Primitive Ontology in a Nutshell

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Abstract

The aim of this paper is to summarize a particular approach of doing metaphysics through physics - the primitive ontology approach. The idea is that any fundamental physical theory has a well-defined architecture, to the foundation of which there is the primitive ontology, which represents matter. According to the framework provided by this approach when applied to quantum mechanics, the wave function is not suitable to represent matter. Rather, the wave function has a nomological character, given that its role in the theory is to implement the law of evolution for the primitive ontology.

1. Introduction

The primitive ontology (PO) approach, first formulated in [Goldstein 1998], and then developed in [AGTZ 2008, 2011, 2014], [Allori 2013a,b], provides a characterization of what it takes for a fundamental theory to be satisfactory when used to ‘read off’ the metaphysics from the physics. Thus, the approach is normative: it tells us what structure proper fundamental physical theories ought to have. In this paper, I will not motivate or defend the view against possible objections because this has been done elsewhere (see [Allori 2013 a,b]). Instead, I will articulate and summarize the main ingredients behind the idea of PO, and I will review what implications this approach has in the case of quantum mechanics.

According to the PO approach, all fundamental physical theories have a common structure, which provides a general explanatory schema with which the theory accounts for what the world is like. According to this approach, any satisfactory fundamental physical theory, if taken from a realist point of view, contains a metaphysical hypothesis about what constitutes physical objects, the PO, which lives in three-dimensional space or space-time and constitutes the building blocks of everything else. In the formalism of the theory, the variables representing the PO are called the primitive variables. In addition, there are other variables necessary to implement the dynamics for the primitive variables: these non-primitive variables could be interpreted as law-like in character. Once the primitive and the non-primitive variables are specified, one can construct an explanatory scheme based on the one that is already in use in the classical framework. This allows determining, at least in principle, all the

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macroscopic properties of familiar physical objects in terms of the PO. This structure holds for classical as well as for quantum theories.

Here is the outline of the paper. In Section 2 I sketch the relationship between physics and metaphysics in this account. Then In sections 3 through 9, I present the various ingredients of the PO approach: first the requirements that the primitive variables are defined in three-dimensional space and on the microscopic level, the distinction between the primitive and the non-primitive variables, the connection between the PO and local beables, how theories with the same evolution of the PO are the same theory, how one can generate new theories mixing up different primitive variables and different evolutions, and how symmetries are properties of the PO. Then, in Sections 10, 11 and 12 I apply the PO approach to quantum theories, focusing on the characterization of the fundamental features of the various theories, the meaning of the wave function, and of Bell’s alternatives. Then I discuss how the PO approach can be helpful in theory evaluation and in constructing a future relativistic invariant quantum theory. I conclude with a section in which I summarize the main ingredients of the PO framework.

2. Physics and Metaphysics
The starting point of this approach is a realist conception of fundamental physical theories, namely the assumption that the picture of reality provided by our best theories is accurate, and thus we can use physics as a guide to metaphysics. The relationship between physics and metaphysics is complicated, and surely deserves more exploration than the one I will provide here. Nevertheless, the main idea behind this framework is that one should not do metaphysics ‘from the armchair,’ so to speak, but rather one should take fundamental physical theories to investigate what the world is like. This does not mean, though, that a fundamental physical theory is completely void of metaphysical commitments to start with. Rather, when building a fundamental physical theory, the theoretician typically already has a metaphysical picture in mind, and if the theory turns out to be empirically adequate, then one will take it to inform us about metaphysics. This relationship is not one way, though. In fact, as it is known, fundamental physical theories are underdetermined by empirical data: many incompatible ontologies can give rise to the same empirical predictions, and thus one cannot rely simply on empirical virtues to select one ontology over another. Metaphysical criteria such as simplicity, explanatory power, or the like can help us with theory selection, so that in this sense metaphysics is also a guide to physics.

3. Primitive Ontology
The minimal requirement to even start this enterprise of inferring what the world is like from a fundamental physical theory is that there has to be something in the formalism of the theory that connects it to the world. That is, any fundamental physical theory should have a clear ontology. If the ontology of a theory is not clear, then it is not clear what entities the theory is assuming to exist, and then it is hard to see how one could
even begin to do metaphysics starting from it. To specify what the ontology of a theory is amounts to selecting which variables, among all of them in the theory, are to be taken as representing what exists in the world. For instance, consider classical mechanics. Arguably, the ontology of this theory is given by particles, with position and momentum.

