An Alternative Interpretation of Probability Measures in Statistical Mechanics

C.D. McCoy*

1 July 2015

Abstract

I offer an alternative interpretation of classical statistical mechanics and the role of probability in the theory. In my view the stochasticity of statistical mechanics is associated directly with the observables rather than microstates. This view requires taking seriously the idea that the physical state of a statistical mechanical system is a probability measure, thereby avoiding the unnecessary ontological presupposition that the system is composed of a large number of classical particles.

1 Introduction

Among the most persistent questions in the philosophy of statistical mechanics is, 'How are probabilities in statistical mechanics to be understood?' Especially in recent years, there have been many studies that have taken up this interpretational question.[1] The common approach taken is to investigate whether some one of the familiar so-called 'interpretations of probability' makes sense of the application of probability in the theory.[2] On the whole, the conclusions of these various studies have not been encouraging.[3] Certainly some accounts of probability receive more attention than others, yet the debate continues and there remains a distinct lack of real consensus on the physical, metaphysical, and conceptual significance of statistical mechanical probabilities.[4]

I adopt a different approach to interpretation than the usual one, for I am skeptical that much light will be shed on the probabilistic nature of statistical mechanics by relying solely on the fruits of conceptual analysis to guide interpretation. I take probability in physics to be essentially a theoretical concept (Sklar, 1979). By an interpretation of probability in a physical theory, I will mean in particular an account of what is randomly

*Department of Philosophy, University of California San Diego, La Jolla, CA, USA. email: cdmccoy@ucsd.edu

[1]For a representative sample of such papers, see (Clark, 2001; Lavis, 2001; van Lith, 2001; Emch, 2005; Winsberg, 2008; Meacham, 2010). The issue is also treated prominently in several books and reviews (Sklar, 1993; von Plato, 1994; Guttmann, 1999; Uffink, 2007; Frigg, 2008; Myrvold, forthcoming).

[2]The interpretation of probability is a distinctly popular and well-traveled topic in philosophy. There are many introductions to the general literature, for example (Gillies, 2000; Galavotti, 2005; Mellor, 2005); one may also usefully consult the encyclopedia article by Hájek (2012).

[3]Winsberg, for example, flatly observes that 'most traditional accounts of objective chance have trouble making sense of non-trivial chances in a deterministic world' (Winsberg, 2008, 873). Although some assessments are sanguine about the prospects of an interpretation, these tend to ignore obvious and serious problems with their favored interpretations.

[4]I set aside in this paper the cluster of interpretations that are characterized by their subjectivist or essentially epistemic character, especially ones based on indifference principles or ignorance. I direct the reader to criticism elsewhere (Albert, 2000; Loewer, 2001; North, 2010; Meacham, 2010). See (Uffink, 2011), however, for a friendly and thorough review of subjectivist interpretations.

determined in the theory, that is, an account of the theory's stochasticity. What is stochastic about a theory can be usefully understood by investigating the answers to two questions:

1. To what are probabilities attached in the theory?
2. What are the random variables in the theory?

Accordingly, the alternative interpretation of statistical mechanical probabilities which I propose locates the stochasticity of the theory in the system's observables. The observable properties of the system are what is random about statistical mechanical systems. Moreover, my main claim can be usefully understood as stating that probabilities are attributed to observable outcomes, where the observables are treated as random variables on the space of possible observable outcomes.

According to this interpretation one must reject the idea that the microstate of the system be understood as the instantaneous state of a large collection of particles (on pain of contradiction); instead one should take seriously the idea that probability measures represent the complete physical states of individual statistical mechanical systems.[5] This latter claim should not be misunderstood—probability measures only represent the physical states of individual statistical mechanical systems *qua* statistical mechanical systems. There certainly may be other accurate descriptions of the system in the terms of a microphysical theory, for example especially in terms of an underlying quantum mechanical state.

Although it is indeed common in the physics literature, especially in the so-called 'Gibbsian' approach to the subject, to regard probability measures as the states of statistical mechanical systems, one still finds rationalizations of an underlying particle ontology by proposing that probability measures represent ensembles of systems (of particles). One of my principal contentions through advocating this view is that one should simply forgo interpreting microstates as necessarily representing an underlying particle ontology of statistical mechanics at all.[6]

Sklar has discussed the possibility of forgoing an ontological interpretation of microstates, but calls such a move a 'radical proposal for revising our basic ontology' (Sklar, 1993, 363). Our present basic ontology, however, surely does not come from classical particle mechanics. The world is, if anything, fundamentally quantum mechanical. Thus, to suppose that statistical mechanical systems are necessarily composed of classical particles is an overly strong ontological and foundational assumption, one which I claim we are better off retreating from in the course of our foundational and philosophical investigations. Besides, recent work in the philosophy of science and physics suggests that forgoing a fundamental ontology for statistical mechanics may not be such a bad thing, as one may usefully regard statistical mechanics epistemologically as a kind of 'special science' (Callender, 2011) and ontologically from a structuralist point of view (Ladyman and Ross, 2007), in which case it makes little sense to presuppose a definite fundamental ontology for such a 'high level' theory, at least without careful investigation of the relations between it and other physical theories.

