Zeno's Paradoxes: A Timely Solution

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Zeno of Elea's motion and infinity paradoxes, excluding the Stadium, are stated (1), commented on (2), and their historical proposed solutions then discussed (3). Their correct solution, based on recent conclusions in physics associated with time and classical and quantum mechanics, and in particular, of there being a necessary trade off of all precisely determined physical values at a time (including relative position), for their continuity through time, is then explained (4). This article follows on from another, more physics orientated and widely encompassing paper entitled "Time and Classical and Quantum Mechanics: Indeterminacy vs. Discontinuity" (Lynds, 2003), with its intention being to detail the correct solution to Zeno's paradoxes more fully by presently focusing on them alone. If any difficulties are encountered in understanding any aspects of the physics underpinning the following contents, it is suggested that readers refer to the original paper for a more in depth coverage.

1. The Problems

Achilles and the Tortoise

Suppose the swift Greek warrior Achilles is to run a race with a tortoise. Because the tortoise is the slower of the two, he is allowed to begin at a point some distance ahead. Once the race has started however, Achilles can never overtake his opponent. For to do so, he must first reach the point from where the tortoise began. But by the time Achilles reaches that point, the tortoise will have advanced further yet. It is obvious, Zeno maintains, that the series is never ending: there will always be some distance, however small, between the two contestants. More specifically, it is impossible for Achilles to preform an infinite number of acts in a finite time.

Distance behind the Tortoise:	5,	2.5,	1.25,	0.625,	0.3125,	0.015625,	….
Time:	1,	0.5,	0.25,	0.125,	0.625,	0.03125,	….

The Dichotomy

It is not possible to complete any journey, because in order to do so, you must firstly travel half the distance to your goal, and then half the remaining distance, and again of what remains, and so on. However close you get to the place you want to go, there is always some distance left. Furthermore, it is not even possible to get started. After all, before the second half of the distance can be travelled, one must cover the first half. But before that distance can be travelled, the first quarter must be completed, and before that can be done, one must traverse the first eight, and so on, and so on to infinitum.

Distance: 1, 1.5, 1.75, 1.875, 1.9375, 1.96885, 1.984425, ….

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Time: 1, 1.5, 1.75, 1.875, 1.9375, 1.96885, 1.984425, ….

or

Distance: 2, 1, 0.5, 0.25, 0.125, 0.625, 0.03125, 0.015425, ….

Time: 2, 1, 0.5, 0.25, 0.125, 0.625, 0.03125, 0.015425, ….

William James' version of the Dichotomy

Time can never pass, as to do so it is necessary for some time interval to go by, say 60 seconds. But before the 60 seconds, half of that, or 30 seconds, must firstly pass. But before that, a half of that time must firstly pass, and so on, and so to infinitum.

60 s, but first 30 s, but first 15 s, but first 7.5 s, but first 3.75 s, but first 1.875 s, but first ….

or

Time: 30, 45, 52.5, 56.25, 58.125, 59.0625, 59.531225, 59.765612, ….

G. J. Whitrow's version of the Dichotomy

A bouncing ball that reaches three quarters of its former height on each bounce, will bounce an infinite number of times, in the same way that distances and times decrease in the Dichotomy. The only difference is that Whitrow uses a factor of three-quarters where Zeno used one half. It also doesn't however matter what fraction is used. The only thing that would change if the balls initial velocity and the distance from the floor of the first bounce remained the same, would be the time in which an infinite numbers of bounces took place.

Height of bounce: 1, 0.75, 0.5625, 0.421875, 0.3164062, 0.2373046, ….

Time: 1, 0.75, 0.5625, 0.421875, 0.3164062, 0.2373046, ….

or

Height of bounce: 1, 0.25, 0.0625, 0.015625, 0.0039062, 0.0009765, ….

Time: 1, 0.25, 0.0625, 0.015625, 0.0039062, 0.0009765, ….

The Arrow

All motion is impossible, since at any given instant in time an apparently moving body (the arrow) occupies just one block of space. Since it can occupy no more than one block of space at a time, it must be stationary at that instant. The arrow cannot therefore ever be in motion as at each and every instant it is frozen still.