Until now, nothing much is new. What changes in the PO approach is that one should also specify, among the elements in the ontology, which variables are primitive and thus constitute the PO of the theory. The discussion in the literature has focused mostly on the situation in quantum mechanics but the approach aims to be general (see, e.g. [Allori 2013 b]). The basic idea is that, when we are using a fundamental physical theory to figure out what the physical world is like, among the variables that represent the ontology of a theory, one can distinguish between primitive and non-primitive variables. In a sense, some part of the ontology is ‘more important’ than other parts: the primitive variables represent matter, what physical objects like tables and chairs are made of, the non-primitive variables in contrast do not. Rather, they are what is needed to complete the theoretical description of the world. In the case of classical mechanics, arguably, particles’ positions are the primitive variables, momenta are not; however one needs momenta to complete the description, to implement the law of motion for the particles. Thus, the qualification ‘primitive ontology’ instead of just ‘ontology’ comes from the idea that the PO does not exhaust all the ontology but it rather just accounts for physical (namely material) objects. As we will see in Section 11, in quantum mechanics the wave function is a non-primitive variable, and it is arguably more similar, in kind, to laws than to material objects.

Let us now focus on the PO itself: the idea of dividing the ontology into primitive and non-primitive is rightfully puzzling and as such requires more exploration. For one thing, this separation does not mean that primitive variables are ‘more real’ than non-primitive ones. Rather, it is a statement about what is material and what is not, without restricting ourselves to say that everything there is has to be material. Other things might exist (numbers, mathematical objects, abstract entities, laws of nature, and so on), and some of them (like natural laws) might be described by other variables, namely the non-primitive variables, in the ontology of a fundamental physical theory. Thus, the primitive variables are primitive in a variety of senses. First, they are ontologically primitive, given that they represent matter and they provide the fundamental entities the theory describes. But also, they are epistemically primitive: representing matter, they are the variables that directly accessible to us, contrarily to non-primitive variables that may represent laws of nature. In addition, they are structurally (or architecturally) primitive, in the sense that they constitute the building blocks of everything else, and in virtue of that and of being in three-dimensional space (or space-time) they ground the
explanatory scheme with which the theory describes and account for macroscopic physical reality.

Notice that the primitive variables are not typically chosen \textit{a posteriori}, once the formalism of the theory has been specified. Rather, there is already a natural interpretation for each mathematical object present in the theory, namely the one the proponent of the theory intended to give them. The scientist's choice of what physically exists in the world will more or less automatically determine the mathematical object to represent it. If one wanted matter to be made, say, of point-like particles, then the natural way to mathematically describe them would be using points in three-dimensional space. Sometimes, though, like in the case of quantum mechanics, the situation is more complicated, since it is not clear what the initial metaphysical hypothesis behind the construction of the theory is. Thus, we find ourselves with the 'bare' formalism, and we are obliged to 'interpret' it \textit{a posteriori}. This is one reason why only in the quantum framework, in contrast with what happens in the classical theories, we have so many possible theories, as we will see in Section 11. The PO approach will allow ruling out at least some of the proposed theories, namely those which will have the wave function as primitive variable. Other considerations come into play in evaluating theories with different POs. For more on this, see [Allori forthcoming 2].

Be that as it may, in the next two sections we will see what it takes for a variable to be a suitable PO and why.

4. Three-Dimensionality
A suitable PO needs to be (1) defined in three-dimensional space (or space-time) rather than being more complicated mathematical objects, and (2) it needs to be microscopic rather than macroscopic. Mathematically, the primitive variables are defined in three-dimensional space or four-dimensional space-time. There can be different kinds of primitive variables: each given type of three-dimensional object can represent a different possible PO for a fundamental physical theory. A point $x$ in $\mathbb{R}^3$, for instance, represents a possible PO, since it can be taken to be represent point-like material particles. This is the case of classical mechanics. Also, a function $f(x)$ defined on $\mathbb{R}^3$ can also be a primitive variable, since it can be taken to represent a matter density field. This, arguably, could be the case of electromagnetic fields in classical electrodynamics, as we will see in Section 7. In addition, points in $\mathbb{R}^4$, when interpreted as space-time, are a possibility, because they can be taken to represent space-time events, or 'flashes,' as the theories we will see in Section 11 will make clear. But do these features make a variable a good PO?

Roughly, the three-dimensionality of the primitive variables allows for a direct contact between the variables in the theory and the objects in the world we want them

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2 For more on the different roles the primitive ontology plays in a fundamental physical theory, see [Allori 2013 b].
to describe\textsuperscript{3}. In fact, a PO represented by an object in a space of dimension $d$, different than 3, would imply that matter - including us - lives in a $d$-dimensional space. Thus, our fundamental physical theory would have to be able to provide an additional explanation of why we think we live in three-dimensional world while we actually do not. It has been argued that this is, at best, undesirable\textsuperscript{4}, in part for reasons connected to another feature that a good PO is supposed to have, namely its fundamentality.