The paper proceeds as follows. First, I discuss the possibilities of characterizing the stochasticity of statistical mechanics, and show how my favored interpretation can be suggested by the formalism of the theory (§1). I then argue that phase-space should be interpreted instrumentally in statistical mechanics (§2), using

[5] The notion that a probability measure is the state of a statistical mechanical system is quite frequently stated—for example: 'By a statistical state of the system we will mean a probability measure μ on the manifold U' (Souriau, 1997, 271). What the interpretational significance of such statements is taken to be, however, varies widely among physicists or is often simply not considered.

[6] To be sure, this idea is not entirely novel since similar views have been suggested before, most notably by Prigogine (See Prigogine (1996) for a popular account of his views). My argument however does not depend at all on the dynamical considerations which Prigogine argues makes a realist interpretation of phase points untenable. See (Batterman, 1991) for a philosophical evaluation of Prigogine's approach. The general view which I advocate here is discussed in the philosophical literature, as far as I am aware, only by Sklar (1993, 7.IV.2, 9.III.1) (he pejoratively calls it the 'radical' or 'revisionist' ontological approach), and he appears to take Prigogine alone as the source of the interpretation.

as an example the phase-space formulation of quantum mechanics, where quantum states are representable on phase-space; clearly in that case one would not assume that points of phase-space represent classical states of particles. In §3 and §4 I argue for the viability of interpreting statistical mechanical observables as the stochastic element of the theory: first by arguing that it provides better coherence between the ontology and epistemology of statistical mechanics; second by showing how a commitment to a fundamental ontology of classical particles can coherently be avoided. I conclude (§6) with some brief comments pointing to potentially fruitful extensions of this work, for example to non-equilibrium statistical mechanics and quantum mechanics.

Statistical mechanics encompasses a fairly diverse array of approaches and formalisms;[7] to avoid excessive complications I restrict my attention to equilibrium statistical mechanics. The case for an alternative interpretation of probability made on its basis is of sufficient interest without immediately considering statistical mechanics in all its diversity.

2 Probability Spaces in Equilibrium Statistical Mechanics

A perspicuous formulation of statistical mechanics should be able to tell us what probabilities do in the theory. This is accomplished in statistical mechanics through its essential incorporation of probability theory as a way of deriving statistical predictions of observables. Precisely how probability is implemented formally in the the theory is however quite flexible, as a consequence of which there exist physically-motivated alternative interpretations that identify different 'things' to which probability numbers are attached and different elements of the framework as random variables—that is to say, what is stochastic in the theory.[8] To be sure, there is a standard view on the matter, namely that probabilities attach to microstates of the system and observables are represented by random variables. My aim in this section is to demonstrate that there exist plausible alternatives to the standard view based on two modifications: (1) where probabilities are understood to attach to observable outcomes; (2) where microstates are understood as random variables.

An equilibrium statistical mechanical system is described by three things: a phase-space Γ, a (macro-)state ρ, and a set of observables $\mathcal{A}$. The notion of a phase-space is borrowed from classical particle mechanics, where it is the space of possible states. Encoded in a classical particle state x of Γ is the spatial configuration of all individual particles as well as their states of motion, that is, their positions and momenta. In statistical mechanics the elements x of Γ are referred to as microstates, and they are standardly interpreted as representing the epistemically-inaccessible but real underlying microphysical states of motion of the system. The macro-state ρ is a probability distribution on Γ, and the observables $A, B, C, \ldots \in \mathcal{A}$ are random variables on Γ, usually represented as maps $\Gamma \to \mathbf{R}$.

A statistical mechanical system described as such naturally describes a probability space, where Γ is the sample space, the set of events $\mathcal{L}$ is conveniently taken to be the set of (Lebesgue) measurable subsets of Γ, and ρ is used to define a probability measure μ_ρ:

$$\mu_\rho(U) = \int_U \rho \, \mathrm{d}\Gamma, \tag{1}$$

[7] 'Unlike quantum mechanics and relativity theory, say, statistical mechanics has not yet found a generally accepted theoretical framework, let alone a canonical formulation. What we find in statistical mechanics is a plethora of different approaches and schools, each with its own programme and mathematical apparatus, none of which has a legitimate claim to be more fundamental than its competitors' (Frigg, 2008, 101).

[8] From a purely mathematical point of view this should be unsurprising, since probability theory is not about probability spaces per se. The same set of statistical outcomes can be modeled by various probability spaces with various choices of random variables. Thus some interpretive work is required to establish which measure space is the 'right' one.

where U is an element of $\mathcal{L}$, and Lebesgue integration is with respect to the natural volume element $\mathrm{d}\Gamma$ on Γ. It is clear, when the theory is formulated this way, that probabilities attach to micro-states, since ρ is undertsood to be a map from Γ to $[0, 1]$.

The empirical content of statistical mechanics is given by statistics of the observables, that is, expectation values, variances, and so on.[9] For example, the expectation value of an observable A is given by treating A as a random variable on Γ associated to the probability measure μ_ρ:

$$\langle A\rangle_\rho = \int_\Gamma \rho\, A\, \mathrm{d}\Gamma. \tag{2}$$

The standard view thus has probabilities associated to microstates and treats observables as random variables. But what about the system is random? Is it the microstates or the observables? The answer is surely not manifest—indeed some interpretive work is required.

