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# Abstract

I examine two claims that arise in Brown's account of inertial motion in *Physical Relativity* (2005). Brown claims there is something objectionable about the way in which the motions of free particles in Newtonian theory and special relativity are coordinated. Brown also claims that since a geodesic principle can be derived in Einsteinian gravitation the objectionable feature is explained away. I argue that there is nothing objectionable about inertia and that, while the theorems that motivate Brown's second claim can be said to figure in a deductive-nomological explanation, their main contribution lies in their explication rather than their explanation of inertial motion.

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## **1** Introduction

Conceptual analysis, at least in the analytic tradition since Frege, is the practice of identifying central features of a concept by revealing the assumptions on which use of the concept depends.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup>This way of expressing the basic idea of conceptual analysis is due to Demopoulos (2000, p. 220).

This approach to conceptual analysis has also been a part of the foundations of physics, at least since Newton. Conceptual analysis in physics is responsible to the body of theory and practice in which the concept is situated and in which it is interconnected with other concepts both physical and mathematical. The identification and explication of these connections, therefore, is a main objective of an analysis.

There is, however, an older tradition in which conceptual analysis does not proceed in this way; concepts are analysed with reference to systems of metaphysical and methodological enquiry. The intuitions that drive this kind of enquiry are held to bear decisively on the analysis of physical theories and the concepts they comprise. This tradition, at least so far as the theory of space and time is concerned, is exemplified in certain arguments offered by Leibniz, Huyghens, Berkeley, Mach, and Einstein.<sup>2</sup> This is not to say that these thinkers did not also pursue conceptual analysis in the sense characteristic of the analytic tradition, but their most famous and influential criticisms of Newton's theory reflect underlying conceptions of substance, action, and causality that are taken at least as seriously as their views about physics. These conceptions, furthermore, are bound up with views about how knowledge of the structure of the world is gained, about the nature of scientific explanation, and about what an empirical theory may legitimately postulate. Harvey Brown's Physical Relativity (2005) belongs to this tradition, and those claims that I will examine are motivated by a number of intuitions about inertial motion that have their source in Einstein. Like Einstein, Brown holds that there is something objectionable about inertial motion in Newtonian theory and special relativity. He calls the objectionable feature 'the conspiracy of inertia', and he claims that the conspiracy is explained away by Einstein's theory of gravitation.<sup>3</sup>

In this essay I will examine Brown's account of inertial motion in Newtonian theory and special relativity. I will argue that there is nothing objectionable about inertia in these theories,

<sup>&</sup>lt;sup>2</sup>This is not to say that Newton's views on space, time, and motion are free of philosophical intuitions. But Newton stands out among these thinkers for offering principles that constitute these concepts independently of any philosophical intuitions one may have about them. In this way, he ensures that the concepts are insulated from such intuitions, even his own.

<sup>&</sup>lt;sup>3</sup>These claims are separable from the principal claim of *Physical Relativity*, namely, that length contraction and clock retardation are in need of a dynamical explanation, and that such an explanation must come from a 'constructive theory', that is, a theory of the forces of cohesion that maintain a body's configuration. This view has been criticised by Norton (2008), Hagar (2008), Janssen (2009), DiSalle (2012), and Nerlich (2013, Chapter 5).

and I will argue that, while there is a sense in which Einsteinian gravitation explains inertia, the main contribution of the theorems that motivate Brown's second claim lies in their explication rather than their explanation of inertia.

The structure of the essay is as follows. In Section 2 I will begin by presenting Brown's account of space-time structure and the metaphor of a conspiracy. In Section 3 I will explore a number of metaphysical principles or intuitions that seem to underlie the conspiracy, and I will argue that they are inimical to our understanding of inertia in Newtonian theory and special relativity. In Section 4 I will consider an implication of accepting Brown's claim that there is something conspiratorial about inertia: I will consider the suggestion that, for a conspiracy theorist, all physical theories are conspiratorial. And I will argue that, even if such a view is defensible, there is little to gain by explaining away the conspiracy of inertia by appealing to Einsteinian gravitation, for one can point to conspiratorial features even in that framework. In Section 5 I will turn to Brown's claim that inertia is explained by Einsteinian gravitation because a geodesic principle can be derived from the field equations. I will review Weatherall's (2011b) challenge to Brown's claim. Weatherall argues that, if there is any sense in which Einsteinian gravitation can be said to explain inertia, then geometrised Newtonian gravitation explains it at least as well. While I agree with Weatherall, I will argue that there is a better way of thinking about the geodesic theorems. Their main contribution lies not in their explanation of inertial motion but in their explication of it.

## 2 The alleged conspiracy

There is a view that can be found in Einstein (e.g., 1922 [1950]; 1924 [1991]) according to which space-time structure explains the motion of free particles. Free particles and light rays run along the 'ruts' and 'grooves' of the affine geodesics of space-time, much as trains run along tracks. On this view, there is a causal inference from the phenomena of motion to space-time structure. This view is sometimes called 'the space-time explanation' or 'the causal-explanatory view'.<sup>4</sup>

<sup>&</sup>lt;sup>4</sup>For an entirely different account of space-time explanation, one that is explicitly non-causal, see Nerlich's *The Shape of Space* (1994b), *What Spacetime Explains* (1994a), and *Einstein's Genie* (2013, Chapter 8).

Brown's account of space-time structure is set against this view. The structure of spacetime is determined by the equivalence-class structure of some particular dynamical theory. For example, the structure of the space-time of Newtonian mechanics is the structure determined by the invariance of the laws of motion under the Galilean group; the structure of the space-time of special relativity is the structure determined by the invariance of Maxwell's electrodynamics (and, as would later be discovered, the laws governing all non-gravitational interactions) under the Poincaré group. In both the Newtonian and special-relativistic frameworks, 'geometric objects' or 'space-time structures', as they are sometimes called, encode the equivalence-class structures determined by the dynamics. For example, an affine geodesic codifies the trajectory of a free particle.

Einsteinian gravitation differs from these theories because the space-times corresponding to solutions of Einstein's equations are not given a priori, but depend on the distribution of mass and energy in the universe. For this reason, the elements of space-time structure that figure in these accounts and the symmetries they admit are not given a priori. In much the same way as in Newtonian mechanics and special relativity, however, elements of space-time structure codify certain important features of physical processes. For example, time-like and null geodesics with respect to the Lorentzian metric codify the trajectories of free massive test particles and light rays.<sup>5</sup> In all of these frameworks, space-time structures do not explain but codify important features of physical processes.<sup>6</sup>

Brown's view is significant for its rejection of the space-time explanation account.<sup>7</sup> But it has another aspect that is just as problematic as the latter view: Newtonian theory and special relativity commit us to accepting something questionable. This is illustrated with a number of

<sup>&</sup>lt;sup>5</sup>Brown (2005, pp. 161-168) appeals to the geodesic theorem to establish that time-like geodesics encode the trajectories of free massive test particles. He notes (2005, pp. 169-177) that the strong equivalence principle must be assumed for null geodesics to encode the trajectories of light rays. The strong equivalence principle is not merely Einstein's equivalence principle, according to which it is impossible to distinguish locally between a homogeneous gravitational field and uniform acceleration. Nor is it that principle together with the further claim that all non-gravitational fields couple to the gravitational field. It is the conjunction of these principles and the principle of minimal coupling, according to which no terms of the special-relativistic equations of motion contain the Riemann curvature tensor. The strong principle ensures that in the neighbourhood of any event the structure of space-time is approximately locally Lorentzian so long as tidal gravitational effects can be ignored.

