**Conditions**

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Any theory of conditions must solve the symmetry problem, i.e. must (i) explain why being a necessary condition and being sufficient condition in many cases are not mutual converses, yet in some cases having to do with the notion of truth they are; and must (ii) explain why being a necessary and sufficient condition is generally non-symmetric, yet in some cases having to do with the notion of truth is symmetric. I explain the problem and propose a solution.

We say that John’s being a man is a necessary condition of his being a father and that his being a father is a sufficient condition of his being a man. What kind of relation is it that John’s being a man and his being a father stand in when one is a condition of the other? When one is a necessary or a sufficient condition of the other? Is the relation ontological or logical? Is the concept of a condition so fundamental as to defy analysis or can we make sense of it?

The literature on these questions is comparatively sparse (see Brennan 2011 for an overview). Summing up the state of the art, we can say that a viable account of conditionship must resolve what might be called the *symmetry problem*: There is, on the one hand, a ‘standard theory’ (Brennan 2011, sec.2) according to which the relation of being a necessary and sufficient condition is symmetric and, on the other hand, an everyday understanding of that relation, implying clearly that it is not. Explicitly, the standard theory can be presented starting from two propositions P and Q, but the presentation is straightforward only with the understanding that ‘P’ and ‘Q’ can, depending on context, be either names of propositions or that-clauses expressing them or noun phrases representing their contents. With this understanding, the standard theory goes as follows. Given a true conditional ‘If P, Q’ it is plausible to say that P is a sufficient condition of the truth of Q and Q is a necessary condition of the truth of P. (Indeed, if the conditional is identified with the material conditional ‘→’, then, given P → Q, from P it follows that Q and from ¬ Q it follows that ¬ P.) Thus, ‘… is a sufficient condition of the truth of …’ and ‘… is a necessary condition of the truth of …’ express converse relations. This suggests that the simpler expressions ‘… is a sufficient condition of …’ and ‘… is a necessary condition of …’ likewise express converse relations. If so, then from P’s being a sufficient condition of Q it follows that Q is a necessary condition of P, and vice versa. And if this is true, then from ‘P is a necessary and sufficient condition of Q’ it follows that Q is a necessary and sufficient condition of P, i.e. being a necessary and sufficient condition is a symmetric relation.

The standard theory is obviously false. We can easily give examples where ‘… is a sufficient condition of …’ and ‘… is a necessary condition of …’ do *not* express converse relations and where ‘… is a necessary and sufficient condition of …’ does *not* express a symmetric relation. E.g., Joe’s making a touchdown is a sufficient condition of his scoring six points, but his scoring six points is not a necessary condition of his making a touchdown. And it may be true that the rain is a necessary and sufficient condition of the cactus flowering and yet be false that the cactus flowering is a necessary and sufficient condition of the rain.[[1]](#footnote-1) In many cases we are unwilling to assent to the claim that if P is a necessary (sufficient) condition of Q, then Q is a sufficient (necessary) condition of P, i.e. we consider these relations not to be mutual converses. Similarly, we don’t think that P is a necessary and sufficient condition of Q iff Q is a necessary and sufficient condition of P, which is to say that we assume this relation to be non-symmetric. The standard theory is grossly mistaken.

But this verdict refers to the simple expressions only, while for the more involved ‘truth of P (Q)’ expressions the standard theory is altogether correct. E.g., these latter expressions can be argued to be mutual converses. Joe’s making a touchdown is a sufficient condition of his scoring six points. Thus, that Joe makes a touchdown is a sufficient condition of the truth of ‘Joe scores six points’. Thus, it is true that if Joe makes a touchdown, then he scores six points. Thus, it is true that if Joe does not score six points, then he does not make a touchdown (assuming that natural language ‘if’ obeys contraposition). And thus it is plausible to say that Joe’s scoring six points is a necessary condition of the truth of ‘Joe makes a touchdown’. It is only the last step (the inference to ‘Joe’s scoring six points is a necessary condition of his making a touchdown’) that fails. In a similar way, in the cactus example we can make it plausible that ‘… is a necessary and sufficient condition of the truth of …’ expresses a symmetric relation. Finally, my opening example of John’s being a man is a special case where the simple expressions themselves are converses and the involved expressions follow suit.

