Unscrambling the Quantum Omelette of Epistemic and Ontic Contextuality: Classical Contexts and Quantum Reality

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Abstract

In this paper we attempt to analyze the physical and philosophical meaning of quantum contextuality. In the first part we will argue that a general confusion within the literature comes from the improper "scrambling" of two different meanings of *quantum contextuality*. The first one is related to an *epistemic* interpretation of contextuality, introduced by Bohr, which stresses the incompatibility (or complementarity) of quantum measurements. The second, is related to an *ontic* notion of contextuality, exposed through the Kochen-Specker (KS) theorem, which focuses on the constraints to discuss about actual (definite valued) properties within the orthodox formalism of QM. We will show how these two notions have been scrambled together creating an "omelette of contextuality" which has been fully widespread through a popularized "epistemic explanation" of the KS theorem according to which: *The outcome of the observable A when measured together with B or together with C will necessarily differ in case* $[A, B] = [A, C] = 0$*, and* $[B, C] \neq 0$. We will show why this statement is not only improperly scrambling epistemic and ontic perspectives, but is also physically and philosophically meaningless. In the second part of the paper, we will analyse the relation between 'classical contexts' and QM. We will show that three accepted presuppositions found within the orthodox literature are, in general, false. Namely: (i) that quantum contextuality does not preclude an *objective* description of physical reality, (ii) that the choice of a context (or basis) restores a classical description of reality, and (iii) that the choice of a (classical) context is a *necessary condition* for accounting for empirical statements in QM.

Keywords: quantum contextuality, epistemic view, ontic view, physical reality.

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1 Epistemic vs Ontic Views in QM

The epistemic and ontic views have played a significant role in the debate about the meaning and interpretation of Quantum Mechanics (QM). In particular, as we shall argue, these two distinct philosophical perspectives have also played a fundamental role in the misunderstanding of one of the main characteristics of the theory, known in the literature as 'quantum contextuality'. In order to address this notion we shall begin our paper by a short review of the role played by both epistemic and ontic viewpoints within the theory of quanta.

The epistemic view of QM goes back to Niels Bohr's interpretation of the theory as a rational generalization of classical mechanics [10]. The Danish physicist stressed repeatedly that the "epistemological lesson" that we should learn from QM is that we are not only spectators, but also actors in the great drama of existence. This idea goes in line with his understanding of physics itself. According to Bohr [6]: "Physics is to be regarded not so much as the study of something a priori given, but rather as the development of methods of ordering and surveying human experience. In this respect our task must be to account for such experience in a manner independent of individual subjective judgement and therefor objective in the sense that it can be unambiguously communicated in ordinary human language." The epistemic view has been taken to its most radical limit by an approach put forward by Christopher Fuchs, Asher Peres, Rüdiger Schack and David Mermin, known as "Quantum Bayesianism", or in short: QBism. As Mermin has clarified —returning in many points to Bohr's epistemic approach— QBism argues explicitly against an ontological reading of QM [39].

The origin of QBism might be traced back to a paper published in the year 2000 entitled *Quantum Theory Needs no 'Interpretation'*. There, Fuchs and Asher Peres [22, p. 70] write explicitly that: "[...] quantum theory does not describe physical reality. What it does is provide an algorithm for computing probabilities for the macroscopic events ('detector clicks') that are the consequences of experimental interventions. This strict definition of the scope of quantum theory is the only interpretation ever needed, whether by experimenters or theorists." Even though the epistemic view has many different proponents, we regard QBism as the most consistent, clear and honest epistemic account of QM (see for a detailed analysis [15]). As remarked by Fuchs, Mermin and Shack [23, p. 2]: "QBism explicitly takes the 'subjective' or 'judgmental' or 'personalist' view of probability, which, though common among contemporary statisticians and economists, is still rare among physicists: probabilities are assigned to an event by an agent and are particular to that agent. The agent's probability assignments express her own personal degrees of belief about the event." According to them: "A measurement in QBism is more than a procedure in a laboratory. It is any action an agent takes to elicit a set of possible experiences. The measurement outcome is the particular experience of that agent elicited in this way. Given a measurement outcome, the quantum formalism guides the agent in updating her probabilities for subsequent measurements." As QBists make explicitly clear: "A measurement does not, as the term unfortunately suggests, reveal a pre-existing state of affairs." Measurements are "personal" and QM is a "tool" for the "user" —as Mermin prefers to call the "agent" [39]. The focus on measurement outcomes of the epistemic view is clearly confronted by the ontic viewpoint and its insistence on the relation between QM and physical reality.

The ontic view, in the context of QM, can be related to Albert Einstein's philosophical position, who confronted Bohr's epistemic understanding of physics. According to this view, it is the physical representation provided by a theory that which expresses what reality is about independently of human choices and conscious beings. As remarked by Einstein [18, p. 175]: "[...] it is the purpose of theoretical physics to achieve understanding of physical reality which exists independently of the observer, and for which the distinction between 'direct observable' and 'not directly observable' has no ontological significance." Unlike positivism, Einstein did not accept observables as "self evident" *givens*.[1] As expressed by Howard [30, p. 206], "he was not the friend of any simple realism". Indeed, for Einstein the interrelation between the description of physical reality and metaphysics was a central aspect of physics itself (see [31]). He also stressed, like Heisenberg, the importance of developing new physical concepts in order to account for new phenomena.

The epistemic and the ontic viewpoints face very different problems and concerns. While the epistemic view concentrates in the way subjects (also called 'agents', 'users', etc.) are capable of relating to observable measurement outcomes, the ontic perspective focuses on the physical meaning and interpretation of the formalism of the theory. In this case, observability —as Einstein himself stressed repeatedly— has no ontological significance; it is only regarded as part of a verification procedure about the empirical adequacy (or not) of the theory in question. While the epistemic proponent might understand the theory as a mere "algorithm" or "economy" of experience, for the ontic viewer a theory and its formalism are telling us something about reality itself. While the former takes the observability of subjects to be the starting point of science and also the end, the latter fundaments the theory in relation to the objective —i.e., subject independent— representation of physical reality. Unfortunately, within QM, these two radically opposite viewpoints —in what respects the presuppositions involved in the meaning of a theory— have been mixed in a "omelette" that we need to unscramble.

