Conventionalism About What? Where Duhem and Poincaré Part Ways

(forthcoming in Studies in the History and Philosophy of Science)

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Abstract

This paper examines whether, and in what contexts, Duhem’s and Poincaré’s views can be regarded as conventionalist or structural realist. After analysing the three different contexts in which conventionalism is attributed to them – in the context of the aim of science, the underdetermination problem and the epistemological status of certain principles – I show that neither Duhem’s nor Poincaré’s arguments can be regarded as conventionalist. I argue that Duhem and Poincaré offer different solutions to the problem of theory choice, differ in their stances towards scientific knowledge and the status of scientific principles, making their epistemological claims substantially different.

Keywords: Conventionalism, neo-Kantianism, Structural Realism, Theory Choice, Underdetermination

1. Introduction

Pierre Duhem’s and Henri Poincaré’s philosophy of science, and especially their stance towards scientific theories, has caused significant debates in the philosophy of science literature. However, a unified and coherent understanding of their views is still to be given. Many regard Duhem and Poincaré as conventionalists (Ben-Menahem (2006), Reichenbach (1958), Popper (1959), Sklar (1974)), others regard them as instrumentalists (Laudan (1968), Stanford (2006)). More recently Duhem and Poincaré have been considered as the founders of structural realism (Giedymin (1982), Worrall (1989), Gower (2000)). However, none of these interpretations provide a fully coherent understanding of the motivations Duhem and Poincaré have in developing their conventionalism and its relevance to their general philosophy of science and especially with their alleged structural realism.

This paper disentangles the different arguments of Duhem and Poincaré, that are usually taken to motivate conventionalism or structural realism, in order to answer two questions: (1) what kind of conventionalism is developed by Duhem and Poincaré, and (2) do they develop structural realism and how does this thesis relate to their conventionalism. I distinguish three different types of conventionalism associated with Duhem and Poincaré: (a) as a position in the scientific realism debate concerned with the aim of science, (b) as a position arising from the problem of underdetermination of theories by the data, and (c) as an epistemological thesis regarding the status of scientific principles. I examine these different understandings of conventionalism in the context of Duhem’s and Poincaré’s arguments and evaluate whether these theses are rightly attributed to them. Last, I examine whether we can attribute a structural realist reading of their views.

The structure of this paper is the following. In section 2, I present Duhem’s and Poincaré’s views on the aim of science and oppose the traditional reading that renders these arguments to establish conventionalism understood as an anti-realist
position with regard to scientific knowledge. I show that the arguments establish structuralism motivated by the history of science. In section 3, I discuss the problem of underdetermination. I show, contrary to what is usually assumed, that Duhem is not led to conventionalism about theory choice. Rather than being arbitrary, for Duhem the choice is settled by a rational faculty he calls ‘good sense’. I also show, contrary to the mainstream literature, that Poincaré’s solution to the problem of underdetermination is driven by the employment of aesthetic values, which he takes to play a regulative role in scientific practice. In section 4, I examine Poincaré’s argument for the conventional and constitutive nature of geometry and mechanics and show how he develops, contrary to Duhem’s epistemological holism, a ‘hierarchy’ of science, which presupposes the Kantian synthetic *a priori*. It is in this context, I argue, that Duhem and Poincaré significantly part ways and as a consequence their structuralisms differ in a crucial way. I argue that according to Poincaré the structure of the world can only be reached by conventional and mind-dependent elements, rendering his structuralism an internalist one. I conclude with an analysis of Duhem’s and Poincaré’s understanding of structuralism and show the limitations of the current literature regarding their alleged structural realism.

2. The Aim of Science

Both Duhem and Poincaré are concerned with the question of what the aim of science is, what scientific theories can teach us about the world, whether there is an unobservable reality beyond the appearances, and whether we can know it. Duhem often seems to take an anti-realist approach to answering these questions – he holds that scientific theories should not search for any underlying ontology beyond what is observable, that theoretical terms such as ‘electrons’ and ‘genes’ should be eliminated from our theories, because we could never observe their alleged referents, and that science in principle should not aim at providing explanations but only representations of observational laws. According to him:

[T]he aim of physical theory is to become a natural classification, to establish among diverse experimental laws a logical coordination serving as a sort of image and reflection of the true order according to which the realities escaping us are organised. (Duhem 1954, 31)

In a similar manner, Poincaré argues that

The object of mathematical theories is not to reveal to us the real nature of things; that would be unreasonable claim. Their only object is to coordinate the physical laws with which physical experiments make us acquainted, the enunciation of which, without the aim of mathematics, would be unable to effect. (Poincaré 1902, 117)

Despite appearances, Duhem and Poincaré are not content with a purely anti-realist picture of science, they both develop several objections to conventionalism and defend the continuity and unity of science.¹

¹ It is important to note a distinction here between conventionalism and anti-realism positions such as instrumentalism. Duhem and Poincaré are often taken to defend a ‘global conventionalism’, equated in the contemporary literature with anti-realist positions such as instrumentalism. On this view, the aim of science is not to provide true descriptions of reality
2.1 Duhem on the Continuity and Unity of Science

Duhem argues that taking scientific theories to simply be convenient tools for prediction does not capture our conviction that scientific theories are continuous and make novel predictions, promoting in this way a more moderate position.

2.1.1 Arguments Against Scientific Anti-Realism

Duhem explicitly articulates two objections against scientific anti-realism, arguing that it cannot account for: (1) the predictive success of science and (2) the unity in science.

The first argument Duhem offers against scientific anti-realism is that it cannot explain the novel predictive power of physical theories. If theories did nothing but classify already known phenomena in some convenient way, how could we explain the fact that they manage to predict phenomena they were not constructed to classify? Duhem argues that physical theories are not artificial classifications but aim to be a 'natural classification' of the observable phenomena. According to Duhem, even though a physical theory cannot reveal the unobservable reality, it can still teach us something of the unobservable world, because “the more complete it becomes, the more we apprehend that the logical order in which theory orders experimental laws is the reflection of an ontological order, the more we suspect that the relations it establishes among the data of observation correspond to real relations among things and the more we feel that theory tends to be a natural classification” (Duhem 1954, 26). For Duhem, the strongest indication of a theory being a natural classification is its ability to make novel predictions (ibid., 28).