<sup>&</sup>lt;sup>6</sup>For a detailed account of how space-time structures encode certain important features of physical processes in Einsteinian gravitation, see Malament (2012, §2.1-2.3).

<sup>&</sup>lt;sup>7</sup>See also DiSalle (1995; 2006a) for a critique of the space-time explanation account.

metaphors, among them, that of a conspiracy among the free particles of the universe. The metaphors can be found in a number of passages, of which the following are representative:

Inertia, before Einstein's general theory of relativity, was a miracle. I ... mean the ... postulate that force-free (henceforth *free*) bodies conspire to move in straight lines at uniform speeds while being unable, by *fiat*, to communicate with each other. (Brown, 2005, pp. 14-15)

A kind of highly non-trivial pre-established harmony is being postulated, and it takes the form of the claim that there exists a coordinate system  $x^{\nu}$  and parameters  $\tau$  such that  $[d^2x^{\nu}/d\tau^2 = 0]$  holds for each and every free particle in the universe. (Brown, 2005, p. 17)

... there is a prima facie mystery as to why objects with no antennae should move in an orchestrated fashion. That is precisely the pre-established harmony, or miracle, that was highlighted above. (Brown, 2005, p. 24)

... force-free particles have no antennae ... they are unaware of the existence of other particles. That is the *prima facie* mystery of inertia in pre-GR theories: how do all the free particles of the world know how to behave in a mutually coordinated way such that their motion appears extremely simple from the point of view of a family of privileged frames? (Brown, 2005, p. 142)

I propose the following as a synthesis of these and other passages that exemplify what I call

*Brown's alleged conspiracy:* As a matter of definition, the free particles of the universe are non-interacting, and thus cannot detect other objects or even determine whether there are any.<sup>8</sup> Yet, they seem to move in a mutually coordinated way. How do they know to move in the way that they do? Newtonian theory and special relativity commit us to thinking that there is a conspiracy among them. These theories assert that there exists a coordinate system  $x^{\mu}$  and parameters  $\tau$  associated with each particle such that the equation  $d^2x^{\mu}/d\tau^2 = 0$  holds.

To put the idea another way, one could say that the free particles of the universe agree not to accelerate and to follow geodesics of the space-time. Furthermore, particles that are themselves composites must satisfy the conservation of momentum. That is, the forces among the constituent particles must be equal and opposite, failing which the composite particles, by their internal forces, will accelerate of their own accord. Therefore, one could say that free composite particles must also conspire to maintain a state of equilibrium.<sup>9</sup> (I will address this in detail in Section 5.)

<sup>&</sup>lt;sup>8</sup>Note that we are considering here only the framework of the laws of motion. In Newtonian gravitation, there is an interaction among all of the particles of the universe—every particle attracts every other with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them.

<sup>&</sup>lt;sup>b</sup>Although the alleged conspiracy may be understood completely in these terms, it is worth noting that, for Brown, there must be more than *four* particles for there to be a conspiracy. This idea is motivated by a Lange-style path-construction proposed by Pfister (2004). While Brown's discussion of Pfister's construction goes part of the way

It is worth noting that the conspiracy metaphor is awkward. Free particles seem to conspire in spite of the fact that they *cannot* communicate. This, presumably, is what is 'miraculous'. But, pressing on with the conspiracy metaphor, one could still ask why the particles are prohibited from conspiring to move according to some law—for example, a law relating their motion to the distribution of mass-energy.<sup>10</sup> Perhaps the metaphor of a pre-established harmony is more apt.

There may be better metaphors or better ways of fleshing out the existing ones with the relevant physics. But I am not at all concerned with the metaphors themselves—I am concerned with the view that underlies them. I will argue that the view in question is driven by a number of metaphysical intuitions that are inimical to our understanding of inertia.

# **3** The conspirators unmasked

I will examine three metaphysical principles or intuitions that seem to underlie the allegation of a conspiracy. I will show that Brown's view, though set against the causal-explanatory account of space-time, is reminiscent of Einstein's view of inertia in Newtonian theory and special relativity. I wish to emphasize, however, that these are plausibility arguments—all of which have textual support, but none of which may account for the alleged conspiracy.

### **3.1** The action-reaction principle

The most notable of these principles can be extracted from Einstein's account of inertial frames in Newtonian theory and special relativity as 'factitious causes' of inertial effects. This is exemplified in his (1916 [1952]) illustration involving two fluid bodies  $S_1$  and  $S_2$  that are alike in size and constitution. The bodies are sufficiently far apart from each other and from other masses that they are subject only to those gravitational forces that arise among their constituent parts.  $S_2$ rotates with constant velocity about the line joining them.  $S_1$  is perfectly spherical while  $S_2$ 

towards explicating the mathematical requirements for inertia, I will argue that it distracts from what really underlies the alleged conspiracy, namely a set of metaphysical principles or intuitions.

<sup>&</sup>lt;sup>10</sup>Or, pressing the conspiracy metaphor further, do free particles conspire or is it the 'parts of space' that conspire to have certain symmetries?

bulges at the equator in the manner of a body subject to a centrifugal force. Einstein asks for the explanation of the difference between these bodies and he replies:

Newtonian mechanics does not give a satisfactory answer to this question. It pronounces as follows: The laws of mechanics apply to the space  $R_1$ , in respect to which the body  $S_1$  is at rest, but not to the space  $R_2$ , in respect to which body  $S_2$  is at rest. But the privileged space  $R_1$  of Galileo, thus introduced, is a merely *factitious* cause, and not a thing that can be observed. (Einstein, 1916 [1952], pp. 112-115)

Newtonian theory fails to give a satisfactory answer because the space R<sub>1</sub>—namely, the inertial frame—is invoked as the cause of the difference between the two bodies. This is philosophically objectionable because something unobservable is being granted a causal role and because this cause acts without being acted upon. Because of these features, Einstein holds the inertial frame to be a factitious cause, one that must be replaced by a genuine cause like the fixed stars. Einstein's illustration bears out what some have called 'the action-reaction principle': For something to be a physical entity it cannot act without being acted upon.<sup>11</sup> Passages supportive of this idea can also be found in *Relativity: The Special and the General Theory* (1916 [1939], pp. 171-173), *The Meaning of Relativity* (1922 [1950], pp. 54-55), 'On the Ether' (1924 [1991], pp. 15-18), and 'The Mechanics of Newton' (1926 [1954], p. 260).<sup>12</sup>

It is significant that Einstein's account of inertial frames in Newtonian theory and special relativity is based on a serious distortion of Mach's and especially Newton's views on rotation.<sup>13</sup>

<sup>&</sup>lt;sup>11</sup>Brown (2005, pp. 140-142) points out that a similar principle can be distilled from Leibniz's philosophy: For something to be a substance it cannot act without being acted upon. See, for example, the *Discourse on Metaphysics* (section 14) and *Monadology* (proposition 61). But Brown and Lehmkuhl (forthcoming, p. 3) question whether the action-reaction principle can be accurately attributed to Leibniz. So, in the end, it is left undecided whether or not the idea has anything to do with Leibniz.

