How to Make Sense of Quantum Mechanics (and More): Fundamental Physical Theories and Primitive Ontology

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Abstract

Quantum mechanics has always been regarded as, at best, puzzling, if not contradictory. The aim of the paper is to explore a particular approach to fundamental physical theories, the one based on the notion of primitive ontology. This approach, when applied to quantum mechanics, makes it a paradox-free theory.

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1. Introduction

There is a basic philosophical question that involves physics, metaphysics, and epistemology: can we describe what the world is like through a fundamental physical theory? This question corresponds to the historic disagreement between scientific realists and antirealists. The position of the antirealist is the one according to which we should not believe that physics reveals us something about reality but rather we should be content with physics to be, for example, just empirically adequate. In contrast, the realist is strongly inclined to say not only that physics tells us about reality, but perhaps also that it is our only way to do metaphysics properly. Quantum mechanics has always been taken to be devastating for the realist program: it is considered a paradoxical theory, full of contradictions and mysteries, and therefore not suitable to describe physical reality. In this paper, I wish to explore a particular approach to fundamental physical theories according to which quantum mechanics comes out paradox-free, and thus the scientific realist will be able to use it just like any other theory to complete her program of reading off the metaphysics from the physics. This approach is based on a particular notion, the one of primitive ontology (PO). This has been sketched in various articles[2]. Roughly, the PO is what in a physical theory represents matter, while the other variables in the theory help to implement the law of temporal evolution of the PO. In this paper, I wish to discuss and analyze the various ingredients of the PO approach and see how, when applied to quantum mechanics, it turns it into a perfectly 'normal' theory.

Here is the outline: in the next section, I discuss the different roles and meaning of the various variables present in a fundamental physical theory. In doing that, I introduce the notion of primitive ontology and of physical equivalence: theories with the same histories of the PO should be considered physically equivalent, thus as different

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[2] See in particular [DGZ 1992, 1995, 1997], [Goldstein 2008], [AGTZ 2008, 2011,2014], [Allori 2013a,b].

formulation of the same theory. In Section 3, I present the measurement problem of quantum mechanics and its standard solutions: Bohmian mechanics, the GRW theory, and the many-worlds theory. Then I argue that the real problem of quantum mechanics is not, as the measurement problem suggests, the existence of macroscopic superpositions, but rather that the wave function is taken to represent physical objects. In fact, the wave function is not defined in space-time, and its role in the theory is different from the one of the PO. In Section 6, after having discussed what it means for a theory to be empirically adequate, I present possible empirically adequate quantum theories in which the wave function is not the PO. These theories are obtained varying the type of PO, its evolution, and the evolution of the wave function. In Section 7, I move to the issue of the meaning of the wave function in these theories, which in this approach is often taken to be similar to a law of nature. In the last two sections, I compare the various theories, showing how some of them display a many-world character, and discussing how the notion of symmetry properties is connected with the PO. In addition, I show how one could possibly classify these theories: in some of them, the PO is independent of the wave function, while in others the PO is a function of it.

2. Mathematical Representation of Physical Objects in a Fundamental Physical Theory

Granted that physics is able to provide information about the world, any fundamental physical theory should clearly specify what exists in the world according to the theory. To put it differently, since any fundamental physical theory is written in terms of mathematical entities that obey given laws of motion, the theory should clearly specify what connection there is between the entities in the mathematics and the entities in the world. In fact, if we do not give any rule of correspondence between the objects of mathematics and the one in the world, we cannot say we have a physical theory, but just a collection of mathematical statements.

Usually, the scientist that proposes a given fundamental physical theory has already a metaphysical hypothesis in mind, namely she already has an idea about what are the fundamental objects there are in the world. For instance, as Newton did, she may think that the world is made of particles. Then, when she puts the theory in mathematical form, she uses the mathematical object that naturally corresponds to her metaphysical hypothesis. Points in $\mathbb{R}^3$ naturally represent particles' positions, vector or scalar--valued functions defined in $\mathbb{R}^3$ naturally represent fields, and so on.

According to classical mechanics, the world is made of particles, whose position is naturally mathematically represented by points in three--dimensional space. Suppose instead that one did not specify any connection between the mathematics and the world. Then it is hard to see what the theory describes and predicts. In fact, take

$\frac{mM}{r^2} + qE(r) = m\frac{d^2r}{dt^2}$: what is this equation describing? One cannot say that it represents the equation of motion of a massive particle because there is no specification of what the different symbols mean. In order for that equation to be more than a string of meaningless symbols, one would need to specify that there are actually particles in the world, whose position in space is represented by a vector r in $\mathbb{R}^3$. We also need to specify that the particle under consideration has a mass, represented mathematically by the natural number m, it a has charge, represented by the natural numbers q, and it evolves in time t according to that particular equation, which involves the constant G, and the electric field, represented by the scalar function E. Only at that point we have a physical theory, otherwise we have simply some nice mathematics.

2.1. Theory Architecture, Primitive Ontology and Local Beables

However, not all variables in a theory are on the same footing. Considering the example above, the particles may have, say, masses, not the other way round. What would it mean to say that a mass has a position? Consider velocities: what would it mean to say that a velocity has a position? In this last case, in particular, the velocity is defined in terms of position, so it seems even more natural to think of velocity as on a different level than position. Therefore, first, we postulate that there are particles, completely specified by their positions, mathematically represented by points in three-dimensional space; then we specify what properties they have. In this sense, the particles are the *primitive ontology*, PO, of classical mechanics. Thus, some mathematical objects in the theory, in this case the three-dimensional points representing particles, which are privileged over the others. The variables in the theory have a hierarchical structure, determined by the role they play in the theory: on the foundation one finds the variable that represent matter, the primitive variable, which in the example above is r, and we have seen are of crucial importance for the theory. Then we have many other variables: some are constants of nature, like G in the example above; some others may be taken as describing properties of the PO, like m or q above, which are also constant but may vary depending on the particle under consideration; some are used to parametrize the trajectory of the PO, as t in the equation above. None of these variables is the PO; particles are. In other words, none of them represents matter: table and chairs are not made of charges, say, they are made of particles with charges.