Duhem does not believe scientific anti-realism can adequately explain the ability of scientific theories to make novel predictions and thus suggests that a more optimistic stance towards successful scientific theories must be adopted. Duhem is nevertheless careful in articulating this optimism about scientific theories being natural classifications by using terms like “we apprehend”, “we suspect”, “we feel”. His position is fallibilist, allowing that even theories that make novel predictions might succumb and be abandoned. His position is also, as is made clear below, not full blown scientific realism.2

but, rather, to save the phenomena. On a ‘localised’ understanding of conventionalism, the thesis regards the status of scientific principles and does not make claims about whether scientific theories can describe successfully the unobservable. As Giedymin (1981) and Ben-Menahem (2006) point out, Poincaré’s conventionalism is local because it concerns only the axioms of geometry and certain constitutive principles in mechanics, while the conventionalism of his contemporaries Abel Rey and Édouard LerRoy is generalized.

2 It is this use of language that has motivated an alternative interpretation of Duhem’s concept of natural classification – neither as a selective realist position or anti-realist position. First developed by Darling (2003) and articulated in more depth by Bhakthavatsalam (2015), a motivational realist reading of Duhem’s concept of ‘natural classification’ claims that his aim is to explain scientific practice rather than, as traditional realists, to offer an explanation for the success of science or to give an account of theory’s approximate truth. This reading correctly points to the language Duhem uses for justifying our conviction, it is a ‘feeling’ an ‘intuition’. However, I disagree that Duhem offers no empirical justification for this conviction. Duhem gives us an empirical reason to justify our belief in a theory being natural classification – when it makes novel predictions. Thus, we can project expectation in the success of the theory by reflection on its past success at acting as a ‘prophet’ for us.
The second argument against instrumentalism is again based on the concept of natural classification and pertains to the ability of theories to be unified with other theories. Maxwell's theory of electromagnetism unified electric and magnetic phenomena into a single framework. Newton's theory of universal gravitation accounted for two distinct sets of phenomena: terrestrial and celestial. Duhem believes that this fact cannot be explained by the conventionalists who believe that distinct phenomena can be saved by different theories even if they are found to be in contradiction. Duhem asks:

Why give up the ideal of a completely unified and perfectly logical theory, when the systems actually constructed have drawn closer and closer to this ideal from century to century? [...] physical theory through its successive advances tends to arrange experimental laws in an order more and more analogous to the transcendental order according to which the realities are classified, that as a result physical theory advances gradually towards its limiting form, namely, that of a natural classification, and finally that logical unity is a characteristic without which physical theory cannot claim this rank of a natural classification. (Duhem 1954, 296-297)

Against his contemporaries Abel Rey and Édouard LeRoy, who believe that physical theories are convenient tools for prediction, suited for increasing empirical knowledge, Duhem argues:

But nature protests against this judgment; it declares that there exists a universal and necessary truth, and that physical theory through the steady progress which extends it continually while rendering it still more unified gives us from day to day a more perfect insight into this truth, so that it constitutes a veritable philosophy of the universe. (ibid., 332-333)

Duhem’s most clear defense of unification is stated at the end of his *To Save the Phenomena*. After a detailed discussion of the methodology of Kepler and Galileo, who claimed that their theories described 'real movements' rather than, like Osiander and Bellarmine held, simply saved the phenomena, Duhem argues that:

Despite Kepler and Galileo, we believe today, with Osiander and Bellarmine, that the hypotheses of physics are mere mathematical contrivances devised for the purpose of saving the phenomena. But thanks to Kepler and Galileo, we now require that they save all of the phenomena of the inanimate universe together. (Duhem 1908, 117)

Despite the fact Duhem disagrees with Kepler and Galileo's scientific realism, he also opposes the instrumentalism of Osiander and Bellarmine because Kepler and Galileo have managed to unify the celestial and terrestrial phenomena in a single mathematical scheme, a unification completed by Newton. This unification, for Duhem, cannot be explained by instrumentalism, according to which scientific theories are a 'rack filled with tools'.

Importantly, the selective-realist reading advocated here, and the motivational realist reading, need not be in conflict, as Bhakhthavatsalam (2015) has insightfully noted.
The above passages show that Duhem is not satisfied with an anti-realist view about scientific theories and wants to explain the historical success of science. Like scientific realists, he does not regard scientific theories as mere useful classifications; he believes the unity and novel predictive power of theories should be accounted for as well as the continuity of science. However, like instrumentalists, he does not believe science can provide us with a ‘perfect theory’, i.e. a complete metaphysical description of the world. Unlike scientific realists, Duhem argues that science cannot uncover the ‘realities’ hidden beyond the observable phenomena. However, he believes that a natural classification can teach us how these realities are classified. The next subsection explores in more detail the notion of ‘natural classification’ and its structuralist interpretation.

2.1.2 ‘Natural Classification’

According to Duhem, physical theory cannot reveal the unobservable ‘realities’ or ‘things’ that cause the phenomena we observe. However, the history of science reveals that despite the discontinuities occurring in theory change, we can reach some knowledge of the unobservable reality – we can learn how these ‘realities’ are related. Duhem distinguishes between explanation and representation and argues that the former belongs to metaphysics while the latter to physics. Taking physical theory to be a representation and not an explanation, Duhem argues that even at the ‘ideal end of science’ a natural classification will not reveal the real causes of the phenomena. We can never “strip reality of the appearances covering it like a veil, in order to see the bare reality itself” (Ibid., p. 7). But the more successful in their predictions our theories are, the more they manage to uncover the relations that hold among these ‘realities’ or ‘things’. According to Duhem, “physical theory through its successive advances tends to arrange experimental laws in an order more and more analogous to the transcendent order according to which the realities are classified” (Ibid., p. 297).

Duhem argues that only a ‘perfect theory’ could reveal the true ontological order hidden behind the appearances. A perfect theory would be a ‘theory of everything’ that saves all the phenomena. However, the scientific method can construct only an ‘imperfect’ theory. Even if science was to reach an end, a ‘perfect theory’, a theory that explains all the phenomena, will not be reached.