<sup>&</sup>lt;sup>12</sup>Further mention of the idea can be found in Einstein's correspondence. See Brown and Lehmkuhl (forthcoming) for references.

<sup>&</sup>lt;sup>13</sup>The distortion to which I am referring can be found in Einstein (1916) and elsewhere (e.g., Rindler, 2006, p. 8). On the distorted reading of Newton, it is the inertial frames (sometimes, the absolute space) of Newtonian mechanics and special relativity that cause the rotating fluid body to bulge at the centre. On the distorted reading of Mach, it is the fixed stars that cause the bulge. But Newton did not regard inertial frames or absolute space as the cause of the bulging—he took this 'endeavour to recede' from the axis of motion as a criterion of rotation. In Mach's work we certainly do find the *suggestion* that some theory other than Newton's might provide a more fundamental or more encompassing account of inertia. This is his suggestion that the fixed stars be treated as causally relevant to inertial motion (Mach, 1901 [1902], p. 234). But his *criticism* of Newton's concepts of absolute space and time is distinct from this suggestion. He argued that claims about absolute motion tacitly refer to the fixed stars and not to absolute space; claims about the uniform flow of absolute time tacitly refer to the motion of some body that travels equal distances in equal times, and Mach proposed that we take as equal time intervals those in which the Earth turns through equal angles. For a careful analysis of Newton's and Mach's views on rotation, see Stein (1967) and DiSalle (2002; 2006a).

However, the principle that is motivated by this account may be considered in its own right. I will consider three objections one might raise against it—independently of anything Brown has written—before considering its bearing on Brown's view.

To begin with, one might argue that the action-reaction principle is neither a metaphysical criterion of physicality nor an epistemological criterion of legitimate postulation but an arbitrary invention or 'mere hypothesis'. One might object, as Norton (1993, pp. 848-849) and Pitts (2006, p. 349) have, that a spurious necessity has been attributed to a principle that derives from a vaguely Leibnizian or Aristotelian metaphysics, and thus is not empirically constrained. Second, one might argue that to think of an inertial frame in Newtonian theory or special relativity as something that acts without being acted upon is to misunderstand its role in these theories. Neither Newtonian space-time nor the inertial frame is being postulated as a theoretical entity that is the cause of inertial effects, for such a theoretical entity would go against Newton's theory in which one is always dealing with interactions in which the participants enter reciprocally. Rather, Newtonian space-time is the structure that is *implicit* in Newton's account of causal influence; Minkowski space-time is the structure that is *implicit* in special relativity. To be sure, Newtonian space-time and Minkowski space-time express constraints on the possible evolution of fields—in just the same way that the Hilbert space structure of quantum mechanics and the configuration space of classical mechanics express constraints on possible states of systems. But such constraints do not represent the action of these structures on the fields in Einstein's sense.<sup>14</sup> Third, one might point out that the action-reaction principle amounts to a principle that excludes a priori the possibility that space-time is flat. But whether or not it makes sense to think of spacetime as flat ought to be an empirical question. For example, the equivalence principle—it is impossible to distinguish locally between a homogeneous gravitational field and uniform acceleration—provides a basis for arguing that space-time is not flat. Therefore, there is certainly a basis for arguing that it does not make sense to think of space-time as unaffected by matter, but that argument is founded on an empirical hypothesis and not an a priori demand. Without the equivalence principle, one would have in Newtonian theory and special relativity space-time theories that are empirically unexceptionable. In such a case, the action-reaction principle would,

<sup>&</sup>lt;sup>14</sup>The recognition of this important point can be found in DiSalle (2002, p. 182), Brown (2005, p. 139), Pitts (2006, p. 349), Nerlich (2013, Chapter 8), and Brown and Lehmkuhl (forthcoming, pp. 2-3).

strictly speaking, express nothing but a metatheoretical or metaphysical preference for a different sort of theory—a sort of theory that, in the absence of the equivalence principle or something like it, would be difficult to motivate empirically.

Brown's view of inertial motion is bound up with the action-reaction principle, though the precise bearing of the principle needs to be carefully identified. A few positions can be found: endorsement (Anandan and Brown, 1995), but without any suggestion that the principle is violated by Newtonian mechanics and special relativity; a more measured stance towards the principle (Brown, 1996; Brown, 2005); most recently, a neutral historical study of the role played by the principle in Einstein's thought (Brown and Lehmkuhl, forthcoming). What is common to these works is the view that the global affine and conformal structures that are presupposed by inertial frames in Newtonian theory and special relativity do not play a causal role as Einstein claimed, and so do not violate the action-reaction principle. These structures are 'a codification of certain key aspects of the behaviour of particles and fields' (Brown, 2005, p. 142).

It is interesting to contrast Brown's view of space-time structures as a codification with Weyl's view. While Einstein held that inertial structure in Newtonian theory and special relativity is a factitious cause, Weyl held that once inertial structure is understood to be inseparable from gravitation it must be recognised as something that 'not only exerts effects upon matter but in turn suffers such effects' (Weyl, 1927 [1949], p. 105). He referred to the inertial structure of space-time as 'the guiding field' in analogy with other physical fields, notably fluids. Brown remarks that 'To appeal ... to the action of a background space-time connection in which the particles are immersed—to what Weyl called the "guiding field"—is arguably to enhance the mystery, not to remove it.' (Brown, 2005, p. 142) Weyl's account of the guiding field, with Brown's emphasis on its fluid aspect, enhances the mystery because free particles do not know what kind of space-time they are immersed in—as Brown has put it, 'they just do what they do' (personal communication).<sup>15</sup> Though Brown does not acknowledge Weyl's view except in these few remarks, it is safe to say that he dismisses the guiding field because it has a measure of explanatory power that is reminiscent of Einstein's causal-explanatory account and that goes

<sup>&</sup>lt;sup>15</sup>But there is another reading of Weyl that focuses on his account of the 'world structure' that is exhibited in inertial motion rather than on his account of the guiding field. See, e.g., DiSalle (2006a, pp. 137-149; 2006b).

against his view that space-time structure should be regarded as a codification or representational framework. His use of 'codification' in fact represents a deliberate deflation of inertial structure as something with explanatory power.

On Brown's account, the global affine and conformal structures of Newtonian theory and special relativity do not figure in a causal explanation of inertial motion. But his account has another aspect: Brown emphasises that these structures are absolute or global-they do not couple to matter. It is this lack of coupling that makes the world-lines of free particles 'wholly unexplained' (Brown, 1996, p. 186) and that 'enhance[s] the mystery' (Brown, 2006, p. 142). So, though Brown claims that the action-reaction principle is satisfied by Newtonian theory and special relativity, we may ask whether there is nonetheless a sense in which the principle is violated: Absolute or global background-structures constrain the evolution of fields without being themselves influenced by them. The proper account of this constraint, if not in terms of 'acting' or 'causing' as usually understood, is an open problem and one that has been examined at length, notably by Nerlich (e.g., 1994a, Chapter 7; 1994b, Chapter 2; 2013, Chapters 8 and 9). But, setting aside the question of how we should understand the relation of space-time structures to physical processes, there is at least a defensible interpretation of Brown's view on which the action-reaction principle is violated by Newtonian theory and special relativity and satisfied by Einsteinian gravitation. What is questionable, if the above arguments are on the mark, is whether the principle, in any sense, should be defended at all.

