Mach's Principle: the original Einstein's considerations (1907-12)*

Enrico Gasco†

Abstract

In this article we present the first Einstein's considerations on Mach's Principle that were published in a little note on 1912. In particular we want to suggest the correct interpretation of the most ambiguous machian sentences and then to show, first how Einstein recognizes in Mach's considerations a fundamental Principle of the future General Theory of Relativity and second which is his first attempt in modelling the relation between inertia and gravity.

1 Introduction

Einstein presented Mach's Principle in the article "*Prinzipielles zur allgemeinen Relativitatstheorie*" on 1918 [6]. He expressed in it his current ideas on gravity and ended a long reflection about the origin of Inertia which took his attention during the most prolific years of his scientific activity.

This reflection has its origin in the reading of Ernst Mach's "*The Mechanics in its logical and historical development*" [13] and especially in some 'disputed' passages present in the criticism of the Newton's bucket experiment, where the Austrian Philosopher seems to suggest the existence of a causal link among the inertial characteristics of matter and the other stellar masses. This interpretation had a great success in the German scientific community and some Physicists, such as Friedländers, Föppl, Hoffman ([14, 7, 8, 16]) and Einstein himself, developed machian ideas either from a theoretical point of view or from an experimental one. We my ask ourselves if this interpretation is the right one, if Mach really meant to suggest a link among Inertia and other stellar masses and, in this case, the reason why he did not deepen the argument anymore. In the following pages we'll try to give an answer to these questions and we'll recall some conclusions of a preceding essay, enriching them with a better interpretation of the most ambiguous passages.

In fact these last ones achieves their correct meaning not only on the light of machian philosophy – as we showed in the above mentioned article – but also

*This article was published in italian language on "Quaderni della Stoia della Fisica" in 2005 [11]

†Zirak Software Dept.: enrico.gasco@zirak.it

keeping in mind the 'variation' technique (or thought experiment) that Mach uses to lead to the extreme consequences the specific conceptual relations. Discussing for instance relation between relative and absolute motion, he introduces some equivocal sentences which we have to analyse in accordance to the particular context they are expressed in, and which do not present any general valour. At the end we'll reach the double conclusion that Mach do not suggest any causal link between inertia and other stellar masses but that this hypothesis is easily deduced from his reasoning and from his philosophical convictions.

As a matter of fact, this is the result of einstenian reflection onMach's work: in the passages of Mechanics he first recognizes the general conjecture of a link between body inertia and the presence of other stellar masses and then he searches for a Theoretical Principle to formalise this hypothesis. He spent some years in reaching a complete formulation of Mach's Principle, though the original considerations on machian ideas can be found in the article on 1912, at the end of the preliminary scalar theory of gravitation (he cites the Austrian Philosopher as his inspiration source). The short essay is of great interest because it represents the first Einstein's step to axiomatize Mach's conclusions on inertia origin.

The task of this article is to suggest with more clearness the correct interpretation of the most ambiguous machian sentences and then to show, first how Einstein recognizes in Mach's considerations a fundamental Principle of the future General Theory of Relativity and second which is his first attempt in modelling the relation between inertia and gravity.

2 The correct interpretation of Mach's passages on Newton's bucket.

Mach's reasonings on vessel experiment and conclusions derived from them can be interpreted correctly only at the light of machian Philosophy. Mach's philosophical programme is summarised substantially in two targets: the first one consists in the attempt to unify science without any reductionism (physicalistic or psicologistic); the second one is represented by the elimination of any metaphysical drift present in the scientific work (such as Newtonian concepts of absolute space and time).

To realise this programme, Mach introduces as a pure methodological entities the elements: they represent something neither physical nor psychic – in this case we'll fall again in a kind of reductionism – but impersonal foundation for the construction of scientific knowledge. These elements can create physical, psychic or physiological entities in accordance with their use or connection. In this way Physics is that particular branch of Knowledge which is involved with relations among spaces, times, sounds, colours while Physiology is that discipline that studies connections among physical relations and the parts of our body. Therefore the elements represent the way to reach the first goal of Mach's programme: the unification of various disciplines without reducing one another.