5. Microscopicality
From what we have said so far, the only limitation to the primitive variables is that they are supposed to be defined in three-dimensional space (or four-dimensional space-time). In principle, they could be microscopic or macroscopic entities. However, a microscopic PO, in which the PO constitutes the building blocks of everything else, is able to ground a scheme of explanation that allows determining the properties of macroscopic physical objects in terms of the behavior of the PO. In fact, consider classical mechanics, a theory that provides an example of a microscopic PO. In this theory, arguably\textsuperscript{5} any physical body (gases, fluids, and solids) is satisfactorily described as a collection of (microscopic, three-dimensional) particles. Once the PO and its temporal evolution are given, everything else follows: in classical mechanics (as well as in classical electrodynamics) we can identify macroscopic properties more or less straightforwardly given how the (microscopic) PO combines and interacts to form (macroscopic) physical bodies. Thus, one could argue (as we did in [AGTZ 2008]) Bohr’s quantum theory can be viewed as a quantum theory in which the PO is given by the macroscopic measurement results. However, such a theory would be unsatisfactory: being macroscopic, the PO hardly can ground the explanatory scheme of the theory.

6. Non-Primitive Ontology
In contrast with the primitive variables, the non-primitive variables have the role of implementing the law of motion for the PO. For this reason, such variables are sometimes called ‘nomological’ variables. Roughly speaking, the primitive variables tell us what there is, and the non-primitive variables tell the primitive variables how to ‘behave.’ In classical mechanics, as we already noted, the complete description of any physical system at a given time is given by the couple given by the position $x$ and momentum $p$ of the particles. The position is the primitive variable, while the momentum allows the equation for the position to be defined. In fact, the evolution of position is given by $\frac{dx}{dt} = \frac{p}{m}$ where $m$ is the mass of the particles. The evolution of momentum itself is given by $\frac{dp}{dt} = -\nabla V(x) = F(x, p)$, where $V$ is the potential and $F$ the force. These two first-order equations can be written in a second-order equation plugging in the second equation into the derivative with respect to time of the first

\textsuperscript{3} For more on this, see [Allori 2013b].
\textsuperscript{4} See, e.g.,[Allori, 2013a,b],[Monton 2006].
\textsuperscript{5} Assuming reductionism to be true.
(assuming that the mass remains constant in time). In this way, we get back the usual Newton’s equation: \( \frac{d^2x}{dt^2} = \frac{F}{m} \).

Thus in this sense the PO is the most fundamental ingredient of the theory. It grounds the ‘architecture’ of the theory: first, we describe matter through the primitive variables, then we describe its dynamics, implemented by some non-primitive variables. Then we are done, all the macroscopic properties are recoverable. This is also connected with the ‘primitiveness’ of the PO: even if the primitive variables do not exhaust all the variables in the ontology, because they describe matter in the theory, and because in principle every macroscopic property can be recovered in terms of them, we can directly compare the macroscopic behavior predicted by the theory to the actual behavior of matter. Not so for the other non-primitive variables, which can only be ‘observed’ indirectly in terms of the ways they affect the behavior of the PO.

7. Local Beables and Primitive Ontology

The notion of ‘local beable,’ introduced in [Bell 1987], share some similarities with the notion of primitive ontology. As Bell puts it, “the beables of the theory are those entities in it which are, at least tentatively, to be taken seriously, as corresponding to something real” [Bell 1987, pag. 234] and that correspond to a given portion of space-time (hence they are local in this respect). Thus, there is a clear connection with the notion of PO. Nonetheless, one can think of fundamental physical theories in which the local beables are not necessarily the PO of the theory. Think for instance to classical electrodynamics. In this theory, in addition to the positions of particles, there are also electromagnetic fields. They are local beables of the theory, since they are mathematically described by functions in three-dimensional space, and thus satisfy the first requirement for being a suitable PO. In addition, one could think of them as microscopic and fundamental: the reality described by classical electrodynamics would then fundamentally be made by particles and fields. In this way, the PO of classical electrodynamics would be both of particles and fields. In other words, the world would contain two kinds of material entities, one mathematically characterized by points, and the other by (suitable) fields. Nonetheless, given the role the fields play in the theory, another option seems more suitable. In other words, in the architectural sense we have been exploring in this approach, the (variable representing the) particles and the (variable representing the) fields do not have the same role in the theory: the fields, in fact, implement the evolution of the particles, not the other way round. In this way, thus, the fields seem to be better interpreted as nomological rather than primitive variables. To put it in another way, the variables representing the ontology, \( O \), thus, can be written in terms of the couple (primitive; non-primitive variables). That is, we can use the symbol of the semicolon “,” to divide in the ontology the primitive from the non-primitive variables,