2 The Quantum Omelette

The philosophical stance that we assume defines the specific problems, the possible questions and (even) answers that fall within our system of thought. But

[1] This can be seen from the very interesting discussion between Heisenberg and Einstein [28, p. 66] were the latter explains: "I have no wish to appear as an advocate of a naive form of realism; I know that these are very difficult questions, but then I consider Mach's concept of observation also much too naive. He pretends that we know perfectly well what the word 'observe' means, and thinks this exempts him from having to discriminate between 'objective' and 'subjective' phenomena."

a limit is also a possibility, an horizon. A particular perspective limits the possibility of questioning but it also constitutes it. Problems are not "out there" independent of philosophical perspectives, they are part of a definite viewpoint with definite metaphysical assumptions, presuppositions, without which they could not be even stated. For example, a reductionistic perspective compared to a pluralist one will radically differ when considering the quantum to classical limit [13]. While a realist considers as necessary the existence of an objective representation which explains what the formalism is talking about, the instrumentalist might remain content with an intersubjective epistemic account of measurement outcomes provided by the formalism. Unfortunately, within the philosophy of QM, we are at a stage where ontology and epistemology, objectivity and subjectivity have been mixed up in an "omelette" that we still need to unscramble. As Jaynes makes the point:

> "[O]ur present [quantum mechanical] formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature —all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that the unscrambling is a prerequisite for any further advance in basic physical theory. For, if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we are talking about; it is just that simple." [33, p. 381]

It is interesting to notice, however, that one of the first scrambling of objective and subjective elements was produced, neither from Bohr nor Heisenberg, but by one of the strongest critics to the theory of quanta. In the famous EPR paper [21], Einstein's definition of what had to be considered an *element of physical reality* —in the context of QM— begun the explicit scrambling between an objective *ontic* definition of reality and the subjective, *epistemic* reference, to the quantum measurement process. According to the famous definition: *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to that quantity.* The problem —as many authors have already remarked— comes with the phrase: "without in any way disturbing the system". Bohr himself stressed the fact, in his also famous reply [5, p. 697], that the "criterion of physical reality [...] contains —however cautions its formulation may appear— an essential ambiguity when it is applied to the actual problems with which we are here concerned." Bohr used this "ambiguity" to reintroduce the epistemic analysis of measurements considering "in some detail a few simple examples of measuring arrangements".

Einstein knew very well that a realist should not define physical reality in epistemic terms for that would involve the improper intromission of a subject, breaking down the very possibility to consider an objective reality detached from subjective choices (we will come back to this important point in section 7). Einstein had accepted the intromission of measurement within his own definition of an 'element of reality' in order to expose the incompleteness of QM. But

Bohr was too smart to accept the deal. In his brilliant reply, he deconstructed Einstein's whole argumentation by concentrating on the measurement process alone. Bohr was not willing to enter the ontological debate proposed by Einstein. As remarked by his assistant Aage Petersen:

> "Traditional philosophy has accustomed us to regard language as something secondary and reality as something primary. Bohr considered this attitude toward the relation between language and reality inappropriate. When one said to him that it cannot be language which is fundamental, but that it must be reality which, so to speak, lies beneath language, and of which language is a picture, he would reply, "We are suspended in language in such a way that we cannot say what is up and what is down. The word 'reality' is also a word, a word which we must learn to use correctly" Bohr was not puzzled by ontological problems or by questions as to how concepts are related to reality. Such questions seemed sterile to him. He saw the problem of knowledge in a different light." [42, p. 11]

Indeed, Bohr was very careful not to enter ontic debates and always remained within the limits of his epistemic analysis. However, Heisenberg was not so careful. He wanted, on the one hand, to support Bohr's epistemological approach, and on the other, his own Platonist ontological scheme. In the book *Physics and Philosophy* [27] Heisenberg begun to create what is known today as "The Copenhagen Interpretation of QM" (see for discussion [32]). This interpretation attempted to bring together not only Bohr's epistemological approach and the necessity of classical language but also Heisenberg's own Platonist realism about mathematical equations in physical theories [28, p. 91]. While Bohr's anti-metaphysical commitment considered the language of Newton and Maxwell as the fundament of all possible physical phenomena, Heisenberg's closed theory approach insisted in the radical incommensurability of the physical concepts used by different theories [7]. This book shows not only Heisenberg's fantastic historical and philosophical knowledge about physics, but also the omelette he created scrambling improperly objective and subjective elements, ontology and epistemology. A good example of the quantum omelette cooked in Heisenberg's book is the following passage:

> "With regard to this situation Bohr has emphasized that it is more realistic to state that the division into the object and the rest of the world is not arbitrary. Our actual situation in research work in atomic physics is usually this: we wish to understand a certain phenomenon, we wish to recognize how this phenomenon follows from the general laws of nature. Therefore, that part of matter or radiation which takes part in the phenomenon is the natural 'object' in the theoretical treatment and should be separated in this respect from the tools used to study the phenomenon. *This again emphasizes a subjective element in the description of atomic events, since the measuring device has been constructed by the observer, and we have to remember that what we observe is not nature in itself but nature exposed to our method of questioning.* Our scientific work in

physics consists in asking questions about nature in the language that we possess and trying to get an answer from experiment by the means that are at our disposal. In this way quantum theory reminds us, as Bohr has put it, of the old wisdom that when searching for harmony in life one must never forget that in the drama of existence we are ourselves both players and spectators. It is understandable that in our scientific relation to nature our own activity becomes very important when we have to deal with parts of nature into which we can penetrate only by using the most elaborate tools." [27, p. 9] (emphasis added)

Of course, the fact we should acknowledge physical representation and experience has been created through concepts and tools specifically designed by us, humans, is completely different from claiming that the choice of a specific experimental arrangement determines explicitly physical reality itself. This situation in QM, regarding the subjective definition of physical reality, was clearly recognized by Einstein who remained always uncomfortable with the epistemological reasoning of Bohr. As recalled by Pauli:

> "*Einstein*'s opposition to [quantum mechanics] is again reflected in his papers which he published, at first in collaboration with *Rosen* and *Podolsky*, and later alone, as a critique of the concept of reality in quantum mechanics. We often discussed these questions together, and I invariably profited very greatly even when I could not agree with *Einstein*'s view. 'Physics is after all the description of reality' he said to me, continuing, with a sarcastic glance in my direction 'or should I perhaps say physics is the description of what one merely imagines?' This question clearly shows *Einstein*'s concern that the objective character of physics might be lost through a theory of the type of quantum mechanics, in that as a consequence of a wider conception of the objectivity of an explanation of nature the difference between physical reality and dream or hallucination might become blurred." [40, p. 122]

Independently of Einstein's efforts to discuss the possible physical representation of quantum reality, the Bohrian approach has become a silent orthodoxy that limits the analysis of QM down to the almost exclusive set of problems which —following Bohr's interpretation— presuppose the representation of reality that results from classical physics. These problems have been entangled with incompatible philosophical stances, producing a lot of confusion not only with respect to the metaphysical presuppositions and standpoints of the problems themselves, but also with respect to their limits of inquiry. In the following section we attempt to discuss the Bohrian metaphysical standpoints which have created the present orthodox agenda of discussion and analysis.

3 Physical Representation and the Two (Bohrian) Dogmas of QM

A physical representation of reality cannot be "whatever". Physical representation allows us to imagine what the theory is talking about. Any physical representation is necessarily constrained by specific metaphysical presuppositions. In the case of classical physics the metaphysical principles playing this basic role are those of classical logic, namely, the Principle of Existence (PE) which allow us to say that "something exists", the Principle of Non-Contradiction (PNC) which allows us to claim that the 'something' in question "possesses non-contradictory properties" and the Principle of Identity (PI) which tells us that the something possessing non-contradictory properties "remains identical to itself through time". Without such metaphysical principles at play it becomes impossible to discuss about an "object" since it is this specific set of principles which explicitly defines the notion in a systematic and rigorous manner.

However, since physical representation is in itself metaphysical, it cannot be constrained, in principle, by a presupposed metaphysical scheme. Assuming such a standpoint would imply that we have finally reached the true metaphysical scheme that describes reality as it is —putting an historical end to the quest of physics itself. The ontic pluralist stance implied by representational realism (see [13]) leaves open the possibility to discuss about multiple (even non-classical) representations of reality. Just like the classical representation of physics was developed through centuries, it is in principle possible to imagine that a non-classical physical representation could be developed in the future through the creation of new (non-classical) physical concepts. As remarked by Heisenberg [29, p. 264]: "The history of physics is not only a sequence of experimental discoveries and observations, followed by their mathematical description; it is also a history of concepts. For an understanding of the phenomena the first condition is the introduction of adequate concepts. Only with the help of correct concepts can we really know what has been observed." Unfortunately, the possibility to develop a non-classical physical representation of QM has been severely constrained by Bohr's imposition of two strong metaphysical presuppositions. These Bohrian (metaphysical) presuppositions have been turned in the present literature into unquestionable dogmas that all interpretations of QM must follow. Let us discuss them in some detail.

The first metaphysical presupposition is the idea that there must exist a continuous "quantum to classical limit" —implying what Bokulich calls an "open theory approach" [7]. This reductionistic idea was put forward by Bohr in terms of his *correspondence principle* [9] and can be condensed in the following statement.

Dogma I. Quantum to Classical Limit: One must find a continuous "bridge" or "limit" between classical mechanics and QM.

However, this is certainly not the only possible way to approach the problem of inter-theory relation. In fact, the inter-theoretic scheme proposed by

Bokulich [8], or our the represenational realist scheme [13], open different non-reductionistic possibilities of analysis, which due to the single-viewed Bohrian perspective have not been discussed nor developed within the literature.

The second metaphysical principle which has guided orthodoxy can be also traced back to Bohr and his claim that physical experience needs to be expressed exclusively in terms of classical physical language [10]. Bohr [46, p. 7] stated that: "[...] the unambiguous interpretation of any measurement must be essentially framed in terms of classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time." In this respect, "it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms."

Dogma II. Classical Representation of Physics: One needs to presuppose the classical representation of physics in order to discuss about *phenomena* and interpret QM.

Both (metaphysical) presuppositions go clearly against a radical non-classical understanding of QM, limiting as well different non-reductionistic and non-classical possibilities of analysis. As we shall see in sections 6 and 7, these two old dogmas have played a fundamental role in the misinterpretation not only of qunatum contextuality but also in the imposition of 'classical contexts' within the analysis of QM itself. But before entering this debate, in the following two sections, we still need to provide an analysis and explanation of the distinct content of the notion of contextuality according to the just discussed epistemic and ontic viewpoints of QM.

4 Bohr's *Epistemic Contextuality*

Bohr's notion of contextuality is directly related to his analysis of the double-slit experiment and the necessary requirement to provide an account of each experimental set up in terms of classical concepts (i.e., the wave-particle duality).[2] The epistemic solution presented by Bohr was provided through his notion of complementarity which allowed the co-existence of mutually incompatible classical representations such as those of 'wave' and 'particle'.

Bohr's epistemic solution was tested by Einstein's attempt to reintroduce an ontic debate within the theory of quanta. As we mentioned above, in the EPR paper Einstein introduced his by now famous definition of what could be considered to be an *element of physical reality* within quantum theory. In this way he was attempting to force Bohr to enter the debate within his own realist terms. But, as already mentioned, Bohr would not accept the deal. Einstein had left a loose end in his definition, an "ambiguity" which allowed Bohr to go back into the safety of his epistemological analysis. He did so through his *epistemic* notion of contextuality based on the incompatibility of (complementary) measurement set ups. According to Bohr:

[2] See for a detailed analysis: [10].

> "This necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical description of physical phenomena.*" [*Op. Cit.*, p. 701] (emphasis in the original)

As stressed by Bohr [*Op. Cit.*], this fundamental distinction "has its root in the indispensable use of classical concepts in the interpretation of all proper measurements, even though the classical theories do not suffice in accounting for the new types of regularities with which we are concerned in atomic physics." Bohr's contextuality implied the need to discuss in terms of 'classical contexts'; i.e. contexts described in terms of classical experimental apparatuses. This (metaphysical) requirement was secured, not by the quantum formalism but, by his insistence in the necessity of the use of the classical physical language of Newton and Maxwell —something we have called in the previous section 'Dogma II'. This metaphysical precondition was in line with his neo-Kantian philosophical perspective according to which phenomena must be necessarily considered as classical space-time phenomena. Bohr never discussed in terms of the mathematical formalism of the theory, which he considered to be a "purely symbolic scheme". Instead, he took as a standpoint the representation of experimental set ups in terms of classical physics and its language.