Finally, we have other variables, like the one representing the electric field E above, mathematically represented by a vector field in three-dimensional space. These 'local beables,' to use the terminology introduced in [Bell 1987], may or may not be taken to be part of the PO. In fact, on the one hand, one may think that they compose tables and chairs, and thus they are part of the PO, together with particles. That would mean, though, that there are fundamentally two kinds of 'substances' in the world: particles, and electric fields. On the other hand, one could notice that the fields are introduced to account for the motion of the particles: they help implementing the motion of the particles. So their role in the theory does not seem the same as the role of the particles, they are non-primitive in this respect. Because of their role in generating the law of evolution of the PO, it seems more appropriate to consider them non-primitive, nomological, variables. Roughly speaking, the PO tells us *what there is*, and the non-primitive variables tell the PO *how to move*. In other words, the reason why there are other variables in the theory other than the primitive ones is to 'close the dynamics' for the PO.

2.2. Macroscopic Properties and Primitive Ontology

As a result of taking (microscopic) particles as its PO, classical mechanics is arguably able to account for each and every (macroscopic) property of physical objects. As long as we can neglect quantum effects, macroscopic solid bodies, gases, fluids (i.e. tables, water, air…) and their properties (the solidity of the table, the transparency of the water, and the temperature of air) can be accounted for in terms of the motion of point-like particles moving in three-dimensional space. More precisely, the behavior of extended rigid bodies can be accounted for only considering the motion of their center of mass under the action of definite forces, after an analysis in terms of them as made of interacting point--particles. Similarly, the behavior of gases and fluids is accounted for considering them as composed by many non-interacting identical particles that collide with one another. Indeed, this is how thermodynamics can be derived from statistical mechanics: what in thermodynamics we call pressure, volume, temperature of a gas are derived in the framework of statistical mechanics from the fact that gases are supposed to be composed of small moving particles.

It is crucial to notice that the ingredients that are necessary for this explanatory schema to succeed are fundamentally two: 1) the PO is *microscopic*; 2) the PO is in *space-time*. Without the first requirement, the objects in the PO will not be the building blocks of every other physical object. This is important because the particles clump together in forming bigger objects, which arguably then behave more or less independently on their initial composition. For instance, protons, neutrons and electrons bind together to form atoms, and they behave according to the laws of chemistry; then atoms bind together to form more complicated molecules that obey the laws of biology, and so on. It is this hierarchy of objects, with at its foundations the fundamental particles, is what allows the explanation of the macroscopic properties in term of the microscopic constituents. This is where the second requirements of spatio-temporality of the PO comes in, since it works only if the fundamental building blocks live in the same space as the macroscopic properties that are explained, namely three-dimensional space, or four-dimensional space-time. If the fundamental building blocks of nature live in a different space, as we will see have been suggested, then there is an additional step to be made, namely to explain how we think we live in three-dimensional space while we actually do not (see Section 4).

2.3. Nomological Variables and Physical Equivalence

In order to clarify the difference between the PO and the nomological variables, we could use the following metaphor [AGTZ 2008]. Suppose we want to write a computer program for simulating a system according to a certain theory. In writing the program, we care about the output: the program is able to generate it through other variables, but they are just of an instrumental value to us. The PO is the output of such program, while the other variables serve as *means* for generating this output: they are internal variables of the program: they may be necessary for doing the computation, but they are not what the user is interested in. We seem to understand classical theories in this way: we want to know what the trajectories of the particles, the PO, are. In contrast, the other variables have just the role of implementing the evolution for the output. This has a very important consequence: there might be different ways of producing the same output, using different internal variables. For example, consider Lagrangian mechanics: it is a reformulation of classical mechanics that many physics students use all the time to solve problems. One rewrites Newton's equation using generalized coordinates and a particular function L, the Lagrangian. In practice, it is easier to find the trajectories using this formulation rather than the usual one, if the generalized coordinates are chosen accurately in such a way they can exploit the symmetries in the system. The Lagrangian formulation is therefore an example of how we can obtain the same output, the trajectories, in a way that is different from solving Newton's law of motion. To consider another example, in classical electrodynamics two fields that differ by a gauge transformation generate the same law for the PO. As we do not regard an electromagnetic potential V and another potential that differs from V by a gauge transformation as a different potential than V, we do not regard classical mechanics and Lagrangian mechanics as different theories: they provide us with the same result for the trajectories of the PO. Therefore, we conclude that in general two theories should be regarded as *physically equivalent* when they lead to the same history of the PO. They are, in a very important sense, the same theory.

3. The Measurement Problem and its Standard Solutions

In contrast with classical theories, it is not clear what the underlying metaphysical assumption is in orthodox quantum mechanics, namely the theory that one finds in physics textbook. What is matter made of, according to the theory? The typical answer is usually that it is impossible to describe the microscopic reality if quantum mechanics is true. Thus, we need to become anti-realist, we have no choice. Accordingly, some have responded that, at best, quantum theory can be a theory that describes the measurement results. It is beyond the scope of this paper to discuss the sociological and historical reasons for this claim; let me just say that the so-called measurement problem, or the problem of the Schrödinger cat, played a crucial role, together with certain readings of the Heisenberg uncertainty principle and the various no-go theorems.

