The Meaning of Relativity and the Liberation of the Relationalists

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Abstract:

We analyse the various conceptual notions that go under the umbrella "relationalism/substantivalism". Our focus will be on evaluating the ontological status of spacetime in General Relativity (GR). To this end we systematically develop the ontological framework that implicitly underlies the traditional debate and common understanding of (classical) physics. We submit that spacetime with its chronogeometric and inertial structure, represented by the triple of the bare manifold, the metric and the affine structure, is best construed as the totality of possible and actual spatiotemporal relations of events. This can explain the non-fundamentality of general relativistic gravitational energy and suggests a non-causal, non-interactional understanding of the interdependence of matter in spacetime in GR.

Key words: Relationalism/substantivalism, General Relativity, Gravitational Waves, Analytic Ontology, Gravitational Energy, Action-Backreaction Principle

1. Introduction

This paper addresses the issue of whether spacetime is a substance or a relation, i.e. whether it exists independently of, "over and above" material things, as the stage on which the drama of the world unfolds[1], the thought being that only a substance can exist independently. More precisely our agenda is twofold[2]:

1. To offer some **semantic clarifications**, esp. w.r.t. the content of GR, i.e. outline an interpretation of the basic elements employed in the formalism of GR[3]: What do the various symbols of the formalism represent?
2. To propose an **ontological underpinning of GR,** and gravitational wave theory, in particular w.r.t. category/status of spacetime: What is the ontological status/category of those entities represented in the GR formalism?

We shall argue that spacetime, represented by $(\mathcal{M}, g_{ab}, \nabla)$, the manifold together with its metric and (metrically compatible) affine structure, is best construed not as a substance, but

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[1] For reviews, we refer to Dasgupta (2015), who focuses on the issue in classical mechanics, Huggett/Hoefer (2015), who are historically oriented. Comprehensive, systematic reviews are Earman (1989) and more recently, Pooley (2013).

[2] We'll specify our slightly idiosyncratic use of the terms shortly.

[3] We'll restrict ourselves to interpreting the standard formalism prevalent in the (textbook and non-mathematical-physics) literature, to which thus it is tailored. The interpretation of a more modern formalism, starting from the fibre bundle G, with total space E and structure group G, and then defining the manifold as $M = G/E$, cf. Socovosky (2011), may lend itself to a different ontology.

as the totality of possible and actual spatiotemporal relations between events; ontologically thus it is secondary vis-à-vis matter (in a sense to be made precise below).

"To resist the suspicion that this corner of the debate is becoming merely terminological"[4], we reassure the reader that the relational/substantival nature of spacetime has a direct implication for strategies of how to tackle a quantum theory of gravity[5] - a topic we take up elsewhere[6].

Here, we shall **proceed as follows**: In section II, we shall explain in what respects the debate so far has remained unsatisfactory –mostly due to terminological imprecisions about (ontological) categories. In section III we therefore compile the conceptual paraphernalia we need for a semantic and ontological analysis, and which we shall put to work in section IV, highlighting the Action-Reaction Principle, the construal of Wheeler's characterisation of the Einstein Equations as encoding interaction between matter and spacetime and the status of vacuum spacetimes. We shall devote special attention to the nature of the effects of gravitational waves and the extent to which our ontological analysis explains the non-fundamentality of gravitational energy in GR. We close (section V) with a comparison of the brand of "liberalized" relationalism we try to defend with more traditional forms.

2. The relationalism/substantivalism debate and its discontents

Before embarking upon both a semantic and ontological analysis, a terminological clarification[7] is in order. By **"semantics"** we mean the interpretation of all theoretical terms, i.e. assigning meaning to the formalism ("What do the symbols of the formalism refer to or represent?"); by **"ontology"** we mean a theory about the most general structures of reality, which explicates what all scientific theories (usually concerned with very specific hypotheses, e.g. the effects of impurities on the band structure of a specific solid) tacitly presuppose - or rather: take it for granted that these concepts are "intuitively clear". Such core concepts are, *i.a.*, "thing", "time" or "space". The ontologist thus intends to rationally (re-)construct a conceptual system ("ontology") that captures what reality ("ontics") is fundamentally made of. Reality is, of course, a given; things, their properties and mutual relations cannot be severed *in re* ("ontically"). As we shall see more clearly presently, rather than around "spacetime realism", i.e. the question of whether spatiotemporal relations exist, the relationalism debate revolves around the question concerning what the status or category of "spacetime" is in the logico-conceptual hierarchy of the ontology: In which category (e.g. a substance or a relation) does spacetime belong? How is this category

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[4] Pooley (2013), p. 63.

[5] As suggested, for instance, by Stachel (2014), Sect. 6.4 and Smolin (2005)

[6] Duerr/Lehmkuhl (2015c)

[7] To avoid misunderstandings: We do not follow the Quinean usage of "ontology" (as opposed to "ideology"), cf. Pooley (2012), p. 26; instead, we follow Bunge's usage, (1967) and (1977), for it captures in our opinion more clearly the questions at issue here (and furthermore is more continuous with common parlance in analytic ontology).

characterized? Which concepts does it presuppose? (Think of the analogous case in mathematics: The definition of an algebra presupposes that of a vector space.) What is the status of certain propositions that involve spacetime and its characteristics: Are they theorems that follow from a given set of axioms? The majority of philosophers of physics (wont to feel more at ease at *analyzing* scientific theories) are reluctant to *construct* a systematic, axiomatic ontology, which starts from certain primitive concepts, axioms and postulates[8]. Such an explicit construction of an ontology is called for, firstly, because no ontology can be readily read off the formalism (to be sure: neither can it be based on *a priori* navel-contemplation)[9]; secondly, without such a systematically developed ontology it seems unlikely be possible to assess unambiguously the ontological status of space and time –a question one might be interested in for several reasons. Analogously again, the very idea of assessing the status of, say, Euclid's Theorem -provable theorem? Postulated axiom?- makes no sense without any at least rudimentarily elaborated arithmetic.

With this in mind, let's articulate our **dissatisfaction about the extent literature** on the relationalism debate:

1. Firstly, **five distinct concepts** go under the umbrella **of relationalism/substantivalism**. At worst, terminological carelessness in this regard renders it difficult to fathom the respective authors' aim and whether they have actually achieved it[10]:
    a) "Realism about spacetime": Are certain structures in the mathematical formalism that are (for certain reasons) associated with "spacetime" indispensable or at least explanatorily potent, and should therefore we take the formalism seriously and regard these structures as representing something real? This spacetime realism is a speial case of scientific realism; it is neutral on whether spacetime is a substance, a relation or whatnot. However, if a theory that has been convincingly equipped with an ontology and interpretation, turns out to be empirically successful, scientific realism encourages us to take seriously this whole "web of beliefs", including its specific ontological framework.

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[8] A highly formalised axiomatised ontology is found in Bunge (1977) and, in the same tradition, an "exact philosophy of spacetime" in Vucetich (2011).

[9] Cf. Esfeld (2013)

[10] To give three examples of such careless terminology that invites misunderstanding:

- Baker, for instance, writes "A relationalist […] (holds) that all spatial and temporal properties are reducible to properties of material objects", Baker (2013), p. 1.
- Or, an even more misleading characterization of relationalism is given by Skow, viz. as "the doctrine that space and time do not exist" (Skow (2007), p. 1). Few relationalists, however, would subscribe to a Berkeleyian/ Borgesian view of spacetime as an illusion!
- Historically, "Mach himself never seems to have kept [Machian and Leibnizian relationalism] apart.", Huggett/Hoefer (2015), sect. 8.1.

b) "*Leibnizean* relationalism"[11]: This is the *ontological* claim that spacetime is the totality of relations, and not itself a substance – a dichotomy that usually presupposes a certain type of ontology, a s.c. substance ontology. However, alternatives to substance ontologies have been proposed in which events or properties and relations are primary and exist independently, constituting e.g. ordinary stuff as bundles of properties or relations. We will elaborate on this in the next section.

c) Spacetime subsistentialism: By that we mean the likewise ontological claim that spacetime can ontically subsist, i.e. whether -*irrespective* of the ontological category, e.g. substance or relation- it can exist independently of matter: Which is ontologically more fundamental, matter or spacetime, or are both on the same footing? In a substance ontology, things are the only entities that carry properties and exist independently[12]; consequently the conjunction of substance ontology and Leibnizean relationalism imply the negation of spacetime subsistentialism. Given the alternatives to substance ontologies, one could be both a Leibnizean relationalist and a spacetime subsistentialist simultaneously, though – a view that (ontic) structural realists supersubstantivalists leanings might find appealing.
A particularly virulent question[13] in the relationalism debate concerns the status of vacuum solutions: Do they exist or are they, as Einstein initially thought, merely formal solutions? – A question for the spacetime subsistentialist.

d) "*Machian* relationalism" aims at finding an explanation for the physical distinction of a class of reference frames identified as inertial frames. The Machian relationalist hopes to find it in a *physical* theory that only uses relative distances between material bodies and their derivatives as the fundamental spacetime-theoretical variables. Machian relationalism, with Barbour's Shape Dynamics[14] as its prime contemporary exponent, thus is a (non-mainstream) research programme in search of a theory to supersede GR; it therefore lies outside our present purpose of analyzing GR.

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[11] Friedman (1983), Ch. VI, coined the terms "Leibnizean" and "Machian relationalism", drawing attention to their distinctness. Huggett/Hoefer (2015) take up this distinction, but call it instead "Mach-lite" and "Mach-heavy", respectively.

[12] Sklar, who introduced the relationalism/substantivalism terminology, seems to implicitly presuppose such a substance ontology (which defines substances essentially by being carriers of properties and by subsistence, cf. Kuhlmann (2013), Ch. 7.3) when he characterizes substantivalism as the view that spacetime "can be said to exist and to have specified features independently of the existence of ordinary material objects" (Sklar (1974), p. 161.

[13] Cf. Earman (1989), Ch. 1.6

[14] Cf. Pooley (2013), Ch. 6.2

e) Eliminability/reducibility of spacetime structure: Our distinction bifurcates once more into "scientific reduction" and "ontological reduction":

   a. Whether a given property or a relation are reducible to other degrees of freedom, is a purely scientific question - once the philosophers have made more precise what is supposed to be meant by "reduction"[15]. (To avoid ambiguity, we merely add the rider "scientific".) We submit that solely by itself general-relativistic spacetime relations do not supervene on other degrees of freedom of matter. To see this, let us recall that a relation R supervenes on a property P, iff (i) each *relatum* of R instantiates P and (ii) the instantiations of P uniquely determine R[16]. Neither condition seems to be satisfied:

   - Firstly, recall[17] that Einstein Equations are a set of partial differential equations with a source density $T_{ab} = \frac{2}{\sqrt{|g|}}\frac{\delta}{\delta g_{ab}}\left(\sqrt{|g|}\mathcal{L}\right)$, which itself contains not only the metric g_{ab}, but also the matter Lagrangian $\mathcal{L}$. Apart from a mild constraint on the functional dependence of $\mathcal{L}$ that implements the (Einstein) Equivalence Principle (and thus ensures the validity of the Bianchi identities $\nabla^{\mathrm{b}}\left(\mathrm{R}_{\mathrm{ab}} - \frac{1}{2}\mathrm{R}\mathrm{g}_{\mathrm{ab}}\right) = 0$), GR does not specify the matter theory; in that sense it is not a fundamental theory[18]: The matter theory must be inserted by hand. Consequently, GR just by itself offers no matter properties P to begin with on which the spatiotemporal relations could potentially supervene.