The epistemic notion of contexuality addressed by Bohr made reference to a measurement situation with classical apparatuses, in which classical phenomena could be addressed. Regardless of the acceptance of such standpoint in the orthodox literature we would like to stress that this definition of *epistemic contextuality* is in itself a metaphysical presupposition which —as we shall argue in section 7— has no direct relation to the orthodox formalism of QM. There is in fact no constraint of the formalism regarding the possibility to understand and discuss quantum contextuality beyond classical concepts.

5 Kochen-Specker's *Ontic Contextuality*

Contrary to Bohr, Simon Kochen and Ernst Specker begun their analysis, taking as a standpoint the orthodox formalism of QM, asking an ontological question which has no epistemic reference whatsoever. *Would it be possible to consider projection operators as actual (definite valued) preexistent properties within the orthodox formalism of QM?* This question led them to a very interesting analysis which we now shortly recall.

In QM the frames under which a vector is represented mathematically are considered in terms of orthonormal bases. We say that a set $\{x_1, \ldots, x_n\} \subseteq \mathcal{H}$ an n-dimensional Hilbert space is an *orthonormal basis* if $\langle x_i | x_j \rangle = 0$ for all $1 \leq i, j \leq n$ and $\langle x_i | x_i \rangle = 1$ for all $i = 1, \ldots, n$. A physical quantity is represented by a self-adjoint operator on the Hilbert space $\mathcal{H}$. We say that $\mathcal{A}$ is a *context* if $\mathcal{A}$ is a commutative subalgebra generated by a set of self-adjoint bounded operators $\{A_1, \ldots, A_s\}$ of $\mathcal{H}$. Quantum contextuality, which was most explicitly recognized through the Kochen-Specker (KS) theorem [37], asserts that a value ascribed to a physical quantity A cannot be part of a global assignment of values but must, instead, depend on some specific context from which A is to be considered. Let us see this with some more detail.

Physically, a global valuation allows us to define the preexistence of definite properties. Mathematically, a *valuation* over an algebra $\mathcal{A}$ of self-adjoint operators on a Hilbert space, is a real function satisfying,

1. *Value-Rule (VR)*: For any $A \in \mathcal{A}$, the value $v(A)$ belongs to the spectrum of A, $v(A) \in \sigma(A)$.
2. *Functional Composition Principle (FUNC)*: For any $A \in \mathcal{A}$ and any real-valued function f, i.e. $v(f(A)) = f(v(A))$.

We say that the valuation is a *Global Valuation (GV)* if $\mathcal{A}$ is the set of all bounded, self-adjoint operators. In case $\mathcal{A}$ is a context, we say that the valuation is a *Local Valuation (LV)*. We call the mathematical property which allows us to paste consistently together multiple contexts of *LVs* into a single GV, *Value Invariance (VI)*. First assume that a GV v exists and consider a family of contexts $\{A_i\}_I$. Define the LV $v_i := v|_{A_i}$ over each A_i. Then it is easy to verify that the set $\{v_i\}_I$ satisfies the *Compatibility Condition (CC)*,

$$v_i|_{A_i \cap A_j} = v_j|_{A_i \cap A_j}, \quad \forall i, j \in I.$$

The CC is a necessary condition that must satisfy a family of *LVs* in order to determine a GV. We say that the algebra of self-adjoint operators is *VI* if for every family of contexts $\{A_i\}_I$ and *LVs* $v_i : A_i \to \mathbb{R}$ satisfying the CC, there exists a GV v such that $v|_{A_i} = v_i$.

If we have *VI*, and hence, a GV exists, this would allow us to give values to all magnitudes at the same time maintaining a CC in the sense that whenever two magnitudes share one or more projectors, the values assigned to those projectors are the same in every context. The KS theorem, in algebraic terms, rules out the existence of GVs when the dimension of the Hilbert space is greater than 2. The following theorem is an adaptation of the KS theorem —as stated in [20, Theorem 3.2]— to the case of contexts:

THEOREM 5.1 (KS THEOREM) *If $\mathcal{H}$ is a Hilbert space of $\dim(\mathcal{H})\rangle 2$, then a global valuation is not possible.*

KS theorem proves there is no GV, and thus, an interpretation of projection operators as *preexistent* properties becomes problematic. The theory cannot be interpreted as representing a *preexistent* Actual State of Affairs (ASA). By an ASA we mean a closed system considered in terms of a set of actual (definite valued) properties. Classical actual properties are metaphysically constrained by the logical and ontological PE, PNC and PI (See [13] for a detailed analysis). There are four main points which are important to stress with respect to our analysis:

I. *KS theorem has nothing to do with measurements.* There is no need of actual measurements for the KS theorem to stand. The theorem is not talking about measurement outcomes, but about the *preexistence* of properties. About the constraints implied by the formalism to projection operators (interpreted in terms of properties that pertain to a quantum system). Quantum contextuality cannot be tackled through an analysis in terms of measurements simply because there is no reference at all to any measurement outcome.

II. *KS theorem is not empirically testable.* As it is well known we cannot measure sets of definite values from a quantum state. In QM we can only measure mean values of observables. The measurement of definite values is restricted to very particular cases.

III. *KS theorem makes exclusive reference to the quantum formalism.* KS's notion of contextuality makes reference only to the quantum formalism. In this sense it is an internal formal statement of the theory independent on any particular interpretation of QM.

VI. *KS theorem presupposes an ontic definition of contextuality.* KS makes reference to the interpretation of the quantum formalism in terms of systems with actual (definite valued) properties. Put in a nutshell, quantum contextuality deals with the formal conditions that any realist interpretation which respects orthodox Hilbert space QM must consider in order to consistently provide an objective physical representation of reality.

In the following section we will show how the two previous definitions of epistemic and ontic contextuality have been scrambled up together into an omelette which we attempt to unscramble.

