Risk as a Consequence

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Because an option’s evaluation reviews the option’s relevant consequences, decision theory depends on a characterization of these consequences. A relevant consequence is an event the agent cares about, but beyond this an apt characterization depends on how evaluations of options use consequences. For some evaluations, the characterization of an option’s relevant consequences includes every consequence the agent cares about, but for other evaluations it omits some of these consequences. Section 1 argues that the aptness of a narrow or broad account of an option’s relevant consequences depends on whether evaluation of options uses only an agent’s preferences or also the agent’s independently defined probability and utility assignments. Section 2 considers an objection to taking an option’s risk as a relevant consequence of the option. Section 3, the final section, argues that an option’s broad evaluation is relative to the agent’s perspective in the world that the option produces and therefore appropriately reviews the option’s risk.

1. Two norms for a decision

In decision theory, a representation theorem such as Savage’s ([1954] 1972) shows that if an agent’s preferences meet certain conditions, then they are “as if” maximizing expected utility; probability and utility functions may be constructed to represent the preferences as maximizing expected utility. Some of the conditions for preferences, such as transitivity, are norms, and others, such as the preference ranking’s completeness, are idealizations.

A representation of preferences among options attaches probabilities and utilities to each option’s possible outcomes. The proof of the representation’s existence and uniqueness (up to positive affine transformations of the utility function) assumes that various options may produce the same possible outcome so that preferences among options constrain the outcome’s probability and utility assignments in a representation that uses expected utilities. Savage assumes that for any consequence, some option produces the consequence in all states of the world. According to this assumption, an option’s risk does not count as a consequence because no option produces only it in all states. Only an option with variable possible outcomes generates risk.

Traditional decision theory, in the style of Daniel Bernoulli ([1738] 1954), advances the principle of expected utility maximization independently of representation theorems. It assumes that an agent has probability and utility assignments that are definitionally independent of her preferences among options and in fact may generate her preferences among options. Her probabilities are degrees of belief, and her utilities are degrees of desire. Assuming that the agent is cognitively ideal and rational, her degrees of belief conform to the probability axioms, and her degrees of desire conform to the expected utility principle: her degree of desire that she realize an option equals a probability-weighted average of her degrees of desire for the option’s possible outcomes.

The traditional principle of expected utility maximization is plausible only if an option’s outcome includes every product of the option that the agent cares about. If it omits a relevant consequence, cases arise in which some option’s expected utility exceeds another more choiceworthy option’s expected utility. For example, suppose that the principle acknowledges only monetary outcomes although an agent cares about newness of dollar bills. Then the agent’s preferring a new dollar bill to an old dollar bill violates the principle. The expected utility principle incorrectly classifies some preferences as irrational if it ignores options’ relevant consequences. Although the project of establishing a representation theorem restricts consequences so that they do not include all an agent cares about, the traditional principle of expected utility maximization is inclusive.

Savage ([1954] 1972: 82–91) admits that for normative accuracy, his utility theory should take an option’s outcome as a grand world that covers everything an agent cares about. However, for practicality, he uses only small worlds that omit some relevant considerations. For example, buying a lottery ticket with a car as a prize has possession of the car as the outcome given the ticket’s selection. This small world outcome does not include the price of gasoline, although the price affects the car’s utility.

Savage’s ([1954] 1972: Sec. 2.7) *sure-thing principle*, a preference axiom for his representation theorem, asserts the separability of the preference order of options’ consequences given a state of the world. According to the principle, the order of two options agrees with the order of their consequences in a state *s*, given that they have common consequences in *s*’s complement. This normative principle applies pairwise to options to order them given *s*.

Because Savage uses small worlds, the sure-thing principle faces difficulties such as Allais’s (1953) paradox. For example, suppose that outcomes specify only monetary consequences although an agent also dislikes risk. The figure depicts a possible decision problem. Two options both yield $10 in a certain state and differ only in the state’s complement. Given its complement, one option yields $10 while the other yields $0 or $20 with equal probability. The agent may prefer the first option because of aversion to risk. Next, suppose that the options yield $0 in the state but have the original payoffs in the state’s complement. Given the change, if the state’s complement is unlikely, the agent may prefer the second option to the first option because they differ little in risk and the second offers a chance for a larger prize. This preference reversal, although justified by aversion to risk, is contrary to the sure-thing principle applied with respect to monetary consequences only. The problem disappears if the sure-thing principle acknowledges consequences besides money such as risk.

s ~s

$10 $10 less risky

$10 $0 or $20

$0 $10

$0 $0 or $20 brighter opportunity

Figure. Preferences Influenced by Risk

Savage’s small worlds make the individuation of outcomes coarse. Grand worlds make the individuation of outcomes fine; a single, minor event may distinguish two outcomes. Broome (1991: 115–17) observes that a fine individuation of outcomes threatens some structural axioms of preference on which representation theorems rely. The axioms require that the same outcomes occur in many contexts, and so hold generally only given a coarse individuation of outcomes. Although using a fine individuation of outcomes may thwart the project to establish a representation theorem, the traditional version of the principle to maximize expected utility aims for accuracy rather than practicality. It should use grand worlds. If an agent is averse to risk, then if an option produces a risk, the risk is a relevant consequence in calculations of the option’s expected utility.