We do not possess this perfect theory, and mankind will never possess it; what we possess and what mankind will always possess is an imperfect and provisional theory, which by its innumerable groupings, hesitations and repentances proceeds slowly toward that ideal form which would be a natural classification. (Duhem 1954, 302)

According to Duhem, science is not an explanation because “it cannot attain this degree of perfection […] it cannot render accessible to the senses the reality it proclaims as residing underneath those appearances” (1954, 8). Science is “a system

Lugg (1990) and McMullin (1990) offer lucid presentations of Duhem's concept of natural classification, but their conclusions are in opposition. While Lugg argues that Duhem’s view is a convergent realism, McMullin argues that Duhem's view is a middle way between conventionalism and scientific realism. However, neither of these readings explains how the two seemingly opposing motivations for realism and instrumentalism can be reconciled into a coherent view.
of mathematical propositions, deduced from a small number of principles, which aim to represent as simply, as completely, and as exactly as possible, a set of experimental laws’ (ibid. 19). Duhem is also convinced that the evidence that science progresses and uncovers the true relations among things:

What is lasting and fruitful [in physical theories] is the logical work through which they have succeeded in classifying naturally a great number of laws by deducing from them a few principles; what is perishable and sterile is the labour undertaken to explain these principles in order to attach them to assumptions concerning the realities hiding underneath sensible appearances. (Duhem 1954, 38)

Worrall (1982, 1989) argues that Duhem’s position is best understood as structural realist, accounting for both the argument from theory change and the 'no miracles' argument. The idea is that at the level of ontology we should be anti-realists and believe that the entities postulated by our current theories will likely be abandoned, as was the case with the caloric, the ether and phlogiston. But we can be realists at the level of structure, which is expressed in the mathematical equations that survive theory change. According to Worrall's account the concept of natural classification can be understood as the structural content of the theory, content which is retained in theory change at least as a limiting case. This reading, which will be examined in more detail in section 4, captures Duhem’s mediating position between anti-realism and scientific realism.

2.2 Poincaré on the Continuity and Unity of Science

Duhem's opposition to anti-realism was inspired by the debate between Poincaré and LeRoy. LeRoy generalizes Poincaré’s argument for the conventional status of geometry and mechanics to defend a generalised conventionalism, termed at the time 'nominalism'. Poincaré opposes LeRoy's conventionalism because he believes that such an anti-realist position has undesired consequences, it does not account for the continuity and unity of scientific theories. In his chapter 'Is Science Artificial' from The Value of Science, Poincaré responds to LeRoy by arguing that if science has a value, because of its ability to make successful predictions, then this value must be due to the fact that theories are more than just practical recipes (Poincaré 2001, 320). Like Duhem, Poincaré sees science as “a classification, a manner of bringing together facts which appearances separate, though they were bound together by some natural and hidden kinship. Science, in other words, is a system of relations. Now we have just said, it is in the relations alone that objectivity must be sought.” (ibid., 347)

Poincaré's main concern is to account for 'the bankruptcy of science', a problem widely discussed during his time.

The ephemeral nature of scientific theories takes by surprise the man of the world. Their brief period of prosperity ended, he sees them

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4 The term 'structural realism' was first introduced by Grover Maxwell in his (1962), referring to the thesis developed by Bertrand Russell in The Analysis of Matter (1927). According to this view, our knowledge of the unobservable world is limited to second-order relations between the structure of our stimuli and the structure of their causes. Structural realism, as a thesis motivated by the problem of theory change, was first introduced by Worrall (1989), who traces the position back to Duhem and Poincaré.
abandoned one after another; he sees ruins piled upon ruins; he predicts
that the theories in fashion to-day will in a short time succumb in their
turn, and he concludes that they are absolutely in vain. This is what he
calls the bankruptcy of science. (ibid., 122)

However, Poincaré is not convinced that this argument from theory change, if
properly understood, shows that there is no progress in science and leads to anti-
realism about scientific theories. Poincaré continues his argument by defending the
structural continuity in theory change, by examining the transition from Fresnel’s
wave theory of light to Maxwell’s. Poincaré argues that while Fresnel understood light
in a different way, he nevertheless identified the correct relations between optical
phenomena and the equations that express these relations are preserved in Maxwell’s
theory. He argues that “[t]he true relations between these real objects are the only
reality we can attain, and the sole condition is that the same relations shall exist
between these objects as between the images we are forced to put in their place. If the
relations are known to us, what does it matter if we think it convenient to replace one
image by another?” (Poincaré 2001, 123)

Poincaré argues that science cannot teach us the nature of ‘things’ but it can
teach us the true relations among things. The evidence for this claim is the history of
science itself. “When we ask what the objective value of science is, that does not
mean: Does science teach us the true nature of things? but it means; Does it teach us
the true relations of things?” (ibid. 347-349).

Using the transition from Fresnel’s elastic ether theory to Maxwell’s theory of
the electromagnetic field, Worrall argues that there is continuity between the two
theories but this continuity is expressed in the mathematical equations rather than
their ontological interpretation. Fresnel and Maxwell attributed different ‘nature’ to
light, but they identified correctly its ‘structure’. The idea of structural continuity
according to Worrall, originates in both Duhem and Poincaré. Zahar (2001) shares
Worrall’s conviction that Poincaré is a structural realist “according to Poincaré, only
the universals, and more particularly the relations occurring in a unified and
empirically successful theory, mirror the ontological order of things. As for the nature
of the relata, it will forever remain hidden from us” (Zahar 2001, 37). Furthermore,
Gower argues that “for both Poincaré and Duhem [...] a defensible scientific realism
must be structural in the sense that it attributes reality to the relational structure of a
scientific theory” (Gower 2000, 86).