Now we have to explain which characteristics belong to connections. Mach is keen on this argument since the first years of his historical-philosophical research and in his work on 1872 [12] he provided an answer that will be a source for his following studies. In order to criticise the reductionism exposed by Helmholtz [15] with extremely precision, in " *Die Geschichte und die Wurzel des Satzes von der Erhaltung der arbeit*" he studies the history and the origin of Labour Conservation Principle (nowadays known as Energy). Mach thinks this Principle is the research foundation of each past great naturalist ([1]) and, from the study of its application in Mechanics development, he provides a quite general definition. Mach notes this principle asserts something that is evident to everyone: that bodies won't rise on their own and they can't make this movement because they are linked to the other bodies by the gravitational attraction. The principle of the absence of Perpetuum mobile is "*the principle of the natural universal relation*"; it expresses "*the experience result: the sensitive elements of the world* α, β, γ . . . *appear as dependent on one another. The experience teaches us that the sensitive elements* α, β, γ . . . *into which the world can be broken up are subject to variations and also that some of these elements are connected to others, as they appear and disappear together, or as the appearance of the elements of a type are linked to the disappearance of the elements of another*".

Therefore the notion of the reciprocal phenomena dependence is the starting point of each scientific knowledge and it will be the task of different Science domains to define and measure the various relations. If it is preferred to indicate this connection of facts by using the term "causality", then we should specify that Mach identifies the Principle of Universal Dependence with the Principle of Causality.

From the short passage reported above, we conclude that connections among sensible elements satisfy two fundamental features: (1) each element is in mutual relation to each others; (2) the connection is functional and not causal, in the sense that each given phenomenon cannot be considered as an *effect* of a *cause*, but as an expression of the mutual relation with all other elements. The second point also specifies which is the true purpose of science research: the *description* of the relations existing among sensible elements in which the experience is resolved. The experience represents the foundation of Mach's philosophy: it *has to be provided* and our job is to describe it with a functional model. We haven't to search for something which goes beyond sensible data, because we risk to conceive fictitious entities and relations that have no counterpart in nature.

We now consider how the Principle of Universal Dependence manifests itself in Mechanics foundations and we are especially interested to machian criticism on absolute space and time concepts.

In contrast with Newton, who thought time unrelated with our conscience,

[1]For example Stevin considers it as an instinctive experience, whose contrary is absurd, and uses it in the study of motion along inclined plane; at the same way Galileo conceives the same principle in the form of the law about which a body, by virtue of falling velocity, rises at the same altitude and uses it in the study of pendulum motion and in the analysis of inertia law too; finally Huygens and Bernoulli brothers use this principle in the work on the "vis viva" conservation

and not directly observable (at least concerning the *true* time), Mach considers this concept as an instinctive notion derived from the strict relation existing between our memory and the sensible perception. The sensation of uselessness of external objects, and consequently the absoluteness of Newtonian concept which derives from this, takes place from this close report with our conscience ([2]). As a matter of fact when we assert that an object A changes in time, we are comparing the states of A with those of another object B whose states are considered as a unity of measurement (for example the Earth motion): so we assumed that external objects have a privileged status in time measurement and that its elapsing is an expression of reciprocal dependence among natural phenomena.

We find similar considerations in the criticism of absolute space. Newtonian Mechanics considers the presence of an acceleration as the sign of a departure from a privileged state (inertia) and explains this fact with the introduction of force concept. The inertial movement comes with the choice of a precise reference system – the inertial one – which is the representation of absolute space in Physics. But for which reason do we describe the body motion relative to a quite abstract reference, when it is easy and immediate to refer it to the same universal bodies? They represent the unique way to determine a reference frame. So the motion is not linked to an absolute system as Newton suggested – even if he often associated it to the system of fixed stars as Mach remembers us – but to the same bodies with reference to which we consider the motion. A frame, in Mach's opinion, represents an abstraction or a short and concise description of the bodies with respect to which we describe the motion. So movement study reduces itself to "description" of the natural dependence that links the body in motion and the set of masses in reference to which it is considered.

Keeping in mind the short presentation of the criticism to Mechanics foundations, we own the conceptual instruments to understand the machian considerations on Newton's bucket experiment and the important conclusions which derive from it ([3]). Newton used this experience and the direct observation of a centrifugal force in the rotating water – the important consideration in his reasoning – to demonstrate the existence of absolute motion. To this argument Mach answers noting that absolute rotation of liquid is nothing else that a relative rotation to fixed stars; they represent the right frame with respect to which we study the rotational motion. In this way the acceleration loses the absolute character which presents in Newtonian treatment and assumes that relative character which yet belong to velocity concept. In Mach's opinion there are no absolute motions, the concept is a metaphysical residue and in Nature there are only relative motion. The accelerations, in particular the inertial ones, now depend on the set of bodies to which we refer the motion and so, at the end, the same inertia depends on the presence of other masses.

