and equation K would place constraints on W_1 that would pull in different directions, the time reversal operator in W_1 alone ought to be underdetermined since we don't know which (if any) other laws in this world could help us establish further facts about how the time reversal operator acts on states of that world. This example thus shows that the time reversal operator in a world like W_1 depends a great deal on the laws we take to be fundamental at that world.

A second important feature of my account that these examples highlight is the importance of the identity claims we make concerning variables that appear in different theories. My analysis of both W_1' and W_1'' allowed me to place constraints on the time reversal operator by adding a new fundamental law, but in each case I needed to identify one of the variables (labeled x_3 in each case) with a variable appearing in other laws (u_1 in each case) by claiming that both correspond to the same property. Had I not done so, neither of the laws would have helped to further constrain the time reversal operator candidates under consideration. My method is extremely sensitive to variations in the identity claims we make concerning these variables, but rather than serving as a vice, I take this sensitivity as a virtue of my account of time reversal as it helps us better understand what disagreements over the form a symmetry takes (or ought to take) really amount to (or ought to amount to): differences in 1) the laws taken to be fundamental and/or 2) which variables in different theories are taken correspond to the same physical property.

But setting aside the lessons to be drawn from the underdetermination of u_1 in W_1, the analysis of time reversal in this world is fairly straightforward: distances of Δx will remain unchanged by time reversal while a duration of Δt will change to $-\Delta t$, giving us a time reversal operator like the ones proposed by Horwich and Albert. That is, if the inhabitants of W_1 were to watch a movie and its time-reversed counterpart, they would see one playing "backwards" at the same speed the other was playing "forwards". What's more, the field u_2, while a function of t, is left unaffected by the time reversal process, as many fundamental fields, at least according to Albert, are wont to do. So my account provides us, at least in this case, with a relatively uninteresting time reversal operator for W_1.

7.2. An Analysis of W_2

Take $W_2 = \langle \text{Chazy}_1, \text{Harry-Dym}_2, \text{K}_3, \text{L}_4; t_1 = x_2 = t_3 = t_4; t_1, x_2, t_3, t_4; t_2 \rangle$. This is the world fundamentally governed by the Chazy equation, the Harry-Dym equation, and Equations P and R where the t_1, x_2, t_3, and t_4 all refer to time while t_2 alone refers to spatial position. It's worth noting here how strange this assignment would be for those who agree with Callender <5> and Skow <24> and think that the time variable must be the variable for which the equation's initial value problem is well-behaved, but, for the moment, assume their view is wrong and that we have very good evidence to believe that the x coordinates really represent time in these equations. Again, we take the fields x_1, u_2, x_3, and x_4 to represent different physical quantities but make no further assertions about them. According to Table 2, the following are the candidates for the time reversal operator that don't result in changes to the spatial coordinates (which, remember, in W_2, is just t_2) and do result in changes to the temporal coordinates (which, remember, in W_2 are t_1, x_2, t_3, and t_4):

$$T_{C1}(t_1, x_1) = (-t_1, -x_1) \tag{99}$$

$$T_{C2}(t_1, x_1) = \left(-\frac{1}{t_1}, t_1^2 x_1 + 6t_1\right) \tag{100}$$

$$T_{C3}(t_1, x_1) = \left(\frac{1}{t_1}, -(t_1^2 x_1 + 6t_1)\right) \tag{101}$$

$$T_{HD2}(x_2, t_2, u_2) = (-x_2, t_2, -u_2) \tag{102}$$

$$T_{HD4}(x_2, t_2, u_2) = \left(-\frac{1}{x_2}, t_2, \frac{u_2}{x_2^2}\right) \tag{103}$$

$$T_{HD5}(x_2, t_2, u_2) = \left(\frac{1}{x_2}, t_2, \frac{-u_2}{x_2^2}\right) \tag{104}$$

