Do Gravitational Waves Carry Energy? - Critique of a Procrustean Practice

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Abstract:

We submit that, contrary to the standard view, gravitational waves (GWs) do not carry energy-momentum. Analysing the four standard arguments on which the standard view rests - viz. the kinetic effects of a GW on a detector, Feynman's Sticky Bead Argument, an application of Noether's Theorem and a general perturbative approach – we find none of them to be successful: Pre-relativistic premises underlie each of them – premises that, as we argue, no longer hold in General Relativity (GR). Finally, we outline a GR-consistent interpretation of the effects and, in particular, the spin-down of binary systems.

Key words:

General Relativity, Gravitational Waves, Gravitational Energy, Binary Systems, Sticky Beads Argument

I. Introduction

The following paper will address the simple question: Do GWs carry energy?

To many physicists the question itself may seem an old hat (well befitting the alleged scientific unworldliness often imputed to their colleagues in the philosophy departments): "A representative monograph on GW physics and astronomy for instance proclaims: Since gravitational waves produce a real physical effect […] – it is clear that the wave must be carrying energy."[1] Such an effect is the orbital spin-up of double pulsars, discovered by Hulse and Taylor in 1974, in highly accurate accordance with the GR prediction[2]; the double pulsars are supposed to demonstrate that the binary partners lose energy by radiating away energy GWs, thereby moving closer .

However, whilst certainly Hulse-Taylor-like binaries provide evidence for the *existence* of GWs, the standard *interpretation of their effects* might deserve a second glance: To begin with, the pulsars are in free-fall[3], i.e. move inertially. Shouldn't therefore their kinematic state remain unaltered and, in particular, shouldn't thus the binaries preserve their energy[4]?

[1] Anderson/Creighton (2011), p. 66
[2] Cf. Sect. 7 of Will (2014)
[3] In their original paper by Hulse and Taylor the pulsars are simply modelled as point-particles, i.e. self-gravitational effects had explicitly been neglected, cf. Petkov (2012), Appendix C.
[4] Cf. op. cit., where the same point is stressed.

More generally, if in GR there is no longer a gravitational force –gravity being "geometrised away" - doesn't this compromise the very notion of a gravitational energy as well[5]?

Closely related is the issue of energy conservation: As we learnt already in our introductory mechanics courses, conservation of energy and momentum are tied up with the homogeneity of time and space, respectively[6]. But doesn't GR teach us that space and time are warped and should this not also affect the validity of conservation laws?

What better opportunity than this year's centenary of his *opus summum,* GR, to recall Einstein's admonition never to stop questioning!

In the following we shall concentrate on GWs, the astrophysically most relevant launching pad for the discussion extended to the more general issue of gravitational energy and conservation laws elsewhere[7]. Here, we shall try to defend the claim that GWs do indeed not carry energy; talk about their energy transport is a mere *façon de parler*, useful for certain practical purposes, but ultimately both dispensable and not fundamental.

Our analysis closes a gap in the literature, where a systematic philosophical analysis of GWs has not appeared so far. The **topic is of utmost interest** in at least the following regards:

1. The existence of GWs is one of the key predictions of GR, confirmed observationally, viz. data from binary systems, with excellent accuracy[8] - evidence that we do, of course not, deny, despite challenging its orthodox *interpretation* in terms of radiative energy losses. To understand GR properly, one certainly needs to grasp the meaning of its characteristic effects.
2. In terms of studying such effects we live in extraordinarily exciting times :
    a. Advanced Virgo, a second generation Michelson-interferometric detector with greatly increased sensitivity (compared to its predecessors, e.g. LIGO), is just about to start its hunt for the *direct* detection of GWs[9]. If it turns out to succeed, not only will this provide yet another important confirmation of GR, but also open up a whole new era of astronomy, astrophysics and cosmology[10]! If on the other hand it turns out to find deviations from the GR results, for instance the onset of a scalar GW mode (propagating frequency-dependently, subluminally and longitudinally) on top of the general-relativistic transversal tensor mode (propagating with the speed of light)[11]:

[5] Cf. op. cit.; see also Norton (2012), Sect. 3.9
[6] Cf., for instance, Landau/Lifshitz (1976), §6.
[7] Cf. Duerr/Lehmkuhl (2015b,c) for a detailed analysis of gravitational energy and conservation laws, and binary systems, respectively. See also Trautmann (1962) and Hoefer (2000).
[8] For a review of the tests of GR, see Will (2014), esp. Ch. 6 for GW-related tests, in whose interpretation gravitational energy drain routinely pops up.
[9] Cf. BBC (2015)
[10] Cf., for instance, Flanagan/Hughes (2005), Sect. 6.
[11] Cf. Berry/Gair (2012) for the case of f(R) Gravity.

Even more interesting, for this would mean that GR must be replaced by another theory!

   b. In fact, strictly speaking, the observational anomalies subsumed under Dark Matter and Dark Energy (cosmic expansion), respectively, already indicate the necessity to go beyond GR[12].

   In either case a deep conceptual understanding of GR, its rivaling theories and their consequences is urgently called for! In particular, we desire to know what singles out GR in the landscape of alternative theories of gravity. The nature of GWs might turn out to be a key here.

3. Last, not least, on a more speculative note, in GR, the topic of GW energy and gravitational energy in general has direct implications for the quest for quantum gravity[13]: For instance, what would the non-existence of gravitational energy imply for the existence and nature of gravitons, i.e. the (hypothetical) gauge bosons in quantum theories of gravity, which are supposed to mediate the gravitational field?

May this for the moment suffice to kindle in the reader a spark of the fascination the topic deserves.

Our current study will **proceed as follows**: In section II we review the standard roads that lead to the assignment of gravitational energy to GWs: considerations of kinetic energy in a ring of test particles, Feynman's Sticky Bead argument, the Noetherian framework and the perturbative approach. In section III we critically examine these approaches to GW energy. Finally, in section IV we propose a new, as we shall argue GR-consistent interpretation of the previously encountered GW-phenomena. In section V we summarise our principal results.

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[12] More precisely, cf. Sotiriou (2007):

- Dark Matter anomalies, e.g. the rotational curves of galaxies, tend to be viewed as questions about matter content of the universe of an unknown nature (although, here also models, such as MOND/TeVeS, that seek to account for these phenomena in terms of modifications of gravity have been considered).
- Cosmic expansion tends to be attributed to the presence of the cosmological constant.
  - The cosmological standard model incorporates not only a cosmological constant, but also a scalar that drives cosmic inflation. In what respects such minimal extension differs from GR *simpliciter* is therefore of interest.
  - The cosmological constant itself is usually interpreted as a vacuum energy, an interpretation, however, that so far has failed spectacularly - with a discrepancy between observation and quantum field theoretical prediction of more than 120 orders of magnitude. Promising attempts have therefore been made to account for cosmic expansion without the cosmological constant, and instead in terms of Dark Geometry, i.e. modifications of GR, for instance by incorporating scalars or higher order curvature terms in the action.

[13] Cf. Duerr/Lehmkuhl (2015c) for some tentative suggestions.

II. Four routes to GW energy[14]

In this section, we shall review briefly the four routes wont to be adduced as evidence for the energy carried by GWs[15]:

- The change in kinetic energy of the atoms in a detector, induced by a GW;
- Closely related, the heating up of such a detector;
- The natural energy-momentum of a GW obtained from a decomposition of the metric into a background and perturbations of it (the latter representing the GW);
- An application of Noether's Theorem yielding conserved energy-momentum currents and charges.

1. Kinetic energy of test masses

The default argument for the energy of GWs considers the kinetic effects of a GW on test particles, otherwise at rest. The GW is described here perturbatively in s.c. linear GW theory[16]. (We shall discuss the perturbative approach in full generality in subsection 3.) Within linearised theory, one assumes that the gravitational field (taken to be represented by the metric) is weak, such that the spacetime geometry (g_{ab}) deviates only slightly from a flat Minkowski background (η_{ab})[17]:

$$g_{ab} = \eta_{ab} + h_{ab}, \text{where } |h_{ab}| \ll 1.$$

It turns out that in this perturbative order h_{ab} can be treated like a symmetric tensor field under global Lorentz transformations (from now on in this subsection, we therefore use Greek indices to denote that we are dealing with approximately Lorentz tensors; indices thus are also raised/lowered w.r.t. the Minkowski metric.); linearised theory thus effectively is a special-relativistic theory of gravity for weak fields[18].