Taking consequences noncomprehensively supports using preference axioms to establish a norm requiring that preferences have an expected utility representation. However, taking consequences comprehensively, so that they include an option’s risk when an agent is averse to risk, supports the traditional principle of expected utility maximization.

2. Against risk as a consequence

Let us assume henceforth the project of advancing the traditional principle of expected utility maximization. For this project, strong reasons support putting an option’s risk in the option’s outcome. Do any considerations count against including it?

According to a view common in economics, which Allingham (2002) describes, aversion to risk is the concavity of an agent’s utility curve for a commodity. This view does not take risk as an option’s consequence. Although its technical definition of aversion to risk simplifies economic theory, a commonplace alternative definition begins by taking a risk as a chance of a bad event and an option’s risk as the combination of chances for bad events that the option produces, including any interaction effects of component risks and other consequences. Accordingly, an aversion to risk targets such chances and their combinations. This account of aversion to risk takes an option’s risk as a relevant consequence of the option. Do objections stand in the way of this account?

Some theorists fear that a comprehensive account of an option’s outcome makes the expected utility principle trivial. The broad account permits rationalizing any preference whatsoever. Whenever an agent appears to violate the expected utility principle, one may add to outcomes features that preserve compliance with the principle. If a shopper buys the more expensive of two identical laundry detergents, one may preserve maximization of expected utility by adding to the purchase’s outcome possession of the detergent’s distinctive packaging.

Fine individuation of outcomes, the objection claims, also trivializes constraints on preferences, such as transitivity, because it may transform any apparent violation of a constraint into a compliant case. Given an agent’s apparent preferences for *A* over *B*, *B* over *C*, and *C* over *A*, one may distinguish *A* compared with *B* from *A* compared with *C* to break the cycle.

As Dreier (1996) notes, however, a fine individuation of outcomes does not plausibly eliminate every violation of transitivity. In the example, if *A*, whether compared with *B* or with *C*, is the same in every feature that matters to the agent, then nothing justifies distinguishing *A* compared with *B* from *A* compared with *C* to eliminate the cycle. Dreier defends, in an agent’s decision problem, an individuation of outcomes as fine as necessary to include all that matters to the agent (even if the individuation makes preferences between some outcomes impractical because the agent can never be in a position to choose between them). This response to the fear of trivialization supports a fine individuation of outcomes. An agent’s desires limit individuation of outcomes to preserve the substance of norms of preference and utility.

Suppose that a person cares only about money and prefers (1) a five-dollar bill and a ten-dollar bill to (2) a ten-dollar bill and another ten-dollar bill. This is contrary to the expected utility principle using comprehensive outcomes. How can one trivially save the expected utility principle? Not by introducing something besides money that the agent cares about. The expected utility principle constrains preferences even if outcomes have a fine individuation.

Although the literature has a good reply to the triviality objection, it lacks a good reply to another objection to taking an option’s risk as its consequence. The objection grants that a gamble produces risk, but holds that a rational agent is indifferent to this consequence so that it is not a relevant consequence.

Suppose that an agent compares an option that yields $10 for sure to an option that yields $10 if a coin toss yields heads and –$10 if the coin toss yields tails. The first option is obviously better than the second, but compare the outcome of the first option with the outcome of the second option given heads. The monetary outcomes are the same—a gain of $10 in each case. This section’s account of risk aversion holds that a risk averse agent prefers the outcome of the first option to the outcome of the second option given heads. He prefers an outcome in which he runs no risk and gains $10 to an outcome in which he runs a risk and thereby gains $10. However, a rational agent likes gaining $10 just as much whether or not risk precedes the gain. It is irrational to care whether a gain issues from a risk. The risk does not matter if it has a successful resolution, the objection maintains.

3. Evaluating a world with risk

A reply to this objection explicates the utility of a world that might be an option’s outcome. The utility adopts the agent’s perspective in the world and furthermore is responsive to features of the world unknown to the agent in the world, as this section explains.

A utility assignment for an agent expresses an option using an indexical. When Smith evaluates taking a walk, he evaluates the proposition that he takes a walk rather than the proposition that Smith takes a walk. In evaluating the outcome of such an option, the object of evaluation is a centered world, that is, a world together with a perspective in the world that fixes the denotation of indexicals, as in Lewis (1979). The utility of a world may vary with the world’s center although the world’s features are constant because an agent evaluates the world’s features from a position in the world.

Parfit (1984: Sec. 64) notes that a person typically cares more about a pain to come than an equally intense pain already past. Kahneman and Tversky (1979) note that a person typically cares more about a loss than a gain of equal magnitude. Rawls (1971: Chap. IX) observes that a person prone to envy compares his situation to the situations of others. In many cases an evaluation of an outcome depends on a reference point or perspective. An agent’s utility assignment to a world is relative to a time so that if the agent’s desires and aversions change, the utility assignment may change, but it is also relative to the agent’s position in the world so that the agent may evaluate differently a pain according to whether it is past or future and may similarly evaluate differently a risk according to whether it is past or future.