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6 For a discussion of the pros and cons of the different choices, see [Allori forthcoming 1].
the primitive ones being on the left of the semicolon. For example, when we write \(O=(a;b)\) we mean that matter is represented by a alone. In classical mechanics, we would have \(O=(x;p)\), where \(x\) is the position of particles and \(p\) their momentum. Since the PO specifies what matter is in a given fundamental physical theory, the specification of the PO and its temporal evolution, or its spatiotemporal history, completely determines the theory. Thus, we have different theories depending on where we place the semicolon: one would have (among other less interesting possibilities) \(T_a = (a;b)\), a theory in which matter is represented by \(a\), \(T_b = (b;a)\), in which matter is represented by \(b\), and \(T_{ab}=(a,b)\), in which matter is both made of \(a\) and \(b\) stuff. In the case of classical electrodynamics, if \(x\) denotes the position of particles, \(E\) the electric field and \(B\) the magnetic field, we can have different theories of classical electrodynamics: CED_{particles&fields}=(x,E,B), in which matter is composed by both particles and electromagnetic fields; CED_{partciles}=(x,E,B), in which matter is made of particles and fields are ‘part of the law of nature.’ The former theory is what Albert has in mind in [Albert 2002], while something close to the second option is what is described in [DGZ 1992].

8. Physical Equivalence

Since the empirical adequacy of a theory is decided by the histories of the PO, there could be empirically adequate theories with the same PO but whose evolution is generated by different non-primitive variables. This leads to the notion of physical equivalence: two different theories that provide the same evolution for the PO, no matter how it is mathematically implemented, describe the same physical world. They are indistinguishable as far as the empirical appearances are concerned. Therefore, on the one hand, if we change the PO and its evolution, we change theory, since we change the way the theory describes matter. On the other hand, however, if we keep the same histories of the PO while we change the way in which this evolution is obtained then we have theory that is physically equivalent to the original theory. For instance if the spatiotemporal histories of the PO in \(T_a\), whose ontology is \(O=(a;b)\), are the same as the spatiotemporal histories of PO in \(T_a'\), whose ontology is \(O=(a;c)\), then \(T_a\) and \(T_a'\) are physically equivalent, even if they have a different non-primitive variables.

The notion of physical equivalence between theories was introduced in [AGTZ 2008] in the framework of quantum mechanics. Nonetheless, it is not necessary to go to quantum theories to give an example of physically equivalent theories. Here is a very simple example of physical equivalent theories. If a force is conservative, it can be defined as the opposite of the gradient of the potential. This particular mathematical
operation involves derivatives, and because of this, it is always possible to find two different potentials that give rise to the same histories of the \( \text{PO} \): any two potentials that differ by a constant will do the trick. In fact, they both give rise to the same force (and therefore the same histories of the \( \text{PO} \)), given that the derivative of any constant is always zero. Hence, two theories with such potentials will be physically equivalent.

9. **Symmetry Properties**

There is an important connection between the \( \text{PO} \) and the symmetry properties of a theory. Roughly put\(^{10} \), since the histories of the \( \text{PO} \) provide the metaphysical picture of the world, if the theory is invariant under a given symmetry this picture should not change under the symmetry transformation connected to the symmetry. Given their role, the non-primitive variables will instead transform under the symmetry in such a way as to ensure that the histories of the \( \text{PO} \) will remain invariant. Invariance is therefore a property of the dynamics of the \( \text{PO} \): changing the \( \text{PO} \) of a theory might change its symmetry properties. Therefore, before asking whether a given theory has a given symmetry, it is necessary to identify its \( \text{PO} \) and see whether the transformed histories of the primitive ontology are still possible histories for the theory. In addition, choosing one \( \text{PO} \) rather than another might make the theory acquire or lose symmetries. In this way, if one considers having a given symmetry a *desideratum* for a theory, symmetries could then be used to select, among other super-empirical virtues such as simplicity or explanatory power, the most desirable \( \text{PO} \) (see Section 13).

10. **Quantum Mechanics**

Except that in the case of quantum mechanics\(^{11} \), no one has extensively discussed the plethora of theory that one could generate selecting the \( \text{PO} \), the non-primitive variables and their respective evolutions to obtain an empirically adequate theory. Presumably, this is connected to the fact that, unlike the case of electromagnetic fields, it is much more difficult to make sense of the wave function as composing matter. In fact electromagnetic fields can be taken as a suitable \( \text{PO} \) for classical electrodynamics, even if it can come with some costs, like the loss of symmetry properties. In contrast, the wave function is not even a local beable: as we will see in the next section, it is defined in a configuration space, a highly abstract space, and it is necessary to close the gap between that space and ordinary three-dimensional space in order for the theory to account for the empirical predictions.

Be that as it may, now that we have outlined the main ingredients of the \( \text{PO} \) approach, let us see how we can apply it to the quantum framework. The so-called orthodox quantum mechanics that one can find in physics textbooks has not a clear ontology at all: is it about the motion of microscopic entities, or is it about the measurement results? Luckily, other quantum theories have a clear metaphysical

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\(^{10}\) For more on this, see [ATGZ 2008].