What happens then to the General Covariance of our pristine general-relativistic equations? The invariance of the latter under infinitesimal coordinate transformations, $x^\mu \to x'^\mu = x^\mu + \xi^\mu(x)$ (where $\xi^\mu(x)$ is an arbitrary function of the same order of smallness as h_{ab}), in linearised theory becomes invariance under gauge transformations of the form $h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} - 2\partial_{(\mu}\xi_{\nu)}$.

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[14] Introductions to GW theory are found in almost every GR textbook, e.g. Hobson et al (2006), Ch. 17 and 18.

[15] A state of the art monograph on the topic, for instance, takes it for granted that "the fact that GWs carry energy and momentum is *already* clear from the discussion of the interaction of GWs with test masses presented above" Maggiore (2008), p. 26, our emphasis. Maggiore seems to articulate the common view that each of the arguments we present is already conclusive, on its own.

[16] Cf., for instance, Hobson et al. (2006), Ch. 17, or Misner et al. (1973), Ch. 35 for introductions

[17] In fact, for a GW, such as the ones we expect to be incident on our GW detectors, $h_{\mu\nu}$ is typically of order 10^{-21}. For comparison, even the far greater values of the gravitational fields in our solar system are still quite small, typically: $|h_{ij}| \lesssim 10^{-6}$, cf. Misner et al. (1973), Ch. 39.

[18] Cf. Ohanian/Ruffini (2013), Ch. 3, where this is systematically spelt out.

Expanding the full GR tensors in powers of the perturbation $h_{\mu\nu}$ yields in leading order the corresponding quantities in linearised theory (denoted by the scripted symbols), such as the linearised Einstein tensor:

$$\mathcal{G}_{\mu\nu} = \partial^2_{\mu,\nu} h + \Box h_{\mu\nu} - \partial^2_{\lambda,\nu} h^\lambda_\mu - \left(\Box h - \partial^2_{\kappa,\lambda} h^{\kappa\lambda}\right)\eta_{\mu\nu},$$

with $h \coloneqq \eta^{\mu\nu} h_{\mu\nu}$ and the flat spacetime d'Alembertian $\Box \coloneqq \eta^{\mu\nu}\partial^2_{\mu,\nu}$. For consistency with the weak field limit, the energy-momentum tensor $T_{\mu\nu}$ on the r.h.s. of the Einstein Equations must likewise be of first order in the perturbations ($T_{\mu\nu} \approx \mathcal{T}_{\mu\nu}$)[19].

Harnessing the gauge freedom for the gravitational field, the linearised Einstein Equations, $\mathcal{G}_{\mu\nu} = -\frac{16\pi G}{c^4}\mathcal{T}_{\mu\nu}$, simplify for a particular gauge, viz. the s.c. TT gauge, to an inhomogeneous wave equation. Its general solution is obtained via Greens functions. For the purposes of GWs it suffices to restrict ourselves to the vacuum case ($\mathcal{T}_{\mu\nu} = 0$) with plane wave packets as solutions, $h_{\mu\nu} = \int d^3\vec{k} A_{\mu\nu}(\vec{k}) e^{ik_\lambda x^\lambda}$ (with a generic, wave-vector dependent function $A_{\mu\nu}$, the s.c. polarization tensor). For concrete astrophysical applications such wave packets and their effects are further evaluated.

One may worry here about the gauge dependence of all results thus derived. However, in the context of the cosmological perturbation formalism[20] it can be shown in a manifestly gauge-invariant manner that only the TT-components of the metric obey a wave-like equation, therefore identified as radiative; the other components satisfy an equation of the Poisson type.

Like an electromagnetic one, a GW has two polarizations. Their respective designations, $\oplus$ and $\otimes$, indicate the axes along which a ring of a test particles is distorted (see below). Figure 1 shows the effect a purely $\oplus$-polarised GW travelling along the z-axis, $h^{\oplus}_{ab} = \cos k(ct - zeab\oplus$, produces in a transverse circle of particles.

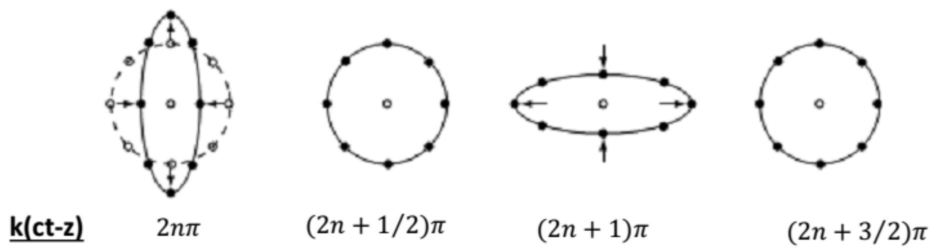

Fig. 1[21]: Deformations of a ring of test particles in the x-y plane or an incident, purely $\oplus$-polarised GW. The initial configuration is shown by the open dots.

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[19] Note that the energy-stress tensor does not so much encode energy-stress of matter *simpliciter*; rather it is a functional/relation depending on both the metric and the matter fields, cf. Lehmkuhl (2010) and Duerr/Lehmkuhl (2015b).

[20] Cf. Flanagan/Hughes (2005). Such a decomposition of the metric into radiative TT-pieces, (unphysical) gauge pieces and gravitational pieces is always possible and unique, cf. op.cit.

[21] Taken from Hobson et al. (2006), p. 506.

For everyday experimental/laboratory practices it is apposite now to adopt the s.c. proper detector frame[22] - a coordinate system in which one fixes the origin, and then uses rigid rulers to delineate coordinates, enforcing thus the geometry measured in these coordinates to be Euclidean.

The GW incident on our above ring of test particles (each assumed to be of mass m) deforms the latter, stretching and squeezing the ring in x- and y-direction, respectively: $\delta x(t) = \frac{h_\oplus}{2}\sin\omega t$ and $\delta y(t) = -\frac{h_\oplus}{2}\sin\omega t$. (Fig.1 depicts the deformations by the black arrows). The particles start moving, thereby changing their **kinetic energy**:

$$E_{kin} = \frac{m}{2}(\delta\dot{x}^2 + \delta\dot{y}^2) = \frac{m}{4}h_\oplus^2\omega^2\cos^2\omega t.$$

Whence the energy gain - so the received wisdom - if not from the GW?

It appears to add consistency to the picture that for an observer in the proper detector frame, the whole deformation scenario also permits a **description in terms of a** well-behaved (albeit solenoidal) **Newtonian force** exerted on the particles; for the coordinate separation vector ξ^μ the geodesic deviation equation yields:

$$\frac{d^2\xi^i}{dt^2} = \frac{1}{2}\ddot{h}_{ij}^{TT}\xi^j$$ [23, 24]

2. Feynman's Sticky Bead Argument

As we already saw in the previous subsection, any claims in GW theory must make sure they are not artifacts of specific gauges/coordinate conditions[25].

During an era where the existence of GWs was still hotly debated, i.e. in the mid 50s, Feynman proposed a simple, qualitative thought experiment[26]. "In fact it was this […] argument […] to convince people that gravitational waves must carry energy."[27]

The setup includes beads on a stick, which serve as a detector: The two beads can "[slide] freely (but with a small amount of friction) on a rigid rod. As the wave passes over the rod, atomic forces hold the length of the rod fixed, but the proper distance between the two beads oscillates. Thus the beads rub against the rod, **dissipating heat**."[28]. The subsequent heating up then seems to demonstrate that the GW can do work. For conservation of energy

[22] Cf., for instance, Maggiore (2008), Ch. 1.3.3

[23] Latin indices i, j, k (also in sums) will in the following always denote spatial components 1,2,3.

[24] Recall that despite adopting the proper detector frame, the Riemann-tensor on the r.h.s., i.e. $\ddot{h}_{ij}^{TT}$, may be evaluated in the TT frame, since the Riemann tensor in linearised theory is invariant, rather than merely covariant.