2. General Comment

It is doubtful that with his paradoxes, Zeno was attempting to argue that motion was impossible, as is sometimes claimed. Zeno would of known full well that in the cases of Achilles and the Tortoise and the Dichotomy (dichotomy in this relation meaning arithmetical or geometrical division), that the respective body apparently in motion would inevitably reach and pass the said impossible boundary in every day settings. Pointing this out does not refute Zeno's argument, as Diogenes the Cynic is apocryphally reputed to have thought he'd done by getting up and walking away. Rather, Zeno is saying through the use of dialectic and by showing that an idea results in contradiction, that an infinite series of acts cannot be completed in finite period of time. If we choose not to believe this we must demonstrate where the fallacy lies and how it is possible. As such, instead of being arguments against the possibility of motion, the paradoxes are critiques of our underlying assumptions regarding the idea of continuous motion in an infinitely divisible space and time. It is the same with the Arrow paradox. We of course know that motion and physical continuity are possible and an obvious feature of nature, so there has to be something wrong with the initial assumptions regarding the paradoxes. But what?

Although Zeno's paradoxes may at first seem like whimsical little puzzles and as though they could be quite easily disposed of without much thought and effort, they show themselves to be immeasurably subtle and profound, as Bertrand Russell once characterised them, when examined in detail, and over the centuries mathematicians, philosophers and physicists have continually argued about them at great length. These people can be divided into two camps: those that think there is no real problem, and those who believe that Zeno's paradoxes have not yet been solved (Morris, 1997).

3. Their Historical Proposed Solutions

Of Zeno's paradoxes, the Arrow is typically treated as a different problem to the others. In fact, all of the paradoxes are usually thought to be quite different problems, involving different proposed solutions, if only slightly, as is often the case with the Dichotomy and Achilles and the Tortoise, with the differentiation being that the first is thought to be expressed in terms of absolute motion, where as the second shows that the same argument applies to relative motion. Although it is not important to the argument, or its possible solution, this is actually incorrect, as any motion necessarily requires relative motion and that a body's position is changing in relation to something else. Therefore, like the paradox of Achilles and the Tortoise, the Dichotomy also involves relative motion, as its position is purported to change over time: in this case, presumably relative to a hypothetical fixed point on earth.

It is usually claimed that the Arrow paradox is resolved by either of two different lines of thought. Firstly, by way of a vague connection to special relativity, where it is argued:

> "The theory of special relativity answers Zeno's concern over the lack of an instantaneous difference between a moving and a non-moving arrow by positing a fundamental re-structuring of the basic way in which space and time fit together, such that there really *is* an instantaneous difference between a moving and a non-moving object, in so far as it makes sense to speak of "an instant" of a physical system with mutually moving elements. Objects in relative motion have different planes of simultaneity, with all the familiar relativistic consequences, so not only does a moving object look different to the world, but the world looks different to a moving object." [2]

However, such arguments are often asserted by those who don't seem to entirely understand relativity and/or its mathematical formalisation, and the reasoning underpinning them is usually of a non-descriptive nature. Indeed, it is difficult to see how special relativity is relevant to the problem at all.

The more popular and common proposed solution is that the arrow, although not in motion at any one instant, when it's trajectory is traced out, it can be seen to be move because it occupies different locations at different times. In other words, although not in motion at any one instant, the arrow is in motion at *all* instants in time (an infinite number of them), so is never at rest. This conclusion stems from calculus and continuous functions (as emphasised by Weierstrass and the "at-at theory of motion"), by pointing out that although the *value* of a function f(t) is constant for a given t, the *function* f(t) may be non-constant at t. Recently, some potential problems with the at-at theory have been noted and revolve around the question of whether it is compatible with instantaneous velocity. [3] Another proposed solution to the Arrow paradox is to deny instantaneous velocities altogether.[4] [*]

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[2] See, *Zeno and the Paradox of Motion*, by Kevin Brown. www.mathpages.com/home/iphysics.html

[3] See, Frank Arntzenius, "Are there really instantaneous Velocities?". *The Monist* , vol 83, no 2, (2000).

