A Defense of Scientific Platonism without Metaphysical Presuppositions

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Abstract: From the Platonistic standpoint, mathematical edifices form an immaterial, unchanging, and eternal world that exists independently of human thought. By extension, "scientific Platonism" says that directly mathematizable physical phenomena – in other terms, the research field of physics – are governed by entities belonging to this objectively existing mathematical world.
Platonism *is* a metaphysical theory. But since metaphysical theories, by definition, are neither provable nor refutable, anti-Platonistic approaches cannot be less metaphysical than Platonism itself. In other words, anti-Platonism is not "more scientifical" than Platonism. All we can do is to compare Platonism and its negations under epistemological criteria such as simplicity, economy of hypotheses, or consistency with regard to their respective consequences.
In this paper I intend to show that *anti*-Platonism claiming in a first approximation (i) that mathematical edifices consist of meaningless signs assembled according to arbitrary rules, and (ii) that the adequacy of mathematical entities and phenomena covered by physics results from idealization of these phenomena, is based as much as Platonism on metaphysical presuppositions. Thereafter, without directly taking position, I try to launch a debate focusing on the following questions:
(i) To maintain its coherence, is anti-Platonism not constrained to adopt extremely complex assumptions, difficult to defend, and not always consistent with current realities or practices of scientific knowledge?
(ii) Instead of supporting anti-Platonism whatever the cost, in particular by the formulation of implausible hypotheses, would it not be more adequate to accept the idea of a mathematical world existing objectively and governing certain aspects of the material world, just as we note the existence of the material world which could also not exist?

0. Introduction

Kant asks himself how the physical laws obtained empirically can take an mathematical form, knowing that mathematics is not based on experience. In this paper I defend – while refusing any direct argumentation of metaphysical order – the response emanating from Platonism.
From the Platonistic standpoint, mathematical edifices form an immaterial, unchanging, and eternal world, that exists independently of human thought. By extension, "scientific Platonism" says that directly mathematizable physical phenomena – the research field of physics – are governed by entities belonging to this immaterial, immutable, eternal, and objectively existing mathematical world.
But how can I defend such an essentially metaphysical approach without ever resorting to metaphysical arguments?
The answer is simple. The direct negation of a metaphysical proposition is in turn a metaphysical proposition. Both propositions – the assertion and its negation – are neither provable nor refutable. All we can do is to compare a metaphysical proposition and its negation under epistemological criteria such as simplicity, economy of hypotheses, consistency and so on.
An ineradicable prejudice alleges that hypotheses reserving the notion of objective existence to the only material world would be "more scientific" than other hypotheses claiming the objective existence of an immaterial reality. But it is not that simple. Since no one can get out of the mental representation he or she has of material reality, no one is able to empirically check whether the material reality supposed to exist objectively coincides with the representations we have about it. (Maddy, 2007, p. 35) Hence Carnap relegates the ontological realism *and its competing theories* like idealism to the same metaphysics. (Carnap, 1931, p. 237) We can certainly question ourselves whether abandoning the hypothesis of the existence of material reality will not lead to an infinity of epistemological and other inconsistencies. But nothing prevents us from taking the same question about the abandonment of the Platonistic hypothesis.

In this paper, I intend to show that *anti*-Platonism claiming in a first approximation

- that mathematical edifices consist of meaningless signs assembled according to arbitrary rules
- that the adequacy of mathematical entities and phenomena covered by physics comes out from the idealization of these phenomena

is based as much as Platonism on metaphysical presuppositions.

Thereafter, I will let each one compare Platonism and anti-Platonism in terms of epistemological criteria such as simplicity, economy of hypotheses, consistency and so on, without directly taking position.

The foundations of mathematics and physics – whatever they could be – are not the same. At first, I oppose consequently the *mathematical* Platonism to its negations. This step will take a good half of the paper.
But it is only on *this* basis that we can later compare Platonism establishing *links between mathematics and physics*, to its negations supposedly "more scientific".