We get to this conclusion also considering the *connections* revealed in experience. In our case the system is formed by the water contained in the bucket

[2] We are not conscious that our temporal measures are linked to the movement of "external objects"

[3] Consider for a detailed exam of machian sentences the articles [17, 10, 9]

and by the set of fixed stars; it interacts generating centrifugal forces in water. Reminding that the *description* of the observed phenomenon has to be *functional*, we deduce that the presence of inertial effects is not to be led back to an hidden cause (or referred to a not observable absolute space), but it is the result of the mutual interaction (of the connection) among the system elements. This reciprocal influence can be expressed in a *more general form* – taking distance from the particular experiment it derives – with the 'conjecture' which asserts that body inertia depends somehow on the presence of other masses and so, in conclusion, it's a borderline case of gravity.

Mach didn't make the last step, although it could be easily deduced from the considerations on bucket experiment, because the main task of machian work was to remove Metaphysical concepts from Mechanics, such as absolute space, and to describe the given experience in a functional way. The achieved generalisation is instead a speculation which goes beyond the particular experience taken in exam and therefore it risks to be a not directly observable conjecture.

In order to get a totally understanding of Mach's ideas on inertia and to reach a complete interpretation of them, we should examine the most equivocal passages of vessel experiment. In particular we analyse two sentences , the most controversial, but before going through a deepen analysis of them, let's specify that Mach uses some reasonings of hard interpretation in both case. As a preliminary consideration we note that he uses the 'variation' method (or thought experiment) to verify, changing some characteristics, the validity of displayed concepts relations.

In the first argumentation Mach asserts that all natural motions are 'relative motions' (also the concepts used for their description, as the force one) and who is convinced of the existence of true motions, forget that the same phenomenon can also be described in a different way (always with relative terms).The Earth behavior, for example, is described either by the Copernican theory or by the Ptolemaic one; the two descriptions are both correct, but the first is more economic and practical than the second one. It's at this point that Mach makes the 'variation'; as in the case of Earth behavior, where we consider Earth motionless or in motion depending on the description used, so in the vessel experiment it is allowed to consider the bucket in rotation and the fixed stars motionless, or the vessel still and the stars in a rotational movement. In fact he asserts, modifying the conditions of Newtonian experience, "*Try to fix the newtonian vessel and rotate the sphere of fixed stars and then prove the absence of centrifugal forces*". Here it is a new description of the same experiment. The passage does not suggest a new experience, which, if it is done (although not realisable), leads to obtain centrifugal forces caused by rotational motion of the stellar vault, but the expression of a different description of the same phenomenon.

After few words, Mach concludes his criticism on bucket experiment (the second argumentation). He asserts "*Newton's experiment with the rotating vessel of water simply informs us that the relative rotation of water with respect to the sides of the vessel produce no noticeable centrifugal forces, but that such forces produced by its relative rotation with respect to the mass of the earth and other celestial bodies*". In this sentence there are some terms, like 'produce' or 'produced', with a clear causal valour which lead to a first interpretation where the fixed stars present a kind of causal influence in the determination of centrifugal forces. Reminding the Principle of Universal Dependence, that represents for Mach the true Causal Principle, we must reject this interpretation because we would risk to insert an hidden cause to explain inertial accelerations, while we have to describe the given experience in functional terms. In the preceding passage Mach describes the experimental data with words: the inertial accelerations are in relation with the masses of Earth and fixed stars, therefore the remote masses, while the sides of the vessel, the nearest masses, have no effect. Then he makes the 'variation' and asks if this conceptual relation is right: what would happen if the vessel sides increase in thickness and mass till they were ultimately several leagues thick? Would the bucket rotational motion induce centrifugal forces in the water? Would the underlined relation still be true, where inertia is determined by distant stars? The answer is given by the same Mach: "*our business is to bring [this fact] into accord with the other facts known to us, and not with the arbitrary fictions of our imagination*".