Several authors have argued that Poincaré clearly opposes anti-realism, especially at the end
of his career when he endorsed the existence of the atom after initially having opposed it.
David Stump (1989) argues that “[Poincaré] does not hold that theoretical objects are
unknowable in principle, rather, he thinks that one must look at the state of experimental
research at the time to see whether or not belief in the existence of a particular theoretical
entity is justified” (Stump 1989, 339). Stump’s interpretation would challenge our current
understanding of Poincaré’s structuralism, which would not allow for the commitment to the
existence and causal detection of unobservable entities. Kripke (1986), on the other hand,
argues that Poincaré's (1913)'s acceptance of the atom did not indicate a shift from
instrumentalism to scientific realism. Rather, the shift concerns the status of the atomic
hypothesis - from ‘indifferent’ it becomes ‘empirical’. Ivanova (2013) argues that Poincaré’s
acceptance of the atom is not in tension with his structuralism because the significance of
Poincaré's argument lies in the fact that he argues against atoms being fundamental entities.
According to this understanding of structural realism science is cumulative; in theory change there are elements of the old theory that are retained in the new one. This preservation is structural, not ontological, and is expressed in the mathematical equations of empirically successful theories. In the transition from Fresnel’s theory to Maxwell’s theory of electromagnetism, the equations of the former theory are completely preserved in the latter theory, receiving a different metaphysical interpretation.

In this section I have highlighted Duhem’s and Poincaré’s main arguments against conventionalism understood as an anti-realist position with respect to the aim of science. While both Duhem and Poincaré argue that science can at best produce approximate, and not true, theories, they do not want to adopt conventionalism. They believe that a more moderate position needs to be adopted if one wants to account for the novel predictive success of theories, their unifying power and the continuity in theory change. Duhem and Poincaré develop an epistemological view according to which our knowledge of the world is limited to its structure, the 'realities' – the unobservable entities that form this structure – remain epistemically inaccessible. The tenability of this reading will be further evaluated in section 5.

3. The Problem of Underdetermination

It is commonly believed that Duhem and Poincaré are lead to conventionalism because of the problem of underdetermination (Ben-Menahem (2006)). In this section I show that the focus of the underdetermination argument for Duhem and Poincaré is different and they develop different responses to it. While Duhem focuses primarily on articulating a solution to the problem of theory choice, Poincaré’s main concern regards the epistemological implications of the problem of underdetermination for the status of geometry. I present Duhem’s articulation of the problem of underdetermination and his solution to theory choice and contrasts it with Poincaré’s articulation of the argument for the underdetermination of geometry and his own understanding of theory choice and epistemic virtues. I show that neither Duhem nor Poincaré can be associated with conventionalism in the context of the problem of underdetermination.

3.1 Underdetermination, Theory Virtues and Good Sense

The problem of underdetermination of theory by the evidence, for Duhem, stems from the holistic nature of confirmation. Given that scientific hypotheses are always tested as a conjunction with auxiliary hypotheses and assumptions, a negative piece of evidence does not refute the testing hypothesis but the whole conjunction. As a consequence, neither deductive logic nor experience can isolate which of the tested hypotheses or assumptions are at fault. The holistic nature of confirmation allows, in light of a negative outcome of an experiment, for the modification of either the auxiliary assumptions used to derive the prediction or the testing theory. For Duhem, scientists faced with the problem of underdetermination have two choices. They could either modify the auxiliary assumptions to accommodate the new evidence or formulate a new hypothesis. However, there is no “absolute principle” which dictates this decision.

[T]he rules of syllogistic logic are not adequate. They must be assisted by a certain sense of soundness that is one of the forms of good sense […]
good sense will intervene at the moment at which one realizes that the consequences of a preconceived idea are either contradicted or confirmed by the experiment. [...] What a delicate task, concerning which no precise rule can guide the mind! It is essentially a matter of insight and ingenuity! (ibid., 23-25)

One traditional way for privileging a theory over its empirically equivalent rival is to employ theory virtues and choose the simplest, most unified, elegant or fertile theory. Duhem, however, does not believe that theory virtues are always sufficient for the resolution of theory choice. This is because different scientists can understand these virtues in different ways, leading to a ‘state of indecision’ in the community. Even if each individual scientist has made up their mind about their preferred theory on the basis of theory virtues, the community might have failed to agree upon a theory.

No doubt the physicist will choose between these logically equivalent theories, but the motives which will dictate his choice will be considerations of elegance, simplicity, and convenience, and grounds of suitability which are essentially subjective, contingent, and variable with time, with schools, and with persons; as serious as these motives may be in certain cases, they will never be of a nature that necessitates adhering to one of the two theories and rejecting the other, for only the discovery of a fact that would be represented by one of the theories, and not by the other, would result in a forced opinion. (Duhem 1954, 288)

What, according to Duhem, guides the scientist’s choice in weighing theory virtues and making the optimal or rational decision is a faculty he calls ‘good sense’. Duhem argues that every scientist possesses good sense, however, some exemplify it more than others – thus the disagreements occurring in science (ibid., 218). Also, he claims that good sense is closely connected to moral virtues, such as impartiality, sobriety, lack of self-interest. He claims that “in order to estimate correctly the agreement of a physical theory with the facts, it is not enough to be a good mathematician and skillful experimenter; one must also be an impartial and faithful judge” (ibid.). He condemns personal interest, stresses the importance of impartiality in science and suggests that developing one’s impartiality would lead to scientific progress. “[N]othing will delay the decision which should determine a fortunate reform in a physical theory more than the vanity which makes a physicist too indulgent towards his own system and too severe towards the system of another” (ibid.). Duhem claims that good sense is cultivated with scientific practice and scientists should aim to develop their good sense in order to accelerate the progress of science:

Since logic does not determine with strict precision the time when an inadequate hypothesis should give way to a more fruitful assumption, and since recognizing this moment belongs to good sense, physicists may hasten this judgement and increase the rapidity of scientific

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6 For a detailed presentation of Duhem’s notion of good sense and its properties in different contexts of scientific inquiry, see Ivanova and Paternotte (2013).
7 The accelerating property of good sense has been the focus of Ivanova and Paternotte’s (2013) interpretation of ‘good sense’. 
progress by trying consciously to make good sense within themselves more lucid and more vigilant. (ibid., 218)

There is an ongoing debate regarding the proper interpretation of Duhem’s notion of good sense. Here I am not interested in evaluating these accounts or defending the tenability of Duhem’s solution. What suffices for the purposes of this section is pointing at the fact that Duhem’s concept of good sense is an attempt to articulate a non rule-governed solution to theory choice. Recognizing the involvement of values and judgments in theory choice and attempting to provide an account that explains how these values and judgments lead to consensus in the scientific community, shows that Duhem is not content with a conventionalist response to the problem of underdetermination. Duhem claims that the notion of good sense can explain the history of science, where Duhem sees rational resolutions of theory choices despite the lack of empirical evidence or conclusive theory virtues. Whether Duhem’s solution is feasible is a subject of ongoing investigation. However, this solution shows that conventionalism about theory choice was not a position Duhem accepted and he believes there is space for rational solution to the problem of underdetermination.