1. Mathematical Platonism and its negations

1.1 Beyond the Hilbert misunderstanding

An entrenched commonplace claims that "in the view of Hilbert" any mathematical edifice would consist of "meaningless signs assembled according to arbitrary rules". Beyond this caricatured vision (comp. Zach 2005, p.31), Hilbert's project takes an very different turn. Hilbert tackles the foundations of mathematics *as if* all mathematical edifices consisted of arbitrary meaningless sign assemblages (Zach, 2005, p.20). For Hilbert, *reconstructing* mathematics *as if* it were an arbitrary system gives mathematics a more solid basis, particularly in regard to the proof process. (Hilbert, 1918/1996, p.1107; Snapper, 1979, p.214) This reconstruction involves the formalization of *given* mathematical edifices by – at least in theory – effectively arbitrary *formal systems*.
A formal system **Sy** *stricto sensu* consists of (i) an alphabet of arbitrary signs **A**, (ii) a set of arbitrary rules **Rm** governing the correct formation of "words" m_i from signs belonging to **A**, (iii) a set of arbitrary rules **Rd** governing the correct deduction of a word m_j from another word m_i, $i \neq j$, and (iv) an arbitrary set **Ax** of axioms Ax_k, i.e. correctly assembled and irreducible words undergoing the following constraints: These Ax_k (i) should not lead to inconsistencies in their consequences and (ii) must cover the entire system. A correctly written word m_u that belongs to a chain of correct deductions $m_i \rightarrow m_j$ going back to the axioms of **Ax** is a called a theorem θ_v of **Sy**.
Let us now consider a *given* mathematical edifice E, whose theorems $\boldsymbol{\theta}_v$ are considered true under a certainly less rigorous proof processus than the "mechanical" deduction of a theorem θ_v belonging to **Sy**. This mathematical edifice E is *formalized* by the formal system **Sy** if and only if it is possible to establish a one-to-one relation, or "bijection" between each theorem θ_v of **Sy** and each theorem $\boldsymbol{\theta}_v$ of E. Let us note this bijection $\mathsf{E} \leftarrow \mathbf{\Phi} \rightarrow$ **Sy**.
What is now the nature of E ? Within $\mathbf{\Phi}$, an edifice E is systematically regarded as "given", yes, but given *how*? Is it just an other formal system ***Sy*** previously constructed – thus another *human made* system – where every sign σ of the alphabet **A** corresponds to one and only one sign σ of the alphabet ***A*** whereas **Rm**, **Rd** and **Ax** are written with the signs σ of ***A***? In this case, the formalization $\mathsf{E} \leftarrow \mathbf{\Phi} \rightarrow$ **Sy** would be reduced to a simple *formal equivalence* ***Sy*** $\leftarrow \mathbf{\Psi} \rightarrow$ **Sy**.
Or, should we consider the edifice E irreducible to a formal system, i.e. irreducible to a human made system **Sy**? In other words, should we consider the edifice E an irreducible reality?

I call "ultra-formal *choice*" the doctrine asserting the reducibility of **E** to ***Sy*** (or of **Φ** to **Ψ**) and "Platonistic choice" the other doctrine claiming that **Φ** is essentially irreducible to **Ψ**.
Both choices Platonistic and ultra-formal are metaphysical options. In absolute terms, the question – The Question according to Barry Mazur (Mazur, 2008, p.2) – whether the ultra-formal choice or the Platonistic choice is the adequate position cannot find a definite answer. But we can now compare the two choices under the criteria of their epistemological consistency.

1.2 Platonism and anti-Platonism in the light of the Hilbert approach

1.21. Some pre-gödelian considerations

Let us formalize a mathematical edifice **E** in terms of **E** $\leftarrow$ **Φ** $\rightarrow$ **Sy**. Is this project compatible with the effective construction of formal systems **Sy** being really arbitrary assemblages of meaningless signs ? That is not certain.
Recall that Hilbert adopts an *as if* strategy. Operating *as if* he used arbitrary **Sy** to formalize **E**, Hilbert implements systems consisting of signs of first order logic governed by their **Rm**, **Rd** and **Ax** to which he adds *specific signs* that depend on the edifice **E** to formalize. (Hilbert, 1927/1996, pp. 228 ff.; Snapper, 1979, pp. 213 ff.) This seems to argue in favor of **Sy** being not arbitrary but fashioned in order to comply with the requirements of *given* **E**. To elucidate this *impression*, we have to introduce the concepts of extension and intension.
For contemporary logic, intension is a property p that determines all the elements belonging to a set **S**, according to the scheme **S** = {e, e $\Rightarrow$ p}. (Carnap, 1947, pp.6, 10, 16) Extension is the corresponding set **S**. For the ancient logic, the extension of a concept κ_i consists of the set of the concepts κ_j to which κ_i "applies" . The intension of a concept κ_j is the set of all concepts κ_i that κ_j "includes". Let us order a set **E** of several concepts κ_u according to their *increasing* extension. To this hierarchization then corresponds a set **I** of concepts κ_u ordered according to their decreasing intension, and vice versa. Our investigation needs a reformulation of the ancient *hierarchizing* conception – rather vague – of intension and extension according to the requirements of contemporary logic. Let P_i be a proposition characterized by a set $\mathbf{p}(P_i)$ of properties. Then we can define the extension $\mathbf{E}(P_i)$ of P_i as the set of the propositions P_j characterized by a set of properties $\mathbf{p}(P_j)$ such as $\mathbf{p}(P_i)$ is included in $\mathbf{p}(P_j)$. In formula: $\mathbf{E}(P_i) = \{P_j, \mathbf{p}(P_i) \subseteq \mathbf{p}(P_j)\}$. The intension $\mathbf{I}(P_i)$ of P_i is then written symmetrically $\mathbf{I}(P_j) = \{P_i, \mathbf{p}(P_i) \subseteq \mathbf{p}(P_j)\}$.
Now let us interpret the set of P_i as the set of propositions of first order logic $\mathbf{L}_{1^\circ}^{+}$ complemented by specific signs dependent on the mathematical edifice **E** to formalize, and the set of the P_j as the set of the propositions of **E**. A $P_i \in \mathbf{L}_{1^\circ}^{+}$ can be applied to a $P_j \in$ **E**, if and only if P_j belongs to the extension of P_i. Or, focusing on $P_j \in$ **E**, we can also say that using P_i to formalize P_j presupposes that P_i belongs to the intension of P_j. Thus we have again the expressions $\mathbf{E}(P_i) = \{P_j, \mathbf{p}(P_i) \subseteq \mathbf{p}(P_j)\}$ and $\mathbf{I}(P_j) = \{P_i, \mathbf{p}(P_i) \subseteq \mathbf{p}(P_j)\}$ which, however, are specified: all P_i belong to the enriched first order logic $\mathbf{L}_{1^\circ}^{+}$ and all P_j, $i \neq j$, to the edifice **E** to be formalized.
Now rewrite the expressions $\{P_j, \mathbf{p}(P_i) \subseteq \mathbf{p}(P_j)\}$ and $\{P_i, \mathbf{p}(P_i) \subseteq \mathbf{p}(P_j)\}$ in the form of implication, which is giving in *both* cases $\mathbf{p}(P_j) \Rightarrow \mathbf{p}(P_i)$.
On a purely formal level, all is said and done. The propositions P_i of the formal system **Sy**, far from being arbitrary, must comply with the propositions P_j of **E**, and the choice of Hilbert to specify **Sy** by $\mathbf{L}_{1^\circ}^{+}$ – first order logic *enriched as required by* **E** – goes exactly in the sense of this constraint. However, in our context, this formal result should be *interpreted*. We then can choose between the following three options:

- We affirm that the mathematical edifice **E** supporting the propositions P_j is *objectively given*, and that according to the implication $\mathbf{p}(P_j) \Rightarrow \mathbf{p}(P_i)$, the system **Sy** supporting the propositions P_i must effectively be configured in order to go with P_j of **E**. We just noticed that this approach is perfectly compatible with the Hilbertian proof-theoretically inspired *as if* -strategy specifying **Sy** by $\mathbf{L}_{1^\circ}{}^+$.

- Or, while accepting that the mathematical edifice **E** is objectively given, we act *as if* **E** had been artificially configured to correspond *via* the implication $\mathbf{p}(P_j) \Rightarrow \mathbf{p}(P_i)$ to the "arbitrary" formal system **Sy**. In this case we must at least ask ourselves whatever could be the point of this inverted Hilbert approach.

- Or, we strongly affirm that any mathematical edifice **E** *is*, like the formal systems **Sy**, an arbitrary construction. This view presupposes – among other things – (i) that the mathematicians from Antiquity to the 19th century, while ignoring the very notion of formal system, already conceived and/or manipulated unknowingly "arbitrary constructions **E**" equivalent to formal systems **Sy**, and (ii) that these "arbitrary constructions **E**" were predestined – thanks to a kind of pre-established harmony anticipating the implication $\mathbf{p}(P_j) \Rightarrow \mathbf{p}(P_i)$? – to meet later the enriched first order logic $\mathbf{L}_{1^\circ}{}^+$ developed by Hilbert.

Let now everyone compare these three options according to the criteria of simplicity, economy of hypotheses, epistemological consistency and so on.

1.22 Are Gödel's incompleteness theorems arguments against Platonism?

First recall the issue and the impact of Gödel's second theorem. A valid formalization of **E** by **Sy** *via* **Φ** presupposes the consistency and completeness of **Sy**. **Sy** is consistent if its axioms $\mathrm{Ax}_k \in \mathbf{Ax}$ do not lead to inconsistencies in their consequences, and complete if no deduction of a theorem θ_v belonging to **Sy** would require the widening of **Ax** by other axioms Ax_l, $l \neq k$. The conditions of consistency and completeness being satisfied, **E** is formalized by **Sy** if and only if there is a bijection **E** $\leftarrow$ **Φ** $\rightarrow$ **Sy**.
Gödel's second theorem shows that the consistency proof concerning all **Sy** as strong as or stronger than formal arithmetic prevents their completeness proof, and vice versa. The establishment of **Φ** between these **Sy** and given corresponding edifices **E** is not possible. Hence we cannot envisage a rigorous Hilbertian formalization of *current* mathematical edifices **E**.
The Gödel disaster as such *does not* constitute an anti-Platonistic argument. Gödel himself is among the most categorical Platonists, (Gödel 1951/1995, pp. 322 ff.) but his approach would lead us too far. Here I limit myself to mention an attempt to circumvent the disaster; attempt claiming openly to be anti-Platonistic, but that, in my opinion, represents in fact and *despite itself* an argument *in favor* of Platonism.
Avigad and Feferman, refering to Gödel, show that we can extract from all the edifices **E** affected by Gödel uncertainty a sub-edifice $\mathbf{E}^{\mathrm{Cm}}$, $\mathbf{E}^{\mathrm{Cm}} \subset \mathbf{E}$, fully mastered not by a Hilbert formalization **E** $\leftarrow$ **Φ** $\rightarrow$ **Sy** but through a process of effective construction. (Avigad & Feferman, 1999, pp. 6 ff; 16 ff. e.a; Zach,2005, pp. 28 ff.) Let us denote by "$\mathbf{M}^{\mathrm{Cm}}$" ("constructively mastered mathematics") the set of all $\mathbf{E}^{\mathrm{Cm}}$ such as $\mathbf{M}^{\mathrm{Cm}} \subset \mathbf{M}$, where "**M**" is for "Gödel affected mathematics". Now, $\mathbf{M}^{\mathrm{Cm}} \subset \mathbf{M}$ in turn gives $\mathbf{p}(\mathbf{M}^{\mathrm{Cm}}) \subset \mathbf{p}(\mathbf{M})$, thus $\mathbf{p}(\mathbf{M}) \Rightarrow \mathbf{p}(\mathbf{M}^{\mathrm{Cm}})$. Unless we consider $\mathbf{M}^{\mathrm{Cm}}$ as an isolated body in **M**, which Avigad and Feferman *do not do*, should the properties of **M** – whatever they may be – not be configured so that the mastered reconstruction of $\mathbf{M}^{\mathrm{Cm}}$ harmonize with **M**? But in this case, is **M** not preceding $\mathbf{M}^{\mathrm{Cm}}$ on the three ontological, logical, and epistemological levels? (Mlika, 2007, p. 39) In other words, is **M** not preceding $\mathbf{M}^{\mathrm{Cm}}$ in a "Platonistic way"?