[4] Albert, D. *Time and Chance.* Chp. 1. Harvard University Press, (2000). For responses to Albert, see David Malament's, "On the Time Reversal Invariance of Classical Electromagnetic Theory" (forthcoming *in Stud. Hist. Phil. Mod. Phys*).

The paradoxes of Achilles and the Tortoise and the Dichotomy are often thought to be solved through calculus and the summation of an infinite series of progressively small time intervals and distances, so that the time taken for Achilles to reach his goal (overtake the Tortoise), or to traverse the said distance in the Dichotomy, is in fact, finite. The faulty logic in Zeno's argument is often seen to be the assumption that the sum of an infinite number of numbers is always infinite, when in fact, an infinite sum, for instance, 1 + 1/2 + 1/4 + 1/8 + 1/16 + 1/32 +...., can be mathematically shown to be equal to a finite number, or in this case, equal to 2.

This type of series is known as a geometric series. A geometric series is a series that begins with one term and then each successive term is found by multiplying the previous term by some fixed amount, say x. For the above series, x is equal to 1/2. Infinite geometric series are known to converge (sum to a finite number) when the multiplicative factor x is less than one. Both the distance to be traversed and the time taken to do so can be expressed as an infinite geometric series with x less than one. So, the body in apparent motion traverses an infinite number of "distance intervals" before reaching the said goal, but because the "distance intervals" are decreasing geometrically, the total distance that it traverses before reaching that point is not infinite. Similarly, it takes an infinite number of time intervals for the body to reach its said goal, but the sum of these time intervals is a finite amount of time.

So, for the above example, with an initial distance of say 10 m, we have,

t = $1 + 1/2 + 1/2^2 + 1/2^3 + \ldots + 1/2^n$ Difference = $10/2^n\ m$

Now we want to take the limit as n goes to infinity to find out when the distance between the body in apparent motion and its said goal is zero. If we define

$$S_n = 1 + 1/2 + 1/2^2 + 1/2^3 + \ldots + 1/2^n$$

then, divide by 2 and subtract the two expressions:

$$S_n - 1/2\, S_n = 1 - 1/2^{n+1}$$

or equivalently, solve for S_n:

$$S_n = 2\,(1 - 1/2^{n+1})$$

So that now S_n is a simple sequence, for which we know how to take limits. From the last expression it is clear that:

$$\lim S_n = 2$$

as n approaches infinity.

Therefore, Zeno's infinitely many subdivisions of any distance to be traversed can be mathematically reassembled to give the desired finite answer.

A much simpler calculation not involving infinitely many numbers gives the same result:

For the Dichotomy:

- A body traverses 10 metres per second, so covers 20 meters in 2 seconds

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* Although correct to question the validly of instantaneous velocity, as we shall see shortly, the real answer to its possible plausibility comes from a different and much more direct source. Furthermore, rather than just being a question of instantaneous velocity, the same applies to the rest of physics and all instantaneous physical values and magnitudes.

For Achilles and the Tortoise:

- Achilles runs 10 metres per second, so covers 20 metres in 2 seconds

- The tortoise runs 5 metres per second, and has an advantage of 10 metres. Therefore, he also reaches the 20 metre mark after 2 seconds

4. Zeno's Paradoxes: A Timely Solution

The way in which calculus is often used to solve Achilles and the Tortoise and the Dichotomy through the summation of an infinite series by employing the mathematical techniques developed by Cauchy, Weierstrass, Dedekind and Cantor, certainly provides the correct answer in a strictly mathematical sense by giving up the desired numbers at the end of calculation. It is obviously dependent however on an object in motion having a precisely defined position at each given instant in time. As we will see shortly, this isn't representative of how nature works. Moreover, the summation of an infinite series here works as a helpful mathematical tool that produces the correct numerical answer by getting rid of the infinities, but it doesn't actually solve the paradoxes and show how the body's motion *is actually possible*. The same fault applies to the Arrow paradoxes proposed solution via Weierstrass' "at-at theory of motion", as a continuous function is a *static* and completed indivisible mathematical entity, so by invoking this model we are essentially agreeing that physical motion does not truly exist, and is just some sort of strange subjective illusion. Furthermore, the above proposed solution also problematically posits the existence of an infinite succession of instants underlying a body's motion. In his book, *Zeno's paradoxes*, Wesley C. Salmon discusses the proposed functional solution:

> "A function is a pairing of elements of two (not necessarily distinct) classes, the do-definition, if motion is a functional relation between time and position, then motion consists solely of the pairing of times with positions. Motion consists not of traversing an infinitesimal distance in an infinitesimal time (before Cauchy's definition of the derivative as certain limit, the derivative was widely regarded as a ratio of infinitesimal quantities. The use of the derivative to represent velocity thus implied that physical motion over a finite distance is compounded out of infinitesimal movements over infinitesimal distances during infinitesimal time spans); it consists of the occupation of a unique position at each given instant of time. This conception has been appropriately dubbed 'the at-at theory of motion." The question, how does an object get from one point to another, does not arise. Thus Russel was led to remark, "Weierstrass, by strictly banishing all infinitesimals, has at last shown that we live in an unchanging world, and that the arrow, at every moment of its flight, is truly at rest. The only point where Zeno probably erred was in inferring (if he did infer) that, because there is no change, therefore the world must be in the same state at one time as at another. This consequence by no means follows…" [5]

What doesn't seem to be realised is that in all of the paradoxes (and proposed solutions to them), it is taken for granted that a body in relative motion has a determined and defined relative position at any given instant, and indeed, that there is an instant in time underlying a body's motion, whether it be an actual physical feature of time itself, and/or a meaningful and precise physical indicator at which the position of a body in motion would be determined, and as such, not constantly changing.

(a). Time and Mechanics: Indeterminacy vs. Discontinuity

Time enters mechanics as a measure of interval, relative to the clock completing the measurement. Conversely, although it is generally not realized, in *all* cases a time value indicates an interval of time, rather than a precise static instant in time at which the relative position of a body in relative motion or a specific physical magnitude would theoretically be precisely determined. For example, if two separate events are measured to take place at either 1 hour or 10.00 seconds, these two values indicate the

[5] For a collection of papers on this matter, and others relating to the paradoxes, see, *Zeno's Paradoxes*. W. C. Salmon (ed). Bobbs-Merrill, New York, (1970).

events occurred during the time intervals of 1 and 1.99999…hours and 10.00 and 10.009999...seconds, respectively. If a time measurement is made smaller and more accurate, the value comes closer to an accurate measure of an interval in time and the corresponding parameter and boundary of a specific physical magnitudes potential measurement during that interval, whether it be relative position, momentum, energy or other. Regardless of how small and accurate the value is made however, it cannot indicate a precise static instant in time at which a value would theoretically be precisely determined, because there is not a precise static instant in time underlying a dynamical physical process. If there were, the relative position of a body in relative motion or a specific physical magnitude, although precisely determined at such a precise static instant, it would also by way of logical necessity be frozen static at that precise static instant. Furthermore, events and all physical magnitudes would remain frozen static, as such a precise static instant in time would remain frozen static at the same precise static instant: motion would not be possible. (Incidentally, the same outcome would also result if such a precise static instant were hypothetically followed by a continuous sequence of further precise static instants in time, as by its very nature, a precise static instant in time does not have duration over interval in time, so neither could a further succession of them. This scenario is not plausible however in the first instance, as the notion of a *continuous progression* of precise static instants in time is obviously not possible for the same reason). Rather than facilitating motion and physical continuity, this would perpetuate a constant precise static instant in time, and as is the very nature of this ethereal notion i.e. a physical process frozen static at an 'instant' as though stuck on pause or freeze frame on a motion screen, physical continuity is not possible if such a discontinuous chronological feature is an intrinsic property of a dynamical physical process, and as such, a meaningful (and actual physical) indicator of a time at which the relative position of a body in relative motion or a certain physical magnitude is precisely determined as has historically been assumed. That is, it is the human observer who subjectively projects and assigns a precise instant in time upon a physical process, for example, in order to gain a meaningful subjective picture or 'mental snapshot' of the relative position of a body in relative motion. [6]