     Suppose, though, we fortuitously stumbled on a model of matter on whose properties spatiotemporal relations would indeed supervene. It seems highly implausible that *any* (classical[19]) matter model would do the trick, so as to capture also the spatiotemporal effects elicited in that matter model! Conversely, it seems safe to assume that at least one (realistic) matter model exists on which spatiotemporal relations do not supervene.

   - Secondly, T_{ab} does not contain all degrees of freedom. At best[20], it encodes the energy-momentum of matter: That need not exhaust all there is to matter, though. In other words, the Einstein Equations are

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[15] A task the conceptual complexity of which should not underestimated, cf. Vanriel/Gulick (2014) for a review.

[16] Cf. Cleland (1985)

[17] Cf. Duerr/Lehmkuhl (2015b) for details

[18] Hoefer (2009), Sect. 4.2 also points this out.

[19] This rider is necessary, for a *unified* quantum description of matter evades the objections offered above. Such unified quantum descriptions of matter have been investigated recently, cf., e.g. Hedrich (2012) for a brief review. In fact, in Duerr/Lehmkuhl (2015d) we explicitly advocate such an emergent spacetime framework as an attractive possible approach to quantum gravity compatible with our results Duerr/Lehmkuhl (2015a,b,c).

[20] In fact, we argue that it does not even represent energy-momentum proper, cf. Duerr/Lehmkuhl (2015b).

    insensitive to matter degrees of freedom other than energy-momentum[21].

  b. Now to the notion of ontological reduction; it is a nonstarter: Relations in general form an *ontologically* irreducible or fundamental category (represented by all predicates other than unary ones), independently of whether certain relations are indeed scientifically reducible or not. This is reflected in the second condition for supervenience: The category of a relation is presupposed their as primitive.

In short: We shall not concern ourselves with Machian relationalism – a research agenda for an alternative to GR – and reject the idea of eliminating spacetime; instead, adopting scientific realism throughout, we shall examine whether a spacetime subsistentialist, Leibniz-relational ontology is defensible[22].

This leads us to a second complaint about the literature:

2. Namely the **tendency not to properly define the ontological categories employed**, first and foremost what exactly one means by the concept of substance/thing[23]. The respective texts restrict themselves to vague, metaphoric paraphrases (such as the examples we gave), obfuscating the debate with cryptic talk about spacetime being a "maybe pseudo-substantial thing"[24]: Recall that ψευδής means "false" or "lying"; consequently, such a substantivalist position would bizarrely be one where spacetime is not a substance after all! An even more misleading claim is that a substantivalist is a realist about spacetime: Whatever brand of realism one may

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[21] This blocks a standard argument against supervenience, invoking the fact that for the same matter energy-momentum, e.g. vacuum, the Einstein Equations can simultaneously yield different spacetimes, e.g. gravitational waves and Minkowski space.

[22] Benovsky (2010) has drawn attention to an interesting point concerning the structural equivalence of substantivalism and relationalism. In light of such structural equivalence he offers as one option (his "strong claim", p. 500) to regard the difference between the two as merely terminological. We take a more conservative stance (Benovsky's "weak claim"): Such an equivalence of two metaphysical positions only indicates similarities in their structure, i.e. the role certain of its elements play in the whole conceptual system at issue; the equivalence does not extend to the nature of the two systems. Such a structural equivalence of two theories that describe unconnected, vastly different phenomena are common in science, e.g. the equations of the Black Scholes option pricing model in essence take the form of the heat equation, while the change of temperature in a body is in no way related to the behaviour of financial markets.

[23] Teller, for instance, diagnoses: "[…] (N)either I nor, as far as I know, anyone else has offered a close analysis of what 'substantival' comes to in this discussion.", Teller (1991), p. 383. Earman's often quoted usage of "substance" in the sense that bodies and space-time points or regions are elements of the domains of the intended models of [space-time theories] of the physical world", Earman (1989), p. 114, somewhat circumvents the ontological problem. Although a powerful tool in other context, such as the issue of Background Independence, cf. Pooley (2015), the model-theoretic perspective is futile, when it comes to ontological questions, since it contains no ontological information whatsoever; it only deals with the mathematical *representation* of certain aspects of the target system. This becomes especially clear in its failure to connect to traditional themes of the (non-Machian) relationalism/substantivalism debate: For instance, the argument of vacuum solutions ceases to be cogently pro-substantivalist, if one precludes vacuum solutions from the notion of intended GR models.

[24] Huggett/Hoefer (2015), sect. 5.2.

advocate, for most human beings - amongst them also the majority of professing substantivalists – relations to their friends or their dog are eminently real. If our best scientific theories happen to contain terms that describe relations, nothing stymies a realist interpretation of them! (Evidently, there is no reason for such a realist to *reify* relations.) Even the more scrupulous accounts suffer from this ambiguity: Lehmkuhl, for instance, in his characterization of the substantivalist's core commitment, defines a substance as "a basic (or fundamental) object that is not derivative of anything else"[25] – a definition that leaves only postpones the crucial points: a) what counts as an object (would a process be one or a property?) and b) what precisely is meant as "derivable" or fundamental[26]? *Obscurum per obscurius.* - As we saw, *ontologically* all relations are irreducible/primitive/fundamental (and whether all relations are *scientifically* reducible is a bold claim!) – as Lehmkuhl himself argued elsewhere himself[27]. In short, our promising definition winds up as one that blurs the ontological difference between substance/thing and relations, rendering it *prima facie* of doubtful value in a debate about the substantival or relational status of spacetime!

Within analytic metaphysics key concepts such as "substance" or "process" have a clearly delineated meaning[28]. If we strive for a lucid and profound ontological analysis, we'd better learn from our metaphysics colleagues! Let's therefore start now with some basic ontological terminology.

3. Ontological framework

Following the Aristotelian-Kantian tradition, we shall adopt an explicitly thing-based or **substance ontology**[29,30], i.e. one that takes the class of things (substances) as the ontologically prior category (a more detailed specification will be given presently). The reasons for this choice are threefold:

- A substance ontology sticks close to our commonsense intuitions, roughly reflecting the background metaphysics most people (including physicists) primarily operate with[31]. Pragmatically, thus, it is a natural starting point.

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[25] Lehmkuhl (2015), p. 5.

[26] To be fair, in sect. 4 Lehmkuhl, loc. cit., elucidates the concepts of reduction (and emergence) in the context of super-substantivalism (whose core commitment consists in the tenet that "spacetime is the *only* (kind of) substance", loc. cit., p. 6).

[27] Cf. Lehmkuhl (2010), esp. sect. 6.

[28] Cf. Robinson (2014)

[29] Cf. Kuhlmann (2013), Ch. 7.3. Ch. 7 gives a brief contemporary introduction to analytic ontology in the context of modern (quantum) physics.

[30] We liberally draw on Mittelstaedt (1981) and Bunge (1977). The former emphasizes the Kantian notion of substance, showing its fertility for contemporary discourse. Bunge tailors his terminology to his original, formalised substance ontology.

[31] A substance ontology may be regarded as an instance of what Strawson calls "descriptive metaphysics", cf. Kuhlmann (2013), Ch. 7.1.

- Most combatants in the relationalism/substantivalism debate seem to have presupposed a substance ontology themselves – albeit only implicitly[32].
- On a more systematical level, a substance ontology turns out to be a natural framework for classical physics, i.e. one close to taking the standard formalism of classical theories at face value: As Scheibe has argued, it is "the modern concept of a physical system which comes nearest to the traditional notion of substance"[33]; he characterizes the latter by independent existence, monadic predication, individuality and completeness. (More on these s.c. ontological category features below.)
  In other words: A "classical ontology" (Mittelstaedt) helps elucidating the conceptual structure of (classical) physics – thus doing exactly what a (classical) ontology should do.

Now to the eponym of our type of ontology: We shall call an entity a **substance** or *substratum* (*ὑποκείμενον*) if it can be ascribed properties[34] - it is predicable. (In the following, "object", "thing", "substance" or "substratum" will be used interchangeably.) On top of being the carriers of properties, substances usually stand in relations to each other. Properties are represented by unary predicates, e.g.: "… is 2 feet tall"; relations by n-ary predicates (with $n \in \mathbb{N}^{\geq 2}$), e.g. "…is married to …". It follows that properties do not have properties themselves[35]: Only their conceptualizations, i.e. the predicates that represent these properties do. (Such a distinction is crucial for ontological consistency: if properties had properties themselves, they would count as substances; consequently the ontological difference between properties and substances would collapse.) For instance, the momentum of an acoustic wave is not differentiable; only the functions that *represent* the density fluctuations and their associated momenta of the air are. Likewise the distinction between intrinsic and extrinsic *properties* is misleading: According to the received terminology a property, e.g. mass, is called intrinsic, if the property holds independently of what other object exist and which properties they possess; an extrinsic property of an object, e.g. weight, by contrast, is one that holds in virtue of its relation to other objects. Within our nominalist/substance-ontological framework, properties don't exist *simpliciter*, though (only to the extent that there exist objects that possess them); hence, *a fortiori* there exist no distinction between intrinsic/extrinsic properties. Given that such a (generally useful) distinction by definition presupposes a relation, anyway, we propose the following way out to save it: Firstly the distinction holds only between *predicates,* i.e. the elements of our conceptual system we employ to represent properties; secondly, extrinsic predicates are only shorthand for or implicit ways of representing a relation (with a non-descript

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[32] Cf. Sklar (1974), p. 161, for instance, whose characterization of substantivalism is manifestly embedded in a substance ontological framework.

[33] Scheibe (1991), p. 215

[34] Moreover, Kant's formulations, which provide an impressive ontological underpinning of classical physics, can be understood as requiring that the predicate structure be Boolean, cf. Mittelstaedt (1981), Ch. 4.

[35] Cf. Bunge (1977), Ch. 4, esp. section 4, where the issue of properties of properties is explicitly discussed, as well as some potential objections.

relatum/a). For example, "Fritz is married." is an implicit way of saying "There exists somebody to whom Fritz is married.". In short, extrinsic predicates are relations in disguise.

Opting for a thing-ontology, we now postulate that things/objects, endowed with their respective properties, make up the fundamental furniture of the world[36,37]: Things exist independently (subsist); properties or relations don't. A property or relation is thus always a property or relation *of something* (*in re*): A universe is easily conceivable in which only a horse exist, "whereas a boundary could not even be imagined without something else whose boundary it is"[38]. Relations without *relata* don't exist. In this sense what we mean ontologically, when we say that a relation has changed, it is really the *relata* that stand in new relations. Substances, properties and relations are each *ontologically* irreducible or fundamental: These categories are disjoint; one cannot "reduce" (whatever that may mean) a generic binary predicate to a unary one – irrespectively of the possibility of a scientific reduction.

Let's take a closer look at properties. Amongst all properties, we posit that one is special, namely **mutability**, i.e. the ability of an object to change its state[39]. In turn, we define the state of an object as the totality of its properties. Scientific laws (provided by the individual sciences) are restrictions of the state space of objects, i.e. the set of all logically possible properties, to certain subsets[40] (representing nomological possibilities). **Energy**, we now stipulate, is a quantitative measure of mutability. (A motivation will be given below.) As such it is exactifies a meta-physical (rather than a merely physical) concept[41], in this respect resembling probabilities as quantifications of propensities[42] (with Kolmogorow's Axioms as the formal desiderata a specific probability measure should satisfy). To propose specific quantifications of metaphysical concepts (such as propensities or mutability) assess their empirical adequacy, their interrelations or even prove their uniqueness is one of the jobs of the individual sciences[43].