One interpretation is that any statistical mechanical system possesses in addition to its macrostate ρ a classical microstate $x(t)$ (at time t) which evolves deterministically as in classical particle mechanics. The only thing that could be random about a statistical mechanical system which is also a classical mechanical system is its initial microstate x_0. That is to say, for any statistical mechanical system there was a random trial that determined the initial microstate x_0, which state determines its future evolution and observable properties just as in classical particle mechanics.[10] But since a statistical mechanical system's microstate is presumed to be epistemically inaccessible (it cannot be determined through measurement because of the large number of degrees of freedom), one can only compute the statistical consequences of this uncertainty in order to derive predictions of the macroscopic observables. In this case the previous equation, the expectation value, should be interpreted epistemically; the system's macroscopic observables are actually determined by classical mechanical observables (real-valued functions on phase-space, in particular of the system's classical trajectory):

$$A(t) = A \circ x(t). \tag{3}$$

A second interpretation is possible in which microstates are understood to represent the state of a system of particles. In this view microstates do not evolve deterministically as in classical particle mechanics. Instead the microstate is itself taken to be stochastic. This can be made formally more manifest by treating the microstate as a random variable on phase-space, and representing the observables in the same way as observables in particle mechanics, namely real-valued functions on phase-space.[11] Define a random variable $X : \Gamma \to \Gamma$ on phase-space which is just the identity map between measure spaces Γ. Then the expectation value of an observable can be written

$$\langle A\rangle_\rho = \int_\Gamma \rho\, (A \circ X)\, \mathrm{d}\Gamma. \tag{4}$$

There is yet another possible interpretation of the expectation value of equation (2): the observables themselves are stochastic. To make sense of this claim, however, one has to give up on the idea that statistical mechanical systems possess deterministically evolving microstates, since if the observables were truly stochastic, then the system would have contradictory observable properties (the observables realized

[9] See also (Wallace, 2013b) for a clear and accessible account of how statistical mechanics is used to describe and predict statistical phenomena.

[10] Strictly speaking the random trial does not need to determine the initial condition; there merely needs to be, at some time, a random trial that determines the system's micro-history.

[11] It seems that this view is also sometimes explicitly adopted in the physics literature: 'As time passes, the system continually switches from one microstate to another, with the result that, over a reasonable span of time, all one observes is a behavior "averaged" over the variety of microstates through which the system passes' (Pathria, 1996, 30).

through the stochastic process and the observables determined by the microstate). This was the case with the previous interpretation as well, but in this case I emphasize that there is also no need to give a realistic interpretation of the microstates.

One may find this last remark puzzling, since expectation values are computed by quantifying over microstates. Insofar as one thinks that realism is not an all or nothing affair with respect to a given formal framework, that is, insofar as one is a selective realist, one is however within one's rights to interpret some aspects of a theory's formalism instrumentally. Thus a selective realist may be completely satisfied to interpret phase-space and its microstates as ontologically insignificant representational structure.

Although I find this position reasonable, it is nevertheless possible to mold the formalism somewhat so that the interpretation of observables as stochastic becomes more manifest. Just as one can treat phase-space as a probability space and the microstate as a random variable on this space (as in the second interpretation), one can treat the image set of any observable as a probability space, and the observable outcomes as random variables on these spaces. By this mild subterfuge one can remove all reference to phase-space and its microstates (should one for example be worried about Quinean quibbles).

Given the probability space $(\Gamma, \mathcal{L}, \rho)$, it is trivial to construct these spaces. Let A be an observable of a statistical mechanical system. The measurable space associated with it is the image set $A[\Gamma]$ of A with measurable sets $\mathcal{L}_A = \{A[U]\}$, for sets $U \in \mathcal{L}$. Let ρ be the state of the system with respect to the measurable space $(\Gamma, \mathcal{L})$. The state ρ_A of the system with respect to measurable space $(A[\Gamma], \mathcal{L}_A)$ is given by $A_*\rho$, the pushforward of ρ under A.[12] One now has constructed a new probability space $(A[\Gamma], \mathcal{L}_A, \rho_A)$.

Expectation values of the system are then computed according to the following formula:

$$\langle A \rangle_{\rho_A} = \int_{A[\Gamma]} \rho_A \, A \, \mathrm{d}A. \tag{5}$$

The statistics predicted from formalizing statistical mechanics in this way, namely on probability spaces $\mathcal{A}[\Gamma]$ of observables, is of course identical to the statistics predicted from formalizing it on phase-space (as a probability space). It is so by construction. Moreover this set of probability spaces suggests a different interpretation which is less apparent in the standard formulation. In this interpretation probabilities are attached to observable outcomes, and those observable outcomes are treated as random variables. Together they formally represent the stochasticity of the observables, and thereby the stochasticity of statistical mechanics.