It might also be contended in a more philosophical sense that a general definition of static would entitle a certain physical magnitude as being *unchanging* for an extended interval of time. But if this is so, how then could time itself be said to be frozen static at a precise instant if to do so also demands it must be unchanging for an extended interval of time? As a general and sensible definition this is no doubt correct, as we live in a world where indeed there is interval in time, and so for a certain physical magnitude to be static and unchanging it would naturally also have to remain so for an extended duration, however short. There is something of a paradox here however. If there were a precise static instant underlying a dynamical physical process, everything, including clocks and watches would also be frozen static and discontinuous, and as such, interval in time would not be possible either. There could be no interval in time for a certain physical magnitude to remain unchanging. Thus this general definition of static breaks down when the notion of static is applied to time itself. We are so then forced to search for a revised definition of static for this special temporal case. This is done by qualifying the use of stasis in this particular circumstance by noting static and unchanging, with static and unchanging as not being over interval, as there could be no interval and nothing could change in the first instance. At the same time however, it should also be enough just to be able to recognize and acknowledge the fault and paradox in the definition when applied to time.

It might also be argued by analogy with the claim by some people that the so-called 'block universe model', i.e. a 4-dimensional model of physical reality incorporating time as well as space, is static or unchanging. This claim however involves the common mistake of failing to recognize that unless there is *another* time dimension, it simply doesn't make sense to say that the block universe is static, for there is no 'external' time interval over which it remains the same. If we then apply the same line of reasoning to the hypothetical case being discussed presently, we could say: It doesn't make sense to say that everything would be static at an instant, (with physical continuity and interval in time not being possible), as there would be no time interval for such an assertion to be relative to, referenced from, or over which such an instant would remain the same etc. This objection is valid. However, as it applies

[6] In a 1942 paper, Zeno of Elea's Attacks on Plurality, *Amer. J. Philology* **63,** 1-25; 193-206, H. Frankel hinted towards this same conclusion: "The human mind, when trying to give itself an accurate account of motion, finds itself confronted with two aspects of the phenomenon. Both are inevitable but at the same time they are mutually exclusive. Either we look at the continuous flow of motion; then it will be impossible for us to think of the object in any particular position. Or we think of the object as occupying any of the positions through which its course is leading it; and while fixing our thought on that particular position we cannot help fixing the object itself and putting it at rest for one short instant."

to the hypothetical case under investigation, it should also be clear that it is not any more applicable or relevant than being a semantical problem of the words one employs to best try to put across a point and as being a contradiction in terms, rather than pertaining to any contradiction in the actual (in this case, hypothetical) physics involved. One could certainly also assert that there were no interval in time, and so if one wishes, there were a precise static instant underlying a physical process, without it being dependent on there actually being interval: as is the case with the hypothetical absence of mass and energy, and the resulting absence of 3 spatial dimensions. [7]

(b). Einstein's Train

The absence of a precise static instant in time underlying a dynamical physical process means that a body (micro and macroscopic) in relative motion does not have a precisely determined relative position at any time. The reason why can be demonstrated by employing Albert Einstein's famous 1905 train and the other theoretical device it is associated with, the thought experiment. An observer is watching a train traveling by containing a young Albert Einstein. At any given time as measured by a clock held by the observer, Einstein's train is in motion. If the observer measures the train to pass a precisely designated point on the track at 10.00 seconds, this value indicates the train passes this point during the measured time interval of 10.00 and 10.00999…seconds. As Einstein's train is in motion at all measured times, regardless of how great or small its velocity and how small the measured time interval (i.e. 10.0000000-10.0000000999...seconds), Einstein's train does not have a precisely determined relative position to the track at any time, because it is not stationary at any time while in motion, for to have a precisely determined relative position at any time, the train would also need to be stationary relative to the track at that time. Conversely, the train does not have a precisely determined relative position at an ethereal precise static instant in time, because there is not a precise static instant in time underlying the train's motion. If there were, Einstein's trains motion would not be possible.