[25] Such worries caused a lot of confusion in the history of GWs, with Einstein famously vacillating between belief and disbelief in their existence, cf. Kennefick (2007), esp. Ch. 5, 6, 11

[26] Cf. Feynman et al. (2002), Foreword.

[27] Letter by Feynman, quoted in Kennefick (2007), pp. 220.

[28] Feynman et al. (2002), p. xxvi.

to hold, we obtain an account agreeing with the previously discussed considerations of kinetic energy: Whence should the gain in thermal energy come, if not from the GW?

The argument can be made **more quantitive** with a simple damped spring[29]. Consider two masses m_1 and m_2 placed on the x-axis and connected by spring of spring constant k. The spring is at equilibrium when the masses are separated by length L. Let x measure the displacement of the masses w.r.t. this equilibrium. If a purely ⊕-polarised GW hits the system, the induced oscillations obey $\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 = -\frac{1}{2}h_{\oplus}L\omega^2\cos\omega t$ with the characteristic frequency of the oscillator $\omega_0 := \sqrt{\frac{k}{\mu}}$, the reduced mass of the system $\mu := \frac{m_1 m_2}{m_1+m_2}$ and the damping parameter $\beta := \frac{b}{2\mu}$, where the dissipative force is assumed to be $F_{\text{diss}} = -b\frac{dx}{dt}$. The work done by the GW on the oscillator, averaged over a cycle of oscillation, can thus be determined to give $\langle W_{GW}\rangle = -\langle E_{\text{kin}} + E_{\text{pot}}\rangle = -\langle\frac{\mu}{2}\left(\frac{dx}{dt}\right)^2 + \frac{k}{2}x^2\rangle = \beta\mu x_{\max}^2\omega^2$, with the resonant amplitude $x_{\max} = \frac{1}{2}\frac{h_{\oplus}L\omega^2}{\sqrt{(\omega_0^2-\omega)^2+4\omega^2\beta^2}}$. This dissipated energy manifests itself as thermal energy.
The changes in total energy of the system, $E_{\text{kin}} + E_{\text{pot}}$, are counterbalanced by changes in the energy of the GW.

3. Perturbative approach[30]

Linearised GW theory was premised on expanding around a flat spacetime background. This idea is generalised within a perturbative approach, where one **decomposes the metric into a slowly-varying background and a fast-varying perturbation** (viz. "the wave"). An object then emerges that *prima facie* naturally qualifies as representing the energy-momentum of the GW.

For simplicity, we restrict ourselves to vacuum solutions of the full Einstein field equations, $G_{ab} = 0$. Let's assume the existence of suitable length (or time) scales of the variation, an assumption that enables us to decompose the metric into a background and small fluctuation components. Applying the standard procedure for perturbation theory for the metric (with the formal book-keeping parameter ϵ),

$$g_{ab} = g_{ab}^{(0)} + \epsilon g_{ab}^{(1)} + \epsilon^2 g_{ab}^{(2)},$$

the Einstein tensor can then be expanded up to $O(\epsilon^2)$ as

$$G_{ab} = G_{ab}^{(0)}\left[g_{ab}^{(0)}\right] + \epsilon G_{ab}^{(1)}\left[g_{ab}^{(0)}, g_{ab}^{(1)}\right] + \epsilon^2\left(G_{ab}^{(1)}\left[g_{ab}^{(0)}, g_{ab}^{(2)}\right] + G_{ab}^{(2)}\left[g_{ab}^{(0)}, g_{ab}^{(1)}\right]\right).$$

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[29] Cf. Anderson/Creighton (2012), pp. 65. In essence, the idea underlies resonant mass/bar detectors, cf. Maggiore (2008), Ch. 8.

[30] We closely follow Maggiore (2008), Ch. 1.4, and Padmanabhan (2010), Ch. 9.5, to which we refer for details.

(Here, the superscript $^{(0)}$ denotes the unperturbed (0^{th} order) quantities and, correspondingly, $^{(1)}$ the 1^{st} order perturbations; $G_{ab}^{(1)}\left[g_{ab}^{(0)}, g_{ab}^{(1)}\right]$ is supposed to indicate that the 1^{st} order terms of the Einstein tensor are built from 0^{th} and 1^{st} order terms of the metric.). The first term describes the unperturbed background geometry, the second the **evolution of the perturbations on the background**, i.e. the wave. The last term is the one of principal interest to our current purposes: It describes how the 2^{nd} order perturbations are related to the background and 1^{st} order perturbations. Recast as

$$G_{ab}^{(1)}\left[g_{ab}^{(0)}; g_{ab}^{(1)}\right] = \frac{8\pi G}{c^4} t_{ab}^{(\text{eff})}$$

with the "effective GW energy-stress tensor" $t_{ab}^{(\text{eff})} = -\frac{c^4}{8\pi G} G_{ab}^{(2)}\left[g_{ab}^{(0)}; g_{ab}^{(1)}\right]$, the third term in the above expansion can be construed as follows: The 2^{nd} order perturbations of the metric are sourced by the effective GW energy-stress, reflecting[31] the back-reaction of the gravitational field upon itself, i.e. the gravitational energy *qua* energy contributing to the its own generation.

The **"effective GW energy-stress tensor"** $t_{ab}^{(\text{eff})}$ has the following **properties** that commend it for an interpretation as encoding the energy-momentum of a GW:

1. It is by construction symmetric.
2. It is quadratic in the dynamical components of the "gravitational fields", the GW field variables $g_{ab}^{(1)}$. In this respect $t_{ab}^{(\text{eff})}$ satisfies what we would expect from energy-stress tensors familiar from other classical field theories, such as the electromagnetism or hydrodynamics.
3. It obeys a conservation law, $\nabla^b t_{ab}^{(\text{eff})} = 0$ where the covariant derivative is defined w.r.t. the background metric $g_{ab}^{(0)}$. (For the non-vacuum case, we have the total energy-momentum balance $\nabla^b \left(T_{ab} + t_{ab}^{(\text{eff})}\right) = 0$ - a continuity equation for the energy-momentum fluxes/densities the integration over which, given certain, supposedly natural conditions, gives rise to globally conserved total energy-momentum.
4. It is regarded[32] to originate in the non-linearity of the Einstein equations, the non-linearity itself being supposed to reflect the fact that "gravity gravitates", i.e. all kinds of energy including gravitational energy itself act as a source for the "gravitational potentials" g_{ab}.

We can also re-formulate the perturbative approach in terms of a variational principle (**"Isaacson's variational approach"**)[33]. The idea is to expand the action $S[g_{ab} + h_{ab}] =$

[31] Cf., for instance, Hobson et al. (2008), Ch. 17.11
[32] Op. cit. pp. 486.
[33] Cf. Schutz/Ricci (2010), Sect. 4.2

$\int d^4x\sqrt{|g[g_{ab}+h_{ab}]|}R[g_{ab}+h_{ab}]$ w.r.t. the perturbations h_{ab} of the background g_{ab}, such that:

$$S[g_{ab}+h_{ab}] = S[g_{ab}] + \int dx^4 h_{ab}\frac{\delta\left(\sqrt{|g|}R\right)}{\delta g_{ab}} + 16\pi\sqrt{|g|}L^{(GW)} + \mathcal{O}(h^3),$$

where the GW Lagrangian (whose exact, complicated form doesn't matter here) is built from terms quadratic in the perturbation. Given this GW Lagrangian, we can define an effective energy-stress tensor associated with the GW by treating the perturbation as a matter field: $T^{ab}_{(GW)} = \frac{2}{\sqrt{|g|}}\frac{\delta}{\delta g_{ab}}\left(\sqrt{|g|}L_{(GW)}\right)$.

In sum, again we have perturbatively arrived at an object complying with our intuitions of an energy-momentum of a GW, an object in particular for which a conservation law in the form of a vanishing covariant divergence holds.