Remark 1: Having **no energy** is not the same as having zero energy: The latter presupposes that it's meaningful to assign the entity in question the respective quantity - albeit only with the quantification 0; whereas the former means that it isn't. An elementary example would

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[36] Cf. loc. cit., Ch. 3.4

[37] Like all other theories, every ontology is only "hypothetical, tentative and fallible" (Popper); there are no *a priori* reasons to prefer one (type of) ontology over another, for instance, one based on substances over one based on processes. In particular, one cannot "read off" an ontology from the formalism; it must be constructed explicitly. Pluralism of ideas is called for here as on all other intellectual turfs! (Recall our remarks in footnote 3.) The value of a proposed ontology is always evaluated *ex post*: Does it elucidate (philosophically) certain theories? Does it help us solve certain problems? Is it heuristically (and/or didactically) fertile? How does it fare in comparison with other proposals? A list of meta-criteria of good ontologies/metaphysics is given in Ch. 8 and 9 of Vollmer (1993).

[38] Kuhlmann (2013), p. 73

[39] Cf. Bunge (1977), Ch. 5, (1981), Ch. I and (2000)

[40] Cf. Bunge (1977), Ch. 3.2

[41] Cf. also Bunge (2000).

[42] Cf., for instance, Bunge (1977), Ch. 4

[43] There is no guarantee, though, that the quantification is unique –arguably one of the lessons from the quest for *the* right probability or entropy measure in statistical mechanics, cf., for instance, Sklar (2015), Sect. 2-4.

be: “The determinant of $\mathbb{Q}$ is 0.”, which is nonsense, whereas “$\mathbb{Q}$ doesn’t have a determinant” is an impeccable, true statement.

Remark 2: One must not be misled by ordinary expressions such as “The concept of marriage changed over the course of history”, which simply means that the same *word* was used for different concepts.

How to motivate now the above identification of energy as a measure of the ontological superproperty of mutability?

- Within a purely Lagrangian framework, it can be shown directly[44]: If we take the Lagrangian (or the action) to represent the network of relations that must hold between events (defined them as changes in the state of an entity, see next section) so as to be attribute them to one substance (or, put differently, so as to warrant dia- and synchronic identity of the underlying substratum), the canonical definition of energy as the Lie derivative of the Lagrangian along a time-like vector field literally evaluates the change of the Lagrangian it acts on along the flow of the time-like vector field. Note that the (arbitrarily chosen) time-like vector field here only serves the purpose of an ordering parameter amongst events. (We’ll elaborate the meaning of coordinates and the definition of a field in the next section.)
- The most general universally accepted definition of energy one finds in the physics literature arguably boils down to “capacity to do work”, thus involving forces. The latter induce a change of locomotion (i.e. of the mechanical state), which via the convertibility (and conservation) of energy extends to non-mechanical states. Note that this argument is independent from the previous one, since some classical scenarios (e.g. those involving electromagnetic radiation) do *not* admit of a classical Lagrangian formulation[45].
- By the same token, energy conservation together with a marker theory of causality – causation as a lawlike relation of change between events with energy transfer – link change and energy.
- Thinking of energy as a measure of mutability fulfills two functions by offering a unifying perspective[46]:
  - It accounts for the centrality and ubiquity of energy *within one* discipline. Here one predominantly studies how changes in one thing are related to changes in either the same or another thing. E.g.: How does the stability of a system change if you increase some of its parameters?
  - Likewise, it accounts for the centrality and ubiquity of energy across the various disciplines, where one predominantly studies how changes of things that belong to the domain of one discipline are related to changes in other

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[44] Cf. Duerr (2015a)

[45] Cf. Galley (2012)

[46] Cf. Bunge (2000). The need and fertility for such a unifying perspective should not be underestimated, especially both for the didactic and methodological training of scientists and science teachers, cf. Coelho (2009).

things or systems that belong to a different ("higher" or "lower") discipline. E.g.: How is an increase in the concentration of certain neurotransmitters (change on a biochemical level) related to the onset of a depression (change on a psychological level)?

Remark: **Substance conservation** and energy conservation are not the same[47]:

- The former is a fundamental ontological feature of substances, called "persistence": Material objects don't get created or destroyed out of the blue. Within all of classical/non-quantum physics, including GR, substances persist[48]. In a geometric spacetime setting, persistence translates into continuity of worldlines (or worldtubes for fields).
- Conservation of energy ontologically means that the extent to which objects are able to undergo change itself does not change: Everything changes, except, as it were, changeability. Conservation of energy does not generically hold[49] in GR: It depends on the presence of certain spacetime symmetries. This translates into variability of mutability.

As the reader will already anticipate, we are already squinting with one eye at exploiting this suggested connection between the non-existence of general-relativistic gravitational energy and relationalism. We shall return to this connection in detail in the next section. For the moment, let us collect the last items in our ontological toolkit.

Are all substances automatically material substances? For an answer let us further partition all substances into two categories: Those whose state space is a point (i.e. those substances which cannot change) and those whose state space contains at least two elements (i.e. those substances which can change). We call the former **material/matter** (or *concreta*), and the latter immaterial substances, or concepts (or *abstracta*). All familiar things such as cats or classical fields count as material substances; whereas Quetzalcoatl, mathematical objects or the beauty of Oriel College count as concepts.

Let us pause for a moment to justify this philosophically loaded (e.g. think of the medieval problem of universals!) terminology. We offer two arguments, both continuous with the main approaches to analytic metaphysics of *abstracta*[50]:

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[47] Bunge (2000) doesn't make this distinction – nor does he realise that energy conservation in general doesn't hold in GR. Interestingly, Kant, on the other hand, seems to have kept a "quantum of substance", which apparently corresponds to a conserved quantity associated with the homogeneity of time, and substance conservation conceptually distinct. Due to the (in his opinion necessarily) Euclidean nature of space and time imply each other, cf. Mittelstaedt (1981), p. 130, footnote 3.

[48] Cf. loc. cit. and Kuhlmann (2013), Ch. 7.3. Quantum physics forces us to revise this aspect of the classical ontology: The law of *conservation* of substance no longer holds, cf. Mittelstaedt (1981), Ch. 2 and esp. 4, and Kuhlmann (2013), Ch. 7.4 and Appendix B2. It can be argued that modern particle physics, i.e. QFT, forces us to jettison the concept of substance altogether (in the sense that QFT entities cannot be understood neither as fields nor particles), cf. loc. cit. Ch. 8 and 9. A fortiori, conservation of substance would no longer hold either.

[49] Cf. Hoefer (2000) or Duerr/Lehmkuhl (2015b)

[50] Cf. Rosen (2012)

- Apart from the cardinality of their respective state space, all the various paradigmatic instances of *concreta* and *abstracta* seem to have nothing in common; the cardinality of their state space is their “greatest common ontological denominator”.
- This ties in well with two common demarcation criteria of *concreta* vs. *abstracta*[51]*:* 1. The first one invokes the causal inefficacy of *abstracta*. Causality is a relation between events, which takes the logical form of a sufficient condition: A change c in one thing implies a change e in another thing. Since according to our definition *abstracta* cannot change, it follows that they can neither causally act nor be acted upon. 2. The second criterion invokes the non- *spatiality* of *abstracta*. We shall argue in the next section that, there are no “irreducible, monadic spatiotemporal properties like ‘is located at…’”[52]; instead, spacetime is the totality of spatiotemporal relations between events. As *abstracta* do not change, there are no events in them that can stand in spatiotemporal relations to begin with. For *any* brand of relationalism the argument takes an even sharper form in favour of the conceptual/abstract status of spacetime (thus ensuring virtuous circularity): As the relationalist regards the latter as the totality of spatiotemporal relations, they *constitute* spatiality (or rather: spatiotemporality). Consequently, spatiotemporal relations cannot stand in spatiotemporal interrelations themselves.

Back to our main line. Following the consensus amongst philosophers of science (barring perhaps those working in the philosophy of mind or mathematics), we adopt a materialist framework; we posit that only material substances really exist “out there”[53] (*ontically* subsist): Only material substances enjoy an *ontically* independent existence, whereas concepts enjoy only an *ontologically* independent existence: A cat, which can turn grey or fat, exists out there, whereas the concept of a cat does not exist *ante rem,* that is: The concept (“token”) does not dwell, as a self-sustaining entity, in a Platonic realm of ideas.

Note that we use concepts, i.e. the *ontically* non-real, (e.g. predicates) to *represent* the ontically real (e.g. properties); a conceptual substance (which by definition has properties) can thus represent a physical non-substance (such as a property), which by definition has none. We must keep this in mind, when trying to tailor an ontology we want to develop to the formalism, lest the conceptual substantiality of an object in the formalism decoy us into naively reifying what it represents.

Using the results developed so far, we can re-phrase a key insight: A **necessary criterion for an object to count as a material substance** is to have energy[54,55]. What about a **sufficient**

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[51] Cf. loc. cit., sect. 3.

[52] Earman (1989), p. 13

[53] Cf. Bunge (1981), Ch. I and V

[54] Cf. loc. cit. and Lehmkuhl (2011).

[55] Most authors who broach gravitational energy in the context of the substantivalism/relationalism debate likewise seem to (at least implicitly) cherish this view. For instance, for Earman and Norton the very categorical difference between substantival spacetime (“container”) and matter (“the content of spacetime”) hinges on energy: “If we do not classify such energy bearing structures [...] as contained within space-time, then we do

criterion then? We propose that possessing energy and satisfying certain energy conditions[56] as *jointly* sufficient and necessary for an entity to count as a material thing/substance[57]. Without going into further detail, let's only cursorily make this plausible: Energy conditions encode certain formalized meta(-)physical assumptions for matter[58]: For instance, the dominant energy condition captures the notion that energy-mass can never be transported superluminally; "if one drops the energy condition altogether, it is possible to construct bits of matter that propagate along *any* timelike curve. […] And if one weakens the (dominant) energy condition […] one can construct bits of matter that propagate along spacelike or null curves respectively."[59] In this sense, energy conditions serve as ontological/metaphysical selection rules for matter models.

For the benefit of the reader we list the ontological apparatus developed so far in the following table:

term/entity/ontological category	**definition/ features**
Substance, object, thing	predicability, ontological subsistence
state (space)	(totality of) possible properties
material substance, matter, *concretum*	mutability ($\lvert\text{state space}\rvert \geq 2$), ontic subsistence, persistence
concept, conceptual substance, *abstractum*	immutability, no ontic subsistence
Energy	measure for mutability
event in material substance ϑ	ordered pair of changes in ϑ: $(\text{state}_1(\vartheta), \text{state}_2(\vartheta))$
the World Θ	Totality of all things: $\{\vartheta : \vartheta \text{ is a material thing}\}$

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not see how we can consistently divide between container and contained", Norton/Earman (1987), p. 519. By *modus tollens*, substantivalism (as the claim that spacetime is the substantival receptacle in which all events of the material world take place) thus implies that whatever has energy counts as a substance.

[56] Cf. Poisson (2007), Ch. 2.1 or Malament (2012), Ch. 2.5 for an introduction.