2.3 Is there a constructivist alternative to the ultra-formal and the Platonistic choice?

So far, we have directly confronted the Platonistic choice to its direct negation, the ultra-formal choice. Alongside these opposite extremes, a third way is proposed, the so-called constructivism. From this standpoint, mathematical edifices are neither an objective although immaterial reality nor arbitrary assemblages of meaningless signs. Mathematics would be effectively constructed, without existing outside of their construction. However, this raises a new question: is constructivism not *despite itself* a kind of strengthened Hilbertian *as if*-strategy: the (at least partial) *reconstruction* of what is already there? I return to this rather complex issue at the end of the paper in a technical note. (See technical note n°1) Here I merely point out that the Avigad-Feferman approach that falls intro constructivism (see above) pleads in favor of the reconstruction thesis. *If* this approach holds, the expressions $\mathbf{M}^{Cm} \subset \mathbf{M}$ and $\mathbf{p}(\mathbf{M}) \Rightarrow \mathbf{p}(\mathbf{M}^{Cm})$ denote a *partial* but *mastered* reconstruction of mathematics within global, not mastered but *given* $\mathbf{M}$. The implication $\mathbf{p}(\mathbf{M}) \Rightarrow \mathbf{p}(\mathbf{M}^{Cm})$ seems to denote that $\mathbf{M}$ precedes $\mathbf{M}^{Cm}$ logically. Everyone is now invited to consider whether, on the ontological level, mathematics $\mathbf{M}$ which are *not mastered* by humans but which allows a human extraction of constructive mathematics $\mathbf{M}^{Cm}$, could be human made. In a more intuitive way, we must ask ourselves whether, since Brouwer, constructivist mathematicians could have constructed presently available $\mathbf{M}^{Cm}$ *from nothing*, instead of reconstructing at least *de facto* $\mathbf{M}^{Cm}$ from *given* $\mathbf{M}$.

2. Mathematics and physics: about Quine's indispensability argument

Now we begin to approach the links between mathematics and physics. These words spontaneously could make us think of the so-called "indispensability argument" according to Quine. The "argument" in fact scattered over at least eight publications – the references are in IEP (anonymous, no publication year, sect. 2) – can be summarized by: "given the indispensability of abstract mathematical *beliefs* within physics, physical *experience* justifies the belief in an objective existence of mathematical entities". This position in favor of a *thin-blooded Platonism* is far from being universally shared. My own approach is different. Without arguing directly, I continue on the basis of Quine's position by *comparing* a *full-blooded* Platonism, certainly metaphysical, to its not less metaphysical negations.

2.1 The indispensability argument in the perspective of the ultra-formal *choice*

Consider a proposition such as "At the time of the dinosaurs, long before various philosophical schools began to compete about the foundations of mathematics, the gravity centers of three stones already formed a triangle with an angular sum equal to two right angles". Fano and Graziani would argue that such "somewhat caricatural" comments fall "obviously" within metaphysical beliefs. (Fano & Graziani, 2011, pp. 21 ff.) Perhaps it is true. But let us pass from the theorem of angular sum of the triangle, still accessible to a pre-mathematical/experimental approach, to physical experimentation *stricto sensu*. The *modus operandi* of physics relies on the *epistemological assumption* that physical laws, perhaps not "eternal", are *reasonably timeless* (see below) since a moment belonging to the immediate neighborhood of the Big Bang. A physical law outdated following a new paradigm still remains as a special case of a broader law. In contrast, the *idea* of physical laws changing anytime would not be operating in physics. Far from being a metaphysical belief, the *epistemological presupposition* of *reasonable timelessness* of physical laws is until further notice empirically confirmed. Since astrophysical observation goes back in time, *intrinsic* variations in physical laws would be *empirically* detected. Consider now a physical phenomenon X, and its mathematical expression – denoted) by $P(x)$ – belonging to $\mathbf{E}$. Let us accept the *reasonably* timeless aspect of $P(x)$ as certified by astrophysical observation. Will there be an objective difference between $P(x) \in \mathbf{E}$ and the theorem of the angular sum of triangles we *suppose* already interpreted at the time of the dinosaurs by three non-aligned stones?

In this regard, we are faced with the following choices:

- Either we consider the edifice E, $P(x) \in \mathsf{E}$, as (i) objectively given, and (ii) as timelessly confirmed by physical phenomena X accessible to astrophysical observation, without of course excluding the Hilbert strategy treating E *as if* it were a human made assemblage of arbitrary signs.
- Or we reject the possibility of an objectively given edifice E by qualifying this idea as metaphysical. But in this case we must also state that (i) the epistemological presupposition of *reasonably* timeless physical laws without which the *modus operandi* of physics would become inoperative, and (ii) the mathematized laws $P(x) \in \mathsf{E}$ timelessly confirmed by astrophysical observation, are in turn metaphysically undermined.