\(^{11}\) With the exception of [Allori forthcoming 1] that discusses the case of classical electrodynamics.
commitment, and thus can be used as a guide to metaphysics. They are Bohmian mechanics (BM), the GRW theory (GRW), and Everettian mechanics (also known as the Many-Worlds theory, MW). Also, in [ATGZ 2008, 2011, 2014], some other possible empirically adequate quantum theories are discussed.\(^{12}\)

Particles are the PO of Bohmian mechanics, and they are specified by their position \(x\) in three-dimensional space. The trajectories of a system of particles are determined by Bohm’s guidance equation.\(^{13}\) This equation involves the wave function \(\psi\). Because of its role in generating the histories of the PO of Bohmian mechanics, the wave function is a non-primitive variable. The wave function in turn evolves in time according to Schrödinger’s equation.\(^{14}\) Schematically, then, we have \(O=(x;\psi)\).

The situation in the GRW theory is more complicated. GRW is a theory in which the wave function evolves in time according to a stochastically modified version of the Schrödinger equation, also called GRW dynamics. Roughly, the wave function evolves according to the Schrödinger equation until a random time, randomly distributed with rate \(N\lambda\) (\(\lambda=10^{-15}\) s\(^{-1}\) is a new constant of nature). Then the wave function undergoes an instantaneous collapse with random center, which is mathematically represented by the multiplication of a Gaussian operator (\(\sigma=10^{-7}\) m, the width of the Gaussian, is also a new constant of nature). Historically, GRW has been taken to be a theory in which matter is described by the wave function. This is not the case in the PO approach. In fact, the wave function is a mathematical object that lives in a very abstract space: the space of all the positions of all the particles in the universe, configuration space. If there are \(N\) ‘particles’ in the universe,\(^{15}\) configuration space has dimension \(M=3N\). Thus, by

\(^{12}\) The notation used in these papers is not very illuminating and indeed can be misleading. In fact, it focuses on the evolution of the wave function that is not a good candidate to be a PO. Rather, a better notation would be one that would focus on the evolution of the PO. If \(X\) denotes a generic PO (\(x\) for particles, \(m\) for matter density fields, and \(f\) for flashes), one could then specify in a subscript the type of law \(u\) for the evolution of the PO (deterministic or random), and with a superscript the law \(F\) for the evolution of the non-primitive variable (again, deterministic or random). That is: \(X_{uF}\). In the case of classical mechanics, we would then have: \(x_{\text{deterministic}}^\text{deterministic}\). The fundamental object (the PO) in fact is \(x\), the particles; the superscript indicates that the particles evolve deterministically (according to \(\frac{dx}{dt}=p/m\)), while the subscript indicates that the momentum evolve deterministically as well (according to \(\frac{dp}{dt}=F\)). Obviously, since there are infinitely different deterministic and indeterministic possible equations, this notation is not precise. However, it would be unreasonable to require this from a notation: an effective notation should be able to provide at glance the fundamental features of a given theory, rather than the precise details, and the new notation certainly does that. In contrast, the old notation, in addition of being equally imprecise, was drawing attention to the wrong object, namely the non-primitive variables, rather than to the PO, as we will see, and thus was potentially misleading.

\(^{13}\) See [Bohm 1952], [Bell 1987], [DGZ 1992].

\(^{14}\) In the notation proposed in the previous footnote, BM would be denoted \(x_{\text{Schrodinger}}^\text{deterministic}\).

\(^{15}\) Strictly speaking, whether there are particles or not (intended as point-like building blocks of every material object in the world) depends on the primitive ontology of the theory: in a theory with just the wave function there will not be any particles in this sense, hence the scare quotes.
definition, the wave function is not a suitable primitive variable. One proposal for a reading of GRW as a theory in which the wave function does not represent the PO has been put forward in [BGG 1995] and later dubbed GRWm in [AGTZ 2008]. In this theory the PO is a three-dimensional matter field \( m(x,t) \) defined in terms of the wave function, which evolves according to GRW dynamics\(^{16}\). Another proposal along similar lines was first suggested in [Bell 1987], then adopted in [Tumulka 2006a,b] and called GRWf in [AGTZ 2008]. In this theory, the PO is represented by points in space-time (events), the ‘flashes.’ These flashes are randomly distributed in space-time in a way determined by the GRW evolving wave function: every flash corresponds to one of the spontaneous collapses of the wave function\(^{17}\). Another possibility is a stochastic particle theory called GRWp3 in [AGTZ 2014]\(^{18}\). In this theory, the motion of the particles is governed by the guiding equation, but here the wave function obeys a GRW-like evolution in which the collapses occur exactly as in usual GRW theories except that, once the time for the collapse has been chosen, the collapse is centered at the actual position of the particle displaced at random.