3 Interpreting phase-space

It may have occurred to the reader that the outcome probability spaces just constructed, that is those of the form $(A[\Gamma], \mathcal{L}_A, \rho_A)$, are objectionably parasitic on the phase-space probability space. That is not really the case, since one could just as well work the other direction, that is, by choosing a ρ_A and pulling it back to phase-space. Certainly the pullbacks are not going to be unique in general, but what is to worry? One just has to insure that the pullbacks of the set $\rho_{\mathcal{A}} = \{\rho_A, \rho_B, \dots\}$ agree, and then one has the resources at hand to do the business of statistical mechanics.

Still, I suspect many find great significance in the fact that statistical mechanics is generally formulated on phase-space, perhaps find it auspicious even.[13] But it may also be historical accident and naiveté. I

[12] You can think of this map as using A to pull back measurable sets of the image set of the observable to Γ and then using ρ to compute the set's probability.

[13] 'In statistical mechanics, we have, instead [of what is the case in quantum mechanics], built into the very mathematics from which the probability distributions are derived the underlying phase-space with its pointlike representatives of exact micro-states' (Sklar, 1993, 291).

certainly grant that the phase-space probability space is instrumentally useful, since having a single state on it unifies the observable content of the system in a single space. Indeed, that is just what a state is supposed to be anyway: a unified representation of the observable content of the system. Once one has it, it is a separate, interpretive matter to determine whether one should use phase-space or some other probability space with enough representational content to coordinate the observational content of the physical system. By exhibiting alternative formal presentations of statistical mechanics in the previous section, I have shown then that the traditional, customary interpretive choice is not uniquely justified merely because of a traditional, customary choice in formalism.

If phase space should not be taken too seriously for purposes of ontology, then why would phase-space ultimately be a good way to represent statistical mechanical states? My considered view is that phase-space has enough representational content that it can be usefully used to represent not only statistical mechanical states but underlying physical structure as well. To explain its utility in this latter respect one would wish to have a microphysical theory that can represent its own states as probability measures on phase-space. Classical particle mechanics, the theory typically thought of as providing the underlying microphysics of classical statistical mechanics, cannot realize macrostates in this way, for a classical mechanical state on phase-space is merely a 'sharply-peaked' distribution.[14] If one therefore wishes to understand from where statistical mechanical states come, one should look elsewhere for the provision of phase-space representations.

Quantum mechanics is one appealing place to look. Indeed there is a well-known, straightforward, and illuminating way to represent quantum states and operators on phase-space, namely through the use of Wigner functions and Weyl transforms.[15] Let ρ be a density operator that represents a (mixed) quantum state, that is,

$$\rho = \sum_s p_s |\Psi_s\rangle\langle\Psi_s|, \tag{6}$$

where p_s is the probability of the system being in a particular pure quantum state $|\Psi_s\rangle$ in a set of pure states indexed by s. The Wigner function W_ρ transforms a quantum state ρ into a function on phase-space. It is a map $W_\rho : \Gamma \to \mathbf{R}$ that may be defined as follows:

$$W_\rho(q, p) = \frac{1}{\pi^{N/2}} \int \mathrm{d}^N\gamma\, e^{2ip\gamma/\hbar} \langle q - \gamma|\rho|q + \gamma\rangle, \tag{7}$$

where local coordinates q (position) and p (momentum) have been introduced on $2N$-dimensional phase-space Γ. If the Wigner function is a probability measure on phase-space (it is not necessarily so), then it may (at least formally) serve as a 'statistical mechanical state'. Quantum mechanical observables may be transformed similarly (such transforms are called Weyl transforms) into functions on phase-space, that is, classical observables. The statistical mechanical expectation value of an operator A, then, is simply given by an integral over phase-space:

$$\langle A\rangle_\rho = \int_\Gamma W_\rho\, A\, \mathrm{d}\Gamma, \tag{8}$$

much as in statistical mechanics.

I give the example of quantum mechanics on phase-space not to make the case that this is precisely how statistical mechanical states come about in general. I mean to show rather that phase-space is plausibly a vehicle for representing underlying microphysical structure in an indirect way, and therefore that there is no mystery about its appearance in the formalism of statistical mechanics. The fact that quantum mechanics

[14] In fact equilibrium statistical mechanics is naturally seen as a more general theory than classical mechanics, in that it can represent classical mechanical states as well as more general statistical states.

[15] See, for example, the textbook by Kim and Noz (1991), or the accessible introduction by Case (2008). The formalism is also described in (Wallace, 2012).

is representable on phase-space, even though quantum mechanics is not a theory of classical microstates, suffices to establish this point.[16] For this reason it also is not necessary to reformulate statistical mechanics to avoid referring to phase-space, as one may wish to do to avoid quantifying over microstates. Phase-space is, I expect, of significant practical and heuristic value in establishing the sort of inter-theoretic relations which are needed to illuminate the foundations of statistical mechanics.