The symmetry problem is the challenge to explain why being a necessary condition and being sufficient condition in many cases are not mutual converses, yet in some cases they are, while the involved ‘truth of’-expressions always express mutual converses; and to explain why being a necessary and sufficient condition is non-symmetric, (though perhaps not asymmetric) while its involved ‘truth of’ counterpart is symmetric. In the literature, the symmetry problem has been treated in one way only (so far as I can see): Discuss away the counterexamples and assimilate the simple expressions to the standard theory (see Gomes, Goldstein et al.). Below, I will make a different proposal, arguing in the opposite direction.

To begin, I borrow some insights from truthmaker theory. Without further justification, I accept the theory’s basic assumption that true propositions are true in virtue of something. More precisely, true propositions have truthmakers in virtue of which they are true, and they are true, still more precisely, in virtue of these truthmakers existing. As will become clear, modal propositions generate special complications and I ignore them here to keep my task manageable. I begin with the question: How are non-modal propositions and truthmakers related? This question can be given at least two complementary answers: (1) a given proposition has many possible truthmakers but at most one actual truthmaker; (2) a given truthmaker makes true many propositions. Ad (1), it is easy to see the one-many relation between a proposition and its possible truthmakers. Let a truthmaker be a part of a situation and a situation a part of a possible world; assume that a proposition is made true by a truthmaker iff it is made true by the situation containing it. (I here assume that b contains a iff a is a part of b.) Then a proposition may be made true by different situations; these situations may be parts of the same world and different worlds, or, alternatively, of different worlds exclusively. Thus, ‘Mary eats an apple’ may be made true by different situations, e.g. her eating an apple on the train or at home, quickly or slowly, etc., thus by situations that may or may not be parts of the same world. By contrast, ‘Mary eats this apple’ may be made true by different situations, each of which is part of a different world – assuming that we consider only worlds where Mary cannot eat a certain apple more than once. It is also easy to realize the one-one relation between a proposition and its actual truthmaker when we take truth to be truth in a world. A proposition P is true in a world iff one of its potential truthmakers exists in that world. If more than one of such potential truthmakers exist, just one of them can be P’s actual truthmaker – assuming that P cannot be made true twice over. Thus, of P’s different potential truthmakers in a certain world only one actually makes P true, in that world; the others are different alternative truthmakers. Still P can be made true simpliciter (not at a world) by a plural of truthmakers – given these truthmakers exist at different worlds. In short: the relation of a given truth and its truthmakers is one-many for truth simpliciter and one-one for truth in a world. Ad (2), the relation of a given truthmaker and propositions is again one-many, as Mary’s eating an apple makes true that Mary eats an apple, that someone eats an apple, that someone eats something, etc. In short: a given truthmaker, if it makes true a certain proposition (at a world), makes true (at that world) all its logical consequences, too.

These simple observations suffice to give and defend a definition of conditionship. The proposal is to define a condition as a relation between an arbitrary truthmaker of some proposition P and a specific truthmaker q. I make two assumptions about the linguistic representation of conditionship, again without further justification. Let ‘*P*’ (italics) express the proposition named by ‘P’ (no italics) in the form of a clause, such that ‘that *P*’ is well-formed (while ‘that P’ is not). When the content of P is nominalized, then let the resulting noun-phrase be ‘p’. I assume (first) that ‘p’ names P’s truthmaker. It may happen that a relation expression has a place that can be filled, grammatically, by either a that-clause or a noun-phrase (‘that *P*’ or ‘p’) or, alternatively, by a noun-phrase only (‘p’). I assume (second) that in the first case the expression (‘that *P*’ or ‘p’), via referring to proposition P, implies reference to an arbitrary truthmaker of P, while in the second case the expression (‘p’) refers to a specific one, namely p. Now, the English relation expression ‘… is a (necessary / sufficient / necessary and sufficient) condition of …’ can be filled, grammatically, by a that-clause or a noun-phrase in its first place, but by a noun-phrase only in its second place. Thus, from our assumptions we can conclude that this relation expression links an expression referring indirectly to an arbitrary truthmaker (first place) and one referring directly to a specific one (second place).[[2]](#footnote-2) To enhance readability, when the noun-phrase ‘p’ occupying that first place is replaceable by the expression ‘that *P*’, I write ‘*p*’ (lowercase, italics) instead of ‘p (or that *P*)’. All in all, our definiendum (conditionship) is the relation represented in English in the form ‘*p* is a (necessary / sufficient / necessary and sufficient) condition of q’. The definition itself now is built on interpreting the reference to an arbitrary truthmaker in the relation’s first place as a tacit quantification and explicate it accordingly. Thus, the full definitions of a necessary and a sufficient condition run:

1. *p* is a necessary condition of q iff only situations wherein *P* (i.e. wherein P is true) are situations containing q; and
2. *p* is a sufficient condition of q iff all situations wherein *P* (i.e. wherein P is true) are situations containing q.

(An equivalent to (i) I will use later is (i’): *p* is a necessary condition of q iff all situations wherein *¬ P* (i.e. wherein ¬ P is true) are situations not containing q.) The quantification implied in both definitions should be understood as being restricted to situations containing q, the conditioned truthmaker and containing situations where salient properties of q are varied. This casual definition of the domain is enough for the present purpose.

My proposal for a definition of conditionship has started from non-modal propositions and it should now be clear why. For arbitrary non-modal propositions P and Q, from a clause expressing P and the name of an individual truthmaker q of Q, definitions (i) and (ii) construct kinds of modal propositions, modal in the sense that each contains a quantification over possible situations. I restrict my discussion to non-modal propositions as inputs for condition claims and ignore the question whether modal propositions can likewise be such inputs, simply because that question threatens to pose extra complications. Here, I will be content when the definitions are found plausible for non-modal propositions. The idea motivating (i) and (ii) is best explicated when we consider the following: All empirical knowledge implies modal knowledge allowing us to structure the set of possible worlds. Some of that knowledge is egocentric: When you learn that *P*, i.e. that P is true in your own world, you learn that your own (our) world is a P-world. By contrast, some knowledge is non-egocentric: When you learn that all situations where *P* are situations where *Q*, you learn that all P-worlds are Q-worlds. (Critics of induction doubt whether we can achieve such knowledge empirically.) Obviously, our egocentric knowledge is very limited, i.e. due to our ignorance about many details of our world, we cannot localize it very well within the set of possible worlds. A condition claim of the form (i) or (ii) can, if true, improve this localization, starting from one individual world q. If (i) is true, then the world where q exists (e.g. our own) is a P-world, and if (ii) is true, then the world where q does not exist is a ¬P-world. Whether, and eventually how, we can obtain knowledge of the form (i) or (ii) is another question but its function of enhancing our egocentric knowledge should be clear.

The test of definitions (i) and (ii) is whether they can help us solve the symmetry problem. We see that conditionship is asymmetric by construction. It is expressed by a relation expression ‘…is a (necessary/sufficient/necessary and sufficient) condition of …’ whose first place is filled by a placeholder for a quantified variable, while the second is filled by a constant. A condition so defined can neither have a condition as its converse, nor be symmetric. Explicitly, from ‘*p* is a sufficient condition of q’ it does not follow that ‘*q* is a necessary condition of p’ because the former is analysed as making reference to an arbitrary truthmaker of P and the specific truthmaker q, while the latter is analysed as making reference to an arbitrary truthmaker of Q and the specific truthmaker p. It is plausible to assume that reference to a specific truthmaker q of Q implies the one to some truthmaker of Q – such that a situation’s containing q implies its containing some truthmaker of Q, which in turn implies that it makes Q true – (call this assumption (\*)) but the reverse assumption is certainly false; and so our reference to an arbitrary truthmaker of P in the sufficient condition cannot lead us to the specific truthmaker p in the necessary condition.

Consider now what happens when, within an expression for a condition, we erase the reference to a specific truthmaker (‘q’ in (i) and (ii)). We can do so, without becoming ungrammatical, by replacing the noun-phrase ‘q’ by the noun-phrase ‘the truth of Q’ in the definienda of (i) and (ii). It is, I think, plausible to assume that a situation contains the truth of Q iff it is a situation wherein *Q* (i.e. wherein Q is true). Thus we have:

1. *p* is a necessary condition of the truth of Q iff only situations wherein *P* (i.e. P is true) are situations wherein *Q* (i.e. Q is true); and
2. *p* is a sufficient condition of the truth of Q iff all situations wherein *P* (i.e. P is true) are situations wherein *Q* (i.e. Q is true) is true.