## 3.2 Global coordinate systems as an artifice of thought

Another philosophical intuition about inertial structure is found in Einstein's view that global coordinate systems are an artifice of our thought. Nature is indifferent to our choice of coordinate systems and does not single out certain kinds.<sup>16</sup> Einstein writes:

<sup>&</sup>lt;sup>16</sup>This intuition is bound up with the principle of general covariance, according to which the possible laws of physics should be restricted to those that admit a coordinate-independent formulation. The satisfaction of the principle of general covariance was supposed to be a philosophical advantage of Einsteinian gravitation, one that eliminated the 'epistemological defect' peculiar to Newtonian theory and special relativity with their global inertial frames. When Kretschmann showed in 1917 that Einsteinian gravitation is not unique in this respect, Einstein (1918 [2002], p. 242; 1951, p. 69) proposed an alternative to the principle that he took to capture the theory's characteristic feature and to surmount Kretschmann's objection: The possible laws of physics should not only admit coordinate-independent formulations but these formulations should also be the simplest and most transparent ones available to them. Einstein claimed that this methodological principle has 'significant heuristic force'. The notion of 'theories that are not the simplest and most transparent in generally-covariant form' means 'theories that, in addition to being generally

What makes this situation appear particularly unpleasant is the fact that there should be infinitely many inertial systems, moving uniformly and without rotation with respect to one another, that are distinguished from all other rigid systems. (Einstein, 1951, pp. 27-29)

There are a number of objections to this intuition. First, as DiSalle (2002, pp. 178-180) has argued, saying that it is inherently mysterious that nature should single out certain kinds of inertial frames and their associated coordinate systems amounts to saying that it is inherently mysterious that space-time should have non-trivial symmetries. It may be that the existence of such symmetries and the dynamical laws that exhibit them are themselves mysterious, but the sense of mystery derives from a philosophical view, whatever that might be. Even if one were committed to such a view-one more akin to a form of apriorism than empiricism-it would be equally problematic that nature distinguishes conservative systems from all other physical systems. Second, one might point out that this intuition is bound up with a mischaracterisation of the relation between dynamical laws and coordinate systems, namely the idea that Newton's laws only 'hold' in special coordinate systems. This idea can be found in the work of various authors (e.g., Einstein, 1951, p. 27; van Fraassen, 1985, p. 116; Cushing, 1998, p. 98), and there are passages in which Brown appears to be making such a claim: 'Inertial coordinate systems are those special coordinate systems relative to which the above conspiracy, involving rectilinear uniform motions, unfolds.' (Brown, 2005, p. 15) To put this another way, a class of special coordinate systems is being postulated in which the laws of motion-the laws that determine the alleged conspiracy-hold. But this is to put the cart before the horse. Newton's laws do not hold in special coordinate systems—they assert the *possibility* of coordinate systems in which all accelerations depend on impressed forces. The possibility of such systems is asserted by Newton's laws; those systems are not prerequisites for the laws of motion.<sup>17</sup>

There are passages of *Physical Relativity* that are reminiscent of the Einsteinian intuition that nature does not single out certain kinds of coordinate systems. There are also passages in which the existence of global coordinate systems seems to be conceptually antecedent to, or virtually the same as, the laws of motion and the states of motion they determine: '... we have

covariant, have other non-trivial symmetries'. In this way, we are returned again to the a priori demand to eliminate theories that assert the possibility of global structure, theories whose status one would prefer to think of as an empirical question.

<sup>&</sup>lt;sup>17</sup>See also DiSalle (2006a; 2002) in this regard.

been looking at the nature of the inertial coordinate system  $x^{*}$ , the existence of which was claimed to be tantamount to Newton's first law of motion.' (Brown, 2005, p. 26) At the very least, these passages raise the question whether global coordinate systems are part of what motivates the conspiracy allegation.

To this, Brown might reply that there could hardly be anything objectionable about global coordinate systems since, even in Einsteinian gravitation, space-time is approximately locally Lorentzian, and so a rigid coordinate system can be associated with it. If anything is objectionable, therefore, it is not these coordinate systems but the privileged state of inertial motion in Newtonian theory and special relativity that permits their use—a state of motion that, before Einsteinian gravitation, had to be postulated; it could not be derived from some deeper theory. The claim that this state of motion is unexplained and in need of an explanation will be examined in the next section.

#### 3.3 Inertia as a concept in need of explanation

There is another assumption underlying the allegation of a conspiracy that should be singled out, namely that inertia in Newtonian theory and special relativity is in need of an explanation. This assumption is expressed in many ways in *Physical Relativity* (2005). See, for example, pp. 14-15 and p. 141.

The idea that a proper theory of inertia should say something about the 'dynamical origins of inertia', and in this way explain the concept, is a persistent one. Such a theory would give an account of how inertial motion arises as the effect of some underlying interaction—for example, it might tie inertia in some way to the stability of atoms. But, if *this* is what is needed, it is significant that inertia is no more explained by Einsteinian gravitation than by Newtonian theory and special relativity.

Leaving aside the question of a dynamical origin in this particular sense, what seems to underlie the allegation of a conspiracy is the idea that inertia is unexplained because there is no more fundamental theory from which it can be derived. But this idea is at best unmotivated. It is useful to recall how the concept of inertia arises in Newton's account of causal interaction. The

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first law defines an ideal force-free trajectory, one from which a particle can be deflected by the action of some force. In this way, the law provides the basis for an account of force. The second law then defines and interprets the concepts of force and mass—the acceleration of mass is the measure of the action of some force. Because the second law expresses a criterion for distinguishing free particles from particles acted upon by a force, it might be argued that it alone is sufficient for articulating the concept of inertia: The first law is a limiting case of the second (where  $\mathbf{F} = 0$ ). The first law, however, serves to coordinate or associate the state of motion of a particle unacted upon by forces with a geometric notion, namely a straight line. The first and second laws, therefore, are interdependent. But the third law—and so the conservation of momentum—is also necessary to distinguish free particles from those acted upon by a force. That is, it is necessary provided that we are interested not only in ideal point-particles but actual composite bodies.<sup>18</sup> What this should establish, if it is not already evident, is that the concept of inertia relies on all three laws for its articulation.<sup>19</sup> Furthermore, it should be evident that the laws of motion are mutually complementary. Only taken together do they constitute the Newtonian dynamics.<sup>20</sup>

The idea that there is something objectionable about inertia because there is no more fundamental theory from which it can be derived rests on a demand for an explanation where none is needed. With the laws of motion Newton is offering mathematically formulated criteria for applying the concepts of force, mass, and inertia.<sup>21</sup> It is enough that the concept of inertia behaves according to these criteria and that the theory founded on that concept succeeds in

<sup>&</sup>lt;sup>18</sup>In Section 5 I address directly the use of the notion of point-particle in interpreting Newton's account of inertia. <sup>19</sup>For this reason, it does not make sense to ask: How is it that Newton can assert the first law of motion—before he has introduced a law that defines and interprets the concepts of inertial mass and force and the relation between them? This question can be found in analyses of Newton's account of inertia by such authors as Rigden (1987), according to whom that account is a 'logician's nightmare', and Pfister (2004), who calls it 'logically fallacious'. There are passages of *Physical Relativity* (2005), for example, pp. 14-16, in which the question seems just below the surface.

<sup>&</sup>lt;sup>20</sup>This important point has been made by many authors. See, e.g., Torretti (1990, Chapter 3).

