Ψ-Epistemic Quantum Cosmology?

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Abstract

This paper provides a prospectus for a new way of thinking about the wavefunction of the universe: a Ψ-epistemic quantum cosmology. We present a proposal that, if successfully implemented, would resolve the cosmological measurement problem and simultaneously allow us to think sensibly about probability and evolution in quantum cosmology. Our analysis draws upon recent work on the problem of time in quantum gravity and causally symmetric local hidden variable theories. Our conclusion weighs the strengths and weaknesses of the approach and points towards paths for future development.

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> *Cosmologists, even more than laboratory physicists, must find the usual interpretive rules of quantum mechanics a bit frustrating.*
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> J.S Bell first words of 'Quantum Mechanics for Cosmologists', 1981

> *Finally, I should mention the semiphilosophical issues arising when one attempts to apply a probabilistic theory to the Universe, of which one has only a single copy. Here I made no attempt to deal with these issues and took a simple-minded approach that the theory describes an ensemble of Universes.*
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> A. Vilenkin (a cosmologist) last words of 'Interpretation of the wave function of the Universe', 1989

Contents

1 The Proposal

The interpretational problems that have long plagued the foundations of quantum mechanics are much exacerbated when we attempt to apply quantum theory at a cosmological scale. In particular, if we apply quantum theory to the whole universe, then we are confronted by the three interrelated problems of making sense of measurement, probability, and evolution. This paper provides a prospectus for a new way of thinking about the wavefunction of the universe: a Ψ-epistemic quantum cosmology. We present a proposal that, if successfully implemented, would resolve the cosmological measurement problem and simultaneously allow us to think sensibly about probability and evolution in quantum cosmology.

Our proposal depends upon two distinct interpretational moves. The first move, discussed in §2, is to implement quantization such that there is evolution in the universal wavefunction. This is in contrast to the more standard 'timeless' Wheeler-DeWitt approach to quantum cosmology, but in line with the 'Relational Quantization' program developed and defended by Gryb and Thébault (2012, 2014, 2015, 2016a,b). The second move, discussed in §3, is to take an epistemic stance regarding both the universal wavefunction itself and the Born probabilities that are derived on its basis. We propose a Ψ-epistemic quantum cosmology in that we hold that the wavefunction of the universe is a bookkeeping device for agential knowledge and not an ontological object. The relevant Born rule probabilities are then also epistemic since they are presumed to be defined in virtue of constraints upon the knowledge of an ideal (classical) epistemic agent embedded within the universe. The viability of this second move is inspired by ψ-epistemic approaches to non-relativistic quantum theory. In particular, we will make reference to local hidden variable approaches that exploit the causal symmetry 'loophole' in the Bell, Kochen-Specker and PBR no-go theorems.[1] These two distinct interpretational moves lead us to a Ψ-epistemic quantum cosmology with both the dynamics of the wavefunction and the projection postulate understood in terms of the changes in the (objective) epistemic constraints placed upon an embedded agent.

We further argue in §2 that there are good reasons to believe that an epistemic interpretation of the universal Born probabilities cannot be consistently applied in a quantum cosmology that is timeless. Our first move, therefore, creates conceptual space for the second. Furthermore, the second move resolves a potential worry about the first: that the evolution of the wavefunction of the universe with respect to a time parameter breaks the general covariance of the classical cosmological model. Since the wavefunction is epistemic, it need not be invariant under the same set of symmetries as our ontology – given by the local hidden variables. Moreover, it might even be taken as a necessary precondition of our states of knowledge that their evolution is defined relative to a simultaneity class. Thus, what might look like a fault in our proposal turns out to be rather an attractive feature.

