Do Renormalization Group Explanations Conform to the Commonality Strategy?

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Abstract. Renormalization group (RG) explanations account for the astonishing phenomenon that microscopically very different physical systems display the same macro-behavior when undergoing phase-transitions. Among philosophers, this explanandum phenomenon is often described as the occurrence of a particular kind of multiply realized macro-behavior. In several recent publications, Robert Batterman denies that RG explanations account for this explanandum phenomenon by following (what I call) the commonality strategy, i.e. by identifying properties that microscopically very different physical systems have in common. Arguing against Batterman's claim, I defend the view that RG explanations are in accord with the commonality strategy.

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1. Introduction

So-called renormalization group explanations (RG explanations, for short) in statistical physics have received a lot of attention in the recent literature on scientific explanation, idealization, reduction and emergence. And rightly so, since RG explanations have several philosophically challenging and puzzling features in being potentially non-causal, highly idealized (in involving limit theorems), and arguably non-reductive explanations.[1] In this paper, I will not address any of these fascinating philosophical issues. My aim is more modest and largely independent of questions surrounding the causal vs. non-causal, highly idealized, and reductive vs. non-reductive character of RG explanations. Against prominent claims to the contrary, I will defend the view that RG explanations are not special and quite intuitive in one crucial respect: RG explanations explain the phenomenon that microscopically different physical systems display the same macro-behavior (in certain circumstances, which I will present below) by referring to features that those physical systems *have in common*, although the physical systems at issue are different in many other respects. I will call this broad explanatory strategy the 'commonality strategy' (borrowing this terminology from Papineau 1993 and Lange 2015).

The plan of the paper is as follows: in section 2, I will sketch the commonality strategy and Batterman's claim that RG explanations do not conform to this strategy. In section 3, I present the core elements of the physics of RG explanations. In section 4, I defend the view that RG explanations conform to the commonality strategy, contrary to Batterman's claim.

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[1] See Batterman (2000, 2002), Butterfield (2011), Morrison (2012), Norton (2012), Menon and Callender (2013), Hüttemann et al. (2015), and Reutlinger (2014a, 2014b, forthcoming).

2. RG Explanations, the Commonality Strategy, and Batterman's Claim

RG explanations are intended to provide the explanation of an astonishing fact: of why microscopically very different physical systems display the same macro-behavior when undergoing phase-transitions. For instance, near the critical temperature, the phenomenology of transitions of a fluid from a liquid to a vaporous phase, or of a metal from a magnetic to a demagnetized phase is (in some respects) the same, although liquids and metals are significantly different on the micro-level. This 'sameness' or – to use a technical term – 'universality' of the macro-behavior is characterized by a critical exponent that takes the same value for microscopically very different systems (for instance, Batterman 2000: 125-126; see Fisher 1982; Cardy 1996; McComb 2004; Strevens forthcoming).[2]

In the philosophical literature, the universality of macro-behavior is typically interpreted in terms of the multiple realizability of a kind of macro-behavior. To take one prominent example, Batterman interprets RG explanations as meeting the "challenge of multiple realizability": "How can systems that are heterogeneous at some (typically) micro-scale exhibit the same pattern of behavior at the macro-scale?" (Batterman 2015: 8). The challenge of multiple realizability encoded in Batterman's why-question goes back to Fodor's famously scandalizing way of articulating the request for an explanation of the fact that there is multiply realized macro-behavior:

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[2] The critical exponent typically figures in an equation describing the order parameter of the physical systems in question (that is, a macroscopic physical quantity such as magnetization), in relation to the so-called reduced temperature.

> “Damn near everything we know about the world suggests that unimaginably complicated to-ings and fro-ings of bits and pieces at the extreme micro-level manage somehow to converge on stable macro-level properties. […] [T]he ‘somehow’, really is entirely mysterious […] why there should be (how there could be) macro level regularities at all in a world where, by common consent, macro level stabilities have to supervene on a buzzing, blooming confusion of micro level interactions.” (Fodor 1997: 161)

Fodor demands an explanation for how it is possible that multiply realized macro-regularities obtain given the “confusion of micro level interactions”. Following Fodor’s approach, Batterman, by and large, equates universality and multiple realizability in stating that the explanandum of RG explanations is why a certain macroscopic property is multiply realized (Batterman 2000: 117; 2002: 72; 2015: 8).