6 Scrambling KS with Measurement Outcomes

In [22], Asher Peres made explicit his instrumentalist perspective arguing, together with Chris Fuchs, that "quantum theory does not talk about physical reality". According to his epistemic view, QM is "an algorithm for computing probabilities for the macroscopic events ('detector clicks')". However, as we have argued above, KS makes no reference at all to measurement outcomes. On the contrary, KS theorem analyses the formalism in terms of (meta-)physical reality. This is the reason why, for anyone attempting to follow an epistemic perspective with respect to QM, the KS theorem presents an uncomfortable situation.

A radical epistemic proponent might consider a theory to be —following Mach— an "economy of experience", or —following van Fraassen— a formal structure capable of "saving the phenomena". But, the KS theorem makes no reference at all to the epistemic realm of measurements. On the very contrary, it reflects about the ontological mode of existence of properties themselves (as being *actual* or *preexistent*). This is of course an analysis which does not require *hic et nunc* observability; this is, on the very contrary, a formal-metaphysical analysis. A study about the formal constraints on projection operators when considered in terms of actual (definite valued) properties. In this respect, as we remarked above, the definition of *actual property* is in itself a metaphysical definition (dependent on PE, PNC and PI). The epistemic viewer might feel quite uncomfortable with the ontic debate presented by Kochen and Specker. How can the epistemic advocate avoid being dragged into such a purely formal-metaphysical analysis which has no reference whatsoever to observable measurement outcomes?

In his excellent book, *Quantum Theory: Concepts and Methods*, Peres provides an "epistemic explanation" of the KS theorem which provides the required re-introduction of the notion of measurement within the analysis:

> "The Kochen-Specker theorem asserts that, in a Hilbert space of dimension $d > 2$, it is impossible to associate definite numerical values, 1 or 0, with every projection operator P_m, in such a way that, if a set of commuting P_m satisfies $\sum v(P_m) = 1$, the corresponding values, namely $v(P_m) = 0$ or 1, also satisfy. The thrust of this theorem is that any cryptodeterministic theory that would attribute a definite result to each quantum measurement, and still reproduce the statistical properties of quantum theory, is inevitably contextual. *In the present case, if three operators, P_m, P_r, and P_s, have commutators $[P_m, P_r] = [P_m, P_s] = 0$ and $[P_r, P_s] \neq 0$, the result of a measurement of P_m cannot be independent of whether P_m is measured alone, or together with P_r, or together with P_s.*" [*Op. Cit.*, p. 196] (emphasis added)

Today, this explanation has become extremely popular and is found in almost every paper which discusses the physical meaning of the quantum contextuality exposed by KS theorem. We can resume this reading in the following definition:

DEFINITION 6.1 EPISTEMIC READING OF KS THEOREM: *The measurement outcome of the observable A, when measured together with B or together with C, will necessarily differ in case $[A, B] = [A, C] = 0$, and $[B, C] \neq 0$.*

I myself accepted this "reading" as a "down to earth explanation" of KS theorem in a recent paper [12]. However, when analyzed in detail it is possible to see that, because of the improper mixture of epistemic and ontic presuppositions, the statement has no rigorous physical nor philosophical content. DEFINITION 6.1 describes an experimental situation which, by definition, cannot be empirically tested. It talks about measurements that cannot be measured! In fact, as we have remarked, KS theorem is not empirically testable in a direct manner, simply because it never talks about measurements. But were we to assume a consistent epistemic perspective, we should try not to make reference to a metaphysical reality beyond measurement outcomes. That would be accepting right from the start an ontic perspective regarding the meaning of a theory in terms of the representation of physical reality. Such ontic perspective would force us to abandon our epistemic viewpoint according to which a physical theory is only an economy of experience, an algorithm that predicts 'clicks' in detectors.

In order to clearly understand this point, it is of deep importance to distinguish between the *ontic* incompatibility of properties and the *epistemic* incompatibility of measurements. The fact that even in classical physics we can find epistemically incompatible measurement situations has been very clearly discussed by Diederik Aerts in [1]. Aerts discusses the example of a piece of wood which might posses the properties of being "burnable" and "floatable". Both (classical) properties are *testable* through mutually incompatible experimental arrangements. Indeed, in order to test whether the piece of wood can be burned (the "burnability" as it were), we must light it up and see if it burns, but then —because in fact it will burn— it will no longer be possible to test whether the piece of wood floats. In order to measure the "floatability" we must place the piece of wood in a suitable container filled with water and see what happens. The case is, that a burned wood will not float, and also, a wet piece of wood will not burn. Hence, both properties cannot be tested simultaneously. These two experiments are *epistemically incompatible*. However, the properties are not *ontically incompatible*, the epistemic realm of measurements does not make any direct reference to the ontic level of properties. In classical physics, all properties can be thought to exist as actual (ontic) properties due to the fact that the formal Boolean structure of propositions allows an interpretation in terms of an ASA. The following two definitions are of importance to make clear our analysis:

Definition 6.2 Epistemic Incompatibility of Measurements: *Two contexts are epistemically incompatible if their measurements cannot be performed simultaneously.*

Definition 6.3 Ontic Incompatibility of Properties: *Two contexts are ontically incompatible if their formal elements cannot be considered as simultaneously preexistent.*

Even though classical mechanics might in principle present an epistemic incompatibility of measurements, due to its commutative (Boolean) structure there is no ontic incompatibility between classical properties. On the contrary, in QM the KS theorem makes explicit the ontic incompatibility between properties. This important result is a consequence of the formalism itself. The epistemic incompatibility in QM appears only when classical contexts are considered. But in such case, since KS does not provide a way to test empirically the statement, the discussion becomes completely metaphysical. Furthermore, as we shall argue in section 7, classical contexts are not even required by the theory in order to produce meaningful operational statements. As we shall see in the following section, even within such classical contexts very deep problems arise and classicality is not certain to have been regained —as it is uncritically assumed within the orthodox literature. The epistemic viewer, when entering the KS debate, seems to have been trapped in a metaphysical net with no reference to observable measurement outcomes.