As the time interval measurement is made smaller and more accurate, the corresponding position the train can be said to 'occupy' during that interval can also be made smaller and more accurate. Momentarily forgetting L_p, T_p and time keeping restrictions, these measurements could hypothetically be made almost infinitesimally small, but the train does not have a precisely determined position at any time as it is in motion at all times, regardless of how small the time interval. For example, at 100km/hr, during the interval of 10^{-25} s Einstein's train traverses the distance of 2.7^{-21} cm. Thus, it is exactly due to the train not having a precisely determined relative position to the track at any time, whether during a time interval, however small, or at a precise static instant in time, that enables Einstein's train to be in motion. Moreover, this is not associated with the preciseness of the measurement, a question of re-normalizing infinitesimals or the result of quantum uncertainty, as the trains precise relative position is not to be gained by applying infinitely small measurements, nor is it smeared away by quantum considerations. It simply does not have one. There is a very significant and important difference.

If a photograph is taken (or any other method is employed) to provide a precise measurement of the trains relative position to the track, in this case it does appear to have a precisely determined relative position to the track in the picture, and although it may also be an extremely accurate measure of the time interval during which the train *passes* this position or a designated point on the track, the imposed time measurement itself is in a sense arbitrary (i.e. 0.000000001 second, 1 second, 1 hour etc), as it is impossible to provide a time at which the train is precisely in such a position, as it is not precisely in that or any other precise position at any time. If it were, Einstein's train would not, and could not be in motion.

On a microscopic scale, due to inherent molecular, atomic and subatomic motion and resulting kinetic energy, the particles that constitute the photograph, the train, the tracks, the light radiation that propagates from the train to the camera, as well as any measuring apparatus e.g. electron microscope, clock, yardstick etc, also do not have precisely determined relative position's at any time. Naturally, bodies at rest in a given inertial reference frame, which are not constituted by further smaller particles in relative motion, have a precisely defined relative position at all measured times. However, as this hypothetical special case is relevant to only indivisible and the most fundamental of particles, whose existence as independent 'massive' objects is presently discredited by quantum physics and the intrinsic 'smearing' effects of wave-particle duality and quantum entanglement, if consistent with these

[7] Please see Lynds (2003) for considerations regarding the resulting negation of the notion of a flowing and physically progressive time, and the reason for natures exclusion if it.

considerations, this special subatomic case would not appear to be applicable. Furthermore, and crucially, because once granted indeterminacy in precise relative position of a body in relative motion, also subsequently means indeterminacy in *all* precise physical magnitudes, including gravity, this also applies to the very structure of space-time, the dynamic framework in which all inertial spatial and temporal judgments of relative position are based. [8] As such, the previously mentioned possible special case isn't actually one, and the very same applies.

The only situation in which a physical magnitude would be precisely determined was if there *were* a precise static instant in time underlying a dynamical physical process and as a consequence a physical system were frozen static at that instant. In such a system an indivisible mathematical time value, e.g. 2s, would correctly represent a precise static instant in time, rather than an interval in time (as it is generally assumed to in the context of calculus, and traceable back to the likes of Galileo, and more specifically, Newton, thus guaranteeing absolute preciseness in theoretical calculations before the fact i.e. $\Delta d/\Delta t=v$). Fortunately this is not the case, as this static frame would include the entire universe. Moreover, the universe's initial existence and progression through time would not be possible. Thankfully, it seems nature has wisely traded certainty for continuity.