Everyone is free to judge.

At present, research on possible variations of universal constants is ongoing. (See Uzan, 2002) The definitive detection of such variations – necessarily very small and until further order hypothetical – could lead to the development of new paradigms in physics.
When appropriated, let each one determine the *epistemological value* of these new paradigms according to whether we refer them

- to objectively existing mathematics,

- or to human made arbitrarily constructed mathematics.

2.2 The indispensability argument in the perspective of constructivism

If we assert that mathematical edifices E are constructed by humans, we should assume consequently that the physical phenomena X expressed by $P(x) \in \mathsf{E}$ are in turn "constructed" following the characteristics of E. On the other hand, it seems reasonable to assume that all humans perceive in the same way all physical phenomena mathematizable by $P(x) \in \mathsf{E}$. This requires that all humans construct physical reality in the same way. To remain coherent, such a vision needs a multitude of metaphysical presuppositions oscillating between the approaches of Kant and Hegel.

3. About the epistemological status of mathematical tools in physics

3.1 What is idealization ?

A mathematized physical law represents an idealization of empirical data. Nobody will deny it. In contrast, when the idealization status characterizing the laws of physics is considered as an argument against the Platonistic conception of a physical world governed by pure mathematics, we encounter great difficulties. Specifically, *both* arguments "It is because of their idealized aspect that the physical phenomena are aligned with mathematics" and "Mathematics arises from the idealization of physical phenomena" are not only problematic but still fall within an *identical* problem.
To be sure to talk about the same thing, let us try to clarify what we mean by "idealization". Consider a physical phenomenon X consisting of sub-phenomena X_i. Any manifestation of X_i is in relation with other X_j. Note $X_i = R_i(..., X_j , …)$, i, j = 1, ... n. Suppose that X is expressed by a mathematical entity $P(\boldsymbol{x}) \in \mathsf{E}$ taking the form $P_i(x_i) = \varphi(..., x_j , …)$, i,j = 1, ... n, where x_i, x_j are – in order to simplify – numerical values in the broadest sense: scalars, vectors, tensors, functions of a Banach and so on. Each $\varphi_i(..., x_j , ...)$ corresponds to a $X_i = R_i(..., X_j , …)$. Now we measure

empirically the phenomenon X. In most cases, we will find for each X_i, X_j a higher or lower deviation Δx_i, Δx_j from the ideal values x_i, x_j.. Let $\boldsymbol{x}_i$, $\boldsymbol{x}_j$ be the respective neighborhoods of x_i, x_j, such as the empirical values $(x_i + \Delta x_i)$, $(x_j + \Delta x_j)$ belong to $\boldsymbol{x}_i$, $\boldsymbol{x}_j$. The expression $P(\boldsymbol{x}) \in \mathsf{E}$ is an *idealization* of X if for a large number of empirical measurements of X, the empirical values $(x_i + \Delta x_i)$, $(x_j + \Delta x_j)$ tend to the limits x_i, x_j. Note that here we can not specifically take account of quantum physics, but this does not lead to specific difficulties: (i) The links between mathematized idealization $P(\boldsymbol{x}) \in \mathsf{E}$ and empirical data affected by deviations $(x_i + \Delta x_i)$, $(x_j + \Delta x_j)$, and (ii) the fact that empirical quantum data conflict with the patterns of the macroscopic world, are the subject of complementary but *different* issues which can be treated separately.

3.2 On what is idealization based?

Let us abbreviate the expressions $(x_i + \Delta x_i)$, $(x_j + \Delta x_j)$ by $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ where the index E is for "empirical" and ask the crucial question: How can we explain the fact that a large number of measures $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ tend systematically towards the limits x_i, x_j? The mention of a "large number" of measures could make us spontaneously think of the law of large numbers, in a *first approximation* the fact that the relative frequency of an often enough repeated event tends to a limit equal to its probability. (A more rigorous definition is given in the technical end note n°2.) The law of large numbers undoubtedly plays a role in the tendency of $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ to approach the limits x_i, x_j, but does not explain all. First of all, the law of large numbers *is not* self-evident. This rather complex issue is shortly treated in the technical end note n°2. On the other hand, the law of large numbers is interpreted by very different physical phenomena: playing heads or tails, roulette, road accidents etc. etc. So we have to see the law of large numbers not as a *physical law* but as a kind of *immaterial logic* being "behind" the material phenomena which express it. Now we are certainly free to consider this *kind of logic* (i) as an *idealization*, i.e. an outcome of a large number of empirical trials, or else (ii) as an *objectively given* framework despite its immateriality. Nevertheless, the choice favoring position (i) confronts us with a thorny question: How can a large number of random phenomena obey the law of large numbers, without a law of large numbers pushing them in this sense?
Regarding the reduction of the law of large numbers to an empirical idealization, we thus face the following dichotomy:

- Either we just note that the law of large numbers governing phenomena different from each other exists objectively as an immaterial law, immaterial precisely because of the diversity of phenomena that this law governs.
- Or we consider the law of large numbers as an idealization. But in this case we must assume the *circularity* featured above: how can a large number of random phenomena tend to limits determined by the law of large numbers without a law of large numbers pushing them in this sense?

