A simplified local-realistic derivation of the EPR-Bohm correlation

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We illustrate an explicit counterexample to Bell's theorem by constructing a pair of spin variables within S^3 that exactly reproduces the EPR-Bohm correlation in a manifestly local-realistic manner.

We begin by defining the detections of spin bivectors $\mathbf{L}(\mathbf{s},\,\lambda^k)$ by the detector bivectors $\mathbf{D}(\mathbf{a})$ and $\mathbf{D}(\mathbf{b})$ {Ref. [1]}:

$$S^3 \ni \mathscr{A}(\mathbf{a},\,\lambda^k) \,:=\, \lim_{\mathbf{s}\,\rightarrow\,\mathbf{a}} \left\{-\,\mathbf{D}(\mathbf{a})\,\mathbf{L}(\mathbf{s},\,\lambda^k)\right\} = \begin{cases} +1 & \text{if} \quad \lambda^k \,=\, +1 \\ -1 & \text{if} \quad \lambda^k \,=\, -1 \end{cases} \tag{1}$$

$$\text{and} \quad S^3 \ni \mathscr{B}(\mathbf{b},\,\lambda^k) \,:=\, \lim_{\mathbf{s}\,\rightarrow\,\mathbf{b}} \left\{+\,\mathbf{L}(\mathbf{s},\,\lambda^k)\,\mathbf{D}(\mathbf{b})\right\} = \begin{cases} -1 & \text{if} \quad \lambda^k \,=\, +1 \\ +1 & \text{if} \quad \lambda^k \,=\, -1\,, \end{cases} \tag{2}$$

where the orientation λ of S^3 is assumed to be a random variable with 50/50 chance of being $+1$ or -1 at the moment of the pair-creation, making the spinning bivector $\mathbf{L}(\mathbf{n},\,\lambda)$ a random variable *relative* to the detector bivector $\mathbf{D}(\mathbf{n})$:

$$\mathbf{L}(\mathbf{n},\,\lambda) \,=\, \lambda\,\mathbf{D}(\mathbf{n}) \iff \mathbf{D}(\mathbf{n}) \,=\, \lambda\,\mathbf{L}(\mathbf{n},\,\lambda). \tag{3}$$

The expectation value of the simultaneous outcomes $\mathscr{A}(\mathbf{a},\,\lambda^k)=\pm1$ and $\mathscr{B}(\mathbf{b},\,\lambda^k)=\pm1$ is then worked out as follows:

$$\mathcal{E}(\mathbf{a},\,\mathbf{b}) \,=\, \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n} \mathscr{A}(\mathbf{a},\,\lambda^k)\,\mathscr{B}(\mathbf{b},\,\lambda^k)\right] \quad \text{within} \;\; S^3 := \text{the set of all unit (left-handed) quaternions} \tag{4}$$

$$= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n} \left[\lim_{\mathbf{s}\,\rightarrow\,\mathbf{a}}\left\{-\,\mathbf{D}(\mathbf{a})\,\mathbf{L}(\mathbf{s},\,\lambda^k)\right\}\right]\left[\lim_{\mathbf{s}\,\rightarrow\,\mathbf{b}}\left\{+\,\mathbf{L}(\mathbf{s},\,\lambda^k)\,\mathbf{D}(\mathbf{b})\right\}\right]\right] \quad \text{(conserving total spin = 0)} \tag{5}$$

$$= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n} \lim_{\substack{\mathbf{s}\,\rightarrow\,\mathbf{a}\\ \mathbf{s}\,\rightarrow\,\mathbf{b}}}\left\{-\,\mathbf{D}(\mathbf{a})\,\mathbf{L}(\mathbf{s},\,\lambda^k)\;\mathbf{L}(\mathbf{s},\,\lambda^k)\,\mathbf{D}(\mathbf{b}) \,\equiv\, \mathbf{q}(\mathbf{a},\,\mathbf{b};\,\mathbf{s},\,\lambda^k)\in S^3\right\}\right] \tag{6}$$