How do RG explanations account for multiply realized or universal macro-behavior? Without going into the details of RG explanations for now (see section 3 for a brief exposition of the physics of RG explanations), one obvious and abstract answer might be that universality is explained by that fact that (a) microscopically different systems S_1 and S_2 exhibiting the same macro-behavior have a property in common, although they are very different in many respects, and that (b) this shared property helps to explain the fact that S_1 and S_2 display the same macro-behavior near criticality. Let me call this general explanatory strategy the ‘commonality strategy’. Batterman ascribes the commonality strategy to Papineau:

> "For Papineau, the reducibility of some special science property to physics requires 'only that there should be some physical property present in all and only [the distinct realizers of the special science property]. The presence of such a common property will then provide 'a uniform physical explanation of why those instances always give rise to a certain sort of result' (Papineau [1993], p. 35)." (Batterman 2000: 135-136)

Papineau illustrates the commonality strategy as follows. Taking thermodynamic behavior as a paradigmatic example of multiply realized macro-behavior in physics, Papineau asks how it is possible that microscopically different gases obey the ideal gas law: "After all, aren't there lots of different ways in which the molecules can be moving around in a gas at a given temperature, thus giving us a heterogeneity of physical states for the single macro-state of having that temperature?" (Papineau 1993: 35). Papineau answers that this case of multiple realizability – like many other cases – can be explained by applying the commonality strategy:

> "[T]here is still something physically in common between all those different physical states, namely, that the molecules have a given mean kinetic energy. It is this commonality that then enables us to explain such things as why an increase in temperature at constant volume always results in an increase in pressure." (ibid.)

In other words, Papienau holds that microscopically different gases share a property – their mean kinetic energy. This property helps to explain why microscopically different gases display the same macro-behavior which is captured by the ideal gas law. This is a paradigmatic instance of the commonality strategy.

It is worth emphasizing right away that (a) the mean kinetic energy provides an explanation of the macro-behavior at issue only in concert with other theoretical resources of statistical mechanics (including bridge laws, statistical principles, and the general dynamical laws of classical mechanics), and that (b) these other theoretical resources of statistical mechanics are applicable to microscopically very different physical systems. I will return to this point in Section 4.

One may add to Papineau's presentation of the example that the statistical-mechanical explanation of the ideal gas law he refers to is not merely available for addressing how various (actual or possible) micro states of *one and the same* gas made up of the same molecules give rise to a certain macro-behavior (the case Papineau explicitly discusses), but the statistical-mechanical explanation also illuminates why different micro states of *two (or more)* gases made up of different kinds of molecules may display the same kind of macro-behavior.

However, Batterman claims that RG explanations "differ significantly in kind from Papineau's explanation of the temperature/pressure relation" (Batterman 2000: 136). Batterman argues that RG explanations do not follow the commonality strategy: "my point has been that the RG account explains the universality of critical phenomena *without finding any such property*" (ibid.; emphasis added). Batterman also endorses the same argument in more recent work (Batterman 2002: 72-73, 2015: 8-9; Batterman and Rice 2014: 373).

Let me briefly pause to add a disclaimer. For Batterman, the claim that the commonality strategy does not apply to RG explanations plays a role in a larger argument for the failure of reduction in the context of RG explanations (see Batterman 2000, 2002, 2015). In this paper, I will not be concerned with the question whether a defense of the commonality strategy with respect to RG explanation supports reductionism (see Reutlinger 2014b for a discussion of the reductive character of RG explanations). My sole aim is to argue that RG explanations, interesting and unusual as they may be in other respects, are not special when it comes to following the commonality strategy for explaining multiply realized macro-behavior.

3. The Core Elements of RG Explanations

How do physicists explain the remarkable fact that there is universal macro-behavior by using RG explanations? My strategy in this section is to focus on the *physics* of RG explanations. The following brief exposition of the relevant physics will be non-technical because the paper is concerned with a non-technical question (Batterman 2000: 137-144; for a more technical exposition see Fisher 1982, 1998; Wilson 1983; McComb 2004).

Since it is not relevant for my argument in this paper, I will not attempt to provide a discussion of which philosophical theory of explanation applies to RG explanations (for instance, Butterfield 2011 and Norton 2012 favor a covering-law approach to RG explanations, while Reutlinger forthcoming argues that a counterfactual theory of scientific explanation applies to RG explanations). For present purposes, I will simply assume that some philosophical theory of explanation adequately captures the explanatory character of RG explanations.

For the sake of brevity, it is useful to understand the workings of RG explanations as consisting of three key elements:

I. Hamiltonians,
II. RG transformations, and
III. the flow of Hamiltonians.

Strictly speaking, there is also a fourth element – the laws of statistical mechanics, including dynamical laws and the partition function – which I will leave in the background, for the sake of brevity (Norton 2012: 227; Wilson 1983).

Let me now present the core elements of the physics of RG explanations in more detail:

I. *Hamiltonians:* The Hamiltonian is a function characterizing, among other things, the energy of the interactions between the components of the system. One characteristic of a physical system undergoing a (continuous) phase transition is that the correlation length diverges and becomes infinite. That is, the state of every component becomes correlated not only with the states of its nearby components but also with the states of very distant components. The correlation length diverges, although each component interacts merely locally with its nearby neighbors (Batterman 2000: 126, 137-138).