It is now easy to see that *p* is a necessary condition of the truth of Q iff only situations wherein *P* are situations wherein *Q*. The latter is the case iff all situations wherein *Q* are situations wherein *P*, which in turn is the case iff *q* is a sufficient condition of the truth of P. Thus, *p* is a necessary condition of the truth of Q iff *q* is a sufficient condition of the truth of P.[[3]](#footnote-3) Thus, ‘is a necessary condition of the truth of’ and ‘is a sufficient condition of the truth of’ are converses. It is similarly straightforward to show that *p* is a necessary and sufficient condition of the truth of Q iff *q* is a necessary and sufficient condition of the truth of P. Thus, ‘is a necessary and sufficient condition of the truth of’ is symmetric. [[4]](#footnote-4) Thus, we see in which sense the standard theory is right, after all. Though conditionship, the relation expressed by special cases of ‘is a condition of’, is asymmetric and does not generate converses, a neighboring relation, the one expressed by special cases of ‘is a condition of the truth of’, is symmetric and does generate converses.

We have shown that *p* is necessary condition of the truth of Q iff *q* is a sufficient condition of the truth of P. Moreover, I have suggested (assumption (\*) above) that a situation’s containing truthmaker q being entails its making true proposition q, but not vice versa. Thus, if *p* is a necessary (sufficient) condition of q, then *p* is also a necessary (sufficient) condition of the truth of Q, but not vice versa. E.g., from the assumption that Joe’s making a touchdown is a sufficient condition of his scoring six points it follows that Joe’s scoring six points is a necessary condition of the truth of ‘Joe makes a touchdown’. But from this it does not follow that Joe’s scoring six points is a necessary condition of his making a touchdown.

**References:**

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1. The first example is adapted from Wertheimer (1968: 358), the second from Sanford (1989: 203). For more examples see Wertheimer ibid., Sanford 1989: 176-178, Brennan 2011: sec.s 3 and 4. I have changed Wertheimer’s example of a pair of generic condition claims into a pair of non-generic claims and Sanford’s example of an if-and-only-if-conditional into a pair of condition claims. Generic condition claims and the conditionals related to condition claims present additional complications for analysis that I have to ignore here. [↑](#footnote-ref-1)
2. In effect, I here assume that, given the first place of ‘… is a (necessary/sufficient/necessary and sufficient) condition of …’ is filled by a noun-phrase ‘p’, the reference is generic, i.e. to an arbitrary truthmaker of P. I thus assume that in every condition claim the first place, if filled by a noun-phrase, is to be interpreted as generic. This should not be confused with generic condition claims (as, e.g., Wertheimer’s (1968, p.358) touchdown and voter examples or Sanford’s (1989, p.177) cyanide case), which, as I said, I have to ignore here. [↑](#footnote-ref-2)
3. This claim bears being rephrased in another form. Recall that ‘*p*’ (lowercase, italics) is an abbreviation for ‘p (or that *P*)’, where p is a name of P’s truthmaker, ‘P’ (uppercase, no italics) is a name of P and ‘*P*’ (uppercase, italics) an expression of P in the form of a clause. Suppose that we artificially forbid the first place of a condition relation to be filled by a noun-phrase, such that only ‘that *P*’ is allowed. Then the claim is: that *P* is a necessary condition of the truth of Q iff that *Q* is a sufficient condition of the truth of P. Or a version entirely without names for propositions: Assume that ‘the truth of P’ and ‘its being true that *P*’ both name the truthmaker of ‘it is true that *P*’, then the claim is: that *P* is a necessary condition of its being true that *Q* iff that *Q* is a sufficient condition of its being true that *P*. [↑](#footnote-ref-3)
4. Cases of necessary and sufficient conditions are constructed naturally from either (i) and (ii) or (iii) and (iv). Explicitly: (i + ii) *p* is a necessary and sufficient condition of q iff all and only situations wherein *P* (i.e. wherein P is true) are situations containing q; and (iii + iv) *p* is a necessary condition of the truth of Q iff all and only situations wherein *P* (i.e. wherein P is true) are situations wherein *Q* (i.e. Q is true). [↑](#footnote-ref-4)