[57] Appealing to certain energy conditions might turn out to be a helpful tool to distinguish between physical and non-physical ("geometric") fields in the context of alternative theories of gravity, esp. scalar-tensor theories, cf. Sotiriou et al. (2007). Generically, the Brans-Dicke scalar, which can be assigned an energy-momentum tensor, violates any energy condition. Thus, according to the above criterion it does not qualify as a material/physical scalar, which exists ontologically on a par with other physically substantival things. Essentially, many of the arguments from our discussion of the classical GR case carry over: The Brans-Dicke scalar represents a relation. With hindsight this makes sense, since the scalar was historically introduced to render the gravitational coupling constant a dynamical variable (thereby incorporating Dirac's Law of Large Numbers, cf. Weinberg (1972), Ch. 7.3); but without matter present gravity cannot couple to anything, so that the existence of the scalar (seen as the dynamically evolving strength of the gravitational coupling constant) (onto-)logically presupposes the existence of physical matter fields, and thus doesn't represent an autonomous entity. We will tackle an ontological analysis of Brans-Dicke theory in a future project.

[58] Cf. Curiel (2014c) for a comprehensive review. Note that the empirical discovery of fields that violate *certain* energy conditions may force us to revise the concomitant metaphysical assumptions (for instance about superluminally propagating causal mechanisms). Physicists already use energy conditions to ontologically categorise the fields in terms of substantival (physical) and non-substantival (geometrical), e.g., in the context of gravitational theories, cf. Sotiriou et al. (2007).

[59] Weatherall (2012), p. 20 (his emphasis)

We now hold in our hands the conceptual tools to get our semantic and ontological analysis to work. Methodically, note that an interpretation must be posited explicitly (via what Bunge aptly calls "semantic axioms"[60]).

4. Semantics and ontology of GR

Given a geometric spacetime theory (neo-Newtonian, neo-Minkowskian or general-relativistic), let's scrutinise now one of the main protagonists, the bare manifold $\mathcal{M}$ itself. Scientific realism demands "semantic completeness": In order to promote the mathematical formalism to an empirical theory each indispensable, non-mathematical/formal symbol must be assigned an unambiguous meaning[61]: What then does a point of the **manifold represent**? Since the representing mathematical entity should ideally somehow reflect the features of the real entity it represents, notice first that a point in a set lacks any intrinsic for properties[62]: Not even dimensionality or topology are available; they are properties of the manifold *set* to which the points belong! Accordingly, what a point represents had better not be a thing (which by definition would have to be predicable). Instead, the candidate that lends itself naturally for the office, is an event, defined as a change of the state of a thing θ, i.e. an (ordered) pair of its states, < state$_1(\theta)$, state$_2(\theta)$>. (It follows that an event is not a thing!) Multiply occurring events are represented by different manifold points. Identifying manifold points with events is in line with common practice in Special Relativity, where a correct understanding of length contraction and time dilation calls for the identification of the proper *events* of the respective situation: It's easy to conceive of experimental setups, such as in the Barn-Ladder-Paradox, where the measured time is contracted, whereas the measured length is dilated[63].

The chart (coordinate system), which maps manifold points $\mathcal{M}$ to $\mathbb{R}^4$, provides a mere labeling catalogue of events, – in and of itself boring book-keeping that, figuratively speaking, only specifies the format of the label (via the dimensionality) and the section of the archive in which to retrieve a file of a given label (via the topology). What ultimately we are interested in though, is the real order "hidden underneath the conventional labeling tags", the objective (invariant) relations between events - the *real* spatiotemporal structures: Spatiotemporal statements such as "The proper length of a between two events,

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[60] Cf. Bunge (1967) Ch. 1 and (1981), Ch. 10.

[61] Cf. Bunge (1967), Ch. 1

[62] Our view thus opposes e.g. Stachel's (2014). He takes the lesson from the hole argument to be that "the points of spacetime have quiddity [i.e. share the same intrinsic properties], but no inherent haecceity [i.e. properties that individuate things of the same quiddity]", p. 39. According to our thing-based ontology, only things possess intrinsic properties; non-things, such as processes or events, have no properties at all. Stachel's position seems to imply the existence of non-intrinsic, i.e. extrinsic properties, and hence relations that individuate otherwise indistinguishable entities. Relations one might, however, argue ontologically presuppose distinct *relata*, so that no longer it is clear how the distinctness of spacetime points can be reconciled with the absence of any individuating properties.

[63] Cf. van der Weele/Snoijer (2005)

A and B, in a particle with the affinely-parameterised curve $x^a(\tau)$ is ℓ: $\int_A^B d\tau \sqrt{|g_{ab}\dot{x}^a\dot{x}^b|} = \ell$." can thus be understood relationally[64] in a straightforward manner: Despite the coordinatisation being a conventional labelling system, it can be used to represent an objective pattern of relations –an objectivity that mirrored in the coordinate-independence of the spatiotemporal statement.

This also leads us to a **natural relational definition of a field**[65], namely as the *relatum* of the complex or network of relations amongst events, represented by the field variables. This network encodes the diatopochronic/spatiotemporal identity of the object, i.e. what connects two events that pertain to the same object. These relations that ground spatiotemporal identity are visualised by the 4D trajectories: A point particle with a worldline; a (local) field with a worldtube or worldcone. Let us spell out this sketch with a simple example: The different values of the scalar field ϕ represent the relations that hold amongst events that pertain to one substance $\mathrm{ref}[\phi]$. Here, to distinguish between these relations ϕ holding amongst events and the substance to which they pertain (i.e. the substance that changes), we have designated the latter by $\mathrm{ref}[\phi]$. Via a coordinisation (diffeomorphism) $\chi: \mathcal{M} \to \mathbb{R}^4$ one spreads labels over the bare manifold $\mathcal{M}$, which represents all possible events in (= changes that happen to) $\mathrm{ref}[\phi]$, thus inducing an (arbitrary) order amongst them. The "field configuration" $\phi \circ \chi: \mathcal{M} \to \mathbb{R}$ now represents the relations amongst possible events in $\mathrm{ref}[\phi]$. The dynamics of ϕ is governed by the field equations, that is they encode the different relations (represented by ϕ) between different events. Note that the use of the term "dynamics of ϕ" (or "evolution", which we deliberately avoided) *prima facie* suggests that ϕ is changing. This is misleading: ϕ represents how changes in $\mathrm{ref}[\phi]$ are related. The field equations tell us what these relations are; alternately, if one wants to speak in terms of change: how the relations between two events *vary* over different pairs of change. Given now some initial value data (taking the initial value problem ϕ to be well-posed), applying ϕ to the initial value data, which serve as reference values for the relations, now yields what future events in $\mathrm{ref}[\phi]$ will occur. Let this brief outline suffice for our present purposes.

What can we learn from such a relational interpretation of coordinates and fields? We submit, there are three immediate lessons:

- It refutes Field's popular argument that, since modern field theory by the very definition of a field assigns properties to spacetime points or regions, spacetime as a whole should count as a substance[66]. So our counterexample shows that, by itself, Field's argument does not conclusively demonstrate the substantiality of spacetime (understood here tentatively as whatever is represented by $\mathcal{M}$).

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[64] Dieks (2001) elaborates on this, discussing also this relational interpretation of coordinates in pre-relativistic settings.

[65] Cf. loc. cit., esp. section 5, for a similar account.

[66] Field (1980)

- In fact, in our whole discussion so far, the question of what $\mathrm{ref}[g_{ab}]$ is in particular has not yet been touched upon at all: More generally, the field theoretical formalism (more precisely: the field configuration) by itself only specifies the relations between events. It does not reveal the substance in which the events occur. For that one needs to supplement the formalism with an interpretation; the latter in turn requires an ontology. For instance, the 2-dimensional wave equation $\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}-\frac{1}{k^2}\frac{\partial^2\varphi}{\partial t^2}=0$ (with some given parameter k in units of a velocity) only relates unspecified events. That $\mathrm{ref}[\varphi]$ is a (homogeneous) membrane, e.g. the skin of a drum, fixed at both ends and with the properties of "linear mass density" (represented by μ) and "tension" (represented by τ), both of which related via $k=\sqrt{\tau/\mu}$), and that φ represents the vertical displacements of the membrane, requires the "semantic axioms" (Bunge) of an interpretation. The latter presupposes some (at least implicit, "intuitive") understanding of the ontological categories "substance" or "properties" that underlie the model with its ontological commitment to a predicated substance, called "membrane".
- Our definition of a field easily satisfies an important *desideratum* of relationalism[67]: No irreducible, monadic spatiotemporal properties *need* appear in a correct analysis of the spatiotemporal idiom[68].

Remark: The identification of coordinatisation as merely unphysical labeling deflates the meaning of the $\mathrm{diff}(\mathcal{M})$-invariance (General Covariance) of GR: Generally covariant equations take on a coordinate-independent form, reflecting, as we saw, that the relations expressed in them are objective. General Covariance of geometric spacetime theories ensures directly the objectivity and reality (independence of conventional labels) of the relations represented by generally covariant equations[69]. A change of coordinates only

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[67] As Earman's "third theme" R3 of traditional relationalism requires, cf. Earman (1989), p. 13.

[68] Dieks (2001) is also explicit about this.

[69] Our position over whether General Covariance has physical significance or is physically vacuous, cf. Norton (1993) for a historical review, is not that *per se* it has physical content. Rather we want to underscore two points:

- General Covariance *ensures* objectivity/reality of the encoded relations. As our focus has shifted to objective geometric characteristics and relations of worldlines, we're interested in the relevant invariants, which do not depend on the conventional choice of coordinate systems.
- General Covariance *inherits* physical significance from the geometrisation paradigm, i.e. the identification of inertial and gravitational effects. The recent drift of the debate towards gauge aspects, cf. Pooley (2015), comes close to our point: The gauge group of geometric objects (as mathematical objects) is indeed $\mathrm{diff}(\mathcal{M})$.

Our appeal to both objectivity and the geometrisation paradigm resembles the one Einstein seems to have had in mind with his "private" version of the point coincidence argument, cf. Stachel (2014), Sect. 2.5, according to which physical events are constituted by intersections of worldlines and therefore such intersections must be preserved, as reflected in General Covariance.

Notice how this sheds light on Kretschmann's trick to make any special-relativistic equation generally-covariant by essentially firstly re-placing the Minkowski metric by a general metric and secondly postulating that the associated Riemann tensor vanish, cf. Norton (1993), Ch. 5. What is going on here? The first step, the "general-covariantisation" of the originally special-relativistic equation, say, for the massive Klein Gordon Equation, i.e. the transition $[\eta^{ab}\,\partial_{\mathrm{a}}\,\partial_{\mathrm{b}}+\mathrm{M}^2]\phi=0\rightarrow[g^{ab}\nabla_{\mathrm{a}}\nabla_{\mathrm{b}}+\mathrm{M}^2]\phi=0$, reflects the fact that the described phenomenon is real/objective (coordinate-independent). So far, however, the general-covariantised Klein-changes the labeling - an intrinsically meaningless prop, anyway: A change in coordinates does *not* correspond to an active shifts. Diffeomorphism transformations only have a passive meaning: We simply use a different labeling system. This denial of equating "Hole Transformations" [70] with *active* shifts, which describe a physically distinct world, **blocks the Hole Argument**[71]. According to our relationalist reading of coordinates, diffeomorphically equivalent models describe the same world[72].

Having identified the manifold points as representations of events, one naturally might wonder: How are two events connected to each other? Here the differences between the metric and other fields will come to fore: The latter, e.g. a scalar field φ, only specify that there is a relation between events, as an otherwise primitive intrinsic pattern amongst events in the same substance $\mathrm{ref}[\varphi]$; the relations we represent via the metric on the other hand display a distinctly extrinsic character, expressing properties that hold in virtue of certain relations[73], rather than depending on the internal state of the objects involved.– a first cue that $\mathrm{ref}(g_{ab})$ may not be some all-pervading substance we call "spacetime". Upon closer inspection, how to connect two events comprises two facets:

1. Are there in some sense natural connections between events? Via which paths can two points be linked? Via which chain of events does a causal influence propagate from one event to another? This is what the geodesic/**inertial structure** (more

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Gordon Equation as it stands holds for every reference frame – obviously far beyond the domain of empirical validity. The second step, the additional postulate $R_{abcd}[g] \equiv 0$, now implicitly picks out a class of physically distinguished trajectories (spatiotemporal relations), namely those that move along geodesics w.r.t. to the Levi-Civita connection of Minkowski spacetime. With the general-covariantised Klein-Klein Equation in conjunction with this additional postulate, the claim that the relations represented by ϕ is restricted to the class of inertial frames.