4. The Noetherian perspective[34]

One of the characteristics of GR -most importantly from a gauge theoretical point of view[35]- is its general covariance (GC), i.e. the **invariance under general diffeomorphisms**. GC as a local symmetry of the Einstein-Hilbert action $\int d^4x\sqrt{|g|}R$ lends itself to an application of Noether's 2nd Theorem[36]. Due to the freedom to add total divergences to the Lagrangian in the Variational Problem, we may restrict ourselves to considering the dynamically equivalent[37] truncated ΓΓ-Lagrangian

$$\bar{\mathcal{L}} = 2g^{ab}\Gamma^{d}_{a[b}\Gamma^{c}_{c]d}.$$

It permits us to treat GR like any other field 1st-order theory (recall: $\Gamma^{\mathrm{a}}_{\mathrm{bc}} = \frac{1}{2}g^{ad}(\partial_c g_{db} +$ $\partial bgdc - \partial dgbc$) and apply **Noether's 2nd Theorem**:

Consider an infinite-dimensional group $G_{\infty,\rho}$ of transformations that smoothly depend on ϱ functions $p_\alpha(x^a)$ and their derivatives[38] $\partial_\beta p_\alpha(x^a)$ and give rise to the variation of the dynamical fields ψ_i(of generic tensorial nature) $\delta\psi_i = \sum_\alpha(a_{\alpha i}\Delta p_\alpha + b^c_{\alpha i}\partial_c\Delta p_\alpha)$, where $a_{\alpha i}$ and $b^\mu_{\alpha i}$ are coefficient functions that depend on x^a, ψ_i and $\partial_c\psi_i$. (The Δp_α's indicate that we are taking infinitesimal p_α's.) Then, if the action $S = \int d^4x\mathfrak{L}$ is ("locally gauge") invariant under G_∞, then the following ρ identities hold:

[34] A more rigorous treatment of the issues here is found in Schutz/Sorkin (1977).

[35] The role of, say, the $U(1)$ gauge group in electrodynamics, in GR is taken over by the (non-Abelian) diff($\mathcal{M}$) group.

[36] Treatments of Noether's Theorem can be found in any field theory textbook, e.g. Maggiore (2005), Ch. 3.2. See also Brading/Brown (2000) for a discussion w.r.t. gauge symmetries in general, and Brown/Brading (2002) for a more specific discussion of general covariance as a gauge symmetry.

[37] Cf., e.g., Hobson et al (2008), Ch. 19.19.

[38] For simplicity, a restriction is made to first derivatives. The extension to higher derivatives is straightforward.

$$\sum_i a_{\alpha i} \frac{\delta\mathfrak{L}}{\delta\psi_i} \equiv \sum_i \partial_c \left(b_{\alpha i}^c \frac{\delta\mathfrak{L}}{\delta\psi_i} \right).$$

Here $\frac{\delta\mathfrak{L}}{\delta\psi_i}$ denotes the variational derivatives, corresponding to the Euler-Lagrange expressions. If now the matter field equations (Euler-Lagrange Equations) hold, i.e. $\frac{\delta\mathfrak{L}}{\delta\psi_i} = 0$, the Noether-current $j_\alpha^c \coloneqq \sum_i b_{\alpha i}^c \frac{\delta\mathfrak{L}}{\delta\psi_i}$ on the r.h.s. is conserved: $\partial_c j_\alpha^c = 0$. An integration yields the associated conserved Noether charge $Q_\alpha = \int d^3x\, j_\alpha^0$.

The derivation hinges on the freedom to choose the p_α and $\partial \mathrm{p}_\alpha$, such that they vanish on the integration boundary. The above Noether-identity constitutes the necessary condition that the integral associated with the interior contribution to the variational problem must vanish independently of the boundary contribution.

The above treatment is unsatisfactory, though, for two reasons: Firstly, because the current is gauge-dependent; and secondly, the discarding of interior contributions to the Variational Problem seems unduly restrictive – especially for GR. We therefore present a little known refined result called the **(Klein-Utiyama) Boundary Theorem**[39], which addresses both issues.

Let the action $S = \int d^4x\mathfrak{L}$ be invariant under an infinite-dimensional Lie group, which depends smoothly on the ϱ arbitrary functions $p_\alpha(x^b)$ and $\partial_b p_\alpha(x^b)$. Then there exist three sets of ϱ relationships:

- $\sum_i a_{\alpha i} \frac{\delta\mathfrak{L}}{\delta\psi_i} \equiv -\sum_i \partial_c \left(a_{\alpha i} \frac{\partial\mathfrak{L}}{\partial(\partial_c\psi_i)} \right)$
- $\sum_i b_{\alpha i}^\mu \frac{\delta\mathfrak{L}}{\delta\psi_i} \equiv -\sum_i \left(a_{\alpha i} \frac{\partial\mathfrak{L}}{\partial(\partial_\mu\psi_i)} + \partial_\nu \left(b_{\alpha i}^\mu \frac{\partial\mathfrak{L}}{\partial(\partial_\nu\psi_i)} \right) \right)$
- $\sum_i \left(b_{\alpha i}^\nu \frac{\partial\mathfrak{L}}{\partial(\partial_\mu\psi_i)} + b_{\alpha i}^\mu \frac{\partial\mathfrak{L}}{\partial(\partial_\nu\psi_i)} \right) \equiv 0$

Remark: For a *global* gauge symmetry ($p_\alpha(x^b) = p_\alpha = const.$), the first identity can immediately be seen to reduce to Noether's *1st* Theorem.

Let's apply now the Klein-Utiyama Theorem to GR[40]. (Einstein himself took this variational route that led to the establishment of the continuity equation given below in his 1916 paper on the Hamiltonian principle in GR[41], i.e. two years before Noether's publication.)The expression relevant for us is the decomposition of the Einstein tensor

[39] Cf. Brading/Brown (2000), also for further references to the history of the theorem. (Beware: The last two equations of the Boundary Theorem therein both contain typos.)
[40] Ohanian (2013), Appendix 1 essentially recapitulates all the steps.
[41] Cf. Einstein (1916). See for a slightly different derivation Schrödinger (1950), Ch. XI. For the ensuing discussions between Klein, Noether, Hilbert and Einstein over the status of energy conservation in generally covariant theories, cf. Brading (2005).

$$-\frac{1}{8\pi G}\sqrt{|g|}G_a^b \equiv \sqrt{|g|}t_a^{\ b} + \partial_c \mathfrak{W}_a^{[bc]}$$

Here $\sqrt{|g|}t_a^{\ b} := -\mathcal{L}\delta_a^b + \left(\frac{\partial\mathcal{L}}{\partial(\partial_b g_{de})} - \partial_c \frac{\partial\mathcal{L}}{\partial(\partial_{b,c} g_{de})}\right)\partial_a g_{de} + \frac{\partial\mathcal{L}}{\partial(\partial_{b,c} g_{de})}\partial_{a,c} g_{de}$ denotes the s.c. Einstein pseudo-tensor (plus contributions from second derivatives, which form a surface term) and $\mathfrak{W}_a^{[bc]}$ a so-called superpotential[42], antisymmetric in its upper indices.

The Einstein pseudotensor[43] corresponds to the canonical gravitational energy-momentum tensor, exactly as we would expect it from other field theories[44]. Using the Einstein Equations, we arrive at

$$\sqrt{|g|}(T_a^b + t_a^{\ b}) = -\partial_c \mathfrak{W}_a^{[bc]}.$$

Thanks to the antisymmetry of the superpotential in its upper indices the last equation implies the **continuity equation** ("local conservation law"):

$$\partial_b\left(\sqrt{|g|}(T_a^b + t_a^{\ b})\right) = \partial_{b,c}\mathfrak{W}_a^{[bc]} \equiv 0$$

Despite being not a tensor equation, this continuity holds in every coordinate system and reflects the solenoidality of the total energy density.

Based on our experiences with other fields with their compact support (and benign falloff conditions), we might rest content and assume that an integration over an isolated system leads to a **global conservation law**[45].

A straightforward calculation[46] reveals that in the Newtonian approximation the Noether charge associated with the above continuity reduces to the sum of rest mass, potential gravitational energy and remaining field energy. In other words: In first order, the Noetherian framework reproduces the energy of Newtonian Gravity.