In other words, the rejection of Platonism is paid by *circularity* which, in physics, is execrated as much as metaphysics.
But anyway, the law of large numbers plays here only a partial role. The deviations Δx_i, Δx_j of effective measures from the ideal values x_i, x_j can perhaps be considered as random phenomena. However, the mere fact that the relative frequencies of random events E_i, E_j subjected to a large number of trials tend toward limits equal to the probabilities $\mathrm{p}(E_i)$, $\mathrm{p}(E_i)$ does not explain why the $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ tend to *specific* limits x_i, x_j, so that the corresponding physical phenomenon X takes *exactly* the expression $P_i(x_i) = \varphi_i(..., x_j, \ldots)$, i, j = 1, ... n. For a mathematized physical law $P(\boldsymbol{x}) \in \mathsf{E}$ to be significant, there necessarily must be a set $\mathbf{C}$ of constraints – as immaterial as the law of large numbers – which act in such a way that the empirical measures $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ sufficiently often repeated tend towards the limits x_i, x_j. Such a formulation is doubtlessly a

metaphysical proposition, and the physicist rightly confines himself/herself to taking note of the correspondence between $P(\boldsymbol{x}) \in \mathsf{E}$ and sufficiently often repeated empirical measures $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$, *without paying attention* to **C** ensuring $P(\boldsymbol{x}) \in \mathsf{E}$ through experimental deviations Δx_i, Δx_j. But, by contrast, a *debate* revolving around the following theses: (i) physical phenomena are mathematizable because of their idealization and (ii) mathematics emerges from the idealization of empirical observations, or – as it is suggested by Penelope Maddy – applied mathematics precedes pure mathematics (Maddy, 2008, esp. pp. 20 ff.; pp.33 ff.), such a debate *cannot* legitimately neglect the issue of **C**.

Let us turn first to the thesis (i):

- If we want a mathematizable physical law expressed by $P(\boldsymbol{x}) \in \mathsf{E}$ to correspond to the limits x_i, x_j of sufficiently often repeated effective empirical measures $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$, we *have the possibility* to postulate the existence of a set **C** of constraints guaranteeing that the $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ repeated sufficiently often tend towards the limits x_i, x_j. In this case **C** must reflect the mathematical edifice E to which $P(\boldsymbol{x})$ belongs. In other words E *must precede* the empirical measures $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ on the ontological, logical and epistemological plan.
- Or we refuse to postulate the existence of **C**. But how then idealize empirical measures in terms of $P(\boldsymbol{x}) \in \mathsf{E}$ without postulating any intrinsic link between $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ and $P(\boldsymbol{x}) \in \mathsf{E}$? In other words, how idealize empirical measures in terms of $P(\boldsymbol{x}) \in \mathsf{E}$ while accepting that the $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ can take *any* values?

Let us now admit with P. Maddy the thesis (ii).

- If we do not want to assume absurdly that mathematical edifices E results from the idealization of empirical phenomena evolving *anyhow*, we should accept the existence of **C** ensuring that sufficiently often repeated empirical measures $x_i^{\mathsf{E}} \in \boldsymbol{x}_i$, $x_j^{\mathsf{E}} \in \boldsymbol{x}_j$ tend towards the limits x_i, x_j. In this case, the set **C** of *immaterial* constraints must *correspond exactly* to the mathematical edifice E containing $P(\boldsymbol{x})$.
- If we deny the existence of the set of immaterial constraints **C** corresponding to E, we fall back into the difficult assumption that mathematical edifices E are resulting from the idealization of empirical phenomena evolving *anyhow.*

A more rigorous elucidation of the epistemological status of idealization intervening in the formulation of physical laws must reflect the very thorny issue of *irreversibility.* The technical note n° 3 at the end of this paper brings some additional developments going in this direction.
This being said, we must recognize that *all* (!) standpoints concerning the role of idealization in the mathematization of physical laws include a lot of metaphysics. Everyone is now invited to determine the standpoint that seems more plausible to him or her and especially more consistent and more economical in initial and subsidiary hypotheses. In other words, everyone is now invited to choose between the following two conceptions:

- Either we accept the objective existence of an immaterial mathematical universe and the assumption that some entities of this mathematical universe govern some of the phenomena of material reality, *just as we note the objective existence of material reality, knowing that this latter could also not exist*. From this perspective, the idealization of the physical laws would consist in our experimental efforts to approach optimally these immaterial, immutable, and timeless laws, that the material aspect of physical phenomena and of all experimentation tend to distort.
- Or we consider this conception too metaphysical and replace it by *another* metaphysics. We

decree that mathematics is human made. But in this case we must also adopt a whole range of hypotheses supposed to address the following questions:

(i) How do physical phenomena, that may well not comply with our human made mathematics, nevertheless comply with it, at least to a certain extent?

(ii) How do experimental results, despite their dispersion that precisely requires operations of idealization concerning the formulation of physical laws, nevertheless converge towards immutable limits supposed to belong to human made mathematical edifices?