The analysis of the two argumentations shows an important fact: in both of them the centre of machian reflection isn't the research of inertial properties origin, but the removal of each reference to the concept of absolute motion and the functional description of observable phenomena. The most ambiguous passages, those that seem to indicate a causal link between inertia and the presence of other stellar masses, have a secondary role and they are reached with a rhetorical technique used by Mach to support his main thesis. Only the second reasoning seems to indicate this link in a confused way, but the intuition is not generalisable because it would be an hypothesis not directly verifiable.

Therefore, in our opinion, Mach didn't suggest any effective relation between inertia and other stellar masses; but if we drop the particular philosophical context where the argumentations are suggested, a such link is easily deducible and this is the case for Einstein and some other German Physicists.

3 The first Einstein's remarks on Mach's Principle.

In the preceding section we explained the reasons why Mach didn't single out the close report between inertia and gravity, but Einstein and some German Physicists (such as the Friedländers and Föppl) recognised the value of machian reasoning and freed it from the particular context where it was been exposed: for them body inertia is due to the presence of other stellar masses. While the Friedländers and Föppl tried to follow an experimental way to verify the generalisation of Mach's conclusions, reaching no satisfactory results, Einstein developed this conjecture from a theoretical point of view until formalising it in a Principle of General Relativity. In this paragraph we'll try to show the first steps to get to this final task, only reached in 1918.

In order to get a better understanding about the possible einstenian reflections on the argument, we try to describe his particular use of thought experiments (such as the 'variations' of Mach) and 'observable facts'. So we consider the well known experience of conductor and magnet which had a fundamental role in the construction of Special Relativity (an experience that few physicists found problematic before 1905). In fact we know that when a current is induced in a conductor by a magnet, it results that the observable current is the same either we fix the conductor or we move the conductor and fix the magnet. The important thing is the relative velocity, but the equations which describe the phenomenon (before 1905) are different in the two cases. In front of this inexplicable asymmetry Einstein thought the only way to overcome the problem was to single out a totally general Principle which permitted to consider both cases as theoretically equivalent, as they lead to the same effect in practise. The conductor and magnet experience becomes source of inspiration for a quite general theoretical principle: we know that Einstein solved the problem of asymmetry with the Special Theory of Relativity and with the assumption of Relativity Principle ([4]).

We are in a similar condition when we consider machian argument on rotating vessel: now we haven't an experience which suggests us a solution of a problematic asymmetry, but we posses a convincing reasoning – expressed with mental experiments – that induces ourselves to think inertia as a borderline case of gravity. Einstein recognised in this conjecture a determinant aspect of a new theory which describes gravitational phenomena, so he tried to reproduce it either in the preliminary scalar theory of gravitation or in the final Theory of General Relativity. But a precise difference exists between the case of 1905 and the following attempt: while at the end of 800 century Einstein had all the tickets of the mosaic (the relativity principle too) and so his concern was their unification and modification to get, finally, a complete vision; after 1905 – with the construction of General Relativity – he was in the unpleasant situation to know which feature pertains to the new theory (the link between inertia and gravity) but not a principle already developed to use and extend.

Now we try to describe the *original* eintenian attempt to formalise this characteristic in the preliminary scalar theory of gravitation, developed between 1907 and 1912. At the end of this period there is the famous note on 1912, pointed out in the Introduction, where Einstein cites Mach as his source of inspiration.

We begin with considering the Special Theory of Relativity. It's based on the fundamental idea that some coordinate systems (the inertial ones) are equivalent for the formulation of physical laws: in them the Inertia Principle and the law about which light propagates with finite velocity are true. But are these systems privileged for some reasons inherent the Nature or for the theoretical structure

[4]Why search a general principle and not limit ourselves to the attitude of 800 Physicists who constructed theory by induction, so with subsequent approximations (we are now referring to the example of the 'ad hoc' hypothesis of lengths contraction in Lorentz's theory) is easily said: as Einstein shows us in his *Autobiographical Notes* he thought that the only way to solve the problems on ether behavior and structure was the '*discover of a formal and universal Principle*'

developed?

In a deeper insight the acceleration concept is the discriminant theoretical element that favours inertial systems, because it allows to distinguish among relative motions and true ones as we understand with the rotating vessel experiment. But we know, after machian criticism, that all natural movements are 'relative' motions, as well as relative have to be the concepts used in their description (included the acceleration one). Therefore the acceleration gives up that centrality in the theoretical structure either of Classical Mechanics or of Special Relativity and supports the extension of the equivalence set of systems used to the description of natural phenomena.