In sum, applying the standard field-theoretic tool of Noether's Theorem, we arrive at an object that a) is structurally completely analogous to the energy-momentum of all other field theories, b) thus also enters a total energy-momentum balance, and c) reduces in the Newtonian approximation to the classical energy-balance of the Newtonian case.

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[42] For our purposes it suffices to regard superpotentials as functions of the given form. For superpotentials more generally, cf. Trautmann (1962).

[43] Dirac (1975), Ch. 32 gives a slightly more handy expression for the Einstein pseudotensor. Notice, however, that Dirac merely defines by *fiat* the Einstein pseudotensor, and then derives the continuity equation via the field equations, i.e. the connection to the Noether Theorem is not made.

[44] Cf. e.g. Maggiore (2005) Ch. 3.2

[45] Cf. Weyl (1922), §33.

[46] Cf. Ohanian (2013), p. 11.

5. Summary

The energy of a GW seems warranted by its ability to change the kinetic and thermal state of a detector. Within a perturbative approach and the Noetherian framework, a systematic account similar to the one familiar from other field theories seems possible - a picture *prima facie* consistent and highly satisfactory.

In the subsequent section we shall critically examine these four approaches just reviewed – pointing out their respective shortcomings. We argue the latter turn out to be so severe that they can no longer be sustained as supporting the notion of GW energy.

III. Analysis: Corpses in the cupboard

In this section we critically examine the preceding four arguments. We shall here argue that

1. The above kinematic and thermal arguments employ an illegitimate blend of relativistic and pre-relativistic concepts and intuitions.
2. The background-perturbation split of the metric, characteristic of the perturbative approach, is fundamentally not tenable.
3. Within GR, pseudotensors, as they emerge from both the perturbative and the Noetherian framework, defy a straightforward interpretation as representing.

1. Linearised Theory: Bastardisation of frameworks

Any representation of a theory, since a choice of such is arbitrary, should admit a consistent interpretation. Whereas above we chose for practical reasons the proper detector frame for describing the effects of a GW on our detector, we are equally entitled to adopt the s.c. TT frame. In the latter the coordinate labels are attached to freely falling test particles – an attractive feature in the face of which one might even argue that the TT frame is preferable vis-à-vis the proper detector frame: Firstly, because by attaching the coordinate labels to physical events, it operationally grounds coordinates (conventional, and thus physically vacuous by nature!) in a natural, physical way; secondly, since the test particles/coordinates represent inertial frames, i.e. follow geodesics, the TT frame is a natural frame to couch one's physical accounts in – just as in classical mechanics one usually describes systems in inertial reference frames, lest the picture of the physics at work be obfuscated by fictious forces, e.g. a Coriolis force. In fact, according to a theorem first formulated by Helmholtz[47],

[47] Cf. Mittelstaedt (1981), Ch. II §3.The first rigorous proof, however, seems to have come from Sophus Lie in 1888. I thank Marco Giovanelli (CalTech) for this hint.

the rigid rulers deployed to delineate the coordinates of the proper detector frame are *not* freely movable in spaces of non-constant curvature[48]; a spatiotemporally variable curvature necessarily inflicts deformations on extended bodies. To sustain the rigidity of the rulers therefore requires forces, i.e. additional structure living on the spacetime, to counterbalance such deformations.

In the TT frame, however, as follows immediately from the geodesic equation, no particle thus changes its coordinate position, $\frac{dx^a}{d\tau} = 0$. From this perspective, with no change in velocity of the particles there seems no change in kinetic energy attributable to the GW.

Let's study more systematically the proper general-relativistic counterparts of kinetic energy.

In classical mechanics, **kinetic energy plays a twofold role**[49]:

i. On the one hand as the remaining part of the total energy, after subtracting the contributions from all dynamical interactions;
ii. on the other hand as the Lagrangian for the equation of motion of a free particle[50].

In the transition to special- and general-relativistic mechanics, these two roles and their respective actors no longer coincide. Let's spell this out and see how the previous kinetic energy considerations in the TT frame fit in.

i. Decomposition of the total energy

We mimic the standard decomposition of energy of a particle (with mass m) as the 0-component of the 4-momentum p^a, relative to an observer with the 4-velocity $u^a_{(\text{obs})}$ - the very decomposition that in SR[51] allows us to obtain the energy-mass-equivalence $E = mc^2$: Subtract from the energy relative to the particle relative to the observer its rest energy

$$E_{\text{kin}} = p_a u^a_{(\text{obs})} - mc^2.$$

We evaluate this further via the length of the norm of the 4-velocity of our observer above, $c^2 = g_{ab}u^a u^b = c^2 g_{00}\left(\frac{dt}{d\tau}\right)^2 + c g_{0i} v^i \frac{dt}{d\tau} + c^2 g_{ij} v^i v^j \left(\frac{dt}{d\tau}\right)^2$, where τ denotes the affine parameter and $v^j = \frac{dx^j}{dt}$ the coordinate velocity.

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[48] I.e. in any space equipped with a geometry other than (pseudo-) Euclidean, hyperbolic or elliptical geometry. By free movability of a rod/clock we mean that there exists an isometric mapping of the region of spacetime points that make up the rod/clock onto itself.

[49] Petkov (2012), Ch. 2 and esp. Appendix B expounds an interesting interpretation of kinetic energy as the (real!) work done by the Inertial Force, as which Petkov identifies the (sign-reversed) r.h.s. of Newton's 2nd Law, $\vec{F} = m\dot{v}$. This allows a link between both facets of kinetic energy: The free Lagrangian, i.e. kinetic energy T corresponds to minus the analogous Inertial Energy; consequently the full Lagrangian –T+U represents the sum of the Inertial and potential energy, the extremalization of which yields the equations of motion.

[50] This is how kinetic energy is occasionally even defined, cf. Landau/Lifshitz (1976), §4.

[51] Cf., for instance, Malament (2012), Ch. 2.4

Now consider a stationary observer, for whom $v^i_{(\mathrm{obs})} = 0$ - a natural assumption, since otherwise the observer would be general-relativistically accelerated, i.e. his worldline would deviate from a geodesic). Furthermore recall from sect. II.1 that the radiative degrees of freedom are contained in the TT-parts of the metric (such that $g_{0i} = 0$). Putting this together, we obtain $u^0_{(\mathrm{obs})} = c\frac{dt_{(\mathrm{obs})}}{d\tau} = \frac{c}{\sqrt{g_{00}}}$ and consequently for the kinetic energy[52]:

$$E_{kin} = mc^2\left(\sqrt{\frac{g_{00}}{g_{00} + g_{ij}v^i v^j}} - 1\right).$$

Having chosen a coordinate system, such as the TT frame, in which the particle positions don't change, $v^j = 0$, the kinetic energy is zero! (Not surprising, in retrospect, for no particle is moving in terms of coordinate distances.) For a coordinate system that is at least as legitimate as the one previously used (if not even preferable!), the argument of section II.1, according to which the change in kinetic energy of test particles vouches for the energy of a GW, breaks down.

Let's now turn to the second aspect of kinetic energy: Does the GW affect it?

ii. Lagrangian of a free particle

The GR Lagrangian whose variation yields the equation of motion for a free massive particle, in generalisation of the special-relativistic case, reads[53]:

$$\mathcal{L}_0 = \sqrt{\left|g_{ab}\frac{dx^a}{d\tau}\frac{dx^b}{d\tau}\right|}$$

Being a scalar with the constant numerical value c (for massive particles; for photons, $\mathcal{L}_0 = 0$), by definition, it doesn't change, even if the metric is one that describes a GW. In other words: If we take kinetic energy to refer to $\mathcal{L}_0$, it does not increase.