The situation thus is completely analogous to GR: The Einstein Equations pick out the distinguished trajectories or relations, whereas General Covariance of a given general-relativistic matter field equation reflects the objectivity of the pertinent phenomenon.

(In more standard parlance the general-relativistic matter field equations and the Einstein Equations/Kretschmannisation pick out the dynamically possible and kinematically possible models, respectively cf. Pooley (2015), pp. 11.)

[70] An active shifting of all physical states of affairs, which would correspond to a displacements of objects plus their worldlines (leaving the coordinate chart untouched), *does* make a physical difference: If shifted trajectories deviate from geodesics, inertial effects occur; but even if the geodesic nature of a body's worldline is not altered, for instance, when we subject the body to a time-translation, an observable effect occurs: Think of shifting two points along great circles on the surface of a sphere; a shift can result in collision of the two points! Only for the special case of pseudo-Euclidean geometry do active shifts not result in discernible situations, cf. Nerlich (1994), Ch. 6 and 9. In short: While in a geometric spacetime setting invariance under passive shifts simply reflects coordinate-independence of the real effects, the claim that "General Relativity is distinguished from other dynamical field theories by its invariance under active diffeomorphisms." Gaul and Rovelli (2000), p. 30, is false: GR is not invariant under active shifts. The latter are physically discernible operations.

[71] Cf. Norton (2015) or Stachel (2014).

[72] Cf. Nerlich (1994), Ch. 6 and 9, for a similar point

[73] Cf. Weatherson (2012), esp. sect. 2.3

specifically: parallel transport), embodied by the –aptly dubbed- connection Γ^a_{bc}, provides[74]: A distinguished path, viz. geodesics, between events.

2. How are distances between events measured? This is **chronogeometric** function of the metric, which equips the distinguished path with a measure.

In the standard formalism our manifold is supplemented by both a metric and metric-affine structure. With the above elucidations[75] we have thus arrived at a natural ontological identification of the status of what both represent, namely relations: The metric-affine structure picks out those connections between events that are physically distinguished, viz. the inertial trajectories; the metric equips this path which a notion of spatiotemporal distance as well as the lightcone-structure.

But why should we postulate such structure in the first place? Let us apply the realist strategy mentioned in section 2:

- A*ll* established spacetime theories take for granted inertial frames as primitive, physically distinguished class of reference frames in order to account for all the inertial effects we experience directly every day. (We'll elaborate on this in a bit.) In geometric spacetime theories geodesics (as defined by parallel transport w.r.t. to the affine-structure) take over this role of distinguished trajectories[76]. Scientific realism now abductively encourages that the postulated inertial frames should be taken seriously in the sense that affine structure may be assumed to represent an element of reality, viz. inertial structure.
- By the same argument, since the metric structure features in the explanation of observable effects[77], such as gravitational redshift, we may again abductively infer that it corresponds to something real. Its geometric interpretation as a measure of spatiotemporal distances is vindicated via the experimentally well-corroborated s.c. Clock Hypothesis[78], which equates the proper time measured by a commoving clock with the length of that cure as determined by the metric.

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[74] Equivalently, of course, one may assign the geodesic structure to a derivative operator ∇ with which the manifold is endowed, cf. Malament (2012), Ch. 1.7. The connection and the derivative operator are uniquely determined by $\nabla_{\partial_a}(\partial_b) = \Gamma^c_{ab}\partial_c$ where ∂_c denotes the unit vectors that span the tangent space.

[75] Axiomatically, the identification of affine structure as inertial and the chronogeometric structure is achieved by "interpretative principles" which specify the conformal and affine/projective structure via their respective physical instantiation in terms of the behaviour of point particles and light rays, cf. loc. cit. (2012), Ch. 2.1. Whether these "interpretative principles" or "semantic axioms" (Bunge), are necessary or whether they follow from GR is a different issue, see footnote 84.

[76] It is worthwhile pointing out that Einstein placed a special emphasis on the status of the geodesic postulate as a generalisation of Newton's 2nd Axiom, cf. Lehmkuhl (2014). The crucial point is that in geometrised spacetime theories, Newtonian or Minkowskian inertial frames morph into affine-structure (the uniqueness of which is guaranteed by the Equivalence Principle in the sense of indistinguishability of inertial and gravitational effects), see also Knox (2013).

[77] Cf. Will (2014) for a comprehensive review of all the different tests GR has been subjected to.

[78] Cf. Fletcher (2013), who also proves that for every timelike curve such a clock of arbtritrary precision exists. One may in fact, reverse the argument to the extent that "the 'geometrical' hypothesis linking the behaviour of

A priori the metric and the affine structure need not be related[79]. Motivated, however, for instance by demanding that (affinely parameterized) geodesics on the manifold, $\dot{x}^b\nabla_{\mathrm{b}}\dot{x}^a = \ddot{\mathrm{x}}^a + \Gamma^a_{bc}\dot{\mathrm{x}}^b\dot{\mathrm{x}}^c = 0$, should also extremalise the worldline element $g_{ab}dx^a dx^b$, the simplest choice would be metric compatibility: $\Gamma^a_{bc} = \frac{1}{2}g^{ad}\left(\partial_c g_{db} + \partial_b g_{dc} - \partial_d g_{bc}\right)$. Thus, metric compatibility, as prevalent in GR[80,81], unifies the chronogeometric and inertial structure[82].

In short, we propose that **spacetime** is represented by the **triple $\left(\boldsymbol{M}, \boldsymbol{g_{ab}}, \boldsymbol{\nabla}\right)$**[83], where the metric represents chronogeometric structure and the metrically compatible connection ∇ represents inertial structure. Both are to be construed as **relations between events**. We'll see in a bit that this claim is consistent with the actual physics and sheds some light on issues surrounding gravitational energy in GR.

Let us pause for a moment to make three observations that make contact with the existing relationalism/substantivalism literature:

- The standard **abductive arguments for substantivalism that appeal to inertial effects**, such as Newton's Bucket Experiment, **are all beside the point**: Neither spacetime substantivalism nor relationalism can *explain* inertial effects! What both *ontological* approaches must *postulate* as a brute fact are a physically distinguished trajectories, i.e. inertial structure[84]. That such a distinguished trajectory may be

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ideal clocks to the […] 'metric' field is in principle dispensable" (Pooley (2015), p. 3): It need not be postulated separately.

[79] For instance, in s.c. Palatini f(R) Gravity, cf. Sotiriou et al. (2007), the connection (with which a derivative operator and the Riemann tensor are defined) and is not the Levi-Civita connection of the metric: Geodesics w.r.t. the connection do not extemalise the line element.

[80] It's a striking feature of GR (and more generally, a certain class of alternative theories of gravity, s.c. "Lovelock Gravity", cf. Padmanabhan/Kothawala (2013), pp. 9., that this unification follows from both the inertial and the chronogeometric structure being treated as dynamical, independent structures, subject to variation in the formulation of GR (the s.c. Palatini variation), cf., for instance, Hobson et al. (2006), Ch. 19.11.

[81] By contrast, this unification gets lost in Newton-Cartan Theory, where the chronogeometric structure is encoded in the two metrics, the temporal and the spatial one, and the inertial structure is encoded in a separate derivative operator, cf. Malament (2012), Ch. 4.1.

[82] Einstein himself viewed the revolutionary core of GR, as represented by $(\mathcal{M}, g_{ab}, \nabla)$, in the unification between inertia and chronogeometry , a unification that manifests itself in the Levi-Civita connection Γ^a_{bc}, as it appears in the the geodesic equation, interpreted as a generalisation of Newton's 2nd Law, cf. Lehmkuhl (2014).

[83] Don't conflate spacetime represented by $(\mathcal{M}, g_{ab}, \nabla)$ and the **universe**! The latter corresponds to the system composed of all material objects –and is a substance itself, cf. Bunge (2007). It's characterized by the tuple (M, T_{ab}).

[84] One might argue though that, since the Einstein Equations imply the Geodesic Principle, i.e. the fact that in absence of external forces bodies follow geodesics, GR indeed explains inertia. Einstein himself thought so for a while. In fact, that the Bianchi identities imply the geodesic equations of motion for relativistic dust is a standard textbook exercise, cf., for instance, Hobson et al. (2008), Ch. 8.8. (Similarly, in the geometric optics approximation it can be derived that electromagnetic waves propagate along null-hypersurfaces, cf. Wald (1984), pp. 70.) Furthermore, the s.c. Geroch-Jang Theorem extends this in a rigorous manner to matter other than dust that satisfies the strong energy condition, cf., for instance, Weatherall (2012), Sect. 3. However, Tamir (2011) has subtly argued against the claim that GR contains the Geodesic Principle, pointing out in particular that all realistic matter models violate the strong energy-condition. Weatherall (2012) takes up the idea, esp. Sect. 4, arguing, however, that if one adopts a modified notion of explanation, namely the one he proposes ("puzzleball view"), sect. 5, GR *does* explain inertia.

assumed, however, is good practice of a scientific realist, as we saw just now: Our best theories cannot eliminate inertial reference systems: “Contemporary space—time theories show a ubiquitous need for inertial structure as a theoretical construct.”[85]; any departure from inertial motion elicits well-corroborated effects. It is the ontological construal of this physically distinguished class of trajectories, where the substantivalist and the relationalist differ: For the substantivalist the physical distinction originates in the intrinsic properties of spacetime – without being able to offer a proper theory that explains this distinction from its intrinsic properties. The relationalist, on the other hand, simply claims that there exists a distinguished chain of relations connecting two pair of events[86]. Both take inertial structure as a primitive. The point of contention between the relationalists and substantivalists is a purely ontological, conceptual one! Here we clearly, see why Leibnizian- relationalism (the ontological issue just mentioned) and Machian relationalism must be kept apart: The latter seeks a theory that can account for, i.e. explain the physical distinction of inertial trajectories – a distinction that the former must accept as a *datum* and give it a satisfactory conceptual analysis. Note, however, that the innovation of GR to treat inertial structure as dynamical (via metric compatibility) is a necessary first step towards a Machian relationalism: If one wants to derive inertial structure, it surely cannot be fixed.

- In principle, one might also consider promoting the inertial structure (absolute acceleration) to the status of a primitive, intrinsic property of a body, as Sklar has proposed[87], emphasizing that this acceleration absolutism does not necessarily conflict with Leibniz-relationalism. His proposal has been criticised elsewhere[88] in detail; we only add what seems to be in favour of our own case:
  - It remains, as he himself admits, only a sketch; crucial details need still to be fleshed out.
  - Our ontological and semantic proposal is close and natural to the formalism, as we outlined above.
  - If in GR the affine and metric structure are unified by metric compatibility, they should have the same ontological status, as it has in our proposal. For

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[85] Teller (1991), p. 377. According to Teller, a substantivalist might then argue that since inertial structure must be structure of something, “this something is precisely what we have in mind when we talk about the manifold of space-time points, substantivally conceived”, loc. cit. Such an argument falls into the error we anticipated above: Even if a conceptual entity has properties, we may not infer that what the latter *represents* is necessarily a substance. In the jargon of the field: Mind the type-token difference!