But there is no escape, KS cannot be empirically tested. It makes reference to definite values of projection operators, not to measurement outcomes. KS does not provide an empirical result, it is a discussion about the limits implied by the formalism of QM to the metaphysical mode of existence of properties (projection operators). KS makes explicit the deep metaphysical problem that any interpretation of QM must face in case it attempts to interpret the theory in terms of an ASA. This is why, an "epistemic reading" of KS theorem is simply untenable. In conclusion, the KS debate is a purely ontic debate, it has no epistemic elements at play.

We shall now turn our analysis to expose the untenability of certain widespread presuppositions regarding the necessity —imposed by Bohr— to make reference to classical contexts within the analysis of quantum theory.

7 Classical Contexts in Quantum Mechanics

In this section, we will present a series of arguments which expose the fact that the following three widespread (metaphysical) presuppositions accepted within the orthodox literature are, in general, false:

i. Quantum contextuality does not preclude an *objective description* of physical reality.

ii. The choice of a context (or basis) restores a *classical description* of physical reality.

ii. The choice of a (classical) context is a *necessary condition* for accounting for empirical statements in QM.

We now turn to the specific analysis of each one of these statements.

7.1 Is 'Contextual Reality' Objective?

Many authors —e.g., G. Bene, D. Dieks, P. Grangier, M. Bitbol, R. Griffiths, V. Karakostas and K. Svozil [2, 3, 4, 18, 24, 26, 35, 36, 45]— who understand very well the contextual nature of QM, have argued repeatedly in different manners that "KS theorem does not preclude an objective description of physical reality". However, instead of restricting their analysis to the notion of ontic contextuality discussed above (section 5), most of them re-introduce in this debate the notion of epistemic contextuality (section 4) creating an omelette of objective and subjective aspects of the formalism (section 2) within the debate of quantum contextuality. In this section we will show that, if we consider actuality as the mode of existence which describes physical reality —as KS theorem and all of these authors implicitly or explicitly do— then, "contextual reality", due to the intromission of the subject within the definition itself of what is considered to be physically real, cannot be considered to be a tenable objective notion.

Before entering the debate, and in order to discuss about the possibility to provide, from the contextual theory of quanta, an "objective description of physical reality", we must obviously agree on the meaning of "objective". Let us provide some definitions which are required for a clear exposition of the content of the debate.

The notion of objectivity was introduced by Immanuel Kant in order to resolve the longstanding debate regarding theory and experience between rationalists and empiricists. On the one hand, the empiricist argued that the fundament of knowledge is observable empirical data; on the other hand, the rationalists claimed that it was reason alone which would provide not only a foundation but also a guide for a true understanding of reality. But while the rationalists were incapable of justifying the external world, the empiricists could not explain the path from empiric observations to theory-description. As Hume had clearly exposed in his analysis of causation, our categorical understanding is not something "we find outside in the world" but something that we subjects impose, shaping intrinsically our experience. We do not see identities walking in the empirical world, we have never observed the principle of non-contradiction. Kant, resting on Hume's critic to causality, developed the notion of objectivity in order to find a middle path.

Objective knowledge was defined in the *Critique of Pure Reason* as the knowledge of the trascendental subject, a knowledge restricted by the categories and forms of intuition (Newtonian space and time). But the recognition of our categorical constraints as subjects implied also restrictions and limits to knowledge itself. Kant distinguished between trascendental (noumenic) reality and objective (phenomenical) reality. Only the second was accessible to physicists.

Definition 7.1 Transcendental Reality: *A reality that is external to the subject, undisclosed in experience, which Kant looked upon as the intrinsically unknowable cause of subjective experience.*

Within the Kantian scheme, transcendental reality amounts to reality *as it is*, also called by Kant *Das ding an sich* [the thing in itself]. Noumenic reality is not accessible to the subject since there is no possible experience of it.[3] Within the Kantian architectonic trascendental reality is that which will remain always "veiled" —to use a term made popular by Bernard D'Espagnat— to the physicist. As a consequence, according to Kant, the physicist must limit his study to objective reality alone.

Definition 7.2 Objective Reality: *A reality which is a product of a mental synthesis based on the spatiotemporal structure of experience, achieved with the help of spatiotemporal concepts, and resulting in an objective reality from which the objectifying subject can abstract itself.*

According to Kant, the physicists must only consider the "reality" related to objective (represented) phenomena. While trascendental reality is an *absolute*

[3] Jacobi [1787: 223] famous remark makes clear the problem: "Without the presupposition [of the 'thing in itself,'] I was unable to enter into [Kant's] system, but with it I was unable to stay within it." As Schopenahuer would also later on make clear, the category of *causality* could not be applied within Kant's system to *noumenic reality*, which lies of course beyond categorical representation and (objective) experience.

notion, independent of any categorical representation; objective reality is a *relative* notion, categorically constrained and shaped by the subject. However, the fact that the categories are subject-dependent does not imply that the subject has a saying in what objective reality amounts to. The subject must be capable of totally abstracting or detaching himself in order to claim that his representation of reality is objective.

Of course, all this was very clear to both Einstein and Pauli, who were part of the neo-Kantian milieu of central Europe at the beginning of the 20th century. As recalled by Pauli, Einstein knew very well that objectivity required an objectifying subject capable of abstracting himself from the description of physical reality itself:

> "[...] it seems to me quite appropriate to call the conceptual description of nature in classical physics, which Einstein so emphatically wishes to retain, 'the ideal of the detached observer'. To put it drastically the observer has according to this ideal to disappear entirely in a discrete manner as hidden spectator, never as actor, nature being left alone in a predetermined course of events, independent of the way in which phenomena are observed. 'Like the moon has a definite position' Einstein said to me last winter, 'whether or not we look at the moon, the same must also hold for the atomic objects, as there is no sharp distinction possible between these and macroscopic objects. Observation cannot *create* an element of reality like position, there must be something contained in the complete description of physical reality which corresponds to the *possibility* of observing a position, already before the observation has been actually made.' I hope, that I quoted Einstein correctly; it is always difficult to quote somebody out of memory with whom one does not agree. It is precisely this kind of postulate which I call the ideal of the detached observer." [38, p. 60] (emphasis added)

The question that can settle the debate about the objective character or not of contextuality in QM is thus the following:

Can the subject be considered as completely detached from a reality described in terms of contextual choices?