<sup>&</sup>lt;sup>21</sup>What is more, the laws of motion express criteria for explicating and applying concepts that were *already* in use. For example, Huyghens saw clearly that determining places and velocities, accelerations and rotations implicitly presupposes a privileged state of uniform rectilinear motion relative to which they can be referred. But it was Newton who first offered systematic criteria for the application of the concept, as part of an empirically adequate dynamical theory. Or take, for example, the pre-theoretical concept of force as something determined by pushing, pulling or pounding some mass. The third law, too, is implicitly presupposed in the work of Huyghens, Wallis, and Wren on collisions.

explaining the phenomena that it sets out to explain. Inertia no more than force or mass is in need of further explanation.

In each of the foregoing lines of argument I have tried to expose and refute an assumption underlying the alleged conspiracy. But even if Brown were to reject some or all of these assumptions or to question the relative support that they lend to his total view, the burden is still on him to explain the philosophical basis that supports the allegation of a conspiracy—in other words, to explain why a paragon of an empirically-successful theoretical concept is in any way lacking in its explanatory credentials.

**4** If inertia is conspiratorial, should a broad class of theories be so characterised? Leaving aside the philosophical basis that seems to motivate the allegation of a conspiracy, one might ask: If in fact there is something conspiratorial about inertial motion in Newtonian theory and special relativity, should a broad class of theories be so characterised? As a way of making sense of why one might read a conspiracy into the motion of free particles, one might suggest that, for a conspiracy theorist, *all* physical theories are conspiratorial to varying degrees. But I will argue that this suggestion, in a sweeping sense at least, deflects attention from the ideas that truly motivate the conspiracy allegation.

It is helpful to consider a few examples that do not bear out the correct sense of 'conspiracy'. For example, one might say: If there is something conspiratorial about inertia, it is remarkable that there is nothing conspiratorial about the conservation of linear momentum, according to which the total momentum in an isolated system is conserved.<sup>22</sup> Take, for example, a system of billiard balls in free space. The total momentum of the balls, before and after a collision, is conserved. One could impute to a conspiracy theorist the view that the balls conspire to interact only with each other and not with their environments, and to transfer momentum among themselves such that their total momentum is conserved.

<sup>&</sup>lt;sup>22</sup>One might think that the conservation of linear momentum is in fact an excellent example since, in the case of an isolated system, the principle of conservation of linear momentum simply *is* the first law.

Or, if inertia is a conspiracy, why is there nothing conspiratorial about the motion of noninteracting charged particles in electromagnetic fields? Every electron interacts with a given electromagnetic field in exactly the same way. One could suggest that a conspiracy theorist might say that the electrons conspire to act in this way. But the conspiracy theorist might reply that the field is a common cause to which their motion can be attributed, and so the alleged conspiracy is explained away.<sup>23</sup> On this line of reasoning, it is the presence of a field that charged particles can 'feel' that distinguishes their movement from the conspiratorial behaviour of free uncharged particles.

To take another example, if inertia is a conspiracy, there is equal reason to think of equilibrium as arising from a conspiracy. There are many ways in which one might formulate such a conspiracy. To take a simple example, consider a rod in uniform translatory motion whose particles are in a stable equilibrium configuration. We might then Lorentz-boost the rod so it travels faster. The rod undergoes an acceleration before settling into a new stable equilibrium configuration. One could imagine that a conspiracy theorist might say that the particles conspire to reassemble themselves into the Lorentz-contracted rod. But no doubt a conspiracy theorist thinks of equilibrium as explicable by locally-acting forces and would therefore reject the comparison.

Though one might attempt to make sense of the conspiracy allegation in this way, this suggestion trivialises Brown's view. Not one of these examples captures the sense of 'conspiracy' or 'pre-established harmony' at issue for him. The principles and intuitions we have considered above reveal a view about absolute or global background-structures, structures that constrain the possible states of a system without being themselves influenced by the system's evolution.

To take an example that does seem to capture the correct sense of 'conspiracy', we might look to the theory of weak interactions. Consider chiral or 'handed' processes, that is, processes

<sup>&</sup>lt;sup>23</sup>For the same reason, there is nothing conspiratorial about the motion of a pair of harmonic oscillators—of similar constitution that are isolated from, and thus unable to communicate with, each other—oscillating at the same frequency. Presumably, there is a common cause in their past for the synchrony of the forces that produce the oscillations.

whose theoretical account displays a left-right asymmetry. If inertia is a conspiracy, there is equally good reason for seeing something conspiratorial in the handedness exhibited by parity violation in the theory of weak interactions. One could ask, why isn't there anything conspiratorial about the beta-decay of, e.g., cobalt 60 atoms? How do all of the cobalt atoms in the universe know to exhibit handedness in the same sense when they are oblivious to one another? One could say that they conspire to do so.<sup>24</sup> This example seems to have the salient feature: The relevant phenomenon, handedness, is tied to a global space-time structure, namely orientation. The natural reply to the conspiracy theorist is, of course, that cobalt 60 atoms display handedness not because they conspire but because parity-violating experiments are part of the evidentiary basis for orientation, one that we have specified in Minkowski space-time.<sup>25</sup> But, for a conspiracy theorist, the orientation of Minkowski space-time ought to be just as problematic as the global affine structure.

What seems to underlie Brown's view is the idea that absolute or global backgroundstructures are what might be called 'unexplained foundations'. They are unexplained in the sense that they cannot be derived from more general assumptions. But the idea that there is a problem with unexplained foundations is itself problematic; it seems to reflect a foundationalism that is difficult to motivate empirically. Even if Brown acknowledges that 'all explanation must stop somewhere', and so his view is not susceptible to any sort of regress, he still has to establish that, e.g., the global affine structures of Newtonian theory and special relativity are in need of an explanation—a view I have argued against in Section 3.3. Furthermore, if the notion of an unexplained foundation is indeed at the root of the conspiracy allegation, then there can hardly be much gain in explaining away the conspiracy of inertia by appealing to Einsteinian gravitation, for one can point to conspiratorial features even in that framework.

One could regard the global topology, metric signature, orientability, and temporal orientation, among other features of Einsteinian gravitation, as having the marks of a conspiracy,

<sup>&</sup>lt;sup>24</sup>This example is suggested by Brown (2005, p. 142; personal communication). A detailed discussion of parity violation in the beta-decay of a cobalt 60 isotope, in the philosophical literature, can be found in Huggett (2000) and Pooley (2003).

<sup>&</sup>lt;sup>25</sup>For details on specifying the orientability and orientation of a manifold, see Malament (2012, §2.1-2.2).

in this more specific sense.<sup>26</sup> Consider the Lorentzian signature of the pseudo-Riemannian metric. The Lorentzian signature of the metric does not come from the field equations—it is a separate assumption, one motivated by the Lorentz covariance of the non-gravitational interactions.<sup>27</sup> Temporal orientation must also be specified. By examining these and other features of Einsteinian gravitation, we find that, while the affine and conformal structures are determined by the distribution of mass-energy, the theory requires the postulation of a number of quantities that do not come from the field equations alone. If there is any sense in which Newtonian theory and special relativity are conspiratorial, then certain features of Einsteinian gravitation can be said to be no less conspiratorial.