3.2 Geometric Underdetermination, Conventions and Epistemic Virtues

Understanding Poincaré’s solution to the problem of underdetermination requires a distinction between his argument for the underdetermination of geometry from experience and the general argument for underdetermination of theory by the data. Poincaré’s alleged conventionalism is rooted in his articulation of the argument from underdetermination in the context of geometry. In chapter 4 of Science and Hypothesis, Poincaré presents his famous argument for the empirical equivalence of Euclidean and non-Euclidean geometries. He argues (2001, 55-56) that the appearances can be described either by employing a non-Euclidean geometry of constant negative curvature or by employing Euclidean geometry and modifying the laws of physics (adding heat forces that distort the measurements). There is, thus, a choice to be made as to which geometry to choose in order to describe the phenomena.

According to Poincaré, one could cultivate an alternative, observationally indistinguishable theory, by changing the geometry and modifying the physical laws. Experience, then, is compatible with alternative geometries, leading to the problem of choosing which geometry to be employed. However, this choice is not arbitrary and is limited to only three alternative geometries of constant curvature (Friedman (1999)). Many have suggested that this argument is a special case of Duhem’s argument for underdetermination of theory by experiment and leads to conventionalism about scientific theories. However, a more convincing interpretation is to take the argument to establish that geometry by itself makes no empirical predictions, that

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8 Stump (2007) and Fairweather (2012) argue that because of his appeal to moral virtues, Duhem’s notion can be understood as a form of virtue epistemology. Ivanova (2010) develops objections to these readings and offers an alternative account that she takes to fit better with Duhem’s general epistemology of science.

9 Ivanova (2010) develops objections to Duhem’s notion by arguing that good sense can justify a preference for a theory only retrospective, after new evidence has supported the theory, which makes the notion redundant.

geometrical knowledge is not empirical.\textsuperscript{11} The conventional epistemological status of geometry is established in the chapter 6 of \textit{Science and Hypothesis}, where Poincaré argues that geometry is neither synthetic \textit{a priori} nor empirical, rendering it conventional. I discuss this argument in detail in section 4.2.

The argument for underdetermination of geometry by experience leads to the problem of choosing which geometry to employ. Contrary to Duhem, who points at the ambiguity problem for aesthetic values, Poincaré argues that simplicity leads to a resolution of theory choice. On several occasions Poincaré claims that we resolve this choice by employing some 'conventions', which suggests that he does not attribute any epistemic significance to these values but treats them merely as pragmatic devices for choice. In the case of geometry, Poincaré believes that we have good reasons to choose one of the alternative geometries and this choice is guided by considerations of simplicity:

Euclidean geometry is, and will remain, the most convenient: first, because it is the simplest, and it is not so only because of our mental habits or because of the kind of direct intuition that we have of Euclidean space; it is the simplest in itself. (Poincaré 2001, 45)

Poincaré argues that we choose between different geometries on grounds of simplicity and convenience, making Euclidean geometry always the most convenient option. Since Poincaré renders the choice between Euclidean and non-Euclidean geometries a conventional one, it is important to clarify whether simplicity is also considered a pragmatic value when employed in choices between competing physical theories or whether it has any epistemic import. Poincaré claims that the question of whether nature is simple or not is irrelevant to our practices.

[T]hose who do not believe that natural laws must be simple are still often obliged to act as if they did believe it. They cannot entirely dispense with the necessity without making all generalisations, and therefore all science, impossible. It is clear that every law can be generalised in a number of ways, and it is a question of choice. The choice can only be guided by considerations of simplicity […] every law is held to be simple until the contrary is proved (Poincaré 2001, 113).

According to Poincaré, simplicity should be employed as a regulative rule in the selection of fact, forming of hypotheses and comparison of theories.\textsuperscript{12} This rule, he claims, is part and parcel of our intellectual capacities: “in formulating a general, simple, and formal law, based on a comparatively small number of not altogether consistent experiments, we have only obeyed a necessity from which the human mind cannot free itself” (ibid., 100). Poincaré gives an interesting justification for using simplicity as a regulative rule; he claims simplicity is linked to the ultimate aim of science, that of reaching a highly unified theory of everything (ibid., 112).

Poincaré believes physical theories can reveal real\textsuperscript{13} relations and are not simply convenient systems for prediction. The considerations we employ to prioritise

\textsuperscript{12} I explore Poincaré’s views on aesthetic values in my [unpublished manuscript].
\textsuperscript{13} In section 4.3 I clarify that for Poincaré what is real is ultimately what is commonly accepted by rational beings (his account is not an externalist one).
a certain theory over its empirically equivalent rivals might be pragmatic considerations leading us to choose the most convenient (simple) rival, but simplicity has an important regulative role for Poincaré insofar as it is linked to the goal of unification. To sum up, both Duhem and Poincaré articulate solutions to the problem of theory choice. For Duhem, the problem of theory choice emerges from the holistic nature of confirmation that results in underdetermination of theories by the data. This problem, which Duhem believes often occurs in scientific practice, does not lead to conventionalism. Despite the fact that epistemic virtues cannot always guarantee the resolution of the underdetermination, the choice is not arbitrary but is grounded in the scientist’s intuitions that guide them to the optimal choices. Poincaré, on the other hand, develops a different solution to the problem. He limits the choice of geometries to three alternatives, but does not regard it as arbitrary. He argues that simplicity is a regulative rule that picks Euclidean geometry as the ultimate option. While not being taken to have epistemic import, simplicity is regulative because it promotes the goal of science. As a consequence, neither Duhem nor Poincaré is content with regarding theory choice to be settled on arbitrary grounds. They both believe a rational choice can be made between competing rivals, disagreeing on the role of theory virtues in that choice.