Lastly, in Everettian mechanics the wave function evolves linearly according to Schrödinger’s equation. Almost all the proponents of Everettian mechanics agree in considering the wave function as the object in the theory that describes physical objects, but this is again incompatible with the PO approach\(^{19}\). There is a theory, originally developed in [Bell 1987], in which the wave function evolves according to the Schrödinger equation and matter is made of particles, like in Bohmian mechanics, but they do not have a continuous trajectory in space-time. Rather, their configuration at different times is distributed according to a \(|\psi|^2\) distribution, and there is no temporal correlation among them. The theory has later dubbed BMW (Bell Many Worlds) in [AGTZ 2008] and later called Sip (S from the Schrödinger evolution of the wave

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\(^{16}\) In the new notation proposed in this paper, this theory would be dubbed \( m^\text{GRW}_{\text{random}} \).

\(^{17}\) Thus, using the notation introduced here, we would call this theory \( f^\text{GRW}_{\text{random}} \).

\(^{18}\) Or \( x^\text{GRW}_{\text{random}} \).

\(^{19}\) They believe they can describe the world using a mathematical object not defined in three-dimensional space like the wave function. They are presumably driven by the idea that to just have the wave function as the ontology of material object would make the theory simpler and more elegant. Indeed, this is one motivation that arguably has led them to believe that Everettian mechanics is the best solution to the measurement problem: there is no additional variable, and no modification of the Schrödinger dynamics, thus no modification of the traditional quantum formalism. This though, comes with the cost of having an extravagant metaphysics of many worlds. In addition, the simplicity of the mathematics is counterbalanced by the complexity of the explanatory scheme required to derive the world as we experience it from the theory: given the wave function lives in configuration space and not three-dimensional space, they need a set of rules to recover the traditional properties of macroscopic objects, like for instance, localization in three-dimensional space. Functional approaches along these lines are being developed (see [Wallace 2003]). The primitive ontologist thinks this is unnecessary complicated and departs heavily from how a fundamental physical theory should be. For more see [Allori 2013 a].
function, i for independent, p for particle ontology) in [AGTZ 2011][20]. This theory has a many-worlds character. In fact, because of linearity, in any theory in which we have a wave function evolving according to Schrödinger’s equation there are superpositions. Thus, the wave function of the universe must presumably be thought of as consisting of several packets that are very far apart in configuration space that correspond to unrealized states of affairs: in the Schrödinger cat example, if when we open the box the cat is alive, the state of affairs corresponding to the dead cat are not realized. Some of the packets will have support in events that did not take place in our time, such as for example the dinosaurs have never become extinct. In Bohmian mechanics, this is not a problem: since configurations are continuously connected in time, it is not possible for the configuration to jump, in an instant, from the support of one wave packet to a macroscopically distinct one. However, in Sip there is no connection whatsoever between what there is at a given instant of time and what there is at the previous or following instant. In this case, thus, the configuration will very probably visit in every second those distant regions supporting the other packets: therefore, at time $t$ there can be dinosaurs and at time $t+dt$ they have disappeared. Therefore, many worlds exist, not at the same time, but one after another. Because of this, the fact that right now there are memories and records of the past does not guarantee that they are actually reliable. Rather, the records are most likely to be false: at one instant, there is a set of what we would call ‘records’ that actually do not reflect in any way truthfully what has happened. [AGTZ 2011] also describe another theory with many-worlds character. Consider a three-dimensional matter field PO, whose evolution is determined by a Schrödinger evolving wave function. This theory has been dubbed Sm (S for the Schrödinger equation and $m$ for the matter density function)[21]. In this theory, the superpositions of the wave function are inherited by the matter density field. By the linearity of the Schrödinger evolution, there are non-interacting mass densities associated with the different terms of the superposition: the live cat and the dead cat do not interact with each other, as they correspond to alternative states of the cat. Thus, they can indeed be regarded as comprising many worlds, superimposed on a single space-time. Since the different worlds do not interact among themselves, they are, so to speak, reciprocally transparent. There are also theories with flashes in which the wave function never collapses. For instance, Sf as described in [AGTZ 2011][22] is a theory of flashes whose distribution is determined by a Schrödinger evolving wave function. Similarly to Sm, different non-interacting families of flashes correspond to different terms of the superposition, and hence the many-worlds character of the theory.

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[20] Here we would call it $x_{\text{random}}^{\text{Schrodinger}}$.

[21] Alternatively, $m_{\text{deterministic}}^{\text{Schrodinger}}$.

[22] Or $f_{\text{random}}^{\text{Schrodinger}}$. 

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In addition to the theory already described, we can imagine a variety of other theories mixing up the various types of PO and the various evolution equations. For instance, stochastic mechanics (SM, dubbed Sp’ in [AGTZ 2008])\textsuperscript{23} is a theory of particles that move stochastically, while the evolution of the wave function is deterministic, given by the usual Schrödinger equation [Nelson 1985], [Goldstein 1987]. For more of these theories, see [AGTZ 2008, 2011, 2014], and [Allori forthcoming 2].