Luckily, we now know better than that: it is not true that we have no other option. In fact, there are at least three counterexamples to this impossibility, namely Bohmian mechanics [Bohm 1952], the GRW theory [GRW1986] and the many-worlds theory [Everett 1957], three quantum theories that a scientific realist can use in her program of doing metaphysics through physics.

These theories have been proposed as a response to the problem of the Schrödinger cat. The problem in a nutshell is as follows [Schrödinger 1936]. The fundamental object of orthodox quantum mechanics is the wave function ψ, that evolves according the Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t}=-\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}\nabla_i^2\psi+V\psi, \tag{1}$$

where is $\hbar$ Planck's constant. It is a mathematical feature of this theory that sums of solutions ψ_i, i.e. $\psi=\sum_i a_i\psi_i$, are also solutions of the equation. That implies that if we allow the wave function to completely describe every microscopic or macroscopic physical system, then these 'superpositions,' initially on the microscopic level, will eventually end up describing macroscopic objects like a dead and a living (i.e. non-dead) cat. Since this is contradictory, one concludes that there has to be something wrong in the theory. Since there seems that there are only three assumptions:

a) The wave function provides the complete description of every physical system;
b) The wave function evolves according to the Schrödinger equation;
c) Macroscopic objects have non-contradictory properties;

then one could solve the problem rejecting one of them [Bell 1987]. Bohmian mechanics is taken to reject the first, the GRW theory to reject the second, and the many-worlds theory to reject the third assumption.

3.1. Bohmian Mechanics

The traditional story says that Bohmian mechanics solves the problem of the Schrödinger cat adding particles to the description of the wave function [Bohm 1952], [Bell 1987], [DGZ 1992]. There are two equations that define Bohmian mechanics; Bohm's equation[3]:

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[3] This equation, for simplicity, does not account for spin. For the more general equation, see [DGZ 1992]. I use the standard notations of probability theory, according to which a capital

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\left[\frac{\nabla_i\psi}{\psi}(Q_1, \dots, Q_N)\right], \tag{2}$$

and the Schrödinger equation (1). Thus, in addition to the wave function, in the theory there are also particles, with configurations Q_i in three-dimensional space that evolve according to (2). Thus, the cat is either dead or alive, depending on where her particles are: if they are 'under' the wave function representing a dead cat, the cat is dead; she is alive otherwise.

3.2. GRW Theory

The GRW theory, or spontaneous collapse theory, was proposed by [GRW 1986], and it is taken to solve the problem of the Schrödinger cat denying the second premise. In this theory, the wave function evolves according to a stochastically modified Schrödinger equation. The wave function evolves according to the Schrödinger equation until a random time T_1 , chosen with exponential probability distribution with rate $N\lambda$. Here $\lambda = 10^{-15}$ s-1 is a constant of nature of the theory, and N is the number of 'particles' in the theory[4]. At time T_1, the wave function undergoes an instantaneous collapse: it localizes around a random point X. The collapse is mathematically implemented by Gaussian operator[5] $\Lambda(x) = \frac{1}{(2\pi\sigma\}^{3/2}} e^{-\frac{(\hat{Q}-x)^2}{2\sigma^2}}$, whose width $\sigma = 10^{-7}$m is a new constant of nature ($\hat{Q}$ is the position operator). When the wave function is ψ, the rate of collapses with center x is given by :

$$r(x|\psi) = \langle\psi|\Lambda(x)\psi\rangle, \tag{3}$$

The rate of collapse depends on N, the number of particles, which thus can be taken as a measure of how big the object under consideration is. Because of that, macroscopic objects, like cats, very quickly collapse either into the state that describe a dead cat or into the one describing the cat who's still alive.

3.3. Many-Worlds Theory

The third standard solution of the Schrödinger cat problem is the so-called Everett interpretation, also known as many-worlds theory [Everett 1957], [Wallace 2002]. This theory does not modify the law of evolution for the wave function, and does not add anything to it. Rather, it is usually characterized as accepting that physical objects possess contradictory properties. The basic idea is that superposition states do not really

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letter is used to denote a random variable, while the values taken by it are denoted by small letters.

[4] I am using the world 'particle' in a loose way to preserve the usual language, given that, strictly speaking, there are no particles in this theory.

[5] I am currently ignoring particle labelling, to simplify the notation. A correct equation can be found, e.g., in [AGTZ 2008].

describe a single object, like a cat, having contradictory properties, like being dead and not dead at the same time. In contrast, they describe states of the world in which there is more than one thing. That is, there are two cats, one dead and one alive, and they are in different 'worlds,' different regions of space-time. We do not see both cats because the universe is bigger than we think; we live in one world, say the one with a living cat, and the others are such that they do not interact with us, and this is why we could not detect them.

4. The Wave Function and Configuration Space

Let us now take a step back. The usual story is that the problem of quantum mechanics is the presence of macroscopic superpositions, and the different solutions either try to get rid of them (Bohm, GRW) or they embrace them (many-worlds). According to the proponents of the PO approach, instead, the problem of quantum mechanics is not that there are macroscopic superpositions. Rather, *the problem is the assumption that the wave function represents physical object*. All the theories we have just seen, as stated, take the wave function as the object that represent matter. The PO approach instead proposes that this is a mistake: one always should deny (a), even in a stronger way than Bohmian mechanics does. That is, one always should add something to the description provided by the wave function [AGTZ 2008]. Or better: one should always drop the wave function, and have something in space-time to represent matter. If not, we run into undesired and unnecessary difficulties.