The "non-gain" of kinetic energy in the TT frame notwithstanding, the **physical (spatial) distances/separation,** $\sqrt{|g_{ij}\xi^i\xi^j|}$, for a spacelike vector $\xi = (0, \xi^i)$, do, of course, **change,** whereas the coordinate distance $\sqrt{|\delta_{ij}\xi^i\xi^j|}$ remains constant. To be sure, one can recover

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[52] In Minkowski space, this reduces to the familiar special-relativistic expression for kinetic energy

$$mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right) = \frac{1}{2}mv^2 + \frac{3}{8}m\frac{v^4}{c^2} + \cdots.$$

[53] See, e.g., Rohrlich (2007), Ch. 8.4. That the variation of $\mathcal{L}_0$ yields the geodesic equation is a routine exercise, cf. for instance, in Hobson et al. (2006), Ch. 3.18.

the rate of change of the physical distances[54] (i.e. and interpret the result as a force law, yielding the same result as in the proper detector frame description). However, within the spacetime setting inherent in Minkowskian physics, *co-moving* coordinate and physical (proper) distances coincide. So in order to describe the effects correctly the distinction between coordinate and proper distances is necessary – a distinction *per se* not available in a pre-GR framework. In other words: If we regard linear theory as a special-relativistic theory of gravity and adopt the physically natural TT frame, we cannot recover straightforwardly a force law for the change of physical distances.

Let us close this subsection with an often rehearsed claim[55] that the full Einstein equations can be recovered (uniquely and without appeal to concepts beyond special-relativistic field theory) from self-consistently coupling a massless spin-2 graviton (on a fixed Minkowski background) to the total energy-momentum tensor –in line again with the intuition that "gravitational energy" should also act as a source for the "gravitational field". The idea of this **"bootstrapping"** iteration schema is as follows[56]: i) Calculate the energy-momentum carried by the 1st order perturbation of the metric. ii) This energy-momentum acts as a source for the 2nd order perturbations of the metric. iii) Calculate the energy-momentum carried by the latter 2nd order perturbations, etc. It turns out that "bootstrapping" doesn't work[57]: One cannot develop GR in a systematic expansion of powers of the metric around a flat background spacetime. This means that, without further assumptions, GR *per se* cannot be interpreted as an effective a force theory on a Minkowski background.

Further criticism of linear GW theory will ensue from the discussion of the more general perturbative approach, to which we turn presently. For the moment, however, let us turn to the Sticky Beads Argument.

2. Sticky Beads

The validity of Feynman's Sticky Bead Argument rests on two premises:

- Firstly, we tacitly assumed that the proper general-relativistic description of the spring detector does not already involve correction terms of the same order of magnitude as the heating up (the induced temperature increase in bar detectors being typically already extremely small, $\sim 10^{-21} J$[58]). It would be desirable, though, to start from the general-relativistic counterparts of friction (and a realistic matter model for the spring) and *then* prove this claim. However, such general-relativisations are hard to come by[59]. Too great confidence in and premature

[54] As is explicitly done in e.g. Ohanian/Ruffini (2013), Ch. 5.2.
[55] E.g. Feynman (2002)
[56] cf. Hobson et al (2008), p. 473
[57] Cf. Padmanabhan (2004).
[58] Cf. Aderson/Creighton (2012), p. 66
[59] For instance, a satisfactory general-relativistic statistical mechanics, as will be necessary for a realistic matter model of the spring, has not yet been developed, cf. Rovelli (2012) for some tentative first steps.

extrapolation of our pre-GR physics easily leads us astray, given how extraordinarily complex the "general-relativising" of classically simple situations can easily become (e.g. for the electromagnetic self-force[60], likely to emerge again, too, in a proper general-relativistic model for friction).

For the more quantitative version of the argument, we assumed, for instance, that the spring constant remains indeed a constant - rather than varying also due to the GW. Such a variation should be expected, however, since the atoms of the spring can no longer move inertially, when the wave a passes through them; consequently, one would suspect the spring to lose binding energy. As Cooperstock and Tieu remark in their criticism of the Sticky Bead Argument, "what has been overlooked is that the bar itself has been presumed to be unaffected by the gravity waves"[61].

- But let's grant that even the properly "general-relativised" sticky bead setup would heat up. From this we inferred the energy *transfer from* the GW, so as to restore the energy balance. However, in GR energy-momentum is no longer conserved in general[62]. Recall that conservation laws are associated via Noether's Theorem with spatiotemporal symmetries, e.g. energy or momentum conservation follow from the homogeneity of time and space, respectively – characteristic symmetries of Minkowski spacetime. Generic spacetimes, such as ones with GWs, lack these symmetries. Schrödinger offers a "singularly striking example"[63] of this generic violation of energy conservation in GR, viz. the decrease of energy in a closed bounded universe; the popular, non-relativistic "explanation" invokes the work of the pressure to push back the adiabatically expanding volume of a "spacetime-gas" – a fictional account: There is no piston nor any boundary through which energy could escape; energy conservation simply doesn't hold.

Of both assumptions underlying the Sticky Bead Argument, viz. adequate knowledge of proper the general-relativistic description of the whole setup and conservation of energy, respectively, the first one teems with uncertainties, and the second one is downright wrong. Feynman's argument thus fails to demonstrate that GWs carry energy.

3. Artificial background-perturbation distinction

By construction, the perturbative approach presupposes a background-perturbation split. Occasionally, such a split is pragmatically justified, when dealing with distinguishable scales, λ and L, over which the background geometry and the perturbation vary, $\lambda \ll L$ - a situation we have, for instance, for primordial GWs on an FLRW background. Yet, from a fundamental

[60] The s.c. Brehme-deWitt Equation, cf. some remarks in Ch. 8.4 in Rohrlich (2007).

[61] Cooperstock/Tieu (2012), p. 85, who also discuss an idealised electromagnetic model of the sticky bead detector as an elastic medium and argue that for this model the detector does not heat up.

[62] Cf. Carroll (2010). For a detailed analysis, cf. Duerr/Lehmkuhl (2015b).

[63] Cf. Schrödinger (1950). Misner et al. (1974), §19.4 also discuss in detail the mass and angular momentum of a closed universe.

point of view, it is artificial: Ultimately, there is only one metric; generically, slowly and fast varying parts cannot be severed. For, instance, at early times during cosmic inflation, the wavelength of GWs is smaller than the Hubble scale ("inside the horizon"); as inflation proceeds, the GW's wavelength redshifts and becomes eventually larger than the Hubble scale ("outside the horizon")[64].

Based on a straightforward comparison of the orders of magnitude, one can show[65] that "one cannot introduce the concept of a gravitational wave of arbitrarily large amplitude but varying at a length scale that is sufficiently small compared with the background scale of variation and develop a systematic perturbation theory".[66] A GW can thus not be fundamentally characterized as such a "ripple on a background".

The best way to regard the perturbative approach is as a transition from a fundamental, true, micro-description to a **coarse-grained macro-description**[67]- a tool convenient for approximations under certain conditions, but not a fundamental account.

Might one not object that discarding the perturbative approach as not fundamental bereaves us of the very concept of a GW altogether? Doesn't a GW presuppose a perturbative approach?- The question can be negated. In its fundamental description, a GW is defined as a wavelike propagating curvature perturbation[68]. Exact wave-solutions, such as the pp-wave metric, are known[69].

But may the talk about wave-like propagation of GWs not deceive us: Unlike sound or electromagnetic waves, GWs do not propagate *through* spacetime; GWs *are* spacetimes[70] of a functional form that satisfies a wave equation.

Let's also briefly comment on some minor points pertinent to the perturbatively defined, **effective energy-momentum tensor**, whose properties, especially in comparison with those of the electromagnetic analogue looked *prima facie* attractive:

- First, note it's not a tensor, but only a pseudo-tensor that transforms tensorially only under tensor transformations of the background metric –a problem we'll turn to in the next subsection. Suffice it here to mention that such a non-tensorial nature raises interpretational difficulties.

