Can time have a more dynamical role in a quantum field?

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Abstract. In the formulation of both classical and quantum theories, time is taken as an independent parameter rather than as a dynamical variable. This is in contrast with the way general relativity treats space-time which is dynamical and interacting with matters. If time and space are to be treated on the same footing as required in relativity, can time play a more dynamical role in a quantum field? Taking time as a dynamical variable, we study a wave with 4-vector amplitude that has vibrations of matter in space and time. By analyzing its Hamiltonian density equation, we find that the system has the same physical structures of a zero-spin bosonic field. This quantized real scalar field obeys the Klein-Gordon equation and Schrödinger equation. The possibility that matter has vibrations in time can lead to the quantization of a bosnoic field. Time can play a more dynamical role in a quantum field.

1. Introduction

In the formulation of classical and quantum theories, time is principally treated as a parameter in the equation of motion. The theories postulate a time parameter with respect to which the dynamics unfold. Time and space are treated separately. On the other hand, space-time in general relativity is dynamical interacting with matter and radiation. There is no globally defined time in the theory. Space-time is weaved as unity. Thus, the treatment of time in quantum theory and general relativity is rather different. The problems created by these differences in approach are striking especially when one tries to reconcile the two basic theories from a single framework [1, 2].

The conundrum that time shall be treated as a parameter in quantum theory can be traced back to Pauli's era. Unlike position and momentum which are operators in the quantum theory, Pauli argued that time cannot be assigned as a self-adjoint operator [3] for any bounded, semi-bounded or discrete Hamiltonian. As he concluded [4], "the introduction of a time operator t must be abandoned fundamentally and that the time t in quantum mechanics has to be regarded as an ordinary real number." This assertion has remained a major influence in the development of quantum theory. However, against this orthodoxy, there are many examples in quantum theory where time seems to play a dynamical role, e.g. dwell time of a particle in a region of space, tunneling time or decay time of an unstable particle [5–15]. In most of these cases, time can be associated with operators through the use of positive operator valued measures (POVMs) instead of self-adjoint operators. Apart from these extensive efforts dedicated to resolve the dynamical nature of time, various classical and quantum models have also been proposed by T.D. Lee that suggest time can be considered as a fundamentally discrete dynamical variable [16, 17].

As there are compelling reasons why time shall play a more dynamical role, we ask a few fundamental questions: in classical theory, the amplitude, $\mathbf{X}$, of a wave with vibrations in space can be defined as the maximum displacement of matter in the wave from its equilibrium coordinate. Since matter can have vibrations in the $\mathbf{x}$ coordinates, can it also has vibrations in the time coordinate t? In fact, if space and time are to be treated on same footing, it is plausible to define an amplitude T for vibration in time [18]. Although it is feasible to construct a wave that has vibrations in both space and time, can its properties have something to do with our real physical world?

Here, we investigate the possibility that time can have a more dynamical role in a quantum field. Instead of taking the typical approach by treating time as an operator, we construct a plane wave with a 4-vector $(T, \mathbf{X})$ amplitude that has vibrations in space and time. We define the amplitude in time of a plane wave as the maximum difference between the 'internal time" of matter within the wave and the 'external time' measured by a stationary inertial observer outside the wave; its meaning will be further elaborated in Section 2. By studying the Hamiltonian density equation of this planes wave in Section 3, we find that a harmonic oscillating system with vibration of matter in proper time can be the generator for the energy of mass. In Section 4, we show that an oscillator with vibration in proper time can only have one unique amplitude. This leads to our subsequent reasoning that a real scalar field describing the vibrations of matter in space and time shall be quantized; it has no classical description. Furthermore, this quantized real scalar field obeys the Klein-Gordon equation and has the properties of a zero-spin bosonic field as shown in Section 5. Basic properties of the system in the non-relativistic limit will be further demonstrated in Section 6. The system with vibrations of matter in space and time can produce the familiar structures of a real physical system.

2. Plane Wave with Vibrations in Space and Time

Consider the background coordinates $(t, \mathbf{x})$ for the flat space-time as observed in an inertial frame 'O'. Time in this background is the 'external time' as measured by clocks that are not coupled to the system under investigation [19–21]. We will first study a plane wave with matter that has vibrations in space and time relative to this background coordinate system.