The wave function is not the right kind of mathematical object that is suitable to naturally describe material objects living in three-dimensional space evolving in time. In fact, the wave function $\psi = \psi(r_1, \dots, r_N)$ is a mathematical object that lives in a very abstract space: the space of all $r_1, \dots, r_N$. In classical mechanics, where these variables represent the positions of *N* particles, this space has been called *configuration space*, for obvious reasons. In a theory with just the wave function, though, strictly speaking there are no particles, but the space kept its name nonetheless. The dimension of such space is *M=3N*. If the wave function is taken to represent physical objects, then physical space, the space in which physical objects live, would be configuration space. If so, one would have to explain why *we think* we live in a three-dimensional world, while we do not. That is, one would have to come up with a story to derive $\mathbb{R}^3$ from configuration space. One has to add to the specification of the theory some rule in order to do so, and this is what has been done in [Albert 1996]. Whether the proposed maps are successful is up for debate [Monton 2002, 2006]. In addition, it seems that this position provides a picture of the world that is too revisionary to be easily accepted [Allori 2013a,b]. Since all there is in this theory is the wave function, the statement "there is a table here" cannot mean literally that there is a table here, since there exists no table, as indeed there exists no three-dimensional object at all. Objects do not exist independently to one another, they are all ‘mashed into’ the description provided by the wave function. And the information regarding each one of them has to be extracted from it with the use of something more than the wave function alone. At best, what one might mean is that the wave function is concentrated in a certain location, the one at which the table is. This implies that tables, chairs, humans and all three-dimensional objects do not exist at all in the way we usually think about them. In this framework, the nice explanatory schema that was developed in classical theories to derive the macroscopic properties in terms of the microscopic constituents has to be dropped, and an entirely new reductionist account has to be developed from scratch. Thus, it seems that this can hardly be regarded as a solution for the problem of making sense of quantum mechanics, especially in presence of much more sensible alternatives (as we will see in the following). Since one does not gain any new understanding in entertaining the idea that we live in configuration space and that we are made of wave functions, but rather we only inherit new problems to solve, then there seem to be no reason why we should take this position[6].

Accordingly, the proponents of the PO approach reject that the wave function can represent matter: for the reasons we just saw, the PO has to be in three-dimensional space, or four-dimensional space-time. Consequently, theories that solve the problem of the Schrödinger cat will be satisfactory not because they deny one of the three assumptions we discussed, but rather because each of them postulates the existence of something different from the wave function to describe physical objects. According to the PO approach, quantum theories, like any other fundamental physical theory, possess a definite architecture in which the various variables play different roles: the primitive variables describe matter, the nomological variables, namely the wave function, spell out the law of motion for the PO. The GRW theory, thus, is never ‘bare,’ it is never a theory about the wave function. Rather, different ‘GRW theories’ have been

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[6] One might think that, given the above conclusion, we should also reject string theory. In fact in string theory physical space is supposed to have 10 (or 11) dimensions; if the complaint against the wave function ontology is that the dimensionality of configuration space is M, where M is not 3, then M=10 (or 11) seems just an instance of the type of theories we should reject. However, there is an important disanalogy: in string theory, the starting point is that the number of dimensions of physical space is greater than three; however, it is assumed that all dimensions except three are ‘compactified,’ curled up on themselves, and thus inaccessible to physical objects. Scientists are still looking for mechanisms that would explain why it is the case. Indeed, string theorists hope to find a unique way of compactifying the redundant dimensions, that is, a unique string theory. Thus, in string theory the extra dimensions are added but they are promptly compactified, in order to keep the world *objectively* always like $\mathbb{R}^3$, so that there is no actual explanatory gap between physical and apparent dimensions. In the case of configuration space, instead, the extra dimensions are equally accessible to matter, and one needs a new mechanism to ‘get back’ to the usual three.

proposed: GRWm, a theory with a three-dimensional field PO, to be identified with the matter density of stuff [BGG 1995], and GRWf, a theory with a PO of space-time events [Bell 1987]. Accordingly, the many-worlds theory is not a theory about the wave function also. We will see these theories in a little more detail in Section 6.

5. Quantum Theory Construction Kit

According to the PO approach, then, all the theories above should be reconsidered: they all have a spatiotemporal PO, and the wave function, just like the electromagnetic fields in classical electrodynamics, has the role of generating the space-time 'trajectories' of the PO. Bohmian mechanics can be naturally understood as a particle PO, while the situation is trickier in GRW theory and many-worlds: there can be different, more or less natural, PO for these theories, as we will see in Section 6.

Indeed, there can be many more theories than those we saw in the previous section. In fact, the PO approach provides us with a set of rules for generating quantum theories:

- Make a metaphysical assumption: select a spatiotemporal PO;
- Select a law of evolution for the PO in terms of some appropriate mathematical object (usually the wave function);
- Select a law of evolution for this object.

This game has been played in [AGTZ 2008, 2011, 2014], in which a variety of theories have been proposed and then analyzed.

5.1. Empirical Adequacy and Effective Empirical Equivalence

A first constraint on a theory is empirical adequacy. In other words, we are free to choose the metaphysical hypothesis (i.e. the PO), and then we are free to play with its evolution and the other objects in it (including the wave function), but we have to make it the case that the theory does not get the empirical predictions wrong.