$$= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n} \lim_{\substack{\mathbf{s}\,\rightarrow\,\mathbf{a}\\ \mathbf{s}\,\rightarrow\,\mathbf{b}}}\left\{-\,\lambda^k\,\mathbf{L}(\mathbf{a},\,\lambda^k)\;\mathbf{L}(\mathbf{s},\,\lambda^k)\,\mathbf{L}(\mathbf{s},\,\lambda^k)\;\lambda^k\,\mathbf{L}(\mathbf{b},\,\lambda^k)\right\}\right] \tag{7}$$

$$= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n} \lim_{\substack{\mathbf{s}\,\rightarrow\,\mathbf{a}\\ \mathbf{s}\,\rightarrow\,\mathbf{b}}}\left\{-\,\mathbf{L}(\mathbf{a},\,\lambda^k)\;\mathbf{L}(\mathbf{s},\,\lambda^k)\,\mathbf{L}(\mathbf{s},\,\lambda^k)\;\mathbf{L}(\mathbf{b},\,\lambda^k)\right\}\right] \tag{8}$$

$$= \lim_{n\,\gg\,1}\left[\frac{1}{n}\sum_{k\,=\,1}^{n} \mathbf{L}(\mathbf{a},\,\lambda^k)\,\mathbf{L}(\mathbf{b},\,\lambda^k)\right] \quad \text{\{cf. Appendix B of Ref. [1]\}}. \tag{9}$$

Here the integrand of (6) is necessarily a unit quaternion $\mathbf{q}(\mathbf{a},\,\mathbf{b};\,\mathbf{s},\,\lambda^k)\in S^3$ since S^3 is closed under multiplication; (7) follows upon using (3); (8) follows upon using $\lambda^2=+1$; and (9) follows from the fact that all unit bivectors such as $\mathbf{L}(\mathbf{s},\,\lambda)$ square to -1. Using $I := \mathbf{e}_x\wedge\mathbf{e}_y\wedge\mathbf{e}_z$ with $I^2=-1$, the final sum can now be evaluated by recognizing that the spins in the right and left oriented S^3 satisfy the following geometrical relations {cf. Appendix A of Ref. [1]}:

$$\mathbf{L}(\mathbf{a},\,\lambda^k=+1)\,\mathbf{L}(\mathbf{b},\,\lambda^k=+1) \,=\, (\,+\,I\cdot\mathbf{a})(\,+\,I\cdot\mathbf{b}) \tag{10}$$

$$\text{and} \quad \mathbf{L}(\mathbf{a},\,\lambda^k=-1)\,\mathbf{L}(\mathbf{b},\,\lambda^k=-1) \,=\, (\,+\,I\cdot\mathbf{b})(\,+\,I\cdot\mathbf{a}). \tag{11}$$

In other words, when λ^k happens to be equal to $+1$, $\mathbf{L}(\mathbf{a},\,\lambda^k)\,\mathbf{L}(\mathbf{b},\,\lambda^k)=(\,+\,I\cdot\mathbf{a})(\,+\,I\cdot\mathbf{b})$, and when λ^k happens to be equal to -1, $\mathbf{L}(\mathbf{a},\,\lambda^k)\,\mathbf{L}(\mathbf{b},\,\lambda^k)=(\,+\,I\cdot\mathbf{b})(\,+\,I\cdot\mathbf{a})$. Consequently, the above expectation value reduces at once to

$$\mathcal{E}(\mathbf{a},\,\mathbf{b}) \,=\, \frac{1}{2}(\,+\,I\cdot\mathbf{a})(\,+\,I\cdot\mathbf{b}) \,+\, \frac{1}{2}(\,+\,I\cdot\mathbf{b})(\,+\,I\cdot\mathbf{a}) \,=\, -\,\frac{1}{2}\left\{\mathbf{a}\mathbf{b}\,+\,\mathbf{b}\mathbf{a}\right\} = \,-\,\mathbf{a}\cdot\mathbf{b}\,+\,0\,, \tag{12}$$

because the orientation λ of S^3 is a fair coin. Here the last equality follows from the definition of the inner product.

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[1] J. Christian, *Macroscopic Observability of Spinorial Sign Changes: A Reply to Gill*, arXiv:1501.03393; arXiv:1211.0784.