To summarize, philosophers of physics and in general scholars interested in the foundations of quantum mechanics have always focused on the wave function, but that was a mistake: the wave function should not be taken as representing material objects. The PO approach says that in quantum theories, just like in any other fundamental physical theories, matter is represented by a variable in three-dimensional space (or four-dimensional space-time). Thus, it is a mistake to think that the fundamental object of quantum mechanics is the wave function. Matter in the quantum world is made of particles, or of three-dimensional matter density fields, or flashes, whose law of evolution is determined by the wave function.

11. The Meaning of the Wave Function
What is the wave function if it does not represent a material object? There are different approaches in the literature. First, the wave function has been taken to be a property of the particles\textsuperscript{24}. Another proposal is that the wave function is just a useful mathematical tool [Monton 2006]. In contrast, the proponents of the PO approach argue that the wave function is best seen as a nomological entity. In other words, the wave function is more suitable to represent a law of nature than a physical object [DGZ 1997], [GT 2000], [GZ 2013]\textsuperscript{25}. The idea is that the wave function is like the Hamiltonian in classical mechanics is the generator of motion.

Several objections have been raised against this view\textsuperscript{26}. First, since the PO represents what physical objects are made of while the wave function does not, either one denies the existence of the wave function or has to admit that something is more real than something else is. However, saying that the wave function is real but not physical does not imply there are different degrees of reality: in fact, they might be two kinds of substances, or entities. After all, the very same objections could be raised (but they are not) to a Platonist in the philosophy of mathematics, a dualist in the philosophy of mind, and a realist with respect to laws in ethics or in philosophy of science. Other objections focus on the disanalogies between the wave function and the general conception of laws. For instance, it is argued that the wave function cannot be regarded as a law because it interacts with the particles and thus seems to be more alike matter

\textsuperscript{23} In our notation it would be $x_{random}^\text{Schrödinger}$.
\textsuperscript{24} See e.g. [Monton 2006] for a proposal, [Belot 2012] for a criticism, and [ELHD 2014] for a defense.
\textsuperscript{25} [Callender forthcoming] has recently motivated this view in the Humean framework of laws of nature.
\textsuperscript{26} See [BW 2005], [Belot 2012].
than laws. One could respond saying that the wave function is similar to the potential in classical mechanics in this respect: the potential interacts with the particles but no one considers it real. Also, it has been argued that the wave function evolves in time, while laws are static. In this case, one could just not be bothered by it [Smolin 2013]. In any case, since the idea that laws of nature are static is a classical intuition, one could maintain that instead of trying to force our classical intuitions onto quantum mechanics, we should realize that quantum mechanics is telling us something new about laws of nature [Callender forthcoming]. Be that as it may, one could notice that there is evidence suggesting that in a future quantum cosmology the wave function would be static [GT 2000], eliminating the problem. Another objection is that the wave function is contingent, in the sense that varies with the subsystem, and in contrast laws are universal. A last complaint could be that the wave function is controllable: we can prepare physical systems in the state that we want. If so, it is difficult to regard the wave function as a law, since we do not seem to have control over them. These last two objections can be taken care of remembering that the wave function we can have control and that changes from system to system is the wave function of the system (the conditional or effective wave function [GZ 2013]), while the one that should be intended as nomological is the wave function of the universe (which is universal and we cannot control).

12. Bell’s Alternatives and The Measurement Problem
In some of the quantum theories seen earlier, namely the ones with a PO of particles, the PO is independent from the wave function. In contrast, in the theories with a matter density or PO of flashes, the wave function appears in the definition of the PO. This is the reason why, in theories like Bohmian mechanics, the state of the system is given by the couple \((x, \psi)\). Instead, in theories like GRW the state seems to be given by the wave function alone (even if upon a closer look one would need to specify also a rule to define the matter density or the distribution of the flashes). Thus, while we were used to distinguish between the different solutions of the measurement problem in terms of Bell’s alternatives, namely either the wave function is not complete or it does not evolve according to the Schrödinger equation, we now see another possible characterization. On the one hand, we have the theories in which the PO is independent on the wave function; on the other hand, we have theories in which the PO is defined in terms of it.

Be that as it may, the PO, namely some microscopic ontology in the three-dimensional space, is necessary to solve the measurement problem in the sense that if one does not do that, an entirely different scheme of explanation is necessary for physics to explain the world around us, along the lines of the ones proposed by the proponents of MW. The non-primitive variables provide the dynamics for the PO, and thus complete the theory: once the dynamics of the PO is provided, a picture of the world is given.
13. Theory Construction and Theory Selection

In the process of theory construction, the scientist has a considerable amount of freedom. In fact, first she has freedom of choosing the kind of PO (particles, fields, flashes,…). Then she is free to choose the PO’s temporal law of evolution, in particular whether it is stochastic or deterministic. In addition, she has the freedom of implementing such a law with the aid of some non-primitive variables evolving (or not) according to their own equation, which can be either stochastic or deterministic. Thus, the histories of the PO have to be such that, macroscopically, will recover the empirical predictions, but all the other choices will be guided by super-empirical virtues like simplicity or explanatory power. For more on PO and theory evaluations, see [Allori forthcoming 2].