More to the point, a theory is empirically adequate if it is able to account for the macroscopic appearances. Since orthodox quantum mechanics is empirically adequate in this sense, all it takes is that the theory under consideration is empirically equivalent (or effectively so) to orthodox quantum mechanics: two worlds, governed by the two theories, share the same macroscopic appearance. That is, a theory may be (exactly) empirically equivalent to orthodox quantum mechanics, in the sense that there is no possible experiment that can in principle distinguish between the two. Other theories instead may be effectively empirically equivalent: their predictions are close enough to the ones of orthodox quantum theory so that no discrepancy has been detected so far, even if it is in principle possible to do so. These theories would still recover the macroscopic appearances well enough to be empirically adequate, and in this case, one may speak of 'effective empirical equivalence' with orthodox quantum theory.

Thus, for effective empirical equivalence with orthodox quantum mechanics all we need to ask is that the probability of getting, say, z as an experiment result, agrees with the distribution predicted by orthodox quantum mechanics. To get z as an experiment result is to observe a pointer to point to z, and the orthodox quantum distributions for z can be obtained from the Schrödinger wave function ψ_t for a sufficiently big system containing the pointer by integrating $|\psi_t|^2$ over all configurations in which the pointer points to z. When this is so, we may speak of an 'effective $|\psi_t|^2$–distribution,' or of macroscopic $|\psi_t|^2$ Schrödinger equivariance [AGTZ 2008]. If a theory is proven to have such property, then it is empirically adequate.

Thus we will discard the theories that are not empirically adequate, and then we will use some superempirical virtues to select the most sensible theory(ies) (see section 7).

6. **Possible Quantum Metaphysics**

In this section, we will see some examples of how different POs can be combined with different evolutions, and different nomological variables. We will consider three types[7] of PO: particles, matter density field, flashes.

6.1. Particles

Let us start simple: let us imagine matter is made of (dimensionless) particles. Thus, they are naturally represented as points in three-dimensional space. This is our starting metaphysical assumption. Then we can construct several empirically adequate theories:

A. Bohmian Mechanics: Actually, we do not even have to build a particle quantum theory for scratch because we already have one that would naturally fit the bill: Bohmian mechanics. In fact, this theory can be read naturally as a theory of particles that move in three dimensions according to (2). That equation involves the wave function, which in turns evolves according to the Schrodinger equation (1). The role of the wave function in the theory is not to represent matter, but rather to tell matter how to move: it is the nomological variable. Bohmian mechanics is (exactly) empirically equivalent to orthodox quantum mechanics, and thus it is an empirically adequate theory.

B. Sip: Another possibility of a quantum theory with a particle PO that has been considered in the literature [Bell 1987] is what has been later dubbed "Sip" in [AGTZ 2008][8]. In this theory, the wave function evolves according to Schrödinger's equation, and the PO is given by instantaneous randomly distributed configurations without any temporal correlation among them.

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[7] Clearly, other choices, such as strings, are possible, but they have not been considered here.

[8] 'S' from Schrödinger evolution, 'i' for independent, 'p' for particles.

C. <u>GRWp3:</u> One can build other theories of particles, playing with their evolution and the one of the wave function. For instance, a theory that combines a particle PO and a GRW-like evolving wave function in an empirically adequate way has been dubbed “GRWp3” in [AGTZ 2014] and was first proposed in [Bedingham 2011]. In this theory, the wave function, as usual in this approach, has the role of implementing the law of evolution for the PO, and in this theory, it is fairly complicated. In fact, the particles evolve according to Bohm’s equation (2) with a given wave function until a random time *T*; from that time onward, they will still evolve according to Bohm’s law but with another wave function, namely one that has undergone a collapse. In fact, at random times *T* the wave function collapses into the actual position of the particle at *T*, but ‘displaced’ at random [Tumulka 2011][9].

D. <u>GRWp6:</u> Another empirically adequate theory, dubbed “GRWp6” in [AGTZ 2014], is such that the particles are the PO and the wave function evolves according to the GRW evolution. The particles evolve according to Bohm’s law (2) between the collapses, like in GRWp3. However, at the collapse center, then, all the particles jump at random.

E. <u>MBM:</u> In the PO approach, the wave function, since it does not represent matter, can in principle be dispensed with, or replaced by another mathematical object. Thus, [AGTZ 2014] propose, as a toy theory to illustrate how one can play this game, what they called “MBM,” master equation Bohmian mechanics, in which the wave function is completely absent. In this theory, the PO is of particles, which evolve to something similar to Bohm’s equation (2): instead of the wave function, though, we have what in quantum mechanics everybody calls a density matrix ϱ. However, here the density matrix, which evolves according to the Limblad equation, is taken to be an object not parasitic of the wave function. It turns out that this theory is not strictly speaking empirically equivalent to Bohmian mechanics, but its predictions slightly differ in a way that is currently undetectable and thus it is empirically adequate[10].

6.2. Matter Field

Alternatively, we could think of matter being continuous instead of discrete. Thus, our metaphysical hypothesis is that matter is represented by a field in three-dimensional space:

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[9] Simpler theories like GRWp2, a theory in which the particles would evolve according to Bohm’s law and the wave function would simply collapse randomly like in the original GRW law; and GRWp4, a theory in which the particles would evolve according to Bohm’s law and the wave function would simply collapse into the particle’s position without any displacement, would not be empirically adequate. See [AGTZ 2014] for more details.

[10] This theory is empirically equivalent to GRWm and GRWf, that we will describe later.

$$m(x,t)=\sum_{i=1}^{N} m_i \int dq_1 \dots dq_N \delta^3(q_i-x)||\psi_t(q_1 \dots q_N,t)|^2, \tag{4}$$

where N is the number of 'particles,' and m_i their masses. Given this PO, one can have several empirically adequate theories.