4. Conventions, Constitutivity and Structuralism

While both Duhem and Poincaré advance the holistic nature of theory testing, Poincaré opposes the epistemological holism associated with Duhem. His view is motivated by his neo-Kantianism and conventionalism about the status of constitutive principles. In this section I present how Poincaré responds to Duhem’s epistemic holism by introducing a ‘layered’ account of sciences. I show the substantial differences between Duhem’s and Poincaré’s epistemologies and the structuralisms that stem from them.

4.1 Duhem’s Epistemological Holism

Duhem famously claims that theories are always tested holistically, and, in light of negative evidence, deductive logic cannot determine which element (or elements) needs to be revised; there is a certain freedom with respect to the revisions we can make to our theories so that they can accommodate the evidence. In this sense, any statement in a theory is equally subject to revision and no sentence bears a special epistemic status. Duhem rejects Poincaré’s distinction between constitutive conventions and empirical statements and claims that “hypotheses that by themselves have no physical meaning undergo experimental testing in exactly the same manner.

It is interesting to note that both Duhem and Poincaré do not only link simplicity and unity of a scientific theory to its utility, but also to its beauty. Duhem claims that “[o]rder whenever it reigns, brings beauty with it. Theory not only renders the group of physical laws it represents easier to handle, more convenient, and more useful, but also more beautiful” (1954, 24). Similarly, Poincaré claims that “the scientist does not study nature because it is useful to do so. He studies it because he takes pleasure in it, and he takes pleasure in it because it is beautiful. […] I am not speaking, of course, of the beauty which strikes the senses, of the beauty of qualities and appearances. I am far from despising this, but it has nothing to do with science. What I mean is that more intimate beauty which comes from the harmonious order of its parts, and which pure intelligence can grasp” (Poincaré 2001, 368).
as other hypotheses (1954, 216). Thus, any seemingly *a priori* or conventional statement could in principle be revised on the grounds of experience.\(^{15}\)

### 4.2 The Hierarchy of Science: Poincaré’s Rejection of Duhem’s Holism

Poincaré rejects Duhem’s epistemological holism and advocates the idea that different parts of scientific theories have different epistemic status – some are more epistemically fundamental than others. He distinguishes between three different types of scientific hypotheses – indifferent, conventional and empirical. Indifferent hypotheses are metaphysical hypotheses that cannot be observationally confirmed. Poincaré’s famous example of an indifferent hypothesis is the atomic hypothesis, which he originally regarded as purely fictional and after Perrin’s experiments admitted that its status changed from an indifferent to an empirical hypothesis (Poincaré 1913). Empirical hypotheses are testable hypotheses, e.g. the law of universal gravitation and the laws of electromagnetism. Conventional statements are statements that are neither apodictically nor empirically true but are fixed by convention and play a constitutive role in a scientific theory; they are, to use Poincaré’s term, ‘definitions in disguise’.

We also see that there are various kinds of hypotheses; that some are verifiable and, when once confirmed by experiment, become truths of great fertility; that others, without being able to lead us into error, become useful to us in fixing our ideas, and that the others, finally, are hypotheses in appearance only and reduce to definitions or conventions in disguise. (Poincaré 1913, 28)

According to Poincaré geometry and the fundamental laws of mechanics have a conventional nature, since they lack properly empirical content. They are, however, constitutive of empirical science,

we now arrive at the physical sciences properly speaking. Here the science changes: we meet with hypotheses of another kind, and we recognize their great fertility. No doubt at first sight our theories appear fragile, and the history of science shows us how ephemeral they are; but they do not entirely perish, and from each of them something remains. It is this something that it is necessary to discover, because it is this, and this alone, that is the true reality. (ibid., 29-30)

Poincaré’s idea is based on what he calls 'the series of sciences', which Friedman (1999) calls a 'hierarchy', according to which scientific knowledge is made possible by constitutive principles of different epistemic status. While Poincaré regards the most fundamental layer of arithmetic as synthetic *a priori*, at the end of the hierarchy we have the purely empirical laws.

\(^{15}\) Quine argues that some statements are more 'entrenched' than others and are thus less likely to undergo modification or abandonment. The notion of entrenchment, however, bears no epistemic weight. Our thinking that some statements have a special or fundamental epistemic status, in comparison to others, is “nothing more than a loose association reflecting the relative likelihood, in practice, of our choosing one statement rather than another for revision in the event of recalcitrant experience” (ibid., 43).
Let us describe how Poincaré understands the status and function of different elements in the hierarchy of science.\(^1\) Poincaré starts his description of the 'series of sciences' by examining the epistemological status of arithmetic. Arithmetic has synthetic a priori status because arithmetical knowledge is based on our reasoning by recurrence, what he calls 'mathematical induction'. This reasoning allows us to show that a given theorem does not hold only for a finite amount of cases, but can be taken to hold of all, infinite, cases. As he suggests, “reasoning by recurrence […] is the only instrument which enables us to pass from the finite to the infinite” (Poincaré 2001, 16). For Poincaré arithmetic is synthetic a priori: whereas it enriches our knowledge, since it takes us from the finite to the infinite (and is thus non-tautological), it is based on our intuitive capacity to represent infinite instances of the very same operation.