4 Epistemology and Ontology in Statistical Mechanics

It is widely presumed that the foundations of statistical mechanics is an enterprise centrally concerned with completing the project of the founders of statistical mechanics (Wallace, 2013b, 2), that is, reducing thermodynamics to the mechanical motion of molecules (Callender, 1999, 2001) - 'nought but molecules in motion' in Maxwell's memorable phrase. Philosophers and physicists regularly adopt the traditional presupposition of a classical ontology of particles moving in space as featured in this traditional foundational project. It is moreover natural to borrow this ontology from classical particle mechanics, since classical statistical mechanics is 'built on top of' classical particle mechanics. Thus individual statistical mechanical systems are presumed to have a classical mechanical state (a 'microstate') that evolves deterministically and has determinate physical properties, just as in classical particle mechanics.

It is a central assumption of statistical mechanics, however, that a system's microstate is in general epistemically inaccessible—if it were accessible, then probabilities would not have to be introduced in the theory.[17] Statistical mechanical probabilities are introduced to classical particle mechanics by associating probability measures to particular aggregations of microstates ('macrostates'), that is, a probability measure is a map $\Gamma \to [0, 1]$. These probabilities are of course crucial for deriving the theory's objective empirical content, yet their role in the traditional interpretation is puzzling, at least on the face of it.[18] Macrostates have no influence on the behavior of individual microstates, as the latter are understood to evolve deterministically and therefore in ignorance of this probabilistic element.[19]

Yet it is the macrostates that have observable consequences in the form of statistical predictions (expectation values, variances,...). So why posit an ontology of particles with additional dynamical constraints, then, when such a posit is beyond our epistemic reach? I take it to be a plausible, defeasible principle that 'ontology should recapitulate epistemology.' What I mean by this is that we should accept some degree of anti-metaphysical 'positivism' in scientific metaphysics, namely by only accepting the existence of entities

[16]Contra Sklar: 'In quantum mechanics, the mathematical apparatus posits no further underlying 'point' phase-space state of the system beyond the probability distribution. Indeed, classical phase-space is rejected altogether and is replaced by a phase-space in which the mathematical representatives of the probability distributions (or, rather, the probability amplitudes from which the probability distributions are constructed by multiplying one of them by its complex conjugate - Hilbert space vectors) are the 'atoms' (Sklar, 1993, 291).

[17]Although some textbooks do not even mention microstates, those that do often point out this epistemic assumption explicitly: '...even though the energy is fixed, the system could exist in any one of a number of different microscopic states consistent with that energy. If we only know the total energy, we have no way of distinguishing one microscopic state from another' (Reichl, 1998, 341); 'A macroscopic physical object contains so many molecules that no one can hope to find its dynamical state by observation' (Penrose, 1970, 2); 'The definition requires one to know the initial positions and velocities of all n particles and to follow these motions for all time. Since n is typically of the order of 10^{24}, this is of course impossible' (Ellis, 2006, 65).

[18]'If the laws are deterministic then the initial conditions of the universe together with the laws entail all facts—at least all facts expressible in the vocabulary of the theory. But if that is so there is no *further* fact for a probability statement to be about' (Loewer, 2001, 610); 'The fundamental problem with understanding the probabilities in statistical mechanics to be objective is that we are meant to posit a probability distribution over a set of possible initial states, while we suppose, at the same time, that in fact only one of these initial states actually obtained' (Winsberg, 2008, 873).

[19]This apparent tension has led to many efforts at establishing a philosophical account of 'objective chances' to resolve it. See, for example, (Loewer, 2001; Hoefer, 2007; Maudlin, 2007; Ismael, 2009).

when we have adequate reasons to believe that they exist, and otherwise remaining agnostic (if not skeptical of unduly speculative metaphysics).

The view of statistical mechanics where microstates faithfully represent actual configurations of particles in motion appears to flout this principle flagrantly. Thus, for the sake of scientific respectability, the microstate reifier must take on the burden of explaining why the ontology of statistical mechanics can diverge from the epistemology of statistical mechanics. I certainly do not suggest that such explanations cannot be provided, but I do insist that it is a significant interpretive cost in comparison to the simpler view that I advocate.

In my view the ontology of statistical mechanics includes statistical mechanical systems, that is, those systems, like boxes of gas and the early universe, that are well-described by the theory. The theory describes systems in terms of statistical mechanical states—probability measures—and stochastic observables, the combination of which can be used to derive accurate statistical predictions of observational outcomes.[20] There is no analogous mystery of what probability is doing in the theory as there is in the traditional view. Probability merely describes the stochastic nature of statistical mechanical systems, which fact is responsible for the statistical observable outcomes of said systems.

On this score my favored interpretation comes out ahead of the traditional presuppositions, yet I certainly do not take this to be a decisive point. My aim in setting it out is primarily to push back against a default presumption in favor of the traditional view.[21] With respect to the interpretation of statistical mechanics, understood as an autonomous physical theory, I urge that on the basis of a plausible philosophical principle there should on the contrary be a default presumption in favor of the ontologically less profligate interpretation. With respect to foundational questions about statistical mechanics, however, indeed something more must be said.

5 Alternative Foundations

In the traditional interpretation a particular foundational relationship is assumed between statistical mechanics and classical mechanics (Wallace, 2013a). One point of view is to take classical particle mechanics as a fundamental theory[22]—at least more fundamental than classical statistical mechanics—and statistical mechanics as a 'higher level' theory that successfully describes a particular set of phenomena—thermodynamic phenomena—despite practical limitations in accessibility to the 'lower level' details. It is also sometimes supposed, however, that statistical mechanics is a fundamental theory of sorts, and that the probabilities are somehow objective features of an otherwise classical mechanical world of particles moving in space (Albert, 2000; Loewer, 2001).