A rational person may care more about a risk to come than about an equally great risk already past because he cares only about results once the risk is past. The sensitivity of an agent’s utility assignment to the agent’s position in a world is not irrational. In particular, sensitivity to the agent’s temporal position relative to the option’s risk is not irrational. It is similar to other rational sensitivities to reference points.

For an agent, the utility of a gamble’s outcome appraises a world that includes the gamble’s resolution, but also appraises the world from the agent’s perspective in the world. The perspective specifies the agent and the time and place of the agent’s choice. Even if in the world the gamble is successful, from the agent’s perspective at the time he accepts the gamble, a period follows during which he runs an epistemic risk, involving a subjective probability, that the gamble is not successful. His evaluation of the world from this perspective may differ from his evaluation of the same world from a later perspective after the gamble’s successful resolution. The change in perspective explains why an agent may assign to a gamble in a world where it succeeds less utility at the time of undertaking the gamble than at a later time after the gamble’s successful resolution. Suppose that the agent assigns after a risk’s resolution the same utility to (1) running the risk and gaining $10 as to (2) not running the risk and gaining $10. Nonetheless, at the time of choice the agent may rationally assign less utility to the first outcome than to the second outcome. The utility of a risky option’s world in expected utility calculations evaluates a world at a time when the option’s risk is to come and not at a later time when its risk is past.

Decision theory assumes that the agent’s reference point is the agent’s position at the time of the agent’s decision problem. Utility evaluates risk from a position in a world, the agent’s position at the time of his decision problem. In expected utility calculations for an option, the world’s utility is at a time when the option’s risk in the world is to come and not the world’s utility at a later time when the risk is past. Utility uses the agent’s perspective in the world. It evaluates an option’s world from the agent’s position in it, from which position the option’s risk proceeds if the agent adopts the option.

The utility of an option’s outcome attaches to a proposition that represents a world. The world, which may be only possible and not actual, differs from a propositional representation specifying for each desire and aversion whether the world realizes it. Although the latter proposition’s utility for an agent is the same from any perspective, the possible world’s utility depends on the agent’s perspective in it because the perspective settles which desires and aversions the world realizes. If an agent is averse to running risks in the future, then whether an option’s world realizes the aversion depends on the agent’s position in a world that has him undergo the option’s risk. The world realizes the agent’s aversion just in case the risk is in the agent’s future in the world. Its realization depends on the agent’s position in the world. A world’s propositional representation specifies whether the world realizes an aversion to the risk, but whether the world realizes the aversion depends on the agent’s position in the world.

The utility of an option’s world considers all features of the world, including the resolution of any risks that arise in the world. Given that a risk has a successful resolution, should not the world’s utility be the same as for a world in which the positive result arrives without risk? Given a world, an agent knows how risks run their course. The agent cannot know the world and not know the results of risks.

A world’s utility attends to the world’s features but does not attribute to the agent in the world knowledge of all the world’s features. The agent’s knowledge at the time of a decision problem settles whether an option produces an epistemic risk and so realizes an aversion to risk. After a choice, when experiencing an option’s risk, the agent does not know the risk’s result. Evaluation of a world with the option’s risk does not attribute to the agent knowledge of the risk’s result. If the agent in the world undergoes uncertainty about the option’s outcome, she may downgrade the world because of an aversion to the epistemic risk she runs in the world. For an agent, a world’s utility attends to the result of a risk the agent runs in the world but does not assume that in the world the agent knows the risk’s result.

According to the traditional principle of expected utility maximization, an option’s risk is a consequence of the option. For an agent, a world’s utility considers the agent’s position in the world and the agent’s ignorance in the world. Because utility’s appraisal of a world has these features, a rational aversion to risk may affect a world’s utility and hence an option’s expected utility.

References

Allais, M. 1953. “Le comportement de l’homme rationnel devant le risque: critique des postulats et axioms de l’école Américaine.” *Econometrica* 21: 503–46.

Allingham, M. 2002. *Choice Theory: A Very Short Introduction*. Oxford: Oxford University Press.

Bernoulli, D. [1738] 1954. “Exposition of a New Theory on the Measurement of Risk.” *Econometrica* 22: 23–36.

Broome, J. 1991. *Weighing Goods.* Oxford: Blackwell.

Dreier, J. 1996. “Rational Preference: Decision Theory as a Theory of Practical Rationality.” *Theory and Decision* 40: 249–76.

Kahneman, D., and A. Tversky. 1979. “Prospect Theory.” *Econometrica* 47: 263–91.

Lewis, D. 1979. “Attitudes *De Dicto* and *De Se*.” *Philosophical Review* 88: 513–43.

Parfit, D. 1984. *Reasons and Persons.* Oxford: Oxford University Press.

Rawls, J. 1971. *A Theory of Justice*. Cambridge, MA: Harvard University Press.

Savage, L. [1954] 1972. *The Foundations of Statistics*. Second edition. New York: Dover.