Together the package of ideas presented in this paper might seem rather too new and ambitious. Our proposal, after all, is to try and solve the measurement problem in quantum cosmology and the problem of time in quantum gravity – *simultaneously*. But perhaps these problems have proved so resistant to solution precisely because they have been approached piecemeal. Our main purpose in this paper is to put forward a new interpretational stance and marshal a range of philosophical and physical arguments in its support. At the least, we take ourselves to have established our ambitious proposal to be *not-implausible*. Full assessment requires the construction of a concrete cosmological model. The final section, §4,

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[1] This loophole is exploited by denying the implicit assumption of strictly forwards-in-time causality (Costa de Beauregard, 1953; Aharonov *et al.*, 1964; Werbos, 1973; Cramer, 1986; Price, 1996) that results from rejecting the assumption of 'measurement independence' in Bell's framework. It is metaphysically distinct from what is sometimes called the 'free-choice' loophole, which also rejects measurement independence, and which leads to superdeterministic local hidden variables approaches (Bell, 1981, 1990; Norsen, 2011). We return to this matter in §3.

sets out the conceptual and formal problems that would need to be solved before such a model could be constructed.

2 Time and Probability in Quantum Cosmology

A fundamental principle in all quantum theories is the principle of superposition: the state of the system $|\psi\rangle$ can consist of a superposition of distinct eigenstates. In a quantum cosmology the quantum state, $|\Psi\rangle$, is (or at least can be) in a superposition of all the degrees of freedom of the entire universe.[2] If we apply standard canonical quantization techniques to general relativity[3] then we get an equation for the quantum state of the whole universe called the Wheeler-DeWitt equation:

$$\hat{H}\,|\Psi\rangle = 0, \tag{1}$$

where $\hat{H}$ is an operator version of the Hamiltonian constraint of canonical general relativity. This equation does not describe any temporal evolution of the quantum state – it describes a timeless universe. The Wheeler-DeWitt equation gives us a nomological restriction on the state of the universe – but nothing more. Thus, the problem of interpreting this equation is a fearsome one: it seems very hard to reconcile its structure with our manifestly temporal and non-superposed phenomenology. This is the conceptual core of the problem of time in quantum gravity.[4]

The particular aspect of the problem of time that is most relevant to our current purpose is the interpretation of probability in the context of a wavefunction for the universe that does not evolve. How can we make sense of universal timeless probabilities? One straightforward response is that we simply can not. Probability simply is not an appropriate notion for understanding the wavefunction of the entire universe and rather is something we can only recover for sub-systems. One of the goals of this paper is to explore the space of coherent alternatives to this plausible view. If there prove to be none, then it is the obvious fallback option. The next option to consider is universal chances – ontic probabilities for the universe. One way of understanding such objects would be to invoke Popper's notion of probability as *propensity* and consider the modulus squared of the universal wavefunction as providing a single case probability corresponding to the propensity of particular timeless state of the universe. However, a propensity-type account of universal chances for a timeless universe would, in our view, have neither explanatory force

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[2] Here, and throughout the paper, we will use Ψ to refer to the quantum state of the entire universe and ψ to refer to the quantum state of a sub-system of the universe.

[3] Here we are referring specifically to the 'Dirac quantization' that is based upon the constrained Hamiltonian theory of constraints (Dirac, 1964; Henneaux and Teitelboim, 1992). Application of constraint quantization to the canonical formulation of general relativity leads directly to an equation of the Wheeler-DeWitt form (DeWitt, 1967; Thiemann, 2007).

[4] The conceptual aspects of the problem of time are compounded by various, equally difficult, technical problems. See Isham (1992); Kuchař (1992); Anderson (2012) for review articles. Two notable attempts to deal with conceptual aspects of the problem of time are the time capsules approach of Barbour (1994, 2003a) and internal time approaches (Rovelli, 1991, 2002; Gambini *et al.*, 2009). In our view, neither adequately deals with the problem of interpreting the probabilities.

nor offer any plausible heuristics for theory development. It is not, therefore, in our view an option worth pursuing.[5] Given this, the other options for characterising timeless universal chances look even more unappealing. Perhaps the most popular modern account of chance is the Humean account (Lewis (1994); Loewer (2001); Frigg and Hoefer (2015)). However, this account explicitly relies upon the existence of spatio-*temporal* patterns of local properties to provide a 'supervenience base' for chances. Such structure simply is not available in the context of the Wheeler-DeWitt equation since Ψ is defined over purely spatial degrees of freedom. This leaves us with the final, clearly inconsistent, option of trying to understand timeless universal chances as long-term frequencies. When we have neither time nor an ensemble of systems, long-term frequencies are evidently not an option. While we have not provided arguments to rule them out in principal, we think it is clear from the foregoing that none of the options available for understanding timeless universal chances are at all appealing.