II. *Renormalization group transformations:* Keeping track of the correlations and interactions between all the components of a system undergoing a phase transition is – given the large number of components and the diverging correlation length – practically impossible. So-called renormalization group transformations (henceforth, RG transformations) deal with this intractability by redefining the characteristic length, at which the interactions among the components of the system at issue are described. Repeatedly applying RG transformations amounts to a re-description of the system, say fluid F, on larger and larger length scales while preserving the mathematical form of the "original" complicated Hamiltonian. The transformed Hamiltonian describes a system (and the interactions between its components) with less degrees of freedom than the original Hamiltonian. In sum, the RG transformation is a mathematically sophisticated coarse-graining procedure eliminating micro-details that are irrelevant for the explanation of universality.

III. *The flow of Hamiltonians:* Suppose we start with the original Hamiltonian H of a fluid F undergoing a phase transition. Then, one repeatedly applies the RG transformation and obtains other more 'coarse-grained' Hamiltonians. Interestingly, these different Hamiltonians "flow" to a fixed point in the space of possible Hamiltonians, which describes a specific behavior characterized by a critical exponent (Batterman 2000: 143). Now suppose there is another fluid F* and its behavior (during phase transition) is described by the initial Hamiltonian H*. Repeatedly applying the RG transformation to H* generates other, more 'coarse-grained' Hamiltonians. If the Hamiltonians representing fluid F* and fluid F turn out to "flow" to the same fixed point, then their behavior, when undergoing phase transition, is characterized by the same critical exponent (Fisher 1982: 85; Batterman 2000: 143).

In sum, these three elements of RG explanations allow us to determine whether systems with different original Hamiltonians belong to the same "universality class" and are characterized by the same critical exponent (Fisher 1982: 87). Two systems belong to the same universality class, if reiterating RG transformations reveals that both systems "flow" to the same fixed point.

4. Defending the Commonality Strategy

Now, let me turn to the central question: are RG explanations in accord with the commonality strategy for explaining the occurrence of universal (or multiply realized) macro-behavior? I believe the answer is 'yes', because RG explanations enable us to understand two things: first, they reveal that systems with different micro-structures (represented by different 'original' Hamiltonians) belong to the *same* universality class. Second, RG explanations also show that and why some systems with different micro-structures in fact belong to *different* universality classes. RG explanations reveal that whether a physical system belongs to some universality class *depends* on features such as the symmetry properties of the order parameter (such as magnetization) and the spatial dimensionality of the physical system in question (Fischer 1998: 675; see also Fischer 1982; Wilson 1983; Cardy 1996; McComb 2004).[3]

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[3] Alternative theories of explanation will analyze this notion of dependence in different ways. Butterfield (2011) and Norton (2012) are likely to do so in a covering-law framework; the counterfactual theory of explanation I favor

RG explanations identify common properties of microscopically quite different physical systems. In positive analogy with Papineau's example of the property of mean kinetic energy, a liquid (such as water) and a piece of iron undergoing phase transitions also have something explanatory in common: namely, the symmetry properties of the order parameter and the spatial dimensionality of the physical system. Continuing the analogy to the example of mean kinetic energy, the relevant symmetry properties and the spatial dimensionality are a part of an explanation of the multiply realized macroscopic behavior in question. The fact that microscopically very different physical systems have these properties in common (partially) explains why these physical systems display the same macro-behavior. Knowing that microscopically different systems share these properties renders the occurrence of universal macro-behavior no longer "entirely mysterious" (to use Fodor's words). Thus, RG explanations conform to the commonality strategy.

Let me add two qualifications. First, I take it that RG explanations follow the commonality strategy independently of whether one takes them to be (a) causal or non-causal, and (b) reductive or non-reductive explanations. Second, in addition to Papineau (1993), Lange (2015: section 4) articulates an excellent general defense of the commonality strategy – or a "common feature" account – for other explanations concerned with multiply realized macro-behavior. Lange's main example is an explanation by the physicist John Herschel. Herschel explains a kind of macro-behavior (here, statistical features of the diffusion through a homogeneous, boundless, two-dimensional medium) relying on the

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interprets dependence in terms of (non-causal) counterfactual dependencies (Reutlinger forthcoming).

rotational symmetry of the micro-laws (Lange 2015: 300-302). Although Lange (2015: 300) mentions RG explanations, he does not engage in a detailed discussion of whether they fall under the commonality strategy. I see my argument in this section as supporting Papineau's and Lange's more general claims.