[86] Cf. loc.cit., Sect. IV. To do justice to the relationalists before the advent of non-geometric spacetime theories, one may summarise the jist of their methodologically attractive and heuristically indeed fertile complaints as a mismatch between the spacetime symmetries and the dynamical symmetries describing matter, cf. Pooley (2013), Ch 4.2. This surplus structure is then criticized as being unobservable. To give a concrete example: The Galilei group as the symmetry group of Newton’s absolute space and time is a proper subgroup of the Leibniz group. The relationalists’ methodological demand that this unobservable structure be minimized, if not eliminated, is realised in geometrised spacetime theories, where gravitation has been incorporated into the spacetime geometry.

[87] Cf. Sklar (1974), pp. 229-332

[88] Cf. Earman (1989), Ch. 6.9

the metric structure, however, it make no sense to consider it as an intrinsic property ('"Sklarise" it'): A well-posed question about a distance involves two points, between which the distance relation holds.

- An independent, more direct argument for the relational status of the metric can be constructed from the following syllogism: (P1) The ontological status (denoted by "[.]") of the objects represented by the l.h.s. and the r.h.s. of an equation must be the same. Take as an example $E = mc^2$, irrespective of its exact meaning[89]: $[E]$ is a property, namely energy, $[mc^2] = [m]$ is likewise a property, namely rest-mass. (P2) Manipulations like differentiation w.r.t. its arguments do not change the ontological status of the object: They merely specify local changes, zooming in as it were on infinitesimal details without altering the ontological category. The (time-dependent) mass ratio of two chemicals, for instance, is a relation; so is its rate of change. (P3) The energy-momentum tensor expresses a relation[90]. (P1-3), together with the Einstein Equations, entail: $[T_{ab}] = [G_{ab}] = [g_{ab}]$, i.e. the metric likewise expresses a relation. A similar argument applies to the affine structure.

The reader might level a natural objection against our claim that spacetime is a relation: Wouldn't, according to our ontological terminology, the non-substantival nature of spacetime entail that it didn't have any properties? What then with curvature?- Indeed, we deny that spacetime possesses any properties[91]! W.r.t. the **curvature**, the distinction between the object and its representation becomes crucial: Curvature is a *mathematical* property only of the mathematical object that represents spacetime; i.e. the Riemann tensor R^d_{abc} is a function *not* of spacetime, but of the metric (as a mathematical object) and its derivatives: $R^d_{abc} = \partial_{[b}\Gamma^{\mathrm{d}}_{\mathrm{c]a}} + \Gamma^{\mathrm{e}}_{\mathrm{a[c}}\Gamma^{\mathrm{d}}_{\mathrm{b]e}}$.

Let us close this section with a remark on pre-GR spacetimes. Essentially all our arguments carry through also for the **Minkowski spacetime** of SR (and neo-Newtonian spacetime of Newton-Cartan-Theory): Minkowskian spacetime is represented by the triple $(\mathbb{R}^{3+1}, \eta_{\mathrm{ab}}, \nabla^{(\eta)})$ and best construed as the totality of possible and actual relations between events. Note that the immutability of Minkowski spacetime is particularly natural to apprehend, since it is "absolute" (in Friedman's first sense of the word[92]).

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[89] The correct interpretation of this s.c. Mass-Energy Equivalence turns out to be not so easy, cf. Fernflores (2012)

[90] Cf. Lehmkuhl (2010)

[91] Bunge (1977) also points this out. Einstein himself seems to have articulated this intuition at some point. In a letter to E. Mach from 1913 he writes: "It seems to me an absurdity to ascribe space physical properties." (quoted in Brown/Lehmkuhl (2013)).

[92] Friedman (1983), p 208 distinguishes three meanings of "absolute": a) independent of a reference frame, as opposed to "relative" (for instance, within Newtonian Mechanics e.g. spatial distance, or within GR rotation relative to a local inertial system); b) non-dynamical (for instance, the speed of light in Special Relativity) and c) substantival (as opposed to relational), on which our discussion here focuses. Our analysis illustrated: A dynamical quantity needn't represent a substance, whereas a substance must always be represented by a variable that is both dynamical and non-relative.

For the benefit of the reader we list the results of our semantical analysis in the following table:

element in the formalism	**…represents…**	**ontological status**
manifold $\mathcal{M}$	totality of possible events in the World	All possible changes
metric structure g_{ab}	chronogeometric structure	spatiotemporal distances
affine structure (parallel transport) ∇	inertial structure	spatiotemporal paths
$(\mathcal{M}, g_{ab}, \nabla)$	Spacetime	Totality of spatiotemporal relations between possible events in the World
Coordinates	ordering/labeling system for events	Conventions
General Covariance of a law	Objectivity of the represented pattern	-

It's instructive to contrast our own relationalism against **metric (field) substantivalism**, which has been argued to be the most defensible form of substantivalism[93]. It states that the pair $(\mathcal{M}, g_{ab})$ represents spacetime and that the latter is a substance.

- Three main virtues commend themselves for metric substantivalism:
  - It accounts for the fact that the bare manifold by itself lacks the salient features of spacetime (e.g. light-cone structure, past-future-distinction, etc.) that constitute explanatory core and the essential the metric plays.
  - It is not bound to the notion of primitive identity of manifold points ("hæcceitism": the postulate that the points of the manifold are individuated in virtue of empirically elusive properties) – a manœuvre the Hole Argument[94] forces the advocate of "manifold substantivalism" to resort to[95].
  - Regarding the metric as a substance, like any other field, chimes with the popular view, natural to quantum field theoreticians[96] with little sympathy for the geometric spirit of GR, reflecting in particular, that it seems to carry energy.

Let us comment on these tenets:

We agree with Hoefer's analysis that the concept of primitive identity of manifold points should be relinquished as an *ad hoc* manœuvre. Like us, Hoefer couches his analysis (albeit implicitly) in a substance ontology: "A modern-day substantivalist thinks that space is a kind

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[93] Cf. Hoefer (1996).

[94] Cf. Norton (2015) or Stachel (2014) for a review.

[95] Cf. op. cit., where the manœuvre is dismissed as *ad hoc*.

[96] A textbook written explicitly in this field-theorectical spirit is Weinberg (1972).

of thing that can […] exist independently of material things […] and which is properly described as having its own properties […]"[97].

However, even though we agree with Hoefer that the manifold together with the metric does represent decisive spatiotemporal features, this is not the whole story: $(\mathcal{M}, g_{ab})$ only captures the chronogeometric structure that defines spatiotemporal lengths; the conceptually distinct -albeit *contingently related*- inertial structure must still be accounted for - by extending the spacetime model to the triple $(\mathcal{M}, g_{ab}, \nabla)$.

We also reject the third argument: Its first part, the common linguistic practice as a *fait social,* is irrelevant for any systematic analysis; its second part, the fact that metric substantivalism accommodates for gravitational energy, becomes moot, since, as we and Hoefer himself argued elsewhere[98], gravitational energy does in fact not exist in GR.

5. Discussion

So far, we have mostly operated conceptually. Time to make a more specific connection to the results previously achieved in our analysis of gravitational energy and gravitational waves:

Elsewhere[99] we had argued that **gravitational energy** (energy attributable to spacetime) **does not exist within GR**. Our main line of interwoven arguments was twofold:

- All known definitions of gravitational energy presuppose background structures and involve *ad hoc* assumptions that do not hold for generic spacetimes.
- An evaluation of these conditions yielded that they considerably narrow down the solution space:
    - Known, numerous and natural counterexamples exist – hardly a surprise for a background-independent theory such as GR.
    - Even our cosmological standard model (ΛCDM) does not comply with these conditions.
    - They are unphysical/unstable: Tiny deviations from suitable models lead to models that violate again the conditions.
    - In themselves, their motivation is dubious (e.g. already the plane gravitational wave violates them).
- We concluded that, since for a concept to be fundamental/essential in the framework of a scientific theory, it must be applicable to a sufficiently large class of

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[97] Hoefer (1996), p. 5. (Note, however, that his claim Einstein had espoused a metric substantivalism is wrong. → DENNIS????)

[98] Cf. Duerr/Lehmkuhl (2015b), which in some respects is a detailed elaboration of Hoefer (2000), where he himself argued that gravitational energy does not exist in GR.

[99] Cf. loc. cit.

typical situations and must not hinge on very specific contingent conditions, gravitational energy is not fundamental to GR.

- Retrospectively this result makes sense: “If gravitational force has somehow been compromised - geometrised away- then we should expect the same to happen to [....] ENERGY and MOMENTUM”[100].

Is there a connection between the non-existence of general-relativistic gravitational energy and the main result of our preceding ontological analysis that spacetime is a relation? We submit that there are two important links:

1) We argued that it is necessary for a material substance to possess energy. (Non-material substances, i.e. *abstracta*, do not ontically exist.) From this angle, the non-existence of gravitational energy is a consistency check for relationalism: If gravitational energy did in fact exist, our relationalism would be inconsistent.

2) Our ontological relationalism can in fact ***explain*** **the non-existence of gravitational energy**.

- Let us first ponder: Does spacetime change? We propose it doesn’t. Consider the epitome of a putatively changing spacetime: a gravitational wave passing through an interferometric detector. The wave changes the proper lengths of the detector arms, thereby inducing a difference in running times of laser pulses travelling back and forth, a difference that gives rise to detectable interference patterns. On the face of it looks as if spacetime has changed with the passage of the gravitational wave. However this is *not* so: We are comparing two *different* pairs of events; the spatiotemporal relations between them are different, too. Saying that spacetime has changed, would require one pair of events and their spatiotemporal relations to be treated legitimately as a reference value. But there is no cogent reason to do that: GR lacks all absolute spacetime background structures that might justify privileging a relation among *one* specific pair of events as the default relation.

  Put slightly differently: The Einstein Eqations serve as a black box giving the right metric, the corresponding Levi-Civita connection of which then picks out inertial frames. (In that respect the Einstein Equations take up the role of the demand that the Riemann tensor vanish in Kretschmannisation (see footnote 64): They pick out and fix, by *fiat*, the class of inertial frames, the class of physically distinguished relations between events.) Each pair of events is assigned a spatiotemporal relation – no change is involved in the assignment of these relations: They are fixed.

  In conclusion, spacetime does not change.
- Now recall that we identified energy as a quantification/measure of changeability. Absent any genuine notion of change of spatiotemporal relations, *a fortiori* the changeability of spatiotemporal relations cannot be quantified: Gravitational energy,

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[100] Norton (2001), p. 21 (Norton’s emphases).

as the energy attributable to spacetime, does not exist. (Recall the difference pointed out in section 3 between "having no energy" and "having zero energy".)

Misner et al. even give a related explanation for the non-definability of gravitational energy in a generic spacetime such as a closed universe: "To weigh something one needs a platform on which to stand to do the weighing"[101] – an illustration that can be rendered non-metaphorical now: The definability of gravitational energy requires background structure as a somehow privileged reference entity, background structure that turns out to be contingent, thus undermining the fundamentality of gravitational energy.

Remark: By contrast, for the relations expressed in matter field configurations (recall our sketch of a relationalist definition of a field in section 4) we do have a natural way to pick out a class of privileged default relations, namely both the dynamical symmetries of the matter field itself and the background spacetime structure (especially when the latter is highly symmetric, as it is for all practical purposes of (quantum) field theories on a curved spacetime).