To such a challenge, a conspiracy theorist might reply that, whenever we can explain the conspiratorial features of a theory by showing how they emerge from dynamics at a lower level, we have improved our understanding to a certain degree, we have shown that something miraculous at one level has a deeper reason. For example, Einsteinian gravitation explains remarkable structural correspondences that were previously taken for granted. If indeed all physical theories are conspiratorial—in the specific sense of having unexplained foundations— such a strategy may be available to Brown. But this still fails to address the more important question of why we should regard the features in question as problematic. For example, though the Lorentzian metric signature, orientability, and temporal orientation in Einsteinian gravitation do not derive from the field equations, they are not brute posits—their application is controlled by empirical criteria. If the notion of an unexplained foundation is what is driving the conspiracy allegation, it is better by far to argue that there are no conspiracies at all.

## 5 The alleged explanation of inertia by Einsteinian gravitation

In this final section I will address Brown's claim that inertial motion is explained by Einsteinian gravitation. I will begin by presenting that claim as well as Weatherall's challenge to it. I will then propose another way of thinking about the theorems that drive the claim.

 $<sup>^{26}</sup>$ A discussion of the metric type (pseudo-Riemannian) and signature can be found in Brown (1997). See also Brown (2009, p. 9), who acknowledges that we might regard the universality of local Lorentz covariance as 'mysterious'.

<sup>&</sup>lt;sup>27</sup>The strong equivalence principle, which incorporates the principle of minimal coupling, is also one of the assumptions that ensures that the structure of space-time is approximately locally Lorentzian—at least to the extent that we disregard matter-free vacuum solutions, where the principle is inapplicable. See Ehlers (1973) for a careful discussion.

Brown claims: 'GR is the first in a long line of dynamical theories ... that *explains* inertial motion.' (Brown, 2005, p. 141) Further on, he writes:

Inertia, in GR, is just as much a consequence of the field equations as gravitational waves. For the first time since Aristotle introduced the fundamental distinction between natural and forced motions, inertial motion is part of the dynamics. It is no longer a miracle. (Brown, 2005, p. 163)

Brown's claim rests on the fact that a geodesic principle—free massive test particles traverse timelike geodesics—can be derived from Einstein's field equations together with other assumptions.<sup>28</sup> The claim seems to presuppose a deductive-nomological scheme: One can take the field equations, energy and conservation conditions, and the resulting geodesic principle as *explanans*, then derive the motion of a free particle as *explanandum*.

There are various geodesic theorems, but Geroch and Jang's (1975) has a claim to being the most perspicuous, and I will limit my attention to it. Their theorem has the advantage of avoiding specific assumptions about the nature of the free massive test particle; it also has the advantage of showing that the particle traverses a curve in space-time rather than a line singularity. In any case, if any geodesic theorem can be said to figure in a deductive-nomological explanation of inertial motion, the Geroch-Jang theorem can be said to do so.

Brown's claim that inertial motion is explained by Einsteinian gravitation in a distinctive way was challenged by Weatherall (2011a), who showed that a geodesic principle can be derived in geometrised Newtonian gravitation. With this theorem in hand, Weatherall observes of inertial motion in geometrised Newtonian and Einsteinian gravitation: 'if *either* theory can be thought to explain inertial motion, then *both* do, in much the same way' (Weatherall, 2011b, p. 280).

A line of objection is available to Brown: Both Geroch and Jang's and Weatherall's theorems proceed from the conservation condition  $\nabla_a T^{ab} = \mathbf{0}$  which is assumed to hold at a

<sup>&</sup>lt;sup>28</sup>This is not quite Brown's claim. Brown claims that the geodesic principle follows *directly* from the field equations, a claim that Malament (2012), in light of the result of Geroch and Jang (1975), has shown to be not so straightforward. Brown grants this (Brown and Lehmkuhl, forthcoming, pp. 19-20) and maintains nonetheless that inertial motion is explained by Einsteinian gravitation.

point.<sup>29</sup> But, in Einsteinian gravitation, the conservation condition follows from Einstein's field equations; in geometrised Newtonian gravitation, it is an independent assumption. This line of objection is undermined by Weatherall, who argues that the conservation condition is a background assumption in both theories. It is an assumption that is more general than Newtonian and Einsteinian gravitation, an assumption about a general feature of the world that these theories and others respect.

Weatherall (2011b) offers several arguments for regarding the conservation principle as a background assumption. But, looking at his analysis more generally, it is important to understand this characterisation in his total view. Weatherall defends the idea that space-time theories ought to be seen as arising from a set of assumptions or constraints—as Weatherall (2011b, p. 280) has put it, from an 'interconnected network of mutually dependent principles'. On this view, Einstein's field equations are not attributed any fundamentality or priority as compared with the conservation principle, nor is their novelty—their non-linear character with its important implications—diminished.<sup>30</sup> These equations of mutual constraint are not taken as the 'fundamental dynamical equations', but as an element of a set of constraints, none of which, taken alone, is sufficient to give an account of the physical systems and processes of interest in gravitational physics.<sup>31</sup> For a sustained argument for this view and a careful study of the relation of energy conditions to the field equations, see Curiel (forthcoming).

<sup>&</sup>lt;sup>29</sup>There is disagreement about whether  $\nabla_a T^{ab} = \mathbf{0}$  has a legitimate claim to being a conservation principle. Some (e.g., Weatherall, 2011a, 2011b; Malament, 2012) refer to it as a conservation principle or condition without qualification, which is not to suggest that they are insensitive to the difficulties surrounding energy-momentum conservation principle in the usual sense, namely an integral conservation principle. Brown (2005, p. 141) notes that the equation is 'as close as anything is' to a conservation principle in Einsteinian gravitation; further on (2005, p. 161) he observes that 'it was appreciated from the very beginning that the presence of the covariant derivative and not the simple partial derivative in the equation makes this reading strictly untenable in curved space-time'. For this reason,  $\nabla_a T^{ab} = \mathbf{0}$  is sometimes (e.g., Stephani, 2004; Pitts, 2010) referred to as a 'balance equation'. See Byers (1999) for a particularly clear survey of the historical background to the problem of energy-momentum conservation in Einsteinian gravitation and the relation to Noether's theorems. See Weiss and Baez (2012) for a clear survey of the technical issues.

<sup>&</sup>lt;sup>30</sup>The nature of the constraint on matter fields that is effected by Einstein's field equations is discussed by Brown (2005, p. 163), who includes references for further reading.

<sup>&</sup>lt;sup>31</sup>It is noteworthy that, at least in the case of the Geroch-Jang theorem, the matter field is not required to satisfy Einstein's field equations. As Weatherall (2011b, pp. 277-278) notes, 'The absence of such a condition indicates that the matter described in the theorem is *test* matter, i.e., it is not a source term in Einstein's equation.' So, while the field equations do in general constrain the account of matter fields in Einsteinian gravitation, the theorem's restriction to test matter removes the difficulties that this particular constraint would otherwise raise.

I agree with Weatherall's observation that, if there is any sense in which Einsteinian gravitation can be said to explain inertial motion, then geometrised Newtonian gravitation can be said to explain it at least as well. But I will argue that the main contribution of these theorems lies not in their *explanation* but in their *explication* of inertial motion. By 'explication', I mean the clarification afforded by these theorems of the conceptual structure of Einstein's theory—of a certain account of matter, of the assumptions required for describing the evolution of that matter, and of the interdependence of these conditions—rather than any importance they might have in some or another philosophical account of explanation. This analysis does not diminish the fact that inertia can be brought under a deductive-nomological scheme, but it offers a deeper understanding of the concept than that scheme can provide.