The amplitude for vibration in space, $\mathbf{X}$, of a classical plane wave is well defined; it is the maximum displacement of matter in the wave from its equilibrium coordinate such as in the case for a flexible string under tension. Similarly, let us define a plane wave's amplitude for vibration in time, T, as the maximum difference between the time of matter inside the wave, t_f, and the external time, t. Therefore, if matter inside the plane wave carries a clock measuring its internal time, an inertial observer outside will see the matter's clock vibrates with time, t_f, as related to his own clock measuring time, t. In other words, we have assumed the matter's internal clock is running at a varying rate relative to the inertial observer's clock. The 'internal time' t_f is an intrinsic property of matter‡. The amplitude $(T, \mathbf{X})$ is a 4-vector such that $T^2 = T_0^2 + |\mathbf{X}|^2$, where T_0 is an amplitude with vibration in proper time.

The vibrations in space and time can be written as

$$t_f = t + T\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = t + \mathrm{Re}(\zeta_t^+), \tag{1}$$

$$\mathbf{x}_f = \mathbf{x} + \mathbf{X}\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = \mathbf{x} + \mathrm{Re}(\zeta_x^+), \tag{2}$$

where

$$\zeta_t^+ = -iTe^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \tag{3}$$

$$\zeta_x^+ = -i\mathbf{X}e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \tag{4}$$

and $\omega^2 = \omega_0^2 + |\mathbf{k}|^2$. Thus, time of matter inside the plane wave has this temporal vibration when observed with respect to the external time. This internal time, t_f, is a function of the external time, t, and a dynamical variable for the system.

‡ Unlike the 'intrinsic time' [19, 20] suggested as a dynamical variable of the studied system (e.g. position of a clock's dial or position of a classical free particle [22]) that can function to measure time, the 'internal time' defined here is an intrinsic property of matter that has vibration in time.

For a plane wave with proper time vibrations only, matter has no vibration in space. In this case, $\omega = \omega_0$, $|\mathbf{k}| = 0$, $T = T_0$, and $|\mathbf{X}| = 0$ with

$$\zeta_{0t}^{+} = -iT_0 e^{-i\omega_0 t}, \tag{5}$$

and

$$t_f = t - T_0 \sin(\omega_0 t), \tag{6}$$

$$\mathbf{x}_f = \mathbf{x}. \tag{7}$$

The internal time passes at the rate $1 - \omega_0 T_0 \cos(\omega_0 t)$ with respect to the external time and has an average value of 1. Matter in this plane wave is stationary in space and will still appear to travel along a time-like geodesic when averaged over many cycles. The nature of this internal time will be further elaborated in Section 4.

We can further define a plane wave,

$$\zeta^{+} = \frac{T_0}{\omega_0} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \tag{8}$$

such that ζ_t^+ and ζ_x^+ in Eqs.(3) and (4) can be obtained from ζ^+ as $\zeta_t^+ = \partial\zeta^+/\partial t$ and $\zeta_x^+ = -\nabla\zeta^+$ respectively. Therefore, the vibrations of matter in space and time for a plane wave can be described by ζ^+.

3. Hamiltonian Densities

Let us investigate the properties of a system in a cube with volume V that can have multiple particles with mass m vibrating in space and time. We will impose periodic boundary conditions at the box walls. Instead of carrying out our analysis in terms of the plane wave ζ^+ and its complex conjugate ζ^-, we make the following ansätze

$$\varphi^{+} = \omega_0 \sqrt{\frac{m}{2V}} \zeta^{+} = T_0 \sqrt{\frac{m}{2V}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \tag{9}$$

$$\varphi^{-} = \omega_0 \sqrt{\frac{m}{2V}} \zeta^{-} = T_0^{*} \sqrt{\frac{m}{2V}} e^{-i(\mathbf{k}\cdot\mathbf{x} - \omega t)}, \tag{10}$$

where T_0 here is taken as a complex amplitude and periodic boundary conditions are imposed on the wave vector $\mathbf{k}$.