This leads us to consider potential epistemic approaches to timeless universal probabilities: probabilities as agential states of knowledge. Here we meet an immediate problem. To talk about probabilities as relating to agential states of knowledge we first need to make sense of the ordinary temporal phenomenology of agents from within a sparse, timeless structure. Without an appropriate account of the transtemporal identity of agents, it is not clear that we can genuinely understand probability in relation to 'agents' at all.[6] An epistemic understanding of probability is further complicated in those timeless approaches in which the Born rule must be invoked to assume the perspective of an agent in the first place: there seems to be a fundamental structural problem in giving explanatory priority to epistemic probability in this context. This suggests a general principle[7] defining a limit to the explanatory role of epistemic probability with regard to the probabilistic structure of a physical theory:

> **Primary Precept (PP*)**: an epistemic notion of probability cannot be used to account for ontic probabilistic features of a theory that are themselves necessary to establish the existence of agents within within the systems described by the theory.

Given this, if we accept that a general and necessary requirement of any timeless quantum cosmology is the recovery of the existence of agents and that the

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[5] For discussion of the propensity notion of probability in the context of quantum mechanics see Suárez (2007).

[6] For critical discussion of this point in the context of Barbour's time capsules see in particular Healey (2002). We should note there is some hope to deal with this problem via the Saunders-Wallace-Parfit strategy for thinking about personal identity in Everettian quantum mechanics: 'Even in classical physics, it is a commonplace to suppose that transtemporal identity claims, far from being in some sense primitive, supervene on structural and causal relations between momentary regions of spacetime...it is the survival of people who are appropriately (causally/structurally) related to me that is important, not my survival per se' (Wallace, 2007).

[7] This principle is essentially the complement to Lewis' Principal Principle (PP): that (in our terminology) effectively means that epistemic probability should be constrained by ontic probability. See Dawid and Thébault (2015) for discussion of the primary precept in the context of decoherence and the probability problem in many worlds quantum theory.

mechanism for this recovery involves probabilistic notions then, by **PP***, these probabilistic notions must be ontic. We can explicitly reconstruct this argument as follows:

P1. A timeless theory of quantum cosmology requires a mechanism to recover the existence of agents with the impression of change *[No Temporal Solipsism]*

P2. *(Plausibly)* Any such mechanism will involve probabilistic notions being applied in the context of the wavefunction of the universe.

P3. An epistemic notion of probability cannot be used to account for ontic probabilistic features of a theory that are themselves necessary to invoke the perspective of agents within the theory. *[Primary Precept]*

C. The reconstruction of the impression of change in timeless quantum cosmology requires a non-epistemic (i.e., ontic) approach to probabilities applied in the context of the wavefunction of the universe.

While we do not rule out the possibility that a coherent interpretation of ontic probabilities in the context of the wavefunction of the universe might be possible, as discussed above we find it difficult to imagine what these ‘universal timeless chances’ might be. In fact, although we will not argue directly for it here, we find the entire notion of universal chances deeply conceptually problematic and it will be a working assumption of our research project that any universal probabilities must be epistemic. Thus, given we cannot accept the conclusion of this argument, we are motivated to either reject one of the premises or move past the framework within which it is formulated.

Given that the *No Temporal Solipsism* and *Primary Precept* premises must be accepted, this leaves two options. One can *either* attack the relatively weak second premise of the argument, and claim that, in actual fact, we can find a non-probabilistic means of recovering time from within a timeless quantum formalism;[8] *or* look for a timeful model of quantum cosmology within which epistemic probabilistic notions become plausible. That is, if time is part of the basic ontology of our theory from the start, then we do not need to give arguments (probabilistic or otherwise) as to why agents have the impression of change – there really is change! In this paper, we will explore the possibilities that lie within this final course, and, in doing so, develop a proposal for a dynamic, Ψ-epistemic approach to quantum cosmology.