But how do we make this extension? Einstein reflects about this problem since 1907 when publishes a review article on Special Relativity [2] and exposes in a deductive way several consequences and applications developed from principles conceived in 1905 [1]. Only in the fifth and last section of the essay he introduces the *Equivalence Principle* which will be the starting point and the foundation of his following research. In *Morgan* manuscript Einstein shows how he conceived the Principle; we note especially that a mental experiment with a 'fact', at that time not completely understood, induces him to a quite general conjecture. The empirical phenomenon, we're talking about, consists in the observation that bodies, independently of their nature, fall in an external gravitational field with the same acceleration. As far as the thought experiment, he considers an observer in free fall in a gravitational field. Einstein underlines how the observer has the freedom to consider himself at rest, because if he lets bodies fall, these ones remain in a rest status or in a uniform motion with respect to him.

Therefore we deduce there is no difference between a rest frame and a system in free fall in an external gravitational field; we can't find the 'absolute' acceleration with which the observer frame moves. This consideration can be easily expressed with the following example too.

In a spatial region completely free from forces fields we construct, following Classical Mechanics, an inertial reference, and consider a body of mass M that can freely move (its motion is uniform and rectilinear). Then we take in exam in the same spatial region an accelerated system K' and observe the motion of M relative to K'; it will be accelerated and not uniform and rectilinear any more. Now we wonder if the system K' has an 'absolute acceleration', as Classical Mechanics suggests, or if we can describe the same phenomenon in a different way. A possible way is to consider system K' 'at rest' – so inertial – in an external gravitational field (caused for example by faraway masses). The *two descriptions are equivalent*; in the first case bodies, independently of their physical and chemical nature, move with the same acceleration determined by the movement of K', in the second case they fall with the same acceleration now caused by the external gravitational field. So there is no instruments to establish if the system K' accelerates or if it is at rest in a gravitational field.

Therefore we conclude that Equivalence Principle extends the set of frames useful for the description of natural phenomena 'at least' to uniform accelerated systems.

Concerning our argument, we are interested especially in noting as the inertial behavior of a body in an accelerated frame is described in a completely equivalent form if we use a system in a 'proper' gravitational field: so we obtain a *first* property which lead back to machian conjecture of a link between inertia and the presence of other stellar masses (the gravitational field caused by them).

The Equivalence Principle with the structure of Special Relativity led Einstein to obtain some very important results, already present in the article on 1907, but described in a more compact form in the essay on 1911. In this article there is the first possible experimental test of the future General Relativity Theory (the deflection of light rays), but we focus on the conclusions concerning the *gravity of Energy*, the *equivalence between inertial and gravitational mass* and finally the *change of light velocity in a gravitational field*.

In Einstein's article on 1911 [3] he uses a model formed by three frames: the first system K immersed in a gravitational field, with acceleration γ direct along the positive z-axis; a second system K' in an accelerated motion along positive z with the same acceleration γ; finally a system K_0 at rest. The laws of Special Relativity are valid in K_0 and can be used for the accelerated system K' if we consider short instants of time and not relevant accelerations. With the Equivalence Principle the obtained results are also assigned to the system K immersed in a gravitational field; with this clever method we have information on the gravitational effects studying the behavior of uniform accelerated frames.

Then Einstein considers two material systems S_1 and S_2 along the z-axis of K' at a distance h, so that among them there is a potential difference of h, and imagines that system S_2 emits towards S_1 a light ray of energy E_2 and frequency ν_2, that is measured with an instrument identical to that used in S_2, obtaining E_1 and ν_1 as a result. With the laws of Special Relativity the link between $\nu_1$and ν_2 is:

$$\nu_1 = \nu_2 \left(1 + \frac{\gamma h}{c^2}\right) \tag{1}$$

which is also valid for system K if we replace γh with the gravitational potential difference Φ between the two points:

$$\nu_1 = \nu_2 \left(1 + \frac{\Phi}{c^2}\right) \tag{2}$$

The (1) and (2) seem to indicate an absurd: if the emission process of light is continuous, we don't understand why the two frequencies must be different at the emission and at the reception. To avoid this 'asymmetry' Einstein thinks to modify the time notion and establish that two clocks with which we measure frequencies in S_1 and S_2 have different heartbeat – to compensate the diversity between ν_1 and ν_2. From this assumption we deduce an important consequence for the construction of the theory: the velocity of light is *no more constant* as in the usual special theory, but it depends on gravitational potential where it's measured (so it's an excellent candidate to describe gravitational field).