The plane wave $\varphi^\pm$ satisfies the equation of motion:

$$\partial_u \partial^u \varphi^{\pm} + \omega_0^2 \varphi^{\pm} = 0. \tag{11}$$

The corresponding Hamiltonian density is

$$H^{\pm} = (\partial_0 \varphi^{\pm *})(\partial_0 \varphi^{\pm}) + (\nabla \varphi^{\pm *}) \cdot (\nabla \varphi^{\pm}) + \omega_0^2 \varphi^{\pm *} \varphi^{\pm}. \tag{12}$$

Let us look at each term on the right hand side (r.h.s.) of this Hamiltonian density equation. From Eqs. (9) and (10), the first term of Eq.(12): $H_1^{\pm} = (\partial_0 \varphi^{\pm *})(\partial_0 \varphi^{\pm}) = m\omega_0^2 T^* T/(2V)$, is a Hamiltonian density for vibrations of matter in time. Indeed, $m\omega_0^2/2$ is an usual term that appears in the Hamiltonian of a harmonic oscillator with mass m except the vibration is in time and not in space. (Note that we have not taken into account the order of multiplication between complex conjugates here but shall be considered when the field is quantized.) Similarly, the second term: $H_2^{\pm} = (\nabla\varphi^{\pm *}) \cdot (\nabla\varphi^{\pm}) = m\omega_0^2 \mathbf{X}^* \cdot \mathbf{X}/(2V)$, has the familiar form of a Hamiltonian density with harmonic oscillation in space.

The plane wave $\varphi^{\pm}$ is a function of T_0 as shown in Eqs. (9) and (10). The third term on r.h.s. of Eq.(12) is a Hamiltonian density related to vibrations of matter in proper time, $H_3^{\pm} = \omega_0^2 \varphi^{\pm *}\varphi^{\pm} = m\omega_0^2 T_0^* T_0/(2V)$. After combining the second and third terms, the total Hamiltonian density is

$$H^{\pm} = H_1^{\pm} + H_2^{\pm} + H_3^{\pm} = \frac{m\omega_0^2}{V} T^* T. \tag{13}$$

The energy generated by the vibration of matter in proper time is of special importance in our study. To better understand its properties, we consider the simple plane waves

$$\varphi_0^{+} = T_0 \sqrt{\frac{m}{2V}} e^{-i\omega_0 t}, \tag{14}$$

$$\varphi_0^{-} = T_0^* \sqrt{\frac{m}{2V}} e^{i\omega_0 t}. \tag{15}$$

Matter inside this plane wave $\varphi_0^{\pm}$ has vibrations in proper time only, i.e. $|\mathbf{k}| = 0$ and $\mathbf{x}_f = \mathbf{x}$. Substitute Eqs.(14) and (15) into Eq.(12), the Hamiltonian density is

$$H_0^{\pm} = \frac{m\omega_0^2 T_0^* T_0}{V}. \tag{16}$$

The energy generated inside volume V is $E = m\omega_0^2 T_0^* T_0$ of a simple harmonic oscillating system in proper time. As discussed in the previous section, the vibration in proper time is an intrinsic property of matter. Energy E shall therefore correspond to certain energy intrinsic to matter. However, we have only consider matter with mass m in this simple harmonic oscillating system without involving any of the various charges or force fields. No other energy is present in this system except the energy of mass m. Here, we will consider this energy as the internal energy of mass.

4. Proper Time Oscillator

The vibration of matter in proper time generates energy. If this energy is the internal energy of mass m, it is necessary on shell. For a single particle system, we have

$$E = m\omega_0^2 T_0^* T_0 = m, \tag{17}$$

or simply

$$\omega_0^2 T_0^* T_0 = 1. \tag{18}$$

In addition to the classical concepts of mass [23], we advocate the possibility that internal energy of a point mass m can be generated by the oscillation with a proper time amplitude of $|\tilde{T}_0| = 1/\omega_0$. Only an oscillator with such amplitude is observable in this single particle system.

Let us first consider the point mass in the plane wave φ_0^+. A point mass m at rest in space with angular frequency ω_0 and amplitude $\tilde{T}_0 = 1/\omega_0$ will have vibration in proper time relative to the external time. The internal time $\tilde{t}_f^+$ of the point mass's internal clock observed in frame O is:

$$\tilde{t}_f^+(t) = t - \frac{\sin(\omega_0 t)}{\omega_0}. \tag{19}$$

We will assume the point mass observed is located at the origin of coordinate $\mathbf{x}_0$,

$$\tilde{\mathbf{x}}_f^+(t) = \mathbf{x}_0. \tag{20}$$

From Eq. (19), the internal time rate relative to the external time for this oscillator is $\partial\tilde{t}_f^+/\partial t = 1 - \cos(\omega_0 t)$. The average of this time rate is 1. Its value is bounded between 0 and 2 which is positive. Thus, the internal time of a point mass moves only in the forward direction. It cannot move back to its past. If we assume this point mass is a typical particle that has high vibration frequency, e.g. $\omega_0 = 7.6 \times 10^{20}$s and $|\tilde{T}_0| = 1.32 \times 10^{-21}$ for an electron, the particle will appear to travel along a smooth time-like geodesic if the inertial observer's clock is not sensitive enough to detect the high frequency and small amplitude of the vibration. In fact, as the angular frequency increases and approaching infinity ($\omega_0 \to \infty$), the amplitude of oscillation becomes negligible ($T_0 \to 0$). Such particle will travel along a near time-like geodesic with no vibration observed.