A. Sm: Schrödinger naturally tried to see whether an ontology based on the wave function was possible: he had his famous equation at the very core of quantum theory, and the wave function was the object this equation was for. He did not propose the wave function to represent matter, but rather that something like (4) should be taken as representing matter, where the wave function was evolving according to the Schrödinger equation. However, this field would inherit the same superpositions of the wave function, and thus he dismissed this possibility. As we saw, superpositions are not the problem, so his rejection was quite too hasty: this theory has a viable PO (see Section 7.1 for what may be regarded as a drawback of this theory). This theory has thus been dubbed "Sm" in [AGTZ 2008].

B. GRWm: A theory with a matter density PO defined by (4) and a GRW-evolving wave function has been proposed in [BBG 1995] and called "GRWm" in [AGTZ 2008]. This theory's predictions differ from the ones of orthodox quantum theory, but they are currently undetectable. Thus, the theory is empirically adequate, and can be proven to be empirically equivalent to MBM [AGTZ 2014].

C. Mm: In analogy with MBM, [AGTZ 2014] propose a matter density theory whose evolution is implemented by a Limblad-evolving density matrix rather than a wave function. It can be proven that this theory is empirically equivalent to GRWm (and thus to MBM).

6.3. Flashes

Last but not least, one could imagine the PO not to be in space, evolving in time, but to be directly located in space-time. In a theory with a particles PO, one looks at the particles' trajectories in space-time. Configurations of the same particle at different times are continuously connected by the trajectory, but this is not necessary. For instance, in Sip one would have configurations jumping from one moment to the next without any connection. Alternatively, one could forget about particles, and simply say that there are certain points in space-time, that one can call 'flashes,' that are non-empty, so to speak, and they represent what matter is made of. Thus, the flashes will be the set $F=\{(X_1,T_1),\dots,(X_k,T_k),\dots\}$, k being a progressive natural number that indicate the time progression of the flashes. It is an unusual but possible picture: in this theory, matter is neither made of particles with world-lines, such as in classical or Bohmian mechanics, nor of a continuous distribution of matter such as in GRWm, but rather of discrete events in space-time, in fact finitely many events in every finite-size space-time.

Given this PO, one can again play with the different possible combinations to obtain various empirically adequate alternatives.

A. <u>Sf:</u> [AGTZ 2008] propose that a flash PO could be combined with a Schrödinger-evolving wave function, and they called this theory "Sf." The space-time locations of the flashes are generated by (3) as in the GRW mechanism, but the wave function never collapses.

B. <u>GRWf:</u> Another theory of flashes was proposed in [Bell 1987] and then adopted in [Tumulka 2006]. This theory was called "GRWf in [AGTZ 2008] because the wave function collapses according to the GRW evolution, every flash corresponds to one of the spontaneous collapses of the wave function, and its space-time location is just the space-time location of that collapse given by (3). The predictions of this theory are in accordance with the one of GRWm (and thus with MBM and Mm). Therefore, the theory is effectively empirically adequate.

C. <u>Mf:</u> Also in analogy with MBM and Mm, we have Mf, a theory of flashes in which the rate of the flashes is not generated by the wave function but by a Limblad density matrix [AGTZ 2014]. This theory turns out to be empirically equivalent to MBM, Mm, and, more importantly, to GRWm, making the theory effectively empirically adequate.

A note about all these theories: if the number *N* of 'particles' is large, as in the case of a macroscopic object, the number of flashes is large, too (if $\lambda=10^{-15}$ s-1 and $N=10^{23}$, we obtain $R=10^{8}$ flashes/second). Therefore, for a reasonable choice of the parameters of the theory, a cubic centimeter of solid matter contains more than 10^{8} flashes per second. That is to say, large numbers of flashes can form macroscopic shapes, such as tables and chairs. At almost every time, however, space is in fact empty, containing no flashes and thus no matter.

7. The Meaning of the Wave Function

Before evaluating these theories, let us focus on the wave function. If the PO of the theory are the building blocks of physical world, they are the stuff in three-dimensional space physical objects are made of, while the wave function is not, what exactly is the wave function? If there is a category one can safely put the wave function in, then arguably it is more like a law of nature than like anything else[11]. In other words, the wave function is more suitable to represent a law of nature than a physical object [DGZ

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[11] Alternatively, some have argued that the wave function should be understood as a dispositional property. See [ELHD 2014] and [Monton 2006] for details.

1997], [GT 2000], [GZ 2013][12]. The idea is that the wave function is similar to the Hamiltonian in classical mechanics: it is the generator of motion.

Several objections have been raised against this view (see [BW 2005], [Belot 2012]). First, since the PO represents what physical objects are made of, while the wave function does not, either one denies the existence of the wave function or has to admit that something is more real than something else is. However, saying that the wave function is real but not physical does not imply there are different degrees of reality: in fact, they might be two kinds of substances, or entities. After all, the very same objections could be raised (but they are not) to a Platonist in the philosophy of mathematics, a dualist in the philosophy of mind, and a realist with respect to laws in ethics or in philosophy of science. Other objections focus on the disanalogies between the wave function and the general conception of laws. For instance, it is argued that the wave function cannot be regarded as a law because it interacts with the particles and thus seems to be more alike matter than laws. One could respond saying that the wave function is similar to the potential in classical mechanics in this respect: the potential interacts with the particles but no one considers it real. Also, it has been argued that the wave function evolves in time, while laws are static. In this case, one could just not be bothered by it [Smolin 2013]. In any case, since the idea that laws of nature are static is a classical intuition, one could maintain that instead of trying to force our classical intuitions onto quantum mechanics, we should realize that quantum mechanics is telling us something new about laws of nature [Callender forthcoming]. Be that as it may, one could notice that there is evidence suggesting that in a future quantum cosmology the wave function would be static [GT 2000], eliminating the problem. Another objection is that the wave function is contingent, in the sense that varies with the subsystem, and in contrast, laws are universal. A last, similar, complaint could be that the wave function is controllable: we can prepare physical systems in the state that we want. If so, it is difficult to regard the wave function as a law, since we do not seem to have control over them. These last two objections can be taken care of remembering that the wave function we can have control of, and that changes from system to system, is the wave function of the system (the conditional or effective wave function [GZ 2013]), while the one that should be intended as nomological is the wave function of the universe (which is universal and we cannot control).