A remark is in order on those cases where an ersatz for gravitational energy in its various incarnations is possible[102], glossing for the time being over our arguments against the *fundamentality* of gravitational energy in general. According to the **Positive Energy Theorem**[103], such gravitational ersatz energy turns out always to be positive – an important property for the coherence (e.g. in terms of stability) to the standard view of spacetime carrying energy. Thus, the spacetime in general and the gravitational wave in particular would satisfy our proposed sufficient conditions for material substantiality. Thus, we can explain why spacetime and gravitational waves so beguilingly look like substances in such cases! In the case of gravitational waves this explanations renders explicit the ontological presuppositions underlying Isaacson's variational approach[104]: Here, from the outset, the gravitational wave is *de facto* treated like an ordinary matter field. In this respect, the gravitational energy is an artifact of the implicit ontology: Material substantiality in – energy out.

Let us turn now to the **status of vacuum solutions**, a notorious bee in any relationalist's bonnet.

According to substance ontologies, for a relation to exist, there must be *relata*, between which such relations hold. Consequently, relations between matter-free spacetime points and vacuum solutions in general, *prima facie* look like stumbling stones for any relationalis: Several authors have claimed to show the spacetime structure a relationalist may according to his own standards legitimately avail himself of, viz. spatiotemporal relations between

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[101] Misner et al. (1974), p. 457.

[102] Cf. Duerr/Lehmkuhl (2015a, b)

[103] Cf. Wald (1984), Ch. 11.2 for further references.

[104] Cf. Duerr (2015)

matter-occupied points, is explanatorily insufficient[105]. It seems, a relationalist is committed to discard globally vacuum solutions as non-physical – a disturbing view for the science-loving realist, since after all gravitational waves are vacuum solutions: Why, if they are non-physical anyway, invest millions of pounds into gravitational wave detectors such as LIGO?

These claims, however, are premised on a "narrow relationalism" (Teller), "the view that the actual space-time relations between actual bodies and events exhaust all the facts about space-time"[106]. According to a "narrow relationalist" only spatiotemporal relations between *actually* matter-occupied spacetime points are ontologically legitimate. We reject this restriction to actual material *relata*, instead adopting a **"liberalised relationalism"** (Teller), which recognises not only the actual, but also the scheme of space-time relations of actual or hypothetical objects to each other.[107] Only relations between matter-occupied points represent *actual* spatiotemporal relations, as opposed to only possible ones represented by relations between matter-unoccupied points: If there *were* two specks of dust with negligible mass-energy at the respective spacetime points, how *would* they be spatiotemporally related?

Is this possibility-admitting *extension du domaine de la lutte* an *ad hoc* strategy? On the contrary; rather the restriction to only actualities is - presumably a relic from an empiricism[108] seeking a way to ascend inductively from observable data to theories. In *any* theory, the mathematical formalism and its formal solutions must be supplemented by additional requirements that select physical solutions, i.e. restrict the *formal* solution space of *possible*, abstract/mathematical model to realistic, *actual* solutions. Consider three examples: 1. As far as we know electric charge comes only in integer multiples of 1/3 e. Nonetheless all electrodynamical equations also carry over to even non-integer multiples, which are nomological *possibilia*. 2. Despite the weirdness of their characteristics, such as imaginary masses, superluminal particles, s.c. tachyons, can formally be treated within Special Relativity[109]; to be sure: given the data, their existence is highly unlikely, though. 3. As a last example, recall that the state vector of quantum many-particle systems *empirically,* turns out to be either symmetric or anti-symmetric, in the bosonic and fermionic case, respectively. *Formally*, however, nothing forbids a mixed symmetric many particle-state[110],

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[105] For example, Friedmann (1983), Ch. VI.3 and VI.4, Maudlin (1993), esp. section 6 or Skow (2007). The latter is an interesting case, since it levels an objection at Sklar's original brand of relationalism that considers absolute acceleration a fundamental, intrinsic property of material bodies. Skow then shows that the initial value problem for Leibnizian initial data (i.e. specification of relative positions, velocities and acceleration between two *actual* massive bodies) is not well-posed.

[106] Teller (1991), p. 364

[107] Cf. loc. cit., Sect. II. It must be pointed out, however, that counterfactuals are problematic in GR. We'll come back to this. For (test) particles with negligible energy, however, no problem occurs.

[108] Cf. Earman (1989), p. 135

[109] Cf. Baez (1993)

[110] Cf. Messiah/Greenberg (1964). Such mixed-symmetric states give rise to the s.c. parastatistics. The fact that the statistical quantum mechanical formalism admits such empirically not realized possibilities is sometimes brushed under the carpet by the transition to Fock space, which by construction contains either only symmetric or only antisymmetric states (of a variable particle number).

i.e. some particles transforming symmetrically and others anti-symmetrically: Nature simply seems not to realize this possibility.

In the same vein, the formal solution space of the Einstein Equations also contains *possible* spatiotemporal relations, i.e. spatiotemporal relations betwen *hypothetical* test particles; *actual* spatiotemporal relations on the other hand are those that are represented by metric or affine connections between matter-occupied manifold points. One could argue that Einstein, in the aftermath of de Sitter's discovery of cosmological solutions of the Einstein Equations, viewed **Mach's Principle** in this sense[111], viz. as an ontological selection rule to demarcate *actual*, physical from only *possible,* formal solutions[112].

Remark: Inserting a **cosmological constant** Λ in the Einstein Equations, $G_{ab} = \kappa T_{ab} - \Lambda g_{ab}$, and interpreting it as the zero point energy induced by a quantum field, rescues the actuality/physicality even of "vacuum" solutions (also for the "narrow relationalist"). These solutions then are, strictly speaking, no longer vacuum solutions, because there would exist a physical (albeit quantum) matter field filling the cosmic voids. Unfortunately, this strategy has so far failed spectacularly as "probably the worst prediction in the history of physics"[113] - with a discrepancy between observation and theoretical prediction of more than 120 orders of magnitude[114]. Note also that the interpretation of the cosmological constant as vacuum density is not cogent (albeit natural)[115]: It could be just a free parameter of the minimal modification of the original Einstein Equations, not describing Dark Energy (in the sense of a material source of unknown type) but, as it were, "Dark Geometry".

Let us reply to two **objections against liberalized relationalism**. The first one concerns the suspicion that liberalized relationalism "serves to obscure the substantive aspects in the (relationalism/substantivalism) debate"[116], the second worries about ontological parsimony. We submit, neither is justified:

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[111] What is commonly known as Mach's Principle was actually Einstein's Mach-inspired hypothesis about the origin of inertia, namely that the global mass distribution uniquely determines the inertia of a body, cf. Earman, Ch. 5.8. Since the Ricci tensor, which enters the Einstein equations, only picks out the tracefree parts of the Riemann curvature tensor, cf. Wald (1984), p. 40, The Einstein Equations alone do not fix the metric. If one correlates the latter to inertia, as Einstein did in his 1921 Princeton lectures on the Meaning of Relativity, cf. Earman (1989), pp. 107, it follows that GR violates Mach's Principle, notwithstanding some Machian effects such as frame-dragging. (The initial value problem fares even with regard to the Machian idea(l) that the spacetime distribution determines the metric, cf. loc.cit.) Machian relationalism perceives this violation of the Mach Principle as a deficit, which calls for a new mechanics that conforms to it.

[112] Cf. footnote 5 on p. 526 in Buchwald et al. (2012).

[113] Hobson et al. (2006), p. 187

[114] cf. Carroll (2001), who discusses the interpretation of Λ as vacuum energy in sect. 1.3

[115] *Pace* Baker (2005) whose argument is multiply confused in firstly proclaiming that Λ must be interpreted as vacuum energy, but simultaneously attributing it to the vacuum energy *of spacetime,* and then using an unclear (and, as he admits, idiosyncratic) notion and alleged inter-relation of general-relativistic acceleration (which on top he conflates with the "accelerated increase" of physical separation) and causality to infer that a Λ-driven cosmic expansion is a "causal argument for substantivalism", p. 3, by which – to conclude the parade of flaws and confusions- he doesn't mean substantivalism in the usual sense, but ontic reduction of spatiotemporal relations.

[116] Earman (1989), p.135

- Earman sharply rejects liberalized relationalism on the grounds of "eroding the difference between relationalism and substantivalism; indeed the notion that space points are permanent possibilities of location for bodies is one plausible reading of Newton's 'De Gravitatione'"[117]. However, in what he proclaims, Earman misses the crucial point of the substantivalism/relationalism debate: As we clarified in section 2, it revolves around the ontological category of spacetime (substance, property or relation).
- Teller concedes: "The ontology of liberalized relationalism may appear to be just as rich as, and perhaps in some sense isomorphic to the ontology of substantivalism."[118] One might therefore anticipate the criticism that it smuggles in all the entities back in through the backdoor that the relationalism had wanted to get rid of in the first place, appealing to **Occam's razor**: Note, however, that the appeal to Occam's razor is misplaced in a twofold way:
    - Firstly, it refers to *actual* entities (*entia*), not *possible* ones (*possibilia*). How could it: Any theory admits an infinitude of possible entities! Obviously, we have not multiplied the *actualia*.
    - Secondly, Occam's razor states demands "entia non sunt multiplicanda *praeter necessitatem"*. As we have seen already, the necessity of postulating the existence of inertial structure as primitive, thus physically distinguishing a class of trajectories in order to account for inertial effects, is beyond any doubt for both parties. So should we regard accounting for possible phenomena as *unnecessary*? As we argued and illustrated, we shouldn't: It is an indispensable part of the explanatory labour of scientific theories also to cover hypothetical cases. After all, how to make any new discoveries, when shackling our theoretical curiosity to the confines of already known *actualia*?

    In sum: Occam's Razor does not excise a "liberalized relationalism".

Let us eventually harvest the last fruits of ontological labours w.r.t. the putative interaction between matter and spacetime, sometimes presented as the take-home message of Einstein's GR.

Causality and whether an object acts upon another, are commonly taken to involve energy transfer from one object (or event) to the other[119]. This distinguishes causal relations ("a ball breaks a window") from non-causal relations ("night follows day") –the Humean tradition, which sought to eliminate causality in philosophical parlance in favour of contingent, correlational regularities, notwithstanding. The lack of "spacetime energy" has three immediate consequences in terms of causality:

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[117] Loc. cit.

[118] Teller (1991), p. 365

[119] Cf. Bunge (1977), Ch. 6.5 and references therein, esp. the monograph on causality by the same author.

1. Wheeler's famous slogan "Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve"[120] must not be understood in terms of a causal influence of spacetime upon matter and vice versa. Instead, the **Einstein equations** should be understood **as mutual constraints**, i.e. an identification of the matter energy-stress tensor as the source term of the dynamical equation for the metric. In this respect, in a dynamical fashion, the Einstein Equations take over the role Kretschmann's $R_{abcd}[g] \equiv 0$ had in picking out the physically privileged metric and affine-structure, i.e. in determining the right chronogeometric and inertial structure of the kinematically possible models.

   Schrödinger makes the same point about a non-causal understanding of the Einstein Equations: "Just in the same way as Laplace's equation $div\,\vec{E} = \rho$ says nothing but: wherever the divergence of $\vec{E}$ is non-zero, we say there is a charge and call $div\,\vec{E}$ the density of charge. Charge does not cause the electric vector to have a non-vanishing divergence, it *is* this non-vanishing divergence. In the same way, matter does not cause the geometrical quantity, which forms the first member of the above [i.e. the Einstein] equation to be different from zero"[121]. Recall our earlier remark that the energy-momentum tensor is a misnomer: A more appropriate, unwieldy name would be the "energy-momentum-related source-density functional of the matter variables and the metric for the dynamics of the metric"[122].