The internal clock of the particle with angular frequency $\omega_0 \to \infty$ is a clock suitable for the observer at spatial infinity. Its near time-like geodesic nature is sensitive enough to detect the varying internal time rate of another particle with lower frequency. However, this clock's mass is infinite ($m = \omega_0 \to \infty$). As pointed out by Salecker and Wigner [24], to obtain infinite accuracy in measuring a clock's time means infinite uncertainty in the clock's mass, and thus the clock's mass needs to reach infinity. Some of the studies regarding quantum clocks in the context of time-energy uncertainty relation can be found in references [20, 21, 25–29].

Eqs. (19) and (20) can be Lorentz transformed to another frame of reference O' with background coordinates $(t', \mathbf{x}')$ where the the particle will have vibrations in time and space with amplitudes $\tilde{T} = \omega/\omega_0^2$ and $\tilde{\mathbf{X}} = \mathbf{k}/\omega_0^2$ respectively. (We have assumed frame O is traveling with velocity $\mathbf{v} = \mathbf{k}/\omega$ relative to frame O' and the particle begins at origin of the $\mathbf{x}'$ coordinates at $t' = 0$). The vibrations in time and space are

$$\tilde{t}_f'^+(t') = t' - \frac{\omega}{\omega_0^2}\sin(\frac{\omega_0^2 t'}{\omega}), \tag{21}$$

$$\tilde{\mathbf{x}}_f'^+(t') = \mathbf{v}t' - \frac{\mathbf{k}}{\omega_0^2}\sin(\frac{\omega_0^2 t'}{\omega}). \tag{22}$$

The internal time $\tilde{t}_f'^+$ is measured with respect to the external time in frame O' and is not the internal proper time of the particle's internal clock. In frame O', the particle travels with a velocity. The internal proper time measured by the particle's clock is $\tilde{t}_f^+ = \sqrt{(\tilde{t}_f'^+)^2 - (\tilde{\mathbf{x}}_f'^+)^2} = t - \sin(\omega_0 t)/\omega_0$ as shown in Eq. (19).

Eq. (22) is the trajectory of the particle observed in frame O'. The particle travels with a velocity

$$\tilde{\mathbf{v}}_f^+ = \frac{\partial \tilde{\mathbf{x}}_f'^+}{\partial t'} = \mathbf{v}[1 - \cos(\frac{\omega_0^2 t'}{\omega})]. \tag{23}$$

Apart from this variation in velocity, the internal time rate also varies. From Eq. (21), the internal time rate relative to the clock of the inertial observer is

$$\frac{\partial \tilde{t}_f'^+}{\partial t'} = 1 - \cos(\frac{\omega_0^2 t'}{\omega}). \tag{24}$$

We can calculate the amplitudes of vibration for a particle. For example, we can estimate the amplitude of spatial vibration for an electron:

$$|\mathbf{v}| = 0.99999 \Rightarrow |\tilde{\mathbf{X}}| = 8.6 \times 10^{-9}\mathrm{cm}, \tag{25}$$

$$|\mathbf{v}| = 0.001 \Rightarrow |\tilde{\mathbf{X}}| = 3.9 \times 10^{-14}\mathrm{cm}. \tag{26}$$

In the second, non-relativistic example, the amplitude of the spatial vibration is approximately equal to the diameter of a nucleus which is tremendously larger than the Planck length. However, this vibration also has a very short time scale ($\approx 10^{-21}s$ for electron).