The geodesic theorems make explicit an assumption that Newton makes in his own account of inertial motion. Current discussions of inertial motion in old-fashioned Newtonian theory focus on the laws of motion and the corollaries to the laws. But there is an underappreciated discussion in the Scholium to the Laws in which Newton shows that the third law-and thus the conservation of momentum-is necessary for the first law to apply to systems that are subject to attractive forces. The passage of interest to us is Newton's demonstration of the third law of motion for attractions. The proof is straightforward. Take any two bodies A and B that attract each other. Place between them an obstacle that impedes their coming together. Suppose, for reductio, that A is more attracted to B than B is to A. That is, suppose that  $F_{B \text{ on } A} \neq$  $F_{A \text{ on } B}$ . Bodies A and B will move towards each other, both eventually reaching the obstacle. The obstacle will be pressed more strongly by body A than by body B, and so will not remain in equilibrium between them. The stronger pressure of A against the system comprising the obstacle and B will make the entire system of the three touching bodies move straight forward in the direction from A to B. In empty space, the system will go on indefinitely with a motion that is always accelerated. But this contradicts Law 1. Hence, our supposition that  $F_{B \text{ on } A} \neq F_{A \text{ on } B}$  must be false. Hence,  $F_{B \text{ on } A} = F_{A \text{ on } B}$ .<sup>32</sup>

<sup>&</sup>lt;sup>32</sup>Newton is anticipating the application of the third law to the Solar System. He envisages the steps that he will take to show that the Solar System is effectively isolated.

Though this demonstration of the third law focuses on a system of bodies, it is significant that the law applies also to a single body that is itself a composite system. The first law, together with the second law which expresses a criterion for determining which particles are force-free, is satisfied only in the case of point-particles. For bodies that are themselves composed of particles, the third law is a necessary condition for inertial motion. That is, the system of particles making up a single body must interact in such a way that every force is balanced, failing which the body will accelerate by its own internal forces and violate the first law. Therefore, the notion of equilibrium—as it pertains to a single, composite body as well as to a system of bodies—enters Newtonian theory only via the third law.

Now one might object that Newton writes 'body' in the first law and not point-particle.<sup>33</sup> Further on, for example, in the proof of Proposition 1 of Book 1 and elsewhere in Book 1, Newton also uses 'body' and 'law 1' in the same breath. But even though he writes 'body', Newton is tacitly treating bodies as point-masses, at least in the early sections of *Principia*.<sup>34</sup> Furthermore, Newton's discussion in the Scholium to the Laws makes it clear that only the third law guarantees that a composite body—a composite of two or more simple particles held together by some force of cohesion—will obey the first law. So, though Newton does not always clearly identify where he is treating bodies as point-particles, the point-particle simplification is integral to his thinking and introducing it into the analysis of *Principia* illuminates the account of inertia.

Though it is often overlooked that the third law is a necessary condition for the first law to apply to systems held together by attractive forces, it was well understood by Newtonians in the eighteenth and nineteenth centuries with whom it was further elaborated and clarified. There are too many to consider individually, but it is important to mention d'Alembert, who, in his *Traité de Dynamique* (1743 [1967]), proposed a rational mechanics founded on laws of impact between perfectly hard bodies. Though d'Alembert was manifestly a Newtonian, the influence of Descartes on d'Alembert's thought can be clearly seen. D'Alembert sought to deduce the laws of mechanics from 'certain dispositions of size, figure and motion', in other words, from a purely

<sup>&</sup>lt;sup>33</sup>I thank a referee for raising this line of objection.

<sup>&</sup>lt;sup>34</sup>This is a feature of what I B Cohen calls 'stage one', before Newton gradually introduces, in 'stage two', further properties of physical systems. See the discussion of this in *A Guide to Newton's* Principia (Cohen, 1999, pp. 159-160).

geometrical account. From such a clearly and distinctly known geometrical basis, his laws of motion would be necessary truths and his mechanics would be a genuine metaphysical discovery. This view led d'Alembert to propose laws of motion that are close to Newton's laws.<sup>35</sup> In spite of its Cartesian aspect, however, d'Alembert's mechanics is a restriction of Newtonian mechanics to the mechanics of rigid bodies.<sup>36</sup>

D'Alembert understood clearly that Newton's third law, and therefore the conservation of momentum, must be assumed to give an account of the transfer of motion from one body to another in a collision. Newton's third law enters d'Alembert's mechanics as 'the principle of equilibrium', and the concept of equilibrium is the core of his 'general principle', which we now know as 'd'Alembert's principle'.<sup>37</sup> The principle asserts that 'all the forces acting on points of the system form, with the reactions against acceleration, an equilibrating set of forces on the whole system' (Thomson and Tait, 1867 [1879], p. 248). This is the culmination of the *Traité de Dynamique*; it represents d'Alembert's attempt to reduce the laws of mechanics to a single principle.

With the general principle in hand, d'Alembert deduced three theorems. The first, which is of greatest interest to us, asserts that '[t]he state of motion or rest of the centre of gravity of several bodies does not change by the mutual action of these bodies among themselves, provided that the system is completely free' (d'Alembert, 1743 [1967], Part II, Ch. 2, Theorem I). In this way, the principle of the conservation of the centre of gravity is recovered from his general principle. He deduced a second theorem, according to which 'if weight or an accelerative force— constant for each body and different, if one wants, for each of them—acts on these bodies following parallel lines, the centre of gravity or rather the common centre of mass will describe the same curve that it would have described if these bodies had been free' (d'Alembert, 1743

 <sup>&</sup>lt;sup>35</sup>D'Alembert was reluctant to write of forces. He eschewed the lingering notion of inherent cause and the vis viva controversy. He insisted that 'force' is only that quantity with which we are acquainted through its *effects*.
 <sup>36</sup>D'Alembert proposes to focus on bodies that act on one another by 'immediate impulse, as in the case of an

ordinary impact' or by 'the interposition between them of some body to which they are attached' (d'Alembert, 1743 [1967], p. 49). He considers attractions to have been sufficiently well examined by Newton, and so sets these actions aside.

<sup>&</sup>lt;sup>37</sup>D'Alembert's own statement of the principle (1743 [1967], p. 51) is not straightforward, but clear statements of its essential content can be found in the work of his successors. Thomson and Tait's statement is one such; other statements are found in Mach (1901 [1902], pp. 335-337). For a good, recent discussion of the principle, see Lanczos (1970).

[1967], Part II, Ch. 2, Theorem II). This theorem generalises the first to encompass those situations in which an isolated system is acted upon by a force that is sufficiently distant for the system to be treated like an isolated or 'near enough' isolated system.<sup>38</sup> A third theorem generalises the first still further to encompass systems subject to a constraint. In the Scholia to the Theorems d'Alembert notes that these theorems are equally true for attractions; so, though he deliberately restricts his attention to rigid-body mechanics, he acknowledges that his principle has wider applicability. D'Alembert's laws of motion and his general principle establish clearly that the total 'quantity of motion' or 'momentum' in isolated systems of interacting bodies is conserved.