$$T_{K1}(t_3, x_3) = (-t_3, x_3) \tag{105}$$

$$T_{K2}(t_3, x_3) = \left(\frac{1}{t_3}, x_3 + 4\ln|t_3|\right) \tag{106}$$

$$T_{L1}(t_4, x_4) = (-t_4, -x_4) \tag{107}$$

$$T_{L2}(t_4, x_4) = \left(\frac{1}{t_4}, \frac{x_4}{t_4^2}\right) \tag{108}$$

$$T_{L3}(t_4, x_4) = \left(-\frac{1}{t_4}, -\frac{x_4}{t_4^2}\right) \tag{109}$$

There are many more transformations left after this step of my analysis than in the previous two cases, and we are left with two candidate time reversal operators for equation K: T_{K1} and T_{K2}. Let's assume that T_{K1} is the "true" time reversal operator. If this is so, then the consistency constraints leave us with the following set of transformations as time reversal operator candidates:

$$T_{C1}(t_1, x_1) = (-t_1, -x_1) \tag{110}$$

$$T_{HD2}(x_2, t_2, u_2) = (-x_2, t_2, -u_2) \tag{111}$$

$$T_{K1}(t_3, x_3) = (-t_3, x_3) \tag{112}$$

$$T_{L1}(t_4, x_4) = (-t_4, -x_4) \tag{113}$$

If we take T_{P2} as the correct time reversal operator, then consistency constraints leave us with the follow set of transformations as time reversal operator candidates:

$$T_{C3}(t_1, x_1) = \left(\frac{1}{t_1}, -(t_1^2 x_1 + 6t_1)\right) \tag{114}$$

$$T_{HD5}(x_2, t_2, u_2) = \left(\frac{1}{x_2}, t_2, \frac{-u_2}{x_2^2}\right) \tag{115}$$

$$T_{K2}(t_3, x_3) = \left(\frac{1}{t_3}, x_3 + 4\ln|t_3|\right) \tag{116}$$

$$T_{L2}(t_4, x_4) = \left(\frac{1}{t_4}, \frac{x_4}{t_4^2}\right) \tag{117}$$

There are, then, essentially two candidates for the time reversal operator. The first, T_1, works as follows (again, taking x, without subscripts, to now refer to the position coordinate and t to now refer to the temporal coordinate):

$$T_1(x) = x \tag{118}$$
$$T_1(t) = -t \tag{119}$$
$$T_1(x_1) = -x_1 \tag{120}$$
$$T_1(u_2) = -u_2 \tag{121}$$
$$T_1(x_3) = x_3 \tag{122}$$
$$T_1(x_4) = -x_4 \tag{123}$$

The second, T_2, works as follows:

$$T_2(x) = x \tag{124}$$
$$T_2(t) = \frac{1}{t} \tag{125}$$
$$T_2(x_1) = -(t^2 x_1 + 6t) \tag{126}$$
$$T_2(u_2) = \frac{-u_2}{t^2} \tag{127}$$
$$T_2(x_3) = x_3 + 4\ln|t| \tag{128}$$
$$T_2(x_4) = \frac{x_4}{t^2} \tag{129}$$

Several things are worth remarking on at this point. First, whether one takes T_1 or T_2 to be the "true" or "best" time reversal operator in W_2, we are left with a time reversal operator that determinately transforms all of the coordinates we have examined here, including x_1, u_2, x_3, and x_4. We are also obviously left with two distinct ways the world could be time reversal invariant. But perhaps the most interesting result about W_2 is provided by T_2, a strange-looking time reversal operator which, odd though it is, still makes a kind of sense as a time reversal operator. As with T_1, displacements of Δx remain unchanged by T_2, but T_2 transforms the temporal coordinates in such a way that the distance between two events (say, t_0 and t_1 with displacement $\Delta t = t_0 - t_1$) changes to $-\frac{\Delta t}{t_0 t_1}$. So, were the inhabitants of W_2 to watch a movie, its T_1 counterpart, and its T_2 counterpart, they would note that, while the T_1 and T_2 counterparts were running backwards relative to the initial movie, some segments of the T_2 counterpart would move more slowly than those corresponding segments in the T_1 counterparts, and others would move more quickly.[20]

T_2, while an entirely different transformation than what we've seen previously, thus has some important similarities with the more intuitively plausible T_1 such as the way it inverts a sequence of events, but by adding in an additional time-dependent scaling factor, the overall transformation looks quite different from the one provided by T_1. I believe this shows an additional virtue of my account of time reversal in that it suggests interesting new ways the world could be time reversal invariant. Of course, W_2 on its own is not necessarily such a world, but when conducting further empirical research, scientists in W_2 should be on the lookout for differential equations whose symmetries would, via consistency, establish either T_1 or T_2 as the time reversal operator in W_2. Intuitive time reversal theorists would likely balk at a time reversal