In addition, as we have seen, there is an important connection between the PO of a theory and its symmetry properties: the symmetry properties of the theory will presumably change when changing the PO, thus requiring a theory to have a particular symmetry will put constraints on the choice of its PO. For instance, it has been (controversially) shown that CED_particles&fields lacks important symmetry properties. If so, it may be a reason to reject that PO for CED in favor of an ontology of particles alone in which the symmetries are restored\(^27\). In the quantum framework, the situation is similar. In fact, it turns out that if one takes the wave function as representing the PO of a quantum theory, then the theory loses important symmetry properties, like Galilei invariance. In fact, in order for the evolution to be Galilei invariant, one would need the wave function to transform in a particular way through the multiplication of a suitable exponential. However, if one regards the wave function as a primitive variable, it seems natural to consider it as a scalar field (on configuration space). As such, it would transform in a very different way, hence making the theory non-Galilei invariant. In contrast, if one assumes that the wave function is not primitive variable, then it will be more appropriate to consider the wave function as a ray (or direction) in Hilbert space. In this way, the theory will then gain back its symmetries, provided that the PO is chosen adequately\(^28\). A proponent of the wave function ontology could bite the bullet and insist that quantum theory is, contrarily to what is commonly believed, not Galilei invariant, but this is implausible since it can be shown it will provide some wrong results in the classical limit [Allori 2007].

Moreover, it is important to stress that the notion of PO is helpful in building new theories. For example, because of the connection between the PO and symmetry properties of the theory, choosing one PO instead of another might make it more or less difficult to build a relativistic invariant quantum theory. [Tumulka 2006] has shown

\[^{27}\text{See [Albert 2002] for the original argument that CED_particles&fields is not time reversal invariant, and [Allori forthcoming 2] for an assessment of the situation.}\]

\[^{28}\text{For more on this, see [Allori ms].}\]
that the GRW theory with a flashes PO can be modified so that it becomes a relativistic quantum theory. Similar results have been obtained also by [Dowker and Henson 2002] for a relativistic collapse theory on the lattice with a PO of lattice locations (see also [Dowker and Herbauts 2004, 2006]). Other proposals for a relativistic quantum theory have been put forward. In a relativistic version of Bohmian mechanics [DMGZ 1998], there is an additional physical object that is fundamental, that is the foliation. Such a foliation divides space-time into space-like hypersurfaces, defines absolute simultaneity and temporal ordering of space-like separated points. If we consider this foliation being part of the PO of this theory then we are exactly in the same scheme as above and one can also analyze the hypothesis of the foliation evolving itself in time.

14. Summary

In the PO approach one needs primitive variables, something that in the theory has the role of representing matter. This PO is in three-dimensional space (or four-dimensional space-time) and it is microscopic. In this way, the PO constitute the building blocks of everything else, and grounds the explanatory schema with which the fundamental physical theory accounts for the world around us. In addition, there are other nomological entities, which in the theory have the role of dynamically implement the temporal evolution of the PO. In past fundamental physical theories such as classical mechanics and classical electrodynamics, a three-dimensional microscopic PO has been implicitly assumed. Using the PO approach in the framework of classical electrodynamics has allowed to see how different variables, in this case positions and fields, have actually different role in the theory: the former has an ontological role, while the latter exhibits a nomological behavior. Nevertheless, in quantum mechanics the idea of grounding the ontology of matter into some three-dimensional microscopic entity seems to fall apart: the wave function is not in three-dimensional space, it shows macroscopic superpositions, and thus it is not obvious at all whether it can represent matter. One point of the PO approach is that there is no necessity of abandoning the explanatory schema developed in the classical picture when dealing with quantum mechanics: this requires postulating a PO, either in addition to the wave function (like for instance in the case of Bohmian mechanics, but also as in the case of GRWp3 or Sip), or defined in terms of the wave function (like in the case of GRWf, GRWm but also Sf and Sm). This provides a complication when compared to the traditional picture in the fact that one adds something. Nonetheless, this complication translates into the simplification of not having to come up with a completely different understanding of what it means for a fundamental physical theory to account for macroscopic phenomena. In this way, one can see that the ‘paradigm shift’ that many have advertised has happened moving from classical to quantum theories is not as radical as one might have thought: arguably, the entities and the explanatory schema has remained the same [Allori 2015].
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References