[64] Cf. Flanagan/Hughes (2005), Sect. 5.2
[65] Cf. Padmanabhan (2004) and Padmanabhan (2010), pp. 419.
[66] Padmanabhan (2010), p. 420
[67] Cf. Maggiore (2008), Ch. 1.4.2
[68] From a mathematical and conceptual point of view, what encodes the GW is the Weyl tensor, as used, for instance, in the Petrov classification, cf. Padmanabhan (2010), Ch. 5.6.3. Expressing the GW in the form of a wave equation for the curvature tensor, as for instance, eq. (6.100) in op. cit., has the additional advantage of being manifestly gauge-invariant (with the electrodynamic analogue being $\Box F^{ab} = 2\pi\partial^{[a}j^{b]}$), thus warranting *en passant* the objective (non-gauge) reality of the GW.
[69] Cf. Misner et al. (1973), Ch.35.9, 35.10 and 35.11.
[70] Cf. Cooperstock/Tieu (2012), Ch. 6.1

- Secondly, "one should not [...] be too firmly guided by the analogy with electromagnetism. The reason why the electromagnetic energy momentum tensor is quadratic in the field variable is that the electromagnetic field [...] does not carry charge and so cannot act as its own source. Indeed, this is the physical reason why electromagnetism is a linear theory."[71] Rather than with the linear Maxwell theory, GR ought to be compared with the likewise genuinely non-linear vectorial Yang-Mills-type theories[72]. Consequently, an appeal to our intuitions based on the electromagnetic analogy, in and of itself, bears little argumentative weight.
- A third point is that that for a flat spacetime background, i.e. within linear theory, the effective gravitational energy-stress pseudo-tensor is not invariant under the gauge transformations of the type $h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} - \partial_{(\mu}\xi_{\nu)}$. To be sure, this lack of gauge-invariance can be cured, however, by averaging over several wave lengths[73], reflecting the definition of perturbations only w.r.t. the typical length/time scales of the background. The averaging – in essence renormalization group transformations[74], a powerful tool for moving between different layers of description- renders the gravitational energy-stress pseudo-tensor eventually gauge-invariant. Note, though, how here the fundamentally untenable background-perturbation split has sneaked in.

Finally, let's close with drawing attention to **two failures** of the perturbative approach[75] from a practical perspective (although we ultimately reject the fundamentality of such a notion, we here adopt the common, convenient *façon de parler* –hence the quotation marks in "gravitational energy-momentum"):

- It does not provide any notion of a "gravitational energy of the system". Consequently, the perturbative approach falls short of concrete applications, such as binary systems, the "gravitational energy" of which is depleted by the GWs.
- It is too crude to deliver the flux of angular momentum[76], important for the correct description of millisecond pulsars, whose rotation rate increases due to a transfer of angular momentum from the accretion disk surrounding the pulsar.

In sum, the perturbatively obtained gravitational energy-momentum is not suitable for astrophysical applications. Furthermore, and more from a fundamental point, it loses its initial appeal upon realizing its non-tensorial nature and the fact that the electromagnetic analogy is not reliable. Most importantly, the split of the metric into a background and a perturbation, presupposed by the perturbative approach, is ultimately artificial.

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[71] Hobson et al. (2006), p. 487.
[72] Cf. Deser (1970). In such Yang-Mills theories energy is localisable, thus shows that the problems with localising gravitational energy do not originate in the nonlinearity of GR as such.
[73] Cf. Misner et al. (1973), §35.15
[74] Mostly known from effective field theories, cf. Peskin/Schroeder (1995), Ch. 12
[75] Cf. Poisson/Will (2014), Ch. 12.2.5
[76] Cf. Op. cit., Ch. 12.2.4

4. Problems with pseudo-tensors

In the Noether approach, ostensibly satisfactory, two problems arise: The first one concerns fall-off conditions (in order to well-define the associated integrals/ Noether charges from the currents for non-compact manifolds) and the occurrence of pseudotensors. We postpone a discussion of the former, known as asymptotic flatness – the technical details of which would divert us too far from our current purposes[77]; instead, we shall focus on the latter.

The first thing to notice is that the **Einstein pseudotensor**, as we derived it, is not symmetric. This renders it *prima facie* unsuitable to define angular momentum. This defect, which the Einstein tensor shares with the canonical energy-momentum-complexes from other field theories, e.g. the electromagnetic one, can be overcome, however, by a standard procedure[78].

Now to the elephant in the room: Do the pseudotensorial expressions properly express physically meaningful gravitational energy-momentum density/flux? We concur with Weyl that "[…] the differential relations [i.e. the local form of the energy-momentum conservation law with a suitable pseudotensor] are without real physical meaning"[79]. Instead, we submit, pseudotensors are merely formal objects; the corresponding local conservation laws are likewise merely formal, lacking any fundamental physical meaning. Let's spell this out.

The **generic problem with pseudotensors** consists in their non-tensorial nature, which obstructs any straightforward interpretation:

- Although the quoted continuity equation holds in every coordinate system, it is not generally covariant. Therefore the question is: Does it represent anything real or is it a merely formal book-keeping device? Following Weyl[80], we suggest, the latter to be the case.
    - A non-tensorial transformation behaviour means that the object is **coordinate-dependent**: "Indeed all the [pseudotensor components] can through a suitable choice of a coordinate system be made to vanish; [...] on the other hand, one obtains [pseudotensor components] that are different from zero in a 'Euclidean', completely gravity-free world by using a curvilinear coordinate system."[81] Whether the represented entity exists or not, would

[77] Cf. Duerr/Lehmkuhl (2015b)
[78] The s.c. Belinfante-Rosenfeld procedure, cf. Ohanian/Ruffini (2013), Appendix A3.
[79] Weyl (1923), p. 273 (translation P.D.).
[80] Cf. op. cit. §37
[81] Op. cit., p. 273 (translation P.D.)

thus depend on the choice of a coordinate system, which in and of itself is an arbitrary convention[82] – clearly against the intuition that what counts as real should be invariant[83].

Note that this argument hinges on the premise that the pseudotensor is supposed to represent a *local* gravitational energy-momentum density – a premise we take for granted here[84]. (By contrast, the affine connection, for instance, which encodes inertial structure[85], is both physical and *nonlocal*, connecting the fibres of the vector bundle over different points of the base manifolds.

  - At the root of all evil with the above-mentioned coordinate dependence is that for pseudotensors to represent anything physically real, they presuppose additional background structure absent from generic spacetimes: Pseudotensors preserve their invariance only within a privileged class of reference frames. As we just saw, Einstein's gravitational pseudotensor yields non-vanishing values even in flat space-time, when adopting a curvilinear coordinate system. The restriction to Cartesian coordinates to avoid that situation amounts to a preferred choice of reference frames – a privilege not justified within GR (in contrast to Newtonian theory, within which inertial motion is rectilinear).

- Closely related is **ambiguity**:
  - Firstly, without changing the validity of a formal continuity equation there is the freedom to add at whim to the Einstein pseudotensor an arbitrary superpotential.
  - Secondly, a whole plethora of alternative, conceptually very different gravitational energy-momentum pseudotensors (and cognate objects) has been proposed, each with different strengths - and drawbacks. One example is the Landau-Lifshitz-pseudotensor. Different energy-momentum complexes could yield different energy distributions for the same gravitational background.[86]

In short: The background structure on which any physical, *local* meaning of gravitational pseudo-tensors hinges is not available in typical spacetimes; more precisely, the objects qualifying for gravitational energy thus constructed are not independent of these highly contingent background structures: They presuppose a "preferred" reference frame.

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[82] For an interpretation of coordinates in the context of spacetime theories, cf. Dieks (2001) and Duerr/Lehmkuhl (2015b), sect. 4.

[83] Cf., for instance, Vollmer (2010) or Nozick (2003), Ch. 2.

[84] Recently, Curiel (2014) has proven that quite generally no tensor exists that satisfies natural *desiderata* for representing gravitational energy-momentum.

[85] Cf., e.g., Stachel (1993), p. 135

[86] Cf. Multamäki et al. (2008), who also give further (annotated) references.

IV. Discussion: Towards GR-consistent interpretations

So far, we conclude that the idea of energy of GWs stands on shaky ground. Elsewhere[87] we analyse gravitational energy and conservation laws in GR in general, arguing that the former is no longer a fundamental notion in GR, and must ultimately be abandoned; likewise, tied to the symmetries of a flat spacetime, energy-momentum conservation, in both a local form (in terms of a continuity equation) and a global form (in terms of a conserved integral quantity), no longer holds in generic spacetimes[88].

What does this imply for interpreting the GW effects encountered in the course of our journey so far?

1. Sticky Beads and localisation of gravitational energy

With energy no longer being conserved, the heating up of the Sticky Bead detector has a straightforward interpretation, namely as evidence of that violation of energy-conservation GR predicts.