8. **Primitive Ontology and Theory Evaluation**

All the quantum theories presented in Section 6 have different POs, therefore they cannot be all true. Because they are either exactly or effectively empirically equivalent

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[12] [Callender forthcoming] has recently motivated the view in the Humean framework of laws of nature.

to orthodox quantum mechanics, one cannot use experiments to settle the dispute over which theory is most likely to be true. The issue will have to be settled based on something else. Let us explore some feature of some of these theories here, before arriving to an evaluation.

8.1. Primitive Ontology and Many-Worlds

Because of linearity, in any theory in which we have a wave function evolving according to Schrödinger's equation there are superpositions. Thus, the wave function of the universe must presumably be thought of as consisting of several packets that are very far apart in configuration space that correspond to unrealized states of affairs: in the Schrödinger cat example, if when we open the box the cat is alive, the state of affairs corresponding to the dead cat are not realized. Some of the packets will have support in events that did not take place in our time, such as for example the dinosaurs have never become extinct.

In Bohmian mechanics, this is not a problem: since configurations are continuously connected in time, it is not possible for the configuration to jump, in an instant, from the support of one wave packet to a macroscopically distinct one. However, in the other "S" theories the situation is trickier. In Sip there is no connection whatsoever between what there is at a given instant of time and what there is at the previous or following instant. In this case, thus, the configuration will very probably visit in every second those distant regions supporting the other packets: therefore, at time *t* there can be dinosaurs and at time *t+dt* they have disappeared. Therefore, many worlds exist, not at the same time, but one after another. Because of this, if this theory is correct, the fact that right now there are memories and records of the past does not guarantee that they are actually reliable. Rather, the records are most likely to be false: at one instant, there is a set of what we would call 'records' that actually do not reflect in any way truthfully what has happened. That means, plainly, that they are not records at all. Still, there is no way to experimentally distinguish that theory from, say, Bohmian mechanics.

Similarly, in Sf and Sm (just like in Mf and Mm), the superpositions of the wave function are inherited by the flashes and the matter density field. By the linearity of the Schrödinger evolution, the flashes and the mass density form independent families of correlated flashes or mass density associated with the terms of the superposition, with no interaction between the families: the live cat and the dead cat do not interact with each other, as they correspond to alternative states of the cat. Thus, they can indeed be regarded as comprising many worlds, superimposed on a single space-time. Since the different worlds do not interact among themselves, they are, so to speak, reciprocally transparent.

Note that the concept of a 'world' is just a practical matter, relevant to comparing the matter density function provided by the theory to our observations. However, this is not a problem: there is no need for a precise definition of 'world,' just as we can get along without a precise definition of 'table.'

8.2. Independent and Dependent Primitive Ontologies

Notice that the matter field m and the flashes F are functionals of the wave function, and they are not, strictly speaking, additional variables, as the positions of particles in Bohmian mechanics. Nonetheless, they are additional assumptions of the theory that needs to be specified in order to have a complete description of the world. There could be many functions, not just the ones specified by equations (3) or (4), that define the matter density or the distribution of the flashes, and one should specify which ones are to be taken as the definitions of m and F.

In philosophical jargon, when there is dependence between two variables, it is said that the dependent variable supervenes on the other. The template for the definition of supervenience is the following: Y supervenes on X if no two possible situations are indiscernible with respect to X while differing in Y. For instance, chemical properties supervene on physical properties insofar as any two possible situations that are physically indistinguishable are chemically indistinguishable. One could notice that the mass density and the distribution of the flashes supervene on the wave function: there cannot be a difference in the mass density or in the distribution of the flashes without a difference in the wave function. As we saw, this is very different to what happens in. e.g. Bohmian mechanics, in which positions are specified independently from the wave function. The mass density and the distribution of the flashes are not specified in addition to the wave function, but rather are determined by it. [Lewis 2006] has argued that there is no need of adding the mass density or the flashes to the wave function in GRW, since they are already present in the wave function. Nevertheless, this unspecified supervenience is not enough to arrive to Lewis' conclusion. An important distinction to see why this is the case is between logical (or conceptual) and natural (or nomic) supervenience. In general, when we have logical supervenience between X and Y, we say that X entails or implicates Y, i.e. Y = f(X). For instance, the description "table" supervenes logically on the configuration of the particles composing the table: the table is just a bunch of particles. A different dependence is given by natural supervenience: "the pressure exerted by one mole of a gas systematically depends on its temperature and volume according to the law $pV = KT$, where K is a constant. [...] it is empirically impossible that two distinct moles of gas could have the same temperature and volume, but different pressure [...] But this supervenience is weaker than logical supervenience. It is logically possible that a mole of gas with a given temperature and volume might have a different pressure; imagine a world in which the gas constant K is larger or smaller, for example. Rather, it is just a fact about nature that there is this correlation" [Chalmers 1996]. Another example of natural supervenience is the one between the charge density and the electromagnetic fields [Maudlin 2007]. The relation between the two is $\varrho = \frac{1}{\varepsilon_0}\nabla \cdot E$, and it is a law of nature. The distinction between logical and natural supervenience can be summarized as follows: if Y supervenes logically on X, then once God has created a world with certain X, the Y comes along for free; if Y supervenes naturally on X, then after making the X, God had to do more work in order to make the Y: he had to make a law relating the X and the Y. Once the law is defined, X will automatically bring along the Y. Nevertheless, one could, in principle, have had a situation where they did not. As we have seen above, for example in Bohmian mechanics the PO does not supervene on the wave function: we need to specify the PO in addition to the wave function. In the case of theories of flashes and mass density (and also in some theories with particles PO, like GRWp6) instead the PO is determined by, supervenes on, the wave function *naturally, and not logically*. That is, it is an additional law of nature.