2. Thus, gravitational waves cannot *cause* anything or *act* on anything. This does not mean, of course, they don't have any effects: A gravitational waves that hits a detector reveals itself as an effect – an effect, however, that, as we argued, reflects the difference in the relations between two different pairs of events! One may even reverse the usual order of the explanatory burden: That two relations between two different pairs of events *differ* is exactly what to expect; their similarity is what requires an explanation. In other words: We mustn't (metaphysically) take the highly symmetrical nature of pseudo-Euclidean geometry for granted, which represents such even a uniformity between all spatiotemporal events[123]. The assignment of all spatiotemporal relations to each pair of possible events is determined by *fiat* via the Einstein Equations, whose status in this regard is closer to a "fundamental principle" (Weatherall)[124].

   Remark 1: The same argument of non-causality applies to the **expansion of the universe**: The latter needs not to be interpreted as a *causal* effect of spacetime; again the exact form of the possible and actual spatiotemporal relations between events is

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[120] Misner et al. (1974), p. 5
[121] Schrödinger (1950), p. 99
[122] Cf. Duerr/Lehmkuhl (2015b)
[123] Cf. Nerlich (1994), Ch. 6
[124] Cf. Weatherall (2012), esp. section 4.

simply dictated via the field equations - a form that predicts for instance cosmic expansion[125].

Remark 2: Similarly, rotation -such as it features prominently in the **Gödel solution** with its closed worldlines[126] (which one may regard as an incarnation of Mach's Bucket Experiment)- fits into the relationalist, general-relativistic framework. One might immediately object that global rotation is conceptually problematic for a relationalist, but the 4-dimensional perspective admits a precise definition in terms of the relative positions of *geodesics* (e.g., whether they intersect or recede from each other) via the geodesic deviation equation or the (counterparts of the) optical scalars (shear, vorticity and expansion): For instance, the extent to which light rays are twisted (think of fibers in a rope) is captured in the vorticity[127]. Rotation thus characterized by the relative positions of the geodesics, is doubly relational in nature: Firstly since it is defined as relative positions of geodesic, and secondly, since what the geodesics represent are spatiotemporal relations between events in (hypothetical) massive bodies.

3. Spacetime and matter do not exchange energy; hence no mutual causal influence or interaction proper (defined as a process with energy exchange)[128]: The **Action-Reaction Principle**, which Einstein himself at some point extolled as GR's most singular virtue[129], thus is not satisfied in GR! However, as Brown and Lehmkuhl remind us: "(N) ote that at [Einstein's] time it was quite common not to draw a clear distinction between causality and determinism"[130]. We therefore propose to attenuate the condition of the Action-Reaction Principle so as to accommodate for dynamical evolution of spacetime. We thus stipulate a distinction between a **strong and a weak form** of the Action-Reaction Principle: The former indeed asserts that spacetime and matter causally interact, i.e. exchange energy; the latter denotes

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[125] By contrast, Baker (2005) has argued that the acceleration the cosmological constant gives rise is yet another (besides gravitational radiation) manifestation of the causal prowess of spacetime. Consequently, we should regard spacetime as a substance. The argument is multiply flawed, however: Firstly, whether Λ can be construed as a feature of the spacetime geometry or whether it stems from unknown source of matter, e.g. as the density fluctuations of a quantum field, is an open debate! Most physicists seem to favour the latter option. Secondly, given it is a free parameter of the Einstein Equations: This doesn't mean it corresponds to any property of spacetime; it would merely be a given parameter in the field equations. Thirdly, his appeal to acceleration is problematic, for two notions must be kept apart in GR: On the one hand the 4-acceleration, and the rate of change of the rate of change of the physical distance. For dust particle (e.g. a distant galaxy) in an FLRW cosmology, the 4-acceleration is zero; the physical distance to the galaxy, however, changes, manifesting itself by the well-known cosmological redshift. Let's assume for the sake of the argument that acceleration is always brought about by a causal mechanism: Still, since the 4-acceleration is the proper general-relativistic counterpart to Newtonian acceleration, the argument collapses.

[126] Cf. Malament (2012), Ch. 3

[127] Cf. Malament (2012), Ch. 2.8.

[128] Cf. Curiel (2000), Lam (2009) and Hoefer (2009), sect. 4.2 for a similar argument.

[129] Cf. Brown/Lehmkuhl (2013)

[130] Footnote 5, loc. cit.

dynamical completeness[131], i.e. the idea that in terms of dynamical evolution, matter fields and the metric interdepend.

According to this terminology, GR only satisfies the weak form of the Action-Reaction Principle.

The main ontological function of the original Action-Reaction Principle carries over, though, to the weak form: Namely that it serves as a reality criterion for the entities involved[132]. Note, however, the difference: Whereas the strong form is a sufficient criterion for the represented entities that satisfy it to count as material substances, the weak form only yields that they are real, irrespective of their ontological category. Applied to GR, this confirms the multiply emphasized fact that the spatiotemporal structure encoded in the metric is real, but not a substance; via metric compatibility this carries over to the affine structure.

Admittedly, our discussion of alleged causal agency of spacetime is limited to the extent that it hinges on a specific theory of causality, employing s.c. causal markers, which some may reject. It will be rewarding to investigate how different accounts of causality[133] fare in GR[134], in particular the Strong Action-Reaction Principle. Of special interest are, of course, counterfactual accounts of causality. They face a severe difficulty, however, as Curiel points out: Modal statements like “How would a light ray move, were a certain object X not there?” don’t have any obvious meaning within GR, “because however we make sense of ‘removing matter’ from a spacetime region, the metric will *eo ipso* be different in that region from what it was”[135]. Thus, “we have no way to conclude on any principled basis ‘what the metric would look then look like’”[136].

A proper analysis causality in various theories and their validity in GR also has a pressing practical aspect: If the effects brought about by the spacetime geometry are not causal, then the usual argument invoking causality to discard advanced wave solutions as unphysical, is strictly speaking no longer applicable.

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[131] The only absolute elements in GR, i.e. an element exempt from dynamical completeness, are the Lorentzian signature of the metric and the volume element, both of which hardly qualify as fully-fledged elements. For an introduction to absolute structures in GR, cf. Straumann (2013), Ch. 3.5 and, in particular, Pooley (2015).

[132] Cf. Brown/Lehmkuhl (2013)

[133] Such as e.g. reviewed in Dowe (2007).

[134] Lam (2009), sect. 4 and 5, discusses how a causal theory of properties, according to which it lies in the nature of properties to elicit certain causal effects, fares within GR. He argues that a causal theory of properties requires certain non-trivial topological conditions to hold, without which it cannot make sense of a plethora of spacetimes. There is a potential loophole in Lam’s analysis, namely “the extent to which the causal theory of properties is applicable to spatiotemporal (and gravitational) properties”, Lam (2009), p. 15 (translation, P.D.). Recall that we argued that spacetime as the network of (actual and possible) spatiotemporal relations does not have properties. Whether an advocate of the causal theory of properties could simply extend the “causal disposition”, which previously he had envisaged for (intrinsic) properties only, upon relations as well, remains to be seen.

[135] Curiel (2014b), p. 2.

[136] Loc. cit., p. 3.

We will pursue some consequences of these results further in a subsequent paper[137]: In particular, we argue that the non-interactional nature of gravity will have some bearing on approaches to quantum gravity. Mainly, however, we will analyse the standard interpretation of binary systems whose increase in orbital frequency is commonly understood to reflect the emission of gravitational wave energy.

6. Conclusion

With our plea that spacetime, as represented by $(\mathcal{M}, g_{ab}, \nabla)$, is best construed as the totality of spatiotemporal relations between events our position naturally appears to qualify as a form of **relationism** in that "bodies alone exhaust the domains of the intended models of [spacetime theories of the physical world]"[138]. Substantivalism, on the other hand, asserts that "both bodies and space are substances in that bodies and space points or regions are elements of the domains of the intended models of [space-time theories] of the physical world"[139].

Let's by way of a summary compare our results with the tenets of "traditional relationalism", which Earman characterizes by three "themes": [140]

R1: "All motion is the relative motion of bodies."[141]

Remark: With the modification in meaning the term "motion" exacts in a general-relativistic setting we fully subscribe to this: Like in all geometrised spacetime theory, in GR the spatiotemporal behaviour of bodies is described four-dimensionally, expressed through paths (trajectories or curves). They represent the (actual or possible) spatiotemporal relations in which one event stands to another, both pertinent to (actual or possible) bodies. If "motion" is understood in this broader sense of trajectories, then all motion is ultimately the relative motion of bodies. Global rotation, as appears to realize Mach's Newtonian Bucket Experiment, does not contradict this, for the concept of global rotation (and other forms of global "motion" such as a global expansion or shear) are defined via relative behaviour of (congruences of) geodesics, which in turn have (actual or hypothetical) test particles as their *relata*.

R2: "Spatiotemporal relations among bodies and events are direct; that is, they are not parasitic on relations among a substratum of space points that underlie bodies or space-time points that underlie events."[142]

Remark: We fully subscribe to this theme, too.

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[137] Cf. Duerr/Lehmkuhl (2015d)
[138] Earman (1989), p. 114
[139] Loc. cit.
[140] Loc. cit., pp. 12
[141] Cf. loc. cit., p. 12
[142] Loc. cit. 9), p. 12

- Spacetime is indeed the totality of direct relations among *events* (represented by the manifold points). The non-existence of gravitational energy played a distinguished role here in our chain our arguments as a super-property: An entity that does not possess energy is not a material substance.
- Note that Earman's quoted definition does not specify, whether the *relata* of the spatiotemporal relations must be actual or possible bodies or events. We opted for Teller's "liberalized" reading to include possible events as legitimate as well, whereas the majority of relationalists seem to have restricted the admissible domain to actual bodies or events.
- Such relations are not 'parasitic' on relations between substantival spacetime points to the extent that a) firstly manifold points *represent* events; and the latter are not substances, but changes of the states in such b) secondly, we argued that as categories spatiotemporal relations are *ontologically* not reducible to substances and c) thirdly, we argued against an *ontic* reducibility of spacetime within (classical) GR.
- By contradistinction, substantivalism regards spacetime as a "substance in that it forms a substratum that underlies physical events and processes, and spatiotemporal relations among such events and processes are parasitic on the spatiotemporal relations inherent in the substratum of space-time points and regions."[143] Recall that the technical term "substratum" amounts to the "ultimate subjecthood of predication" (Kuhlmann), i.e. being the carrier of properties. The ontological framework we adopted was based on such a substratum-type notion of substance. With the relational nature of spacetime, we explicitly embraced the consequence that it does not possess any properties: only its mathematical $(\mathcal{M}, g_{ab}, \nabla)$ does. *Tout court*: We indeed deny substantivalism.

R3: "No irreducible, monadic spatiotemporal properties like 'is located at…'appear in a correct analysis of the spatiotemporal idiom."[144]

Remark: Again, we fully subscribe to this: With spacetime as a totality of relations, by our very definition there aren't any "monadic spatiotemporal properties". In fact, spacetime, we argued, has no properties at all. The statement the spacetime has vanishing curvature does not refer to a property of spacetime, but merely implicitly characterizes the metric structure by which it is partially represented as satisfying $R_{abcd}[g] \equiv 0$; the metric structure, however, is a *given* structure on the differentiable manifold, the specific form of which is given by the Einstein Equations.

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[143] Loc. cit., p. 11

[144] Loc. cit., p. 13

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