[20]Those wishing to invalidate T_2 as a candidate time reversal operator could justify this decision by relying on the fact that this transformation "stretches" the interval in question in a way that, intuitively, time reversal should not. That is, in light of the analysis in this section, one may wish to supplement my list of intuitive assumptions regarding T with the requirement that, for $T(t) = f(t)$, where t is the variable in our theory referring to time, it must be the case that $|f(at + b)| = |af(t) + b|$ or some similar constraint that keeps the time reversal operator from stretching out temporal intervals. Likewise, others may object that this candidate time reversal operator contains an artificial singularity at the point $t = 0$ and so wish to add a "no artificial singularities" constraint to the list of intuitive assumptions previously articulated. However, without an explanation of why it is that we ought to accept these as necessary conditions for a symmetry to constitute time reversal, I see no reason to exclude T_2 as a time reversal operator in W_2 out of hand. My thanks to Bryan Roberts for a helpful discussion about this time reversal operator.

operator like T_2, but I see no reason to do so without any further information; again, the fundamental laws are invariant under this transformation, and it inverts sequences of events as required, so there seems no principled reason to reject fascinating and bizarre transformations like T_2 as candidate time reversal operators out of hand.

8. Conclusion

In concluding this paper, I should take a moment to compare my analysis to the stances towards time reversal in these four possible worlds adopted by token fictional representatives of both the intuitive and theory-relative archetypes. I have pointed out along the way already numerous places where my account diverges from intuitive accounts like Albert's, but it's worth remarking on the differences once again. Albert's account would have us take as candidates for time reversal only transformations T such that $T(x) = x$ and $T(t) = -t$. As I've shown above, transformations that satisfy this constraint are among the most common, appearing in W_1 and W_2 both (though Albert and others like him would doubtless want more information about how to interpret the fundamental fields in these worlds before agreeing with my analysis of how time reversal transforms these fields). However, my analysis of W_2 suggests that there may be another sort of time reversal (the one I call T_2) which acts on t in a different way. My account challenges intuitive accounts to justify their intuitions that rule out T_2 as a viable time reversal candidate, and, regardless of whether or not intuitive accounts can provide a compelling defense of these intuitions, I believe forcing the intuitive theorist to engage this challenge will result in progress in better understanding time reversal.

Theory-relative accounts are likewise challenged by my account, though the challenge they face is quite different from the one faced by intuitive accounts. Looking back over the four examples above, it is clear how the "intuitive" criteria I lay down in step 3 parts B and C (as described in the previous section) are essential to paring down the number of candidate time reversal operators for each world, and it is equally clear how essential this paring down of symmetry candidates is for making progress both in determining what the time reversal operator looks like and how one might best draw metaphysical conclusions from such a result. The challenge to theory-relative accounts, then, is to explain where the "intuitive" criteria laid down by my account go wrong, for it is these criteria that allow my account to provide more specific details than theory-relative accounts can about the behavior of the time reversal operator in each world. Again, I don't think my challenge reveals a fatal flaw in the reasoning of theory-relative accounts of time reversal, but it is a challenge that must be met if theory-relative accounts wish to show themselves superior to my account.

Though I have applied my method to determine the time reversal operator for various possible worlds governed by small subsets of the differential equations I have catalogued here, there is still much more work to be done in fleshing out my account of time reversal and, perhaps, extending it to physical laws of greater interest. Though my analysis of W_1 and W_2 provides interesting examples of ways the world could be, the real test for my account will come when my methodology is applied to a set of differential equations that more aptly represents what we take to be our best available fundamental physical theories or when is it extended to a more general mathematical framework that allows it to weigh in on physical theories that don't satisfy the previously articulated constraints. Unfortunately, such an analysis is beyond the scope of this work, and it may be some time before the relatively new work of Hydon inspires mathematicians and physicists to provide us with a more physically interesting set of differential equations all of whose discrete symmetries we know, but even so, my analysis of these examples does provide an important next step towards assessing the prospects of this new account of time reversal.

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