If we consider reality to be metaphysically described in terms of an actual state of affairs —as it is the case of all classical physics including relativity theory—, then this *necessary condition* for objective description, due to the KS theorem, is violated. Indeed, as we have seen above, the ontic reading of contextuality implies the impossibility to describe reality in terms of such actual state of affairs. Einstein was correct to dismiss Bohr's complementarity approach as an objective account of QM. Indeed, the epistemic proposal of the Danish physicist did not accomplish the basic objectivity condition. And this is the reason why Bohr had to shift from *objectivity* to *intersubjectivity.*

Intersubjectivity relates to the mutual communication of observations between subjects.[4] But contrary to what it is repeatedly claimed in the orthodox literature, there is nothing objective in the notion of intersubjectivity. The notion of objectivity requires an 'object' of which one can be objective (subject independent) about. 'Objective' refers always to an 'object'. If one considers only subjective observations with no reference but themselves, there is nothing to be objective about. A personal experience obviously cannot be considered as being "objective" since, by definition, it is subject-dependent. The Bohrian claim —taken to its extreme limit by QBism (see for a detailed discussion [15])— that an algorithm of experience is "objective" because there is no subject playing a role within the quantum algorithm misses completely the point at stake. The requirement of an objective statement is not only that subjects can be detached, it is also that we are talking about something beyond subject observation, namely, an object of study. Without an object of study —call it physical reality, the micro-world, quantum particles, etc.— an objective statement looses completely its fundament as well as its content.

Neither Bohr's complementarity scheme nor the proposals of many authors earlier addressed provide an objective description of physical reality in accordance with the orthodox formalism of QM. All these proposals require an intervening subject who is "not only a spectator, but also an actor in the great drama of quantum existence". The choice of the context (or basis) breaks down the *necessary condition* for an objective description of physical reality. We could say that this choice amounts to breaking down the *ontic incompatibility* of properties. The introduction of a *subjective choice* within the representation of reality itself precludes the possibility of producing an objective description of such subject-independent physical reality.

As we have remarked already, given what we know up to the present, there seems to be no escape from KS ontic contextuality. Either we must change the formalism of QM in order to recover a classical understanding of *what there is* in terms of an ASA, as the hidden variable project attempts to do; or we must come up with a different metaphysical understanding of physical reality itself, one that goes beyond the constraints imposed by the actual realm in terms of PE, PNC and PI.

7.2 Are Quantum Contexts Really Classical?

It is commonly accepted that the introduction of a classical context within the formalism of QM provides a bridge to recover the classical Boolean structure of propositions common to all classical physical theories. It is constantly repeated within the literature that discusses foundational issues about QM that the Boolean structure allows us to restore a classical discourse about actual properties that pertain to physical systems. The claim is that once the context has been fixed (chosen) classicality is finally regained. In this section we will present an argument that proves the untenability of such unjustified metaphys-

[4] D'Espagnat has called such statements *weakly objective* statements. See: [17].

ical claim.

Our argument makes use of the existence of quantum superpositions within supposedly "classical contexts". Quantum superpositions are of course one of the most important formal elements when discussing about the theory of quanta. Their treatment, together with entanglement (i.e., the pasting of superpositions), has allowed physicists to advance the most outstanding technical and experimental developments in quantum information processing. In order to be very clear about the discussion at stake we provide, following [14], the following contextual definition of quantum superpositions:

Definition 7.3 Quantum Superposition: *Given a quantum state, Ψ, each i-basis defines a particular mathematical representation of Ψ, $\sum c_i \; |\alpha_i\rangle$, which we call a quantum superposition. Each one of these different basis dependent quantum superpositions defines a specific set of meaningful operational statements. The set of meaningful operational statements are related to each one of the terms (found in a particular quantum superposition) through the Born rule. The notion of quantum superposition is contextual for it is always defined in terms of a particular experimental context (or basis).*

In a typical Stern-Gerlach experiment the description of the quantum state of the particle is, in general, of the following type:

$$\alpha \mid \uparrow_i\rangle + \beta \mid \downarrow_i\rangle$$

This is in no way different to the famous cat state imagined by Schrödinger after interacting with an atom described by QM as being partly 'decayed' and partly 'not decayed', can be at the same time 'dead' and 'alive' [44].

It is this type of state which is used in the new quantum technological era of information processing that is taking place today. But as it is well known, the formal features of this state do not allow us to introduce a classical type interpretation of each term in the superposition. Given a context, a quantum superposition (of more than one term) within that context cannot be described in classical terms. As stressed by Paul Dirac [19]: "The nature of the relationships which the superposition principle requires to exist between the states of any system is of a kind that cannot be explained in terms of familiar physical concepts. One cannot in the classical sense picture a system being partly in each of two states and see the equivalence of this to the system being completely in some other state." So it is simply not true that with the choice of the context classicality is restored. The fact that a set of projection operators pertain to a Boolean algebra does not mean that they can be interpreted in classical terms as it is generally implied. It is false that once a context is fixed we recover a classical discourse to interpret the quantum formalism. Let us see this is some detail.

The problem to interpret quantum superpositions in classical terms comes from a set of well known facts to the community which discusses foundational issues about QM:

I. Given a quantum superposition of the type '$\alpha \mid \uparrow_i\rangle + \beta \mid \downarrow_i\rangle$', each one of the states $\mid \uparrow_i\rangle$ and $\mid \downarrow_i\rangle$ is related to a meaningful operational statement which is empirically testable. This provides a realist ground to talk about all terms as elements of physical reality (see [13]). As Griffiths makes the point: [25, p. 361]: "If a theory makes a certain amount of sense and gives predictions which agree reasonably well with experimental or observational results, scientists are inclined to believe that its logical and mathematical structure reflects the structure of the real world in some way, even if philosophers will remain permanently skeptical."

II. The probabilities, $|\alpha|^2$ and $|\beta|^2$, which provide the expectation values of the states, $\mid \uparrow_i\rangle$ and $\mid \downarrow_i\rangle$, through the Born rule, cannot be interpreted epistemically (see [43]). Notice that it is this mathematical fact which is also responsible for creating the infamous measurement problem.[5]

III. The main difficulty, as Schrödinger implicitly expressed in [44], is that a quantum superposition of the type '$\alpha \mid \uparrow_i\rangle + \beta \mid \downarrow_i\rangle$' describes the state of a system which possesses simultaneously two contradictory properties (see for a detailed discussion [11]). Obviously classical objects cannot be described in terms of such paraconsistent formalization.