Batterman anticipates my line of argument and replies:

> "the RG analysis also demonstrated certain physical features – the spatial dimension and the symmetry property of the order parameter – that are shared by the systems in the universality class. These properties are *not sufficient* for a system to exhibit the upper level universal behavior" (Batterman 2000: 136, emphasis added; also Batterman and Rice 2014: 361).

This response falls short of being a convincing criticism of the commonality strategy. First and foremost, it strikes me as uncharitable to read Papineau's idea that multiple realization is explained by reference to a "physical property present in all and only" the microscopically different physical systems exhibiting the same macro-behavior as the claim that this common property also has to be *sufficient for explaining* the macro-behavior in question. Moreover, the requirement that the common properties be *sufficient* for explaining the macro-behavior is unnecessarily strong and an advocate of the commonality strategy need not accept it. Even if properties such as spatial dimensionality and the symmetry properties of the order parameter alone are not sufficient for explaining universal behavior, this fact does not highlight a difference between

RG explanations and our paradigm of the commonality strategy, the statistical mechanical explanation of the ideal gas law. Mean kinetic energy alone is also not sufficient for explaining the macro-behavior described by the ideal gas law (a point I already mentioned in Section 2). Instead, a bridge law[4] connecting mean kinetic energy and temperature, statistical principles, and the general (dynamical) laws of statistical mechanics are further non-redundant parts of a larger statistical mechanical explanation of the ideal gas law. (Note that the bridge laws, statistical principles, and the general dynamical laws apply to microscopically different systems.) This larger statistical mechanical explanation is sufficient for explaining the macro-behavior.

The analogy between an RG explanation of universality and the statistical-mechanical explanation of the ideal gas law suggests that a proponent of the commonality strategy need not (and should not) require that the common property be sufficient for the macro-behavior in question. It is more reasonable to demand that the common property be a non-redundant part of an explanation for the macro-behavior. As Lange has recently expressed this point, a proponent of the commonality strategy "need not say that simply citing the common features suffices; it can require that an explanation show how the common

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[4] As a referee remarked, the correct interpretation of bridge laws is crucial for deciding whether RG explanations are reductive explanations and, moreover, whether RG explanations support the claim of reductive physicalism (in which Papineau is interested). However, my aim in this paper is not to get involved in debates on reductive explanations and, even less, on reductive physicalism. For this reason, I rely on Dizadji-Bahmani et al.'s (2010: 404) minimalist account of bridge laws according to which bridge laws are interpreted as correlations between macroscopic and microscopic physical quantities. This account of bridge laws has two advantages: (a) the account is neutral with respect to reductive physicalism, and (b) Dizadji-Bahmani et al.'s account of bridge laws is compatible with multiple realization – one of Batterman's main qualms with respect to bridge laws (Dizadji-Bahmani et al. 2010: 406-407).

features result in the macrobehavior." (Lange 2015: 299) If this general point regarding the commonality strategy is true, then the dimensionality and the symmetry properties of the order parameter are best understood as a non-redundant (but not sufficient) part of a more encompassing and ultimately sufficient statistical-mechanical explanation – that is, an explanation involving the elements of the RG explanans plus the bridge laws, the general (dynamical) laws, and further statistical principles of statistical mechanics (see Section 3). (Note once more that the bridge laws, the general dynamical laws, and the statistical principles apply to microscopically very different systems.) Hence, Batterman's objection can be refuted, and one can subsume RG explanations under the commonality strategy in analogy with the statistical-mechanical explanation of the ideal gas law.

Let me conclude by pointing out a strength of understanding RG explanations as an instance of the commonality strategy. The commonality strategy is compatible with one central insight of RG theory, namely, that many micro-details are not relevant for the explanation of the universal macro-behavior in question (a point that Batterman emphasizes repeatedly; for instance, Batterman 2000: 128). To put it in Papineau's words, the commonality strategy is not based "the absurdly strong requirement that the instances of the reduced category should share all their physical properties" (Papineau 1993: 35) – such a strong requirement would indeed be incompatible with RG explanations. However, the commonality strategy fortunately allows for vast microscopic differences among the physical systems exhibiting the same macro-behavior.

5. Conclusion

RG explanations are often taken to explain universal or multiply realized macro-behavior. Batterman has repeatedly argued that RG explanation do not work by conforming to the commonality strategy, i.e. such explanations do not account for a kind of universal or multiply realized macro-behavior by referring to properties that microscopically quite different systems have in common. Following Papineau's (1993) and Lange's (2015) general defense of the commonality strategy, I opposed Batterman's claim by defending the view that RG explanations do in fact conform to the commonality strategy. I have claimed that, according to RG explanations, the properties common to microscopically different systems (such as the symmetry properties of the order parameter and spatial dimensionality) are a relevant part of a larger explanation of universal macro-behavior (involving Hamiltonians, RG transformations, the flow of Hamiltonians, and the laws and postulates of statistical mechanics).

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