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More technical end notes

1. About mathematical constructivism as an alternative to Platonism

Any discussion about mathematical constructivism requires lengthy developments. The following lines are just trying to raise some difficulties about constructivism that cannot simply be swept aside.
As everyone knows, from the standpoint of constructivism, only the set of natural numbers **N** is given. Any other mathematical edifice **E** would be effectively constructed by humans on the bases of **N**. On the other hand, Troelstra and van Dalen say that "(...) *mathematics does not consist in the formal manipulation of symbols* (...)" (Troelstra and van Dalen, 1988, p. 4). For Brouwer, the approach to be followed presupposes "(...) *completely separating mathematics from mathematical language*. (...)" (Brouwer in Iemhoff, 2014, sect. 2.1). However, if we admit that a mathematical *language* ***L***, far from being reducible to an assemblage of meaningless signs, performs the function of a significant denoting as signified an edifice **E**, should this latter not be there at least potentially? Would the "effective construction" of **E** not be in fact a *reconstruction*, i.e. an *as if* strategy to what the constructivism just imposes a set of rules and constraints different of the Hilbert approach? Certainly, the "logical *foundations*" of classical mathematics are transformed – notably by the "Brouwer-Heyting-Kolmogorov-interpretation" – intro construction rules such as "$\vee$: to prove $P \vee Q$ we must either have a proof of P or have a proof of Q"; "$\wedge$: to prove $P \wedge Q$ we must have both a proof of P and a proof of Q", etc. These rules are necessarily incompatible with the *tertium non datur*; thereby, from the constructivist view, several theorems of classical mathematics cannot be validated. But how to avoid that these rules govern the construction of anything from anything? In this regard the constructivism introduces *axioms,* an approach which, from my standpoint, pleads for the *reconstruction* thesis. Moreover, some constructivist axioms are supposed to ensure continuity between achieved constructions of edifices **E** and what remains to be constructed in the future. For this purpose Kreisel introduced the so-called **CS** or **IM** axiomatic (for "creating subject" or "idealized mathematician"); the mathematics community in flesh and blood whose members reciprocally correct their mistakes, knowing that over time the younger generations succeed the more ancient, is *idealized* by an immortal mathematician, free from error, but not omniscient; he or she or it must proceed in stages. These **CS** axioms are **(CS1)** $\square nA \;\square\; \neg\square nA$, **(CS2)** $\square mA \rightarrow \square m{+}nA$, **(CS3)** $\exists n\, \square nA \leftrightarrow A$. **(CS1)** can be neglected here. The interpretation of **(CS2)** is "That what the **CS** assert as true at the moment n, he or she or it maintains it as true at all moments m + n, n ≥ 0." In other words, the **CS** accumulates his/her/its truths over time. But for such a accumulation to remain consistent over time, the discoveries made at the moment m should harmonize with each future discoveries made at any moment m + n. Under these conditions, does what is effectively constructed at the moment m, not presuppose the potential presence already at the moment m of all what will be constructed at any future moment m +n ? The other axiom **(CS3)** denotes " What *is* true will be discovered by the **CS** at a moment n as being true." (Comp. Iemmhoff, 2014, 2.2) But can we expect to *discover* in the future something that is not potentially given?
Anyway, if we classify Platonism rightly into metaphysics while considering constructivism as a negation of Platonism, then constructivism is neither more, nor lees metaphysical than that it denies. Let everyone decide which of these two metaphysics is more consistent.

2. The law of large numbers and its epistemological status

Kolmogorov, after having admitted "for long time" that (i) "the frequency approach based on the idea of a *limiting frequency* as the number of trials tends to infinity does not give a foundation for the applicability of the results of probability theory to practical questions, where we deal with a finite number of trials; [and that] (ii) the frequency approach in the case of a large but finite number of trials cannot be developed in a rigorous purely mathematical way", adopted later a more nuanced view of the point (ii), while maintaining his position on the point (i). (Kolmogoroff, 1963, pp.176 f.) To take account of the fact that we always deal with a finite number of trials, we have to formulate the law of large numbers in terms of meta-probabilities. Indeed, for a finite number N of trials, the difference between the relative frequency $n(e_i)/N$ of an event e_i and its probability $p(e_i)$ cannot be predicted with absolute certainty. All we can do is to determine the meta-probability $\mathbf{p}$ that this difference, after N trials, is less than or equal to ε, where ε is an arbitrarily small real number. *If and only if* there is a factual probability law $\mathbf{p}(\mathbf{e})$ over the set $\mathbf{e}$ of events e_i which allows to know *effectively* $p(e_i)$, we can express this meta-probability by $\mathbf{p}[\,|n(e_i)/N - p(e_i)| \leq \varepsilon\,]$. On this basis and introducing a second arbitrarily small real number δ, the law of large numbers takes the form $\forall i, \forall(\varepsilon, \delta), \exists N_0, \forall N \geq N_0, \mathbf{p}[\,|n(e_i)/N - p(e_i)| \leq \varepsilon\,] \geq (1 - \delta)$. (Mugur, 2006, pp. 224 f.). Now the simple reading of the law of large numbers denotes that the latter, instead of "founding" the factual probabilities $p(e_i)$, *presupposes* their *given* existence in order to acquire significance.

3. Idealized physical laws and irreversibility

Formulating idealized physical laws means – among other things – to ignore all the factors generating irreversibility, such as friction, energy perdition, experimental perturbations and so on. So far, there is no problem. It is however questionable to use the idealized aspect of physical laws as an anti-Platonistic argument.