Comparing Eqs. (14) and (15), the plane wave φ_0^+ with a particle traveling forward in time is mathematically equivalent to plane wave φ_0^- with a particle traveling backward in time - time reversal symmetry, a property of an antiparticle [30, 31]. The internal clock of this antiparticle shall read

$$\tilde{t}_f^-(t) = -t + \frac{\sin(\omega_0 t)}{\omega_0}. \tag{27}$$

Thus, the internal time rate relative to the external time for the oscillator with amplitude $\tilde{T}_0^* = 1/\omega_0$ is $\partial \tilde{t}_f^-/\partial t = -1 + \cos(\omega_0 t)$. The average of this time rate is -1. Its value is bounded between 0 and -2 which is negative. Thus, the internal time of this antiparticle moves only in the backward direction.

5. Field Quantization

The amplitude of a classical harmonic oscillator with a point mass vibrating in space can take on different values. This is unlike the case for a simple harmonic oscillator with vibration in proper time. The condition that mass is on shell imposes a constraint allowing only an oscillator with proper time amplitude $|\tilde{T}_0| = 1/\omega_0$ to be observed. The classical harmonic oscillator has no such constraint.

As shown in Eq. (16), the amplitude T_0 of plane wave $\varphi_0^{\pm}$ determines the amount of energy in volume V. In the above analysis for a plane wave with proper time vibrations, we have assumed a system with only one particle (antiparticle). For a many-particle system, it can have n integer number of oscillators. We can generalize condition (18) as

$$\omega_0^2 T_0^* T_0 = n, \tag{28}$$

which is a Lorentz invariant. The number of particles observed in the system shall remain same under Lorentz transformations. Taking the point mass as a particle(antiparticle) with de Broglie's mass/energy ($m = \omega_0$) in Eq.(16) is $H_0^{\pm} = n\omega_0/V$. The energy in this plane wave with vibrations in proper time is quantized with $n = 0, 1, 2, ...$

Under a Lorentz transformation, $\varphi_0^{\pm} \rightarrow \varphi^{\pm}$. Instead, let us consider a plane wave $\varphi_n^{\pm}$ which is normalized in volume V when $n = 1$,

$$\varphi_n^{\pm} = \gamma^{-1/2}\varphi^{\pm}, \tag{29}$$

where $\gamma = (1 - |\mathbf{v}|^2)^{-1/2} = \omega/\omega_0$. Replace $\varphi^{\pm}$ with $\varphi_n^{\pm}$ in Eq. (12), the Hamiltonian density for plane wave $\varphi_n^{\pm}$ is $H_n^{\pm} = \gamma H_0^{\pm} = n\omega/V$. The energy in this plane wave $\varphi_n^{\pm}$ is quantized with n particles (antiparticles) of angular frequency ω in volume V.

We can obtain a real scalar field by superposition of plane waves,

$$\varphi(x) = \sum_{\mathbf{k}} \varphi_{n\mathbf{k}}^{+}(x) + \varphi_{n\mathbf{k}}^{-}(x) = \sum_{\mathbf{k}} (2V\omega)^{-1/2}(\omega_0 T_{0\mathbf{k}} e^{-ikx} + \omega_0 T_{0\mathbf{k}}^{*} e^{ikx}), \tag{30}$$

which satisfies the Klein-Gordon equation. The corresponding Hamiltonian density equation is $H = 1/2[(\partial_0\varphi)^2 + (\nabla\varphi)^2 + \omega_0^2\varphi^2]$. Since φ is the superposition of plane waves $\varphi_{n\mathbf{k}}^{\pm}$, the energy observable in this real scalar field is necessary quantized.

In quantum field theory, the transition to a quantum field can be done via canonical quantization. Similarly, we can treat $\varphi(x)$ and H as operators. Condition (28) can be extended to the quantized field with $N_{\mathbf{k}} = \omega_0^2 T_{0\mathbf{k}}^{\dagger} T_{0\mathbf{k}}$ as the particle number operator. Ordering between $T_{0\mathbf{k}}$ and $T_{0\mathbf{k}}^{\dagger}$ shall be taken into account. We can also define the annihilation operator $a_{\mathbf{k}}$ and creation operator $a_{\mathbf{k}}^{\dagger}$ as $a_{\mathbf{k}} = \omega_0 T_{0\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger} = \omega_0 T_{0\mathbf{k}}^{\dagger}$ such that $N_{\mathbf{k}} = a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}$. Comparing these results with quantum field theory, the real scalar field with vibrations in space and time has the same physical structures of a zero-spin bosonic field.