The assumption of a necessary relationship between classical and statistical mechanics in these ways should, it seems to me, strike us as somewhat odd—again, the world is quantum mechanical, so far as we know, not classical mechanical.[23] Assuming that the ontological foundation of classical statistical mechanics

[20]It may seem we have lost a spacetime picture by speaking of the theory in these terms, but that is not necessarily so. The phase-space formulation can be used, for example, to represent external constraints like the volume of the system.

[21]'As far as things look now, nothing in statistical mechanics forces us to abandon the appropriate exact micro-states posited by the underlying dynamical theory, be it classical or quantum in nature' (Sklar, 1993, 418); 'Classical theorists who want to remain realists about exact states and unique trajectories need not be terribly moved by [Prigogine's] argument. They may claim that it just begs the question. It seems to me that nothing short of bona fide no-hidden-variables proof can genuinely compel one to give up the concept of an exact state' (Batterman, 1991, 260).

[22]'Mechanics is a completely general theory, that is it ought to give a complete description of any physical situation to which it applies' (Clark, 2001, 271).

[23]'Why should we consider quantum issues when working in the foundations of statistical physics? The simple (too simple) answer is that classical physics is false. If our purpose, in doing foundational work, is to understand the actual world, it is necessary to use a

is the same as found in classical particle mechanics not only faces the epistemological challenge raised already, but it is quite unnecessary, since there are these days other potential fundaments for the former besides the latter.[24]

A key virtue of adopting the alternative interpretation of statistical mechanics which I am advocating is that it avoids any unwarranted and necessary commitment to a particular fundamental ontology and foundation for the theory. Sklar finds this 'revisionist ontology' objectionable precisely because it fails to complete the historical project of statistical mechanics. As he says, 'denying the existence of the exact micro-states...doesn't seem to help us at all in resolving the most crucially puzzling questions such as those centered around the origin of time-asymmetry' (Sklar, 1993, 366). On the contrary, I believe denying a particle ontology for statistical mechanics will help solve those problems, namely by re-directing foundational efforts from the fruitless search for explanations mired in the classical thinking of outmoded science to other avenues of explanation based on our best understanding of current physics. Avoiding ontological commitments does not force one to be agnostic about foundational matters; rather the interpretation encourages the fruitful reorientation of foundational investigations into statistical mechanics from the intra-theoretic to the inter-theoretic.

How can this alternative interpretation coherently avoid the commitment to a particular foundational viewpoint? One appealing way is to understand statistical mechanics as a 'special science' (Callender, 1997; Callender and Cohen, 2010; Callender, 2011).[25] For Callender statistical mechanics is a theory like those found in biology or economics:

> It's a new special science, one that grounds and unifies a lot of macroscopic behavior. It too is restricted to certain kinds of systems, in this case macroscopic systems whose energy and entropy are approximately extensive. (Surely a better characterization can be given, but this will do for now.) The claim is that the [statistical mechanical] probabilities only kick in when we have systems meeting such a description. Once they develop, one uses [statistical postulates] to great effect. But one does not use it to describe and explain the frequency with which such systems develop in the first place. The science of statistical mechanics is about systems with certain features. Yet it's no part of the science to say anything about the frequencies of these systems themselves (Callender, 2011, 110).

That is just to say that it's a job for another theory with greater explanatory resources. Indeed, it may be a job for other *theories*, since it is certainly open that the probability measures which serve as statistical mechanical states may be mutliply-realized in distinct physical structures, such that reductive and emergent relations between those structures and their statistical mechanical descriptions are system-dependent.

Thinking of the theory as a special science gives reason to avoid unnecessary commitments to a particular fundamental ontology, at least from the point of view of the special science. Structuralism encourages the further view that it is unnecessary to posit any theoretical ontology over and above the structure given by the theory in the theory itself. Structuralism, in its most benign form, should certainly not be seen as an effort to 'get rid of' ontology altogether, but rather as an effort to proof our metaphysical thinking of naive ontological prejudices—especially such as that the world is made up of individual classical particles).[26]

theory which validly describes that world' (Wallace, 2001, 1).

[24] The founders of statistical mechanics had an excuse, as they were not so spoiled for choice in alternatives!

[25] The particular interpretation advocated here differs somewhat from Callender's own position, at least in the details. For example, he says that 'the special-sciences view doesn't claim that macroscopic systems aren't composed of particles that always evolve according to microdynamical laws. They are' (Callender, 2011, 110). While I certainly do not make the claim that macroscopic systems are *not* composed of particles, neither do I claim that they necessarily are so composed.