Intuition and common sense suggest that irreversibility is the direct negation of reversibility. Note however the following detail: For any factor A to be a direct negation of the factor B, both factors A and B should refer to the same type of system. Certainly, the common sense still s*eems* to indicate that irreversibility satisfies this condition: A gas constituting an isolated system Σ characterized by its entropy variation $\Delta S \geq 0$ *appears* to remain the "same" system Σ between its initial state Σ_I corresponding to a low entropy S and its final state Σ_F with entropy maximum. And *if* the entropy of the system Σ returned later to its initial minimum value – which of course never happens – should we not still mention the "same" system just evolving differently?

In fact, contrary to the false evidences of "common sense", the initial Σ_I state and the final state Σ_F of a *same* irreversible *process* does *not* engage a system remaining the "*same*" throughout the process.

Consider an ideal watch without internal frictions etc. whose needles turn by their own inertia at a constant speed. This system, as long as nothing disturbs it, is reversible in terms of the spatial configuration of its needles; it will return to any configuration it occupies at a given moment. Under these conditions, the system (i) is characterized by an entropy variation $\Delta S = 0$ and (ii) "remains the same" because it conserves its *functioning mode*. Now let us create an irreversible situation by projecting the system violently to the ground. This time the entropy variation gives $\Delta S > 0$, while the system – reduced to fragments – *does not* conserve its *functioning mode*. Nobody would seriously say that the fragments scattered on the ground are the "same" system as the ideal watch in operating condition. In a more general way, the ideal reversibility of a physical system $\Sigma^{IR} \equiv P_i(x_i) = \varphi(\ldots, x_j, \ldots)$, i,j = 1, ... n, *presupposes* the conservation of the functioning mode $fm(\Sigma^{IR})$ while its real irreversibility *is* simply the non-conservation of $fm(\Sigma^{IR})$. The deviations $(xi + \Delta xi)$ $(xi + \Delta xj)$ the real system Σ expresses relatively to the ideal Σ^{IR} system are due to entropy generating factors such as friction, energy perdition, experimental perturbations etc. The irreversibility of these phenomena *overlays* the ideal reversibility of Σ^{IR}. The ideal conservation of $fm(\Sigma^{IR})$ can be formalized in a very general way by the Klein four-group $\mathbf{V} = (\{I, e, f, g\}, \perp)$. The four elements I, e, f, g are transformations. $\perp$ symbolizes the transformation of any $a \in \{I, e, f, g\}$ into $b \in \{I, e, f, g\}$. "I" designates the identity transformation such as $\forall a \in \{I, e, f, g\}, I \perp a = a \perp I = a$. For e, f, g, we set $\forall a \in \{I, e, f, g\}, a \perp a = I$ and $e \perp f = f \perp e = g$, $f \perp g = g \perp f = e$, $g \perp e = e \perp g = f$. It is easy to show that $I \perp e \perp g \perp f = I$. In fact, $I \perp e = e$, so $(I \perp e) \perp f = e \perp f = g$. Since $I \perp e \perp f = g$, $(I \perp e \perp f) \perp g = g \perp g = I$.

The sentence $I \perp e \perp g \perp f = I$ *formalizes any system remaining* identical *through all its transformations*. Hence the reversibility of an ideal system Σ^{IR} can be written $fm(\Sigma^{IR}) \Rightarrow \mathbf{V}$. Any form of irreversibility in contrast expresses the transition $\mathbf{V} \rightarrow$ non-$\mathbf{V}$. The functioning mode $fm(\Sigma^{IR})$ of the system Σ^{IR} is obviously an intrinsic property of Σ^{IR}, whereas the transition $\mathbf{V} \rightarrow$ non-$\mathbf{V}$ occurs *independently* of the intrinsic properties of the concerned system Σ^{IR}. Consider several systems with very different properties: a watch, a car engine, our Earth with its moon revolving around it etc. Let us smash this watch against a wall, forget to put oil in the car engine, imagine that a mega-meteorite pulverizes our Earth. The result implies in all cases the transition $\mathbf{V} \rightarrow$ non-$\mathbf{V}$. However, this transition $\mathbf{V} \rightarrow$ non-$\mathbf{V}$ is not sufficient for determining the three respective $fm(\Sigma^{IR})$ of the watch, the car engine and the Keplerian Earth-moon system. The entropy production due to the deviation $(xi + \Delta xi)$ $(xi + \Delta xj)$ that a real system Σ marks relative to the corresponding ideal system Σ^{IR} is only a special case of the transition $\mathbf{V} \rightarrow$ non-$\mathbf{V}$ affecting any system independently from its intrinsic properties. In these conditions, does the thought pattern of a physical phenomenon Σ ideally governed by $P(\boldsymbol{x}) \in \mathbf{E}$ but *de facto* altered by an irreversibility being superposed on $P(\boldsymbol{x}) \in \mathbf{E}$, does this thought pattern not imply that the ideal law $P(\boldsymbol{x}) \in \mathbf{E}$ exists independently from the irreversibility factors affecting it?

- If we postulate that $P(\boldsymbol{x}) \in \mathbf{E}$ results from the idealization of irreversible phenomena, we must deny irreversibility because in a material world where irreversibility expressing the transition $\mathbf{V} \rightarrow$ non-$\mathbf{V}$ makes all ideal entities tend to degradation, the emergence of an ideal law $P(\boldsymbol{x}) \in \mathbf{E}$ from degraded phenomena would go against irreversibility.
- If we refuse to deny irreversibility denoted by experience, we must accept that the ideal law $P(\boldsymbol{x}) \in \mathbf{E}$ is *initially given*.

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