If we indicate with c_0 the velocity of light measured by a clock U_0 in the origin of system K immersed in a gravitational field, and with c the same velocity of light but measured with a clock U_1 in a point at Potential Φ with respect to the origin, the relation among two velocities, reminding the different heartbeat of U_0 and U_1, is:

$$c = c_0 \left(1 + \frac{\Phi}{c^2}\right) \tag{3}$$

Therefore (3) shows that velocity of light does not remain constant if we consider space-time regions with variable gravitational potential: so we deduce that *Lorentz's transformations lose their universal applicability* and must be replaced by most general laws. In 1912 [4] Einstein constructs approximately the right transformations, using the model of three Reference frames and imposing the invariance of spherical shell in expansion at light velocity. We only underline which is the change on velocity of light shifting from a system to another: naming with $K(x, y, z, t)$ an accelerated system which moves along the x-axis and with $\Sigma(\xi, \eta, \zeta, \tau)$ a rest frame oriented in the same way of K, the transformation law for c is:

$$c = c_0 + ax \tag{4}$$

where c is the velocity of light measured in K, c_0 the same velocity measured in Σ and finally a represents acceleration of K origin relative to Σ. Now let us remember that c depends on space coordinates (in our case the only x) and not by time, so that the study of the c changes is equivalent to study the behavior of a static gravitational field. In order to get a covariant theory is fundamental to obtain an equation which describes the c field and that remain constant in form in shifting from a system to another. The most simple equation consistent with (4) is:

$$\triangle c = 0 \tag{5}$$

At this point Einstein returns to study the behavior of a material point free to move in the c field. He comes to the fundamental equation:

$$\frac{d}{dt}\left[\frac{1}{c^2}\left(1 - \frac{q^2}{c^2}\right)\right] \tag{6}$$

where $q = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$represents particle velocity. The (6) can be written as:

$$E = \frac{mc}{\sqrt{1 - q^2/c^2}} \tag{7}$$

which represents the Energy Principle for the considered system. If we approximate (7) for a motion slow enough, we get an important relation for our discussion:

$$E = mc + \frac{m}{2c}q^2 \tag{8}$$

The last term at second member of (8) is in first approximation the particle *Kinetic Energy*: we underline it depends *explicitly* by the c-field (so by gravitational potential) and so it's a clear *interaction term.* In the ordinary Classical Mechanics the Gravitational Potential Energy and the Kinetic one present two different behaviors, which Einstein underlined as a motive of dissatisfaction, because the first one is a totally relative expression (there's the distance among interacting particles), instead the second one depends only by particle velocity and not by other bodies with respect to which motion is considered (they are absent). The new aspect of (8) consists in the presence of an interaction term with mass which generates the field; in this way we obtain a kinetic energy *relative* to masses present in the system, as a typical machian spirit. This consideration shows another contact point between Einstein's reflection and the corresponding conjectures of the Austrian Philosopher (we can use only relative concepts for physical description).

It also represents the determinant point to completely understand the note on 1912 [5]. In this essay Einstein try to demonstrate the existence of an effect similar to the electrodynamical induction in the case of a static gravitational field; in fact he asks himself if a massive body in accelerated movement can induce an inertial force on a small mass in its neighbors. Einstein constructs a model formed by a material spherical shell K and a massive point P free to move in its interior; then he research the force induced on P by an accelerated motion of K. It's easy to note that Einstein here suggests again the machian arguments on vessel experiment; for example we remember the sentence in which Mach asserts: "*No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick*" ([5]). To determine the induced force, Einstein first demonstrates that the two bodies of the System are in interaction. He uses the scalar theory of gravitational field developed in that period because the spherical shell K creates a static gravitational field. Besides he knows either the value of gravitational Potential inside K (which is equal tokM/r where k is the universal gravitational constant, r the distance from shell centre) or the form of equations which govern massive point motion (6). But the major interest of Einstein is the use of the Kinetic Energy obtained from (8):

$$L = \frac{m}{2}q^2\frac{c_0}{c} \tag{9}$$

(q is the particle velocity, c_0 the light velocity at infinity). The (9) represents an excellent candidate to demonstrate that point P interacts with mass M. In (9) the only unknown quantity is the light velocity which depends on the