[26] 'We are not 'anti-ontology' in the sense of urging a move away from electrons, elementary particles etc. and towards 'observable structures' or the S-matrix or whatever; rather, we urge the reconceptualization of electrons, elementary particles and so forth in

Such ontological prejudices can impede progress in understanding the foundations of physics, and may even impede physics beyond foundational matters, since as Cao (2003, 5-6) observes, 'an ontological commitment made in a scientific discipline also dictates its theoretical structure and the direction of its evolution.' An ontological commitment to classical particles dictated the theoretical structure of statistical mechanics, and has since largely dictated the direction of its evolution both within physics and especially philosophy. We now know more and better, and therefore a foundational account that presupposes a particle ontology should be viewed as the depreciated metaphysical fiction of a former time. The alternative account proposed here provides one coherent starting point to move forward beyond it.

6 Concluding Remarks

The prejudice in foundational work on statistical mechanics towards interpreting microstates as representing the states of systems of particles has unfortunately suppressed consideration of alternative interpretations of the theory and its probabilities.[27] The only criticisms of this account have been overstated as well.[28] Although the limited discussion of this alternative has so far centered around Prigogine's work, the general view, independent of his specific proposals, is quite plausible on the philosophical and theoretical grounds promoted here. No doubt much work remains to flesh out the view, but I hope to have established that it can and should be taken seriously as a viable alternative to the traditional attitudes.

As I have argued, to metaphysically reify the microstates of phase-space is to make suppositions about an underlying ontology, suppositions that have no grounds in the empirical or even the theoretical facts of statistical mechanics. There is no need to be alarmed at the deferral of foundational questions to other theories or the rejection of a particular fundamental ontology by rejecting this move. If one understands statistical mechanics structurally and as a sort of special science, then there is no need for it to justify itself or to seek answers within its own framework. There is surely some story as to why statistical mechanical systems behave as they do, have the states that they do, and are well represented in a phase-space picture. But I have insisted that this story plausibly involves more fundamental theories in an essential way.

Adopting this interpretation of statistical mechanical states opens up many new possibilities for foundational and philosophical work. I will mention a couple; the reader can no doubt envision more. In the first place the possibility of interpreting statistical mechanics in this way makes for a stronger connection to the interpretation of quantum mechanics and the measurement problem. There are interesting analogies (and dis-analogies) between popular interpretations of quantum mechanics and the ways of interpreting the stochasticity of statistical mechanics and its statistical predictions. Secondly, taking probability measures seriously as statistical mechanical states also suggests that the Hamiltonian dynamics of classical particle mechanics is inappropriate for describing the dynamics of non-equilibrium statistical mechanics. Since the action of a Hamiltonian defined on phase-space on probability measures defined on phase-space is non-dissipative, no statistical mechanical state will ever approach equilibrium. Although particle-based intuitions lead to non-equilibrium equations of motion like the Fokker-Planck equation (Kadanoff, 2000, Ch. 6), the interpretation I advocate would seem to suggest viewing these intuitions as heuristic. It may therefore open up alternative ways of understanding the utility of such equations in non-equilibrium statistical mechanics.[29]

---

structural instead of individualistic terms' (French and Ladyman, 2003, 37).

[27] Sklar (2009) for example does not mention any such alternatives, such as the one developed here, in his Stanford Encyclopedia of Philosophy entry on the philosophy of statistical mechanics.

[28] 'All in all, the positing of genuine tychistic states for systems and of the probability distributions invoked by statistical mechanics as representing the individual states of such systems seems pretty implausible' (Sklar, 1993, 292).

[29] 'These equations are just convenient to use; we know how to use them but do not fully understand why they can be so useful' (Ma, 1986, 367).

References

Albert, David. *Time and Chance*. Cambridge, MA: Harvard University Press, 2000.

Batterman, Robert. “Randomness and Probability in Dynamical Theories: On the Proposals of the Prigogine School.” *Philosophy of Science* 58: (1991) 241–263.

Callender, Craig. “What is ‘The Problem of the Direction of Time’?” *Philosophy of Science* 64 (Proceedings): (1997) S223–234.

———. “Reducing Thermodynamics to Statistical Mechanics: The Case of Entropy.” *The Journal of Philosophy* 96: (1999) 348–373.

———. “Taking Thermodynamics Too Seriously.” *Studies in History and Philosophy of Modern Physics* 32, 4: (2001) 539–553.

———. “The Past Histories of Molecules.” In *Probabilities in Physics*, edited by Claus Beisbart, and Stephan Hartmann, Oxford University Press, 2011, 83–113.

Callender, Craig, and Jonathan Cohen. “Special Sciences, Conspiracy and the Better Best System Account of Lawhood.” *Erkenntnis* 73: (2010) 427–447.

Cao, Tian Yu. “Structural Realism and the Interpretation of Quantum Field Theory.” *Synthese* 136: (2003) 3–24.

Case, William. “Wigner functions and Weyl transforms for pedestrians.” *American Journal of Physics* 76: (2008) 937–946.

Clark, Peter. “Statistical Mechanics and the Propensity Interpretation of Probability.” In *Chance in Physics*, edited by Jean Bricmont, Detleff Dürr, Maria Galavotti, Giancarlo Ghirardi, Francesco Petruccione, and Nino Zanghi, Springer, 2001, 271–281.

Ellis, Richard. *Entropy, Large Deviations, and Statistical Mechanics*. Springer, 2006.