(c). The Solution over 2500 Years Later

To return to Zeno's paradoxes, the solution to *all* of the mentioned paradoxes then,[9] is that there isn't an instant in time underlying the body's motion (if there were, it couldn't be in motion), and as its position is constantly changing no matter how small the time interval, and as such, is at no time determined, it simply doesn't have a determined position. In the case of the Arrow paradox, there isn't an instant in time underlying the arrows motion at which it's volume would occupy just "one block of space", and as its position is *constantly* changing in respect to time as a result, the arrow is never static and motionless. The paradoxes of Achilles and the Tortoise and the Dichotomy are also resolved through this realisation: when the apparently moving body's associated position and time values are fractionally dissected in the paradoxes, an infinite regression can then be mathematically induced, and resultantly, the idea of motion and physical continuity shown to yield contradiction, as such values are not representative of times at which a body is in that specific precise position, but rather, at which it is *passing* through them. The body's relative position is constantly changing in respect to time, so it is never *in that* position at any time. Indeed, and again, it is the very fact that there isn't a static instant in time underlying the motion of a body, and that is doesn't have a determined position at any time while in motion, that allows it to be in motion in the first instance. Moreover, the associated temporal (t) and spatial (d) values (and thus, velocity and the derivation of the rest of physics) are just an *imposed static* (and in a sense, arbitrary) *backdrop*, of which in the case of motion, a body *passes* by *or through* while in motion, but has no inherent and intrinsic relation. For example, a time value of either 1 s or 0.001 s (which indicate the time intervals of 1 and 1.99999….s, and 0.001 and 0.00199999…. s, respectively), is never indicative of a time at which a body's position might be determined while in motion, but rather, if measured accurately, is a representation of the interval in time during which the body *passes* through a certain distance interval, say either 1 m or 0.001 m (which indicate the distance intervals of 1 and 1.99999….m, and 0.001 and 0.0019999….m, respectively). Therefore, the more simple proposed solution mentioned earlier to Achilles and the Tortoise and the Dichotomy by applying velocity to the particular body in motion, also fails as it presupposes that a specific body has precisely determined

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[8] For further detail, please see Lynds (2003).

[9] Zeno conceived another paradox, often referred to as the Stadium or Moving Rows. Unlike the paradoxes of Achilles and the Tortoise, the Dichotomy, the Arrow, and their variations however, the stadium is a completely different type of problem. It is usually stated as follows: Consider three rows of bodies, each composed of an equal number of bodies of equal size. One is stationary, while the other two pass each other as they travel with equal velocity in opposite directions. Thus, half a time is equal to the whole time. Although its exact details, and so also its interpretation, remain controversial, the paradox is generally thought to be a question of relative velocity, and to be addressed through reasoning underpinning Einstein's 1905 theory of special relativity. I would suggest however that, if the argument is to be accepted as it has been set forward above, it doesn't actually pose a paradox (and that Special Relativity has no direct relevance to it either), but rather that Zeno has failed to recognise that the time taken for the each moving row to pass the other would be *half* the time required to pass a row of the same length if it were stationary, rather than being (in any sense) equal, which in some ways, is the intuitive view. That is, Zeno couldn't decide if the time required was equal or a half, as both intuitively seemed to make equal sense.

position at a given time, thus guaranteeing absolute preciseness in *theoretical* calculations before the fact i.e. $\Delta d/\Delta t=v$. That is, a body in motion simply doesn't have a determined position at any time, as at no time is its position not changing, so it also doesn't have a determined velocity at any time.

Lastly, and to complete the mentioned paradoxes, William James' variation on the Dichotomy is resolved through the same reasoning and the realisation of the absence of a instant in time at which such an indivisible mathematical time value would theoretically be determined and static at that instant, and not constantly changing. That is, interval *as represented by a clock or a watch* (as distinct from an absent actual physical progression or flow of time) is *constantly* increasing, whether or not the time value as indicated by the particular time keeping instrument remains the same for a certain extended period i.e. at no time is a time value anything other than an interval in time and it is never a precise static instant in time as it assumed to be in the paradoxes.

5. Closing Comment

To close, the correct solution to Zeno's motion and infinity paradoxes, excluding the Stadium, have been set forward, just less than 2500 years after Zeno originally conceived them. In doing so we have gained insights into the nature of time and physical continuity, classical and quantum mechanics, physical indeterminacy, and turned an assumption which has historically been taken to be a given in physics, determined physical magnitude, including relative position, on its head. From this one might infer that we've been a bit slow on the uptake, considering it has taken us so long to reach these conclusions. I don't think this is the case however. Rather that, in respect to an instant in time, it is hardly surprising considering the extreme difficulty of seeing through something that one actually *sees* and *thinks* with. Moreover, that with his deceivingly profound and perplexing paradoxes, the Greek philosopher Zeno of Elea was a true visionary, and in a sense, over 2500 years ahead of his time.

Very helpful and thoughtful discussion and comment received from J. J. C. Smart, C. Grigson, A. McDonald and W. B. Yigitoz in relation to the contents of this paper are most gratefully acknowledged.

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