Obviously, a classical system cannot posses contradictory properties. The existence of quantum superpositions is by itself a prove of the fact that quantum contexts are not at all "classical". Leaving aside many worlds interpretations which do provide a realist account of superpositions, the orthodox literature —following Bohr— has systematically avoided the discussion and analysis about the conceptual meaning of quantum superpositions (see [14] for a detailed discussion).

7.3 Are Classical Contexts Really Necessary?

The idea that classical contexts are a necessary requirement to discuss about quantum phenomena was imposed by Bohr and his insistence in the need to explain QM as a rational generalization of classical mechanics [10]. This idea has become an unquestionable dogma which every interpretation seems to presuppose in order to make reference to measurement outcomes. However, dogma II is clearly not a *necessary condition* for all imaginable interpretations.

In order to address the need (or not) of classical contexts in order to make reference to empirical statements in QM we first need to define what is the meaning of such classical context. Firstly, we must recognize the fact that a "classical context" is in itself part of a representation provided by classical physics. Secondly, we also need to agree about te meaning of empirical statements within

[5]QBism does interpret quantum probability in epistemic terms but at the hight cost —at least for a realist— of having to argue that "quantum mechanics itself does not deal directly with the objective world" [23]. The proposal of Everettian many worlds interpretation which also makes use of epistemic probability has also found serious difficulties to justify the fact that rational agents use the Born rule in order to choose between different possibilities [34].

a theory. In order to be able to continue with our analysis, let us provide the following two definitions:

Definition 7.4 Classical Context: *A classical context is a situation described in terms of classical objects composed by definite valued properties. Classical objects and its properties are restricted to a classical ontology which must respect PE, PNC and PI.*

Definition 7.5 Meaningful Operational Statements (MOS): *If given a specific situation a theory is capable of predicting in terms of definite operational statements the empirical outcome of a possible measurement, then we consider such statement as meaningful relative to the theory.*

It is important to remark that MOS are statements which do not necessarily imply a coherent representation of the phenomena in question. Only in case one is a realist about physical theories one should also search for a coherent representation which allows to understand MOS beyond mere measurement outcomes. QM is a good example of the fact that a theory can produce MOS without having a convincing physical representation to discuss about.

In QM, we know that a quantum state, Ψ, gives rise to definite empirical statements regarding observables through the Born rule. In fact, that is the way that physicists actually use the theory in order to measure the observable quantities:

Definition 7.6 MOS in QM: *Given a vector in Hilbert space, Ψ, the Born rule allows us to predict the average value of (any) observable O.*

$$\langle \Psi | O | \Psi \rangle = \langle O \rangle$$

These predictions are independent of the choice of any particular basis. They are context-independent predictions.

Thus, if we consider the MOS addressed by quantum theory, we see there is in fact no reference to any particular context, neither to a classical experimental apparatus. *The MOS about quantum observables are context independent, they do not make reference to a particular experimental situation.*

So where is the necessity of classical contexts coming from? As we have remarked above, this fundamental presupposition can be traced back to the second Bohrian metaphysical presupposition according to which all *phenomena* must be necessarily described in terms of the classical representation of physics (section 2). As we remarked above, the very definition of a 'classical context' in terms of the representation of classical physics, as an apparatus represented as a well defined object possessing definite valued properties through time, is in itself a metaphysical definition. We should be also aware that such idea, according to which our classical representation of physics is the only possible representation capable of accounting for experience (or observations) contains in itself, implicitly, other strong metaphysical presuppositions at play. An important (metaphysical) implication is that —according to this perspective— we have already reached the final conditions of human understanding. That the only way to "observe" is according to the theories of Newton and Maxwell. That would also imply the very impossibility of the creation of new fundamental theories.

The acceptance of the necessity of classical contexts precludes the possibility of producing, through new physical theories, a new physical experience, a new way of observing. The reason that sustains this idea rests on the dogmatic belief that: "[...] the unambiguous interpretation of any measurement must be essentially framed in terms of classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time." We know that this classical understanding of observability has been severely questioned within the philosophy of science literature. The debate in philosophy of physics regarding the theory ladenness of physical observation has made very clear that observability in physics cannot be understood as the ground of physical theories. On the very contrary, many authors —Einstein and Heisenberg in between them— have argued that it is in fact quite the opposite, "It is only the theory which decides what we can observe" [28, p. 63].

The failure of the orthodox project to explain the relation between QM and classical reality is also an exposition of the weakness of dogma II. Today QM is used by physicists in an instrumentalist fashion without any coherent reference to a representation of (quantum) physical reality. This also makes explicit the fact that the need of making reference to classical contexts has no justification beyond a dogmatic perspective which assumes right from the start a relation between QM and classical physics that has not been, up to the present, adequately explained (see for discussion [13]).

Conclusion

In this paper we have argued that there are two different definitions of quantum contextuality that have been scrambled together creating an omelette that we need to unscramble. While the first epistemic notion is due to Bohr's analysis of the measurement process, the second ontic definition relates to the intrinsic contextual aspect of the quantum formalism as exposed through the KS theorem. We have shown how these two different understandings of contextuality have been mixed in the literature by a misreading of the KS theorem in terms of measurement situations and outcomes. We have explained why the widespread Bohrian epistemic understanding of contextuality is untenable when discussing about KS ontic notion of contextuality. In the second part of the paper we have discussed, firstly, why ontic contextuality precludes an objective description of physical reality in terms of define valued properties. Secondly, we have presented arguments which show that the widespread idea according to which, 'once the context is fixed, classicality is regained', is untenable within the orthodox formalism due to the existence of quantum superpositions and entanglement. Finally, we have shown that the idea that classical contexts are a *necessary condition* to account for quantum measurements is, in general, false. This idea explicitly depends on the metaphysical (Bohrian) presupposition according to which phenomena must be necessarily framed in terms of the classical representation provided by Newton and Maxwell.

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