[5]The bucket sides correspond to spherical shell, while the water particles are described by massive point P. The rotation of vessel supposed by Mach is now represented by the accelerated motion of K

space coordinates. Einstein determines its value inside the sphere supposing the motion of P is relatively slow and obtains the Kinetic Energy:

$$L = \frac{m}{2} q^2 \left(1 + \frac{kM}{Rc_0^2}\right) \tag{10}$$

where we observe that *inertial mass* of the particle is now:

$$m' = m + \left(\frac{kM}{Rc_0^2}\right) \tag{11}$$

From this expression we deduce a very important result: "*the presence of the inertial shell K increases the inertial mass of the material point P inside the shell. This suggests that the entire inertia of a mass point is an effect of the presence of all other masses, which is based on a kind of interaction with the latter*" (in a footnote Einstein cites Mach as his source of inspiration). Once again it's important to underline that a mental experiment leads Einstein to make a generalisation and to express with major conviction that inertia and gravity are linked.

With (10) and (11) Einstein demonstrates that the two System parts are in reciprocal interaction and admits in the last point of article that they exchange accelerations one another, as the *Universal Principle of Mach* suggests ([6]). In this way the K accelerated motion induces a force on P equal to

$$\frac{3}{2} \frac{kMm}{Rc^2} \Gamma \tag{12}$$

where Γ is the acceleration of K. The (12) is the final point of Einstein's essay.

We now summarise our conclusions. Between 1907 and 1912 Einstein is interested on the extension of Relativity Principle to systems in not uniform motion, in order to develop a more general theory which includes the Special Theory on 1905 and embodies gravitational phenomena. He knows that the new theory has to justify a principle obtained from the reading of machian pages and that establishes a close connection between inertia and gravity. He has not reached a totally formalised Principle yet – in fact he will spend some years to make a complete formulation of this idea (1918) – but in the period 1907-12 he explores and obtains some right results in this research. Among them the most important is the Equivalence Principle which indicates clear links between inertia and gravity and leads him to consider a simple model for clarifying the conjecture of a causal connection between inertia and the presence of other bodies of Universe.

[6] See the article [10, 9]

References

[1] Einstein A. *Annalen der Physik*, (17):893, 1905.

[2] Einstein A. *Jarbuch der Radioaktivitat und Elektronik*, (4):411, 1907.

[3] Einstein A. *Annalen der Physik*, (35):898, 1911.

[4] Einstein A. *Annalen der Physik*, (38):355, 1912.

[5] Einstein A. *Vierteljahrsschrift fur gerichtliche medizin und offentliches Sanitatswesen*, (44):37–40, 1912.

[6] Einstein A. *Annalen der Physik*, (55):241–244, 1918.

[7] Foppl A. *Akademie der Wissenschaften, Munchen, Mathematisch-WissenschafttlicheKlasse, Sitzungs*, (5):5–28, 1904.

[8] Foppl A. *Akademie der Wissenschaften, Munchen, Mathematisch-WissenschafttlicheKlasse, Sitzungs*, (6):5–28, 1904.

[9] Gasco E. Mach's contribution to the origin of inertia. *http://philsci-archive.pitt.edu/1259/*, 2003.

[10] Gasco E. Il contributo di mach sull'origine dell'inerzia. *Quaderni di Storia della Fisica*, 2004.

[11] Gasco E. Il principio di mach: le prime considerazioni di einstein (1907-12). *Quaderni di Storia della Fisica*, 2005.

[12] Mach E. *Die Geschichte und die Wurzel des Satzes von der Erhaltung der arbeit.* 1872.

[13] Mach E. *Die Mechanik in ihrer Entwickelung. Historisch-kritisch dargestellt.* Leipzig, 1883.

[14] Friedlaender I Friedlaender B. *Absolute oder Relative Bewegung?* 1896.

[15] Helmholtz H. *Uber die Erhaltung der Kraft.* 1847.

[16] Hofmann. *Kriticsche Beleuchtung der beiden Grundbegriffe der Mechanik: Bewegung und Tragheit und daraus gezogene Folgerungen betreffs der Achsendrehung der Erde und des Foucault'schen Pendelversuchs.* 1904.

[17] Norton J. Mach's principle before einstein. In *From Newton's bucket to quantum gravity.* 1995.