More precisely, note first that the detector itself is held together by inner (mostly electromagnetic) forces. If a GW passes through it, its constitutive atoms are **prevented from following inertial worldlines**: Work is being done by the inner forces; the binding energy of the detector decreases (neglecting other forms of energy). After the GW has passed through the detector, the latter has therefore changed. Some of its binding energy has been converted into heat. If one now registers that the total energy balances before and after the GW incidence do not match, this is merely a manifestation of the (quantifiable) violation of energy-conservation in GR.

The conclusion to abandon energy-conservation, together with the abolition of gravitational energy altogether, puts us in a position to **resolve a long-standing paradox** concerning the non-localisability of gravitational energy, too. Consider Butcher's poignant observation: "[…](E)ven after fifty years, it has not been possible to explain where in spacetime this gravitational energy resides […] In spite of this difficulty […]: when gravity and matter interact, the exchange of energy can be localized. To see this, we need look no further than the [GW] detector: here the energy exchange is certainly localized in so far as it takes place

[87] Cf. Duerr/Lehmkuhl (2015b), drawing on pioneering work by Hoefer (2000).

[88] More precisely, $\nabla_b T^{ab} = 0$ only yields conserved quantities, if the spacetime possesses a Killing field, cf., for instance, Straumann (2013), Ch. 3.4. Minkowski spacetime has ten of them, corresponding to the 10-dimensional Lie-algebra of the Poincaré group: the four generators of translations and the six generators of the homogeneous Lorentz transformations.

only within the confines of the detector."[89] Our solution offers a simple explanation of these facts:

- *Non-gravitational* energy is indeed localisable, whenever there is matter present: Assigning matter, such as an electromagnetic field, localisable energy-momentum, poses no problems.
- Such energy is no longer conserved in generic, viz. non-flat spacetimes – a non-conservation that, in the standard view, is attributed to the *neglect* of gravitational energy. Rejecting energy conservation thus explains why it *appears* that "when gravity and matter interact, the exchange of energy [between spacetime and matter] can be localized": One scrutinises all the localizable non-gravitational energy and ascertains whether it is conserved or not.
- To look for the energy losses/gains due to dissipation (or absorption) via a GW indeed amounts to looking for the right answer to the wrong question, as Misner et al. (whom Butcher cites) put it[90]: Absent any matter, in whose energy content we could register any de- or increase, and, furthermore, rejecting any fundamental concept of gravitational energy, eventually we can also explain "why it has not been possible to explain where in matter-free spacetime this gravitational energy resides" - simply because there is none!

2. Binary Systems

The details of the various approaches and approximation schemes in the treatment of binary systems deserve an analysis in their own right, in particular the status of the point particle limit in GR (or inevitable divergences in the standard, s.c. PPN approximation scheme)[91]. Here, following the ubiquitous textbook custom, we restrict our discussion to interpreting the binary systems, modelled as point particles. Our main point is the following:

When interpreting the inspiralling orbits of binary neutron stars in terms of gravitational energy loss via GWs, one presupposes that only the influence of a dissipative force, viz. gravitational radiation, can account for preventing orbits to be bound. Within classical celestial mechanics, this assumption is surely justified for Kepler potential. In GR, the situation is different though: No bound solutions of the 2-body problem exist[92]; the Einstein field equations do not admit of periodic solutions. As Cooperstock and Tieu put it, "on this basis, the period-changing binary pulsar is simply manifesting its conformity with the mathematical demands of Einstein's General Relativity rather than the preconceptions regarding energy."[93] In other words, the instability of orbits in the general-relativistic 2-body

[89] Butcher (2014), pp. 1. Note, though: While Butcher still clings to the notion of gravitational energy, we abandon it.
[90] Cf. Misner et al. (1973), p. 466
[91] Cf. Ehlers et al. (1976), who asked seminal questions.
[92] Cf. Papapetrou (1957,58)
[93] Cooperstock/Tieu (2012), p. 83

problem is a geometric brute fact, rooted in the non-linearity of GR. If GR - the theory more fundamental than Newtonian Gravity - forbids bounded orbits in principle, then it is neither necessary nor legitimate to try to explain such unstable orbits in terms of a dissipative mechanism – a mechanism alien to a strictly GR framework, anyway.

The common talk of gravitational energy loss via gravitational radiation is an artefact of identifying those "conservative" parts of the metric giving rise to bound orbits as the gravitational energy of the system, and those "dissipative" parts that correspond to GWs. Already on formal grounds, such a separation from a certain expansion order on breaks down (in fact, the whole expansion *per se* blows up!); furthermore, as we saw, one cannot fundamentally split the metric into a background and GW-ripples on it is within GR; *tout court,* there is only one metric. If for pragmatic reasons, one only looks at low order terms, such a split may often be possible. Slicing up such an approximation of the full metric into conservative and dissipative parts allows one to translate the resulting phenomena into a familiar Newtonian framework; the (heuristic or didactic) utility of such a translation, though, comes at the price of fundamentality.

3. Noetherian conservation laws

Abandoning both energy-momentum conservation and gravitational energy, the identities obtained from a Noetherian approach take on a "negative" meaning as expressing the lack of energy conservation.

Although energy-momentum is not conserved – the straightforward interpretation of $\nabla_{\mathrm{b}}\mathrm{T}^{\mathrm{ab}} = 0$[94] - , one can, as we saw, restore a continuity equation by inserting pseudo-tensorial quantities, such as Einstein's. However, such quantities are merely formal, lacking any inherent physical meaning. Formally possible as such a restoration of energy-momentum conservation certainly is, in Hoefer's words: "If energy is not conserved quite generally, there is no need to make up a story about where it has gone when a system loses it."[95]Rather, the Noetherian continuity equations quantify the extent to which energy-momentum conservation is violated. Thereby, one circumvents the mentioned interpretative difficulties with pseudotensors.

Moreover, given that field-theoretically, i.e. on a very fundamental level, (canonical) energy-momentum is defined as the Noether currents associated with time-translation invariance, equations such as $\sqrt{|g|}(T_a^b + t_a{}^b) = -\partial_c \mathfrak{W}_a^{[bc]}$, which we encountered in II.4, can also be understood as expressing the generic *discrepancy* in GR between energy-momentum and the source term tensor T_a^b of the Einstein Equations[96]. In other words, T_a^b does not represent an energy stress tensor *sensu stricto* – a thought taken up elsewhere[97].

[94] Cf. Ohanian (2013), p. 3
[95] Hoefer (2000), p. 296
[96] Szabados (2009), sect. 3.1.2 makes this point.
[97] Cf. Duerr/Lehmkuhl (2015b), sect. 4.2.

V. Summary

In this paper we examined the four standard arguments on which the claim rests that GWs carry energy. We found none to be able to substantiate that claim. For the benefit of the reader, let us recapitulate our principal points:

- Invoking the kinetic effects of a GW on test particles incurred an illegitimate bastardisation of relativistic and pre-relativistic concepts; instead, these phenomena can be understood geometrically as tidal effects due to geodesic deviation.
- Invoking the heating up of a Sticky Bead setup presupposes energy conservation, which no longer holds in GR, and ignores the fact the atoms of the detector can no longer move inertially. The increase in thermal energy is a manifestation of the fact that the atoms of the detector are prevented from inertial motion; the binding energy is change and partially released as heat. If the sum total of energy changes, this is an effect of the non-flat spacetime geometry.
- The continuity equations obtained from a Noetherian perspective feature pseudotensors; their interpretation as physical entities requires physically distinguished background structure GR intrinsically lacks, though. These continuity equations are to be understood as quantifying the degree to which energy-momentum conservation is violated in GR. The pseudotensors are merely formal "book-keeping devices" (Hoefer) to quantify the non-conservation.
- The perturbative approach rests on a fundamentally untenable separation of the metric into a background and a GW. Rather than a fundamental description, it is a tool for a coarse grain description of suitable spacetimes.

The increase in orbital frequency of binary must no longer be interpreted in terms of the system losing energy via gravitational radiation; rather, the inspiralling trajectories of the binary partners is a geometric brute fact of the general-relativistic 2-body problem.

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