Having established the synthetic a priori status of arithmetic, Poincaré turns to the status of mathematical magnitude (2001, 24). He argues that we create the mathematical continuum by applying the law of non-contradiction using as a raw material the idea of the physical continuum, which we build from our sensory experience. Because the law of non-contradiction plays an essential part in creating the notion of mathematical continuum, Poincaré concludes, as in the case of the synthetic a priori status of arithmetic, that “this notion has been created entirely by the mind, but it is experience that has provided the opportunity.” (ibid.) Poincaré's next step is to introduce a way to measure magnitudes on the continuum and he does this by introducing the addition operation (ibid., 27). Friedman (2001) argues that even though Poincaré regards the theory of mathematical magnitude as a synthetic a priori, the addition operator required to make the continuum measurable requires the introduction of purely arbitrary elements, which shows the first introduction of a convention into the hierarchy. On the next level of Poincaré's hierarchy of science he places geometry – he takes the one-dimensional mathematical continuum, applies it to many dimensions and adds a metric.

Space is another framework which we impose on the world. Whence are the first principles of geometry derived? Are they imposed on us by logic? Lobatschewsky, by inventing non-Euclidean geometries, has shown that this is not the case. Is space revealed to us by our senses? No, for the space revealed to us by our senses is absolutely different from the space of geometry. Is geometry derived from experience? Careful discussion will give an answer – no! We therefore conclude that the principles of geometry are only conventions; but these conventions are not arbitrary (ibid., 6)

The laws of Newtonian mechanics are found on the next level of Poincaré’s hierarchy bearing the same epistemological status as geometry – that of a convention.\(^2\) The three laws of motion, according to Poincaré are not empirical but conventional because they serve as implicit definitions of the concepts involved in them – time, motion, mass, force, and without these definitions the laws have no empirical applicability.

\(^{16}\) Poincaré’s hierarchy of science is developed in Friedman (1999), chapter 3.
\(^{17}\) Interestingly, while Poincaré regards both the laws of mechanics and the axioms of geometry as constitutive conventions, their origin is different. While geometry is ultimately constructed by our mind by the a priori concept of a group (Poincaré 2001, 59), the laws of mechanics are idealized from our experience (ibid., 105, Poincaré 1913, 123-124). For Poincaré’s two-type conventionalism, see Stump (2015), chapter 3.
Having fixed the laws of mechanics, the next level of Poincaré's hierarchy contains all properly empirical sciences. Here, laws are allowed to be modified in light of experiment, since the constitutive framework has been fixed so all their central terms have been defined. As Poincaré explains:

We now come to the physical sciences, properly so called, and here the scene changes. We meet with hypotheses of another kind, and we fully grasp how fruitful they are. No doubt at the outset theories seem unsound, and the history of science shows us how ephemeral they are; but they do not entirely perish, and of each of them some traces still remain (ibid., 6)

The hierarchical picture of scientific theories shows that some principles bare different status. As Friedman (2001) stresses, this view strictly opposes the Duhem-Quine holism, according to which all sentences in a scientific theory have the same (empirical) status. On this framework, the likelihood of revision is established not on epistemological grounds but on purely intentional grounds – by how likely it is that the scientific community will want to revise a given statement. This is a very important conclusion; it shows where Duhem and Poincaré part ways – while for Duhem science is seen in a holistic way, where all sentences have the same epistemic status, for Poincaré science is a hierarchy where some elements have an asymmetric (constitutive) role and cannot be revised unless the whole theoretical framework is modified. For Poincaré the synthetic a priori and conventional elements serve a different function within the theory and are not on equal epistemic footing.

4.3 Where Duhem and Poincaré Part Ways

One of the most important tasks in evaluating Duhem’s and Poincaré’s place in the scientific realism debate regards whether the terms ‘structuralism’ and ‘realism’ can be attributed to their positions. From the passages I discuss in section 2, we can establish that Poincaré and Duhem defend some kind of cumulative structuralism. They believe that the content of a scientific theory amounts to its structural claims – claims about the relations among 'things' and not about the 'nature' of those things – and that those claims tends to survive theory change. Poincaré explicitly states that:

The aim of science is not the things in themselves, as the dogmatists in their simplicity imagine, but the relations between things; outside those relations these is no reality knowable. (1902, xxiv)

Both Duhem and Poincaré believe that the mathematical content of the theory is the only content that captures true relations and hold that these relations survive theory change. The structuralist idea at first glance appears compatible with conventionalism, in particular the idea of conventions as implicit definitions. Implicit definitions focus on the relational content between the elements that satisfy them just like the structural realist focus on the mathematical relations between the elements that satisfy them and not on the ‘nature’ of these elements. Two developments in mathematics particularly influenced Poincaré’s articulation of a structuralist position: Hilbert’s axiomatisation of geometry and Dedekind’s interpretation of arithmetic. According to Hilbert, geometrical concepts, such as lines and points, are defined only implicitly as whatever satisfies the axioms of geometry. The same idea is articulated in arithmetic, where
numbers are implicitly defined as elements that stand in a structure. However, contrary to the account of implicit definitions defended by Dedekind and Hilbert, Poincaré’s conventions are offered as a response to the question of how we link mathematical structure to the appearances.

While it is clear that both Duhem and Poincaré develop a structuralist account of theories, it is controversial whether their position is ‘realist’. The question at stake here is not about justification of belief in unobservable entities. Duhem clearly opposed such realism with his lifelong rejection of atomism. While Poincaré eventually accepted the atomic theory, showing that experimental evidence can shift the status of hypotheses concerning the unobservable, he did not defend an entity-type realism. However, the question at stake here is whether Duhem and Poincaré claim that science gives us epistemic access to a mind-independent world. Poincaré’s opposition to such realism is clear. When talking about the possibility of knowing a mind-independent reality, he argues that:

a reality completely independent of the mind which conceives it, see it or feels it, is an impossibility. A world as exterior as that, even if it existed, would for us be forever inaccessible. But what we call objective reality is, in the last analysis, what is common to many thinking beings, and could be common to all; this common part, we shall see, can only be the harmony expressed by mathematical laws. It is this harmony then which is the sole objective reality, the only truth we can attain. (Poincaré 2001, 14)

In another place in *The Value of Science*, Poincaré argues that:

It will be said that science is only a classification and that a classification cannot be true, but convenient. But it is true that it is convenient, it is true that it is so not only for me, but for all men; it is true that it will remain convenient for our descendants; it is true finally that this cannot be by chance. In sum, the sole objective reality consists in the relations of thing whence results the universal harmony. Doubtless these relations cannot be conceived outside of a mind which conceives them. But they are nevertheless objective because they are, will become, or will remain, common to all thinking beings. (Poincaré 2001, 350)

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18 According to Dedekind (1888), structuralism in arithmetic allows for the elimination of numbers from one’s ontology. Numbers are regarded as ‘free creations of the mind’ that have no properties over and above the properties they receive as elements in the system of arithmetic. The mathematical structuralism articulated by Dedekind is eliminative (see Gower (2000)).