Emch, Gérard. “Probabilistic issues in statistical mechanics.” *Studies in History and Philosophy of Modern Physics* 36: (2005) 303–322.

French, Steven, and James Ladyman. “Remodelling Structural Realism: Quantum Physics and the Metaphysics of Structure.” *Synthese* 136: (2003) 31–56.

Frigg, Roman. “A Field Guide to Recent Work on the Foundations of Statistical Mechanics.” In *The Ashgate Companion to Contemporary Philosophy of Physics*, edited by Dean Rickles, London: Ashgate, 2008, 99–196.

Galavotti, Maria Carla. *Philosophical Introduction to Probability*. Stanford: CSLI Publications, 2005.

Gillies, Donald. *Philosophical Theories of Probability*. London: Routledge, 2000.

Guttmann, Yair. *The Concept of Probability in Statistical Physics*. Cambridge University Press, 1999.

Hájek, Alan. “Interpretations of Probability.”, 2012. `http://plato.stanford.edu/archives/win2012/entries/probability-interpret/`. Edward Zalta (ed.).

Hoefer, Carl. "The Third Way on Objective Probability: A Sceptic's Guide to Objective Chance." *Mind* 116: (2007) 549–596.

Ismael, Jenann. "Probability in Deterministic Physics." *The Journal of Philosophy* 106: (2009) 89–108.

Kadanoff, Leo. *Statistical Physics*. Singapore: World Scientific, 2000.

Kim, Young, and Marilyn Noz. *Phase Space Picture of Quantum Mechanics*. World Scientific: Singapore, 1991.

Ladyman, James, and Don Ross. *Every Thing Must Go*. Oxford University Press, 2007.

Lavis, David. "The Concept of Probability in Statistical Mechanics." In *Frontiers of Fundamental Physics 4*, edited by Burra Sidharth, and Mikhail Altaisky, Springer, 2001, 293–308.

Loewer, Barry. "Determinism and Chance." *Studies in History and Philosophy of Modern Physics* 32: (2001) 609–620.

Ma, Shang-Keng. *Statistical Mechanics*. Singapore: World Scientific, 1986.

Maudlin, Tim. "What could be objective about probabilities?" *Studies in History and Philosophy of Modern Physics* 38: (2007) 275–291.

Meacham, Christopher. "Contemporary Approaches to Statistical Mechanical Probabilities: A Critical Commentary—Part I: The Indifference Approach." *Philosophy Compass* 5, 12: (2010) 1116–1126.

Mellor, David. *Probability: A Philosophical Introduction*. London: Routledge, 2005.

Myrvold, Wayne. "Probabilities in Statistical Mechanics." In *Oxford Handbook of Probability and Philosophy*, edited by Christopher Hitchcock, and Alan Háyek, Oxford University Press, forthcoming.

North, Jill. "An empirical approach to symmetry and probability." *Studies in History and Philosophy of Modern Physics* 41: (2010) 27–40.

Pathria, Raj. *Statistical Mechanics*. Butterworth-Heinemann: Oxford, 1996.

Penrose, Oliver. *Foundations of Statistical Mechanics*. Pergamon Press: Oxford, 1970.

Prigogine, Ilya. *The End of Certainty*. The Free Press, 1996.

Reichl, Linda. *A Modern Course in Statistical Physics*. John Wiley and Sons: New York, 1998.

Sklar, Lawrence. "Probability as a Theoretical Concept." *Synthese* 40: (1979) 409–414.

———. *Physics and Chance*. Cambridge University Press, 1993.

———. "Philosophy of Statistical Mechanics.", 2009. `http://plato.stanford.edu/archives/sum2009/entries/statphys-statmech/`. Edward Zalta (Ed.).

Souriau, Jean-Marie. *Structure of Dynamical Systems*. Springer, 1997.

Uffink, Jos. "Compendium of the Foundations of Classical Statistical Physics." In *Philosophy of Physics*, edited by Jeremy Buttefield, and John Earman, Elsevier, 2007, volume Part B.

———. “Subjective Probability and Statistical Physics.” In *Probabilities in Physics*, edited by Claus Beisbart, and Stephan Hartmann, Oxford University Press, 2011, chapter 2, 25–49.

van Lith, Janneke. “Ergodic Theory, Interpretations of Probability and the Foundations of Statistical Mechanics.” *Studies in History and Philosophy of Modern Physics* 32: (2001) 581–594.

von Plato, Jan. *Creating Modern Probability*. Cambridge University Press, 1994.

Wallace, David. “Implications of quantum theory in the foundations of statistical mechanics.”, 2001. `http://philsci-archive.pitt.edu/id/eprint/410`.

———. *The Emergent Multiverse*. Oxford University Press, 2012.

———. “Inferential vs. Dynamical Conceptions of Physics.”, 2013a. `http://arxiv.org/abs/1306.4907`.

———. “What Statistical Mechanics Actually Does.”, 2013b. `http://philsci-archive.pitt.edu/id/eprint/9846`.

Winsberg, Eric. “Laws and chances in statistical mechanics.” *Studies in History and Philosophy of Modern Physics* 39: (2008) 872–888.