The centrality of the conservation principle to Newtonian theory was equally well understood by Thomson and Tait in their *Treatise on Natural Philosophy* (1867 [1879]). In their discussion of Newton's laws, they observe that

Of late there has been a tendency to split the second law into two, called respectively the second and third, and to ignore the third entirely, though using it *directly* in every dynamical problem; but all who have done so have been forced *indirectly* to acknowledge the completeness of Newton's system, by introducing as an axiom what is called D'Alembert's principle, which is really Newton's rejected third law in another form. Newton's own interpretation of his third law directly points out not only D'Alembert's principle, but also the modern principles of Work and Energy. (Thomson and Tait, 1867 [1879], p. 240)

That the conservation of momentum is a necessary condition for the inertial motion of composite systems was noted in the same year by Maxwell in *Matter and Motion* (1867 [1888]):

... Newton goes on to point out the consequence of denying the truth of [the third law of motion]. For instance, if the attraction of any part of the earth, say a mountain, upon the remainder of the earth were greater or less than that of the remainer of the earth upon the mountain, there would be a residual force, acting upon the system of the earth and the mountain as a whole, which would cause it to move off, with an ever-increasing velocity, through infinite space. (Maxwell, 1876 [1888], p. 48)

<sup>&</sup>lt;sup>38</sup>Theorem II reveals d'Alembert's understanding of Newton's Corollary VI to the laws of motion. What is puzzling, however, is his suggestion that the forces may be different for each body. It may be that d'Alembert states Theorem II in the way that he does because he wants to acknowledge that Corollary VI contains an explicit (restrictive) hypothesis 'If bodies are ... urged by equal accelerative forces along parallel lines...' that is never strictly satisfied, except in the trivial case of zero accelerative forces. This reading seems to be supported by Sklar's (2013, pp. 120-121) interpretation of Theorem II as a generalisation of the principle of the conservation of the centre of gravity 'to include systems of particles all subject to the same external accelerating force, either constant and acting along parallel lines or directed to a point and distance-dependent'.

This vivid illustration of the application of the third law to a body that is itself a composite system establishes in still another way the fundamental role of the conservation of momentum.

What we find in the work of D'Alembert, Thomson and Tait, Maxwell, and others is a deliberate attempt to give a perspicuous account of the necessity of the conservation of momentum for the inertial motion of composite systems, a relation that is manifest in Newton's work but not sufficiently appreciated. As the Geroch-Jang and Weatherall theorems bear out, this relation is equally essential to Einsteinian gravitation and geometrised Newtonian gravitation. In old-fashioned Newtonian theory no less than in geometrised theories the account of inertial motion is not determined by any single principle susceptible of separate explanation but by an interdependence of physical principles that must be assumed together. Old-fashioned Newtonian theory and the geometrised theories are in fact analogous in their accounts of inertial motion: The third law of motion is to old-fashioned Newtonian theory as the conservation principles are to the geometrised theories. This analogy is clearly exhibited by the geodesic theorems, and it highlights the sense in which their contribution to our understanding does not lie in their explanation of inertial motion but in their explication of it.

One might object that the analogy is strained.<sup>39</sup> One might argue that, in the Geroch-Jang theorem, it is assumed that  $\nabla_a T^{ab} = \mathbf{0}$  represents the total energy-momentum only at a point. But what this assumption reflects is not a straining of the analogy, but the recognition of the inherent limitations of the theorem and an appreciation of the idealisations that it requires. What we want to ensure is that the free massive test particle that figures in the theorem is indeed free, that is, that its internal energy-momentum is conserved, that it is not exchanging energy-momentum with external fields, and that the background space-time metric can be kept fixed. To be sure, this represents a severe restriction on allowable models for the theory. But noting this restriction does not undermine the dependency of inertia on the conservation of energy-momentum.

The sense of 'explication' at issue has nothing to do with our ability to derive a previously unprovable proposition from a new theory, though, in the cases that concern us, the proofs contribute to that explication. Nor does this sense of explication have anything to do with any

<sup>&</sup>lt;sup>39</sup>I thank a referee for raising this line of objection.

particular philosophical account of scientific explanation, and so it is independent of the success or failure that attaches to such an account. Rather, the explication is the outcome of an *analysis* that began with the question, on what assumptions does our use of the concept of inertia depend? The analysis reveals that, in both old-fashioned Newtonian theory and in geometrised theories, inertia depends fundamentally on the conservation of momentum. Far from a concern with explaining the causal or dynamical origin of inertia, the geodesic theorems explicate the concept of inertia by explaining the connections between it and other concepts.

# **6** Conclusion

I set out to evaluate Brown's account of inertial motion in Newtonian theory and special relativity, in particular his claim that there is something objectionable—something conspiratorial—about inertia in these theories. I presented and clarified the conspiracy allegation, and I argued that it is motivated by a commitment to a number of philosophical principles or intuitions that are reminiscent of Einstein's view, namely the action-reaction principle, the idea that global coordinate systems are an artifice of thought, and the idea that inertia in Newton's theory is in need of an explanation. These principles reveal that the conspiracy allegation is bound up with a view according to which there is something problematic about absolute or global space-time structures. I argued that, even if Brown were to reject some or all of these principles, the onus would still be on him to explain why there is anything problematic about inertial motion in Newtonian theory and special relativity.

I proceeded to ask, if there is something conspiratorial about inertia, should a broad class of theories be so characterised? I considered the seemingly natural suggestion that, for a conspiracy theorist, all physical theories are conspiratorial. I examined and rejected a sweeping sense of 'conspiracy' that trivialises Brown's view. I then examined a narrower sense that is bound up with the notion that absolute or global background-structures are an unexplained foundation, and I pointed out that Einsteinian gravitation also has such features. I argued that, if indeed the conspiracy allegation is driven by this idea, then it is better to argue that there are no conspiracies at all.

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Last, I addressed Brown's claim that inertia is explained by Einsteinian gravitation because a geodesic principle can be derived from the field equations. I reviewed Weatherall's (2011b) challenge to Brown's claim. Weatherall argues that, if there is any sense in which Einsteinian gravitation can be said to explain inertia, then geometrised Newtonian gravitation explains it at least as well. While I agree with Weatherall, I argued that there is a better way of thinking about the geodesic theorems. That is, their main contribution lies not in their explanation of inertial motion but in their explication of it. This explication is independent of any philosophical account of explanation under which inertia can be subsumed; it is concerned with clearly exhibiting the assumptions on which our use of the concept depends.

I argued that the geodesic theorems of Geroch and Jang (1975) and Weatherall (2011a) explicate inertial motion by making perspicuous the dependency of inertial motion on the conservation of momentum. This is manifest, though under-appreciated, in Newton's own account of inertia, and I argued that the work of his successors—notably d'Alembert, Thompson and Tait, and Maxwell—represents a deliberate attempt to establish the fundamental importance of the conservation principle. In spite of their important differences, old-fashioned Newtonian theory, geometrised Newtonian gravitation, and Einsteinian gravitation are analogous in their accounts of inertial motion.

#### Acknowledgements

I wish to thank Harvey Brown, Jeremy Butterfield, Erik Curiel, Michael Friedman, James Ladyman, Stuart Presnell, Chris Smeenk, David Wallace, Jim Weatherall, and especially Robert DiSalle, Bill Demopoulos, and Wayne Myrvold. I also thank two anonymous referees for the *British Journal for the Philosophy of Science*. This work was supported by the Social Sciences and Humanities Research Council of Canada.

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