19 Duhem refused to accept the atom on methodological grounds due to atoms being unobservable entities. “The school of the neo-atomism […] have taken up again with supreme confidence the method we refuse to follow. This school thinks its hypotheses attain at last the inner structure of matter, that they make us see the elements as if some extraordinary ultra-microscope were to enlarge them until they were made perceivable to us.” (Duhem 1996[1913], 238). An account of this rejection is developed in Achinstein (2007), Ivanova (2013) and Koko (2015).

20 For a presentation of the different epistemic and metaphysical presuppositions of scientific realism, see Chakravarthy (2007).
With the help of the discussion of Poincaré's hierarchy of science we can now evaluate where Duhem and Poincaré stand in the scientific realism debate. According to scientific and structural realism, reality has a mind-independent structure and our best theories provide true descriptions of this reality. These claims clash with the above-mentioned claims that a mind-independent reality cannot be known. Given that Poincaré's thesis is based on the Kantian premise that we cannot discover facts about the world that are independent of our cognitive apparatus, it directly clashes with the realist claim that that there is a mind-independent reality that scientific theories discover. According to Poincaré's epistemology, there is no direct access to the nature or structure of the world because scientific knowledge is conditioned on the conventions we decide to employ.\(^{21}\)

Despite the fact that Poincaré articulates the argument from continuity against the ‘bankruptcy of science’, an argument that Worrall (1989) famously takes to support structural realism, it is difficult to see how Poincaré’s overall view on the structure of scientific theories can be seen in a realist way. Whilst in a broad sense divorced from the synthetic \textit{a priori} in the context of geometry, Poincaré's view has Kantian elements. Poincaré still relies on the synthetic \textit{a priori} in arithmetic and claims that the construction of geometry, while guided by our experience, still had mind-dependent foundation, since the concept of a group from which we construct spatial geometry ‘preexists in our mind’. He makes knowledge of the world conditioned on our choice of geometry and constitutive principles.

This discussion necessarily leads us to consider the role and status of mathematics in Duhem’s and Poincaré’s epistemologies and this investigation requires another paper. Here I note that little work has been done in explicating Duhem’s approach to mathematical knowledge. While Poincaré emphasizes the constitutive and \textit{a priori} nature of mathematics, for Duhem “[m]athematics is the instrument necessary to construct all physical theory. But it is only a means, not an end” (1996, 25). Mathematics, for Duhem, helps us to classify phenomena in a concise way, to represent the phenomena in an elegant and convenient manner. While arithmetic and geometry are revisable on Duhem’s framework, for Poincaré they are synthetic \textit{a priori} or conventional. In \textit{German Science}, Duhem argues that mathematical axioms are ‘self-evident’ (Duhem (1991, 6-11)), in his (1906) he rejects Poincaré’s geometric conventionalism and hierarchical approach to sciences by arguing that any element is revisable in light of experience, even geometry and arithmetic. The important lesson to draw is that while Duhem takes mathematics as a representational too, for Poincaré it plays a constitutive role in our experience and access to the world.

To sum up, despite the fact the structural realist interpretation captures correctly Duhem’s and Poincaré’s aims to account for the argument from theory change and the argument for the novel predictive and unifying power of scientific theories, the metaphysical and epistemological claims of structural realism do not represent Poincaré’s view. By arguing that scientific knowledge is built on conventional and synthetic \textit{a priori} principles, Poincaré opposes the possibility of knowing the mind-independent structure of the world. Duhem, on the other hand, just as Quine after him, renders all statement in a conjunction of beliefs empirical and revisable in light of new evidence. As a consequence, despite the fact Duhem and

\(^{21}\) I explore this problem further in Ivanova (2015).
Poincaré seem to both advance a structuralist position, their views substantially differ with respect to how knowledge of the structure of the world is reached\(^2\).

5. Conclusion

This paper analyses whether and in what contexts Duhem’s and Poincaré’s views can be seen as conventionalist or structural realist and gives a novel, and hopefully more coherent, presentation of their views whilst also highlighting the essential differences between them. While it has been generally noted that Duhem and Poincaré are conventionalists because they do not believe that theories can reveal the underlying nature of the world, on a closer look their aim is to balance the pessimistic meta-induction with the no-miracles argument and show that we can have knowledge of the structure of the world. With regard to the problem of underdetermination, their views have also been misconstrued. Duhem’s solution to underdetermination is not a conventionalist one; he develops an account of rational theory choice despite his belief that aesthetic values are not always sufficient for choice. For Poincaré, on the other hand, choices are led by simplicity which promotes the ultimate goal of science, that of unification. Most importantly, when it comes to their epistemology, Duhem and Poincaré adopt very different views. Duhem’s epistemological holism renders all knowledge empirical and does not assign special epistemic status to any theoretical postulates. Poincaré, on the other hand, develops a hierarchical approach to science, where empirical theories are constituted by synthetic \textit{a priori} and conventional principles. While both express a form of structuralism, Duhem’s and Poincaré’s different understandings of the status of scientific principles makes their claims regarding structural knowledge substantially different.

Acknowledgments

I am grateful to two anonymous referees for this journal for their constructive comments.

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\(^2\) The tension of equating Duhem’s and Poincaré’s positions is noted by Bonnet and de Calan (2009) who claims that: "even by any stretch of the imagination, comparing Duhem and Poincaré seems rather far-fetched given their radical disagreement as far as the tenets on which the foundation of physical theory is based are concerned, that is to say the nature of the mathematical reasoning, the role of definitions, the existence of a continuity or not between the mathematical and physical methods, and the status of hypotheses in physics" (2009, 122).


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