6. Wave Function

To study the case in the non-relativistic limit, we make the ansatz:

$$\psi_{\mathbf{k}} = \frac{\omega_0 T_{0\mathbf{k}}}{\sqrt{V}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_c t + \chi)} \approx \left[\frac{\omega_0^2}{\sqrt{V}} e^{i(\omega_0 t + \chi)}\right] \zeta_{\mathbf{k}}^{+}, \tag{31}$$

where

$$\omega_c = \frac{\mathbf{k}\cdot\mathbf{k}}{2m} \approx \omega - \omega_0, \tag{32}$$

and $e^{i\chi}$ is an arbitrary phase factor. Periodic boundary conditions for a cube with volume V are imposed on the wave vector $\mathbf{k}$. Here, $T_{0\mathbf{k}}$ is considered as a function and not an operator. We will show that $\psi_{\mathbf{k}}$ has properties of the wave function in quantum mechanics.

Schrödinger equation - $\psi_{\mathbf{k}}$ is a solution for the Schrödinger equation of a free particle, $-i\partial\psi_{\mathbf{k}}/\partial t = (2m)^{-1}\nabla^2\psi_{\mathbf{k}}$. The superposition principle holds such that

$$\psi = e^{i\chi} \sum_{\mathbf{k}} \frac{\omega_0 T_{0\mathbf{k}}}{\sqrt{V}} e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_c t)}, \tag{33}$$

is also a solution for the linear and homogeneous Schrödinger equation.

Probability density - The product of $\psi_{\mathbf{k}}$ and its complex conjugate $\psi^*_{\mathbf{k}}$,

$$\psi^*_{\mathbf{k}}\psi_{\mathbf{k}} = \frac{\omega_0^2 T^*_{0\mathbf{k}} T_{0\mathbf{k}}}{V} = \frac{n_{\mathbf{k}}}{V}, \tag{34}$$

is a particle number density. In a quantum wave, the location where a particle can be observed is indeterminate. Only a probability can be assigned. For a plane wave, the probability density has an uniform distribution which is also the particle number density from Eq. (34). The amplitude $\alpha_{\mathbf{k}} = \omega_0 T_{0\mathbf{k}}/\sqrt{V}$ in Eq. (31) is a probability amplitude.

Unobservable overall phase - It is commonly believed that a matter wave can only have a probabilistic interpretation because the overall phase of a wave function is unobservable. As we have shown, the introduction of the arbitrary phase factor $e^{i\chi}$ in Eqs. (31) and (33) does not change the the probability density $\psi^*\psi$ or the result that ψ satisfies the Schrödinger equation. In fact, the theory developed with wave functions ψ shall be invariant under global phase transformation χ but the relative phase factors are physical. Thus, the overall phase of ψ is unobservable. Function ψ is not required to have the same phase as ζ that describes the physical vibrations in space and time.

7. Conclusions and Discussions

In this paper, we treat time as a dynamical variable. Instead of considering proper time as an operator, for example in references [26, 29], we study the possibility that matter not only can have vibrations in space but can also have additional degrees of freedom with vibrations in time. We show that the harmonic oscillator in proper time can be the generator for the energy of mass. However, the energy of a mass is necessary on shell meaning only one unique amplitude for the proper time harmonic oscillator can be observed, $|\tilde{T}_0| = 1/\omega_0$. This is unlike a classical harmonic oscillator with vibration in space that can take on different values as its amplitude. (There is no condition analogous to mass on shell that restrict amplitude of vibration in space to an unique value.) The Hamiltonian of the system is quantized and can only correspond to those generated by n integer number of oscillators. The real scalar field φ does not have a classical description but rather shall be treated as a quantized field. In addition, this real scalar field satisfies the Klein Gordon equation and has the properties of a zero spin bosonic field. The possibility that matter has vibrations in time can lead to the quantization of a bosnoic field. Time can play a more dynamical role in a quantum field.

As shown in Section 4, a particle traveling with velocity $\mathbf{v}$ has vibrations in space and time. A particle in the plane wave with angular frequency ω and wave vector $\mathbf{k}$ will have fluctuations in time and space with amplitudes $\tilde{T} = \omega/\omega_0^2$ and $\tilde{\mathbf{X}} = \mathbf{k}/\omega_0^2$ respectively. These vibrations are the results when time is taken as a dynamical variable by treating space and time on the same footing. Although the amplitude of the vibration in space is small (size of a nucleus in the non-relativistic example given in Section 4), its effect may be observable if nature has something to do with these vibrations of matter in space and time.

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