We have previously seen that the measurement problem is the problem of the inadequacy of the wave function as the PO of quantum mechanics. Therefore, possible solutions of the measurement differ not in the fact that either we add something or we let the wave function evolve to an equation that is different form Schrödinger's evolution. Rather, different solutions are characterized by whether the PO is independent of the wave function or it is defined in terms of it. Therefore, we have:

1) PO independent of ψ: the PO is genuinely an additional variable, chosen independently of the nomological variable (usually the wave function). This is the case of Bohmian mechanics, GRWp3, MBM, Sip.
2) PO functional of ψ: the PO is defined in terms of the nomological variable (usually the wave function) in a particular way. This is the case of theories with flashes and mass density, and of GRWp6.

8.3. Primitive Ontology and Symmetries

As we saw already, because of the view that the various solutions to the Schrödinger cat are ultimately not about wave functions but about histories of a PO in space-time, the law of evolution of the wave function should no longer be regarded as playing the central role in the theory. The wave function is a nomological variable: it helps implementing the law of motion for the PO.

Because of this, the symmetries of a theory are determined by the PO, not by the wave function. Roughly put, to say that a theory has a given symmetry is to say that the possible histories of the PO (those that are allowed by the theory), when transformed according to the symmetry, will again be possible histories for the theory; and the possible probability distributions on the histories, those that are allowed by the theory, when transformed according to the symmetry, will again be possible probability distributions for the theory [AGTZ 2008].

Therefore, the invariance of a quantum theory directly concerns the law for the PO; it concerns the invariance of the law for the wave function only indirectly, contrary to what is often, erroneously, believed. Changing PO could (probably will) change the symmetry properties of the theory. In the literature, there are some examples of this, for instance, GRWf has been modified to make it relativistic invariant [Tumulka 2006], a relativistic extension of Sm has been proposed in [AGTZ 2101], while GRWm still awaits a relativistic generalization.

9. Final Remarks

The different features of the quantum theories analyzed in this paper are summarized in table 1. Let us conclude with few considerations about theory selection.
If you like particles, just go Bohmian. What is the point of complicating the theory with a non-linear evolving wave function, as in GRWp3? Why make the trajectories discontinuous, like in GRWp6? More difficult to justify is why not use the density matrices. Presumably, one would reject it on the basis that it is unnecessarily complicated. Sip, as we saw, has a many-worlds character but also implies that our memories are entirely unreliable, so does not seem to be a reasonable choice. Thus, if you like many-worlds, a PO of particles should not be your choice.

Theory	Primitive Ontology			PO independent of wave function	Many-Worlds Character	Empirical Equivalence with orthodox quantum theory	Relativistic Version
	Particles	Matter field	Flashes				
Bohmian Mechanics	√			√		√	√ (foliation)
Sip	√			√	√	√	
GRWp3	√			√		√	
GRWp6	√					√	
MBM	√			√			
Sm		√			√	√	√
GRWm		√					
Mm		√			√		
Sf			√		√	√	
GRWf			√				√
Mf			√		√		

Figure 1

If you like continuous matter, GRWm seems to be the best choice, unless you like many-worlds, in which case you would pick Sm. Mm instead seems unnecessary complicated. Arguably, though, the choice of an extravagant metaphysics such as many-worlds, which is neither natural nor mandatory, is difficult to support in absence of an independent justification.

If you like flashes, GRWf seems to give the best balance of mathematical simplicity and metaphysical sensibleness, since it does not possess a many-world character as Sf, and it is not mathematically complicated as Mf.

However, which PO is actually better? Particles are more familiar, in the sense that previous, well-developed, and well-known theories like classical mechanics had a PO of particles. Arguably, the PO of continuous fields is less developed and thus requires more work. Flashes are surely a more exotic choice of PO, but they seem to be connected to relativistic invariance [Tumulka 2006], so presumably are worth taken seriously because of that, since other theories, like Bohmian mechanics, can be made relativistic invariant only adding a foliation in space-time [DMGZ 1998].

As a side note, one may wonder why we did not have so many theories in the pre-quantum era. The answer is that, in contrast with what happened in quantum mechanics, in classical mechanics it was clear from the very beginning what the metaphysical hypothesis was. Accordingly, its law of evolution was selected as the simplest among the (infinite) alternatives. Newton started from the hypothesis that the world is made of particles and then continued accordingly.

In any case, if we follow the PO approach and we get the wave function out of the picture as a possible entity to represent matter, quantum mechanics just becomes a 'regular' theory (with the qualifications that we have discussed): none of the theories considered so far is contradictory, or riddled with paradoxes. If something is puzzling in quantum mechanics then, it is nonlocality, but to properly discussing it, we would have to write another paper.

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