The Wheeler-DeWitt formalism is introduced above via a conditional statement: *if* we apply standard canonical quantization techniques to general relativity *then* we derive a timeless equation for the wavefunction of the universe. What if we *do not* apply standard canonical quantization techniques? Are there alternative views

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[8] This option would seem to be in the same spirit as the approach of Valentini (see Valentini and Westman (2005); Towler *et al.* (2011); Colin and Valentini (2015); Underwood and Valentini (2015) and also Struyve (2010)) within which the Born rule is taken to have a dynamical origin. The extent to which such a framework can be successful implemented in the context of Wheeler-DeWitt cosmologies remains to be seen.

on the quantization of general relativity that lead to timeful rather than timeless quantum cosmologies? One popular option is to move away from the canonical formalism altogether. Rather, many contemporary approaches to quantum gravity are based upon a path integral type quantization of the covariant formulation of gravity. Approaches along these lines are, for example, causal set theory (Bombelli *et al.*, 1987; Dowker, 2005; Henson, 2006), causal dynamical triangulation (Loll, 2001; Ambjørn *et al.*, 2001), spin foams (Baez, 1998; Perez, 2013), or functional RG approaches (Lauscher and Reuter, 2001). There is much diversity within this broad family of 'covariant approaches' to quantum gravity, and evaluation of the extent to which each of them do or do not lead to a genuinely timeful quantum cosmology is a large project that we will not undertake here. We would, however, suggest that those that are genuinely timeful might be amenable to a Ψ-epistemic interpretation along the lines of the proposal discussed in the following sections.

Our present project is concerned with interpreting a recently proposed timeful approach to the canonical formalism. This option is largely unexplored despite a number of appealing and attractive features. In particular, given that the standard canonical quantization procedure can be amended such that non-trivial time evolution of the wavefunction remains, we would then have a formalism for quantum cosmology with unitary 'Schrödinger evolution' of a form analogous to that found within non-relativistic quantum theory. One of the great benefits of retaining such evolution is that a greater number of the candidate interpretations of quantum mechanics could then be extended to the cosmological realm. Given the depth of some of the conceptual problems with the foundations of quantum gravity, allowing a range of new perspectives to be explored may prove fruitful. The particular option that we will explore in the subsequent sections is the 'causally symmetric' two-time boundary approach endorsed by Price (1996) and Wharton (2010b). Such an interpretation of the universal wavefunction requires *some* notion of Schrödinger evolution for the universe, so that more than one temporal boundary can be defined.[9] How then can this move to an amended canonical quantization procedure be justified? Following the work of Gryb and Thébault (2012, 2014, 2015, 2016a,b) we can put forward an argument towards timeful canonical quantum cosmology based upon three interpretational and formal steps.

The first step draws upon a particular 'moderate relationalism' about time. In 'radical relationalism' about time we assert that what it means for a physical degree of freedom to change is for it to vary with respect to a second physical degree of freedom; and there is no sense in which this variation can be described in absolute, non-relative terms. This radical relationalism about time is closely associated with the work of Rovelli (1990, 1991, 2002, 2004, 2007, 2014). Formally, we can capture the essence of radical relationalism very concisely in terms of a prescription for the fundamental Hamilton-Jacobi equation. According to Rovelli the hallmark of a relational system of mechanics is that the Hamilton-Jacobi principal functional,

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[9] We should note here that even in path integral type approaches to causally symmetric quantum mechanics, such as that discussed by Wharton (2013), there is an effective Schrödinger evolution for the probability amplitudes for a transition from an initial to final states.

$S(t, q, Q)$, can be identified with the characteristic functional, $W(q, Q)$.[10] That is, rather than solving an equation of the form:

$$H\Big(q, \frac{\partial S(t, q, Q)}{\partial q}\Big) = \frac{\partial S(t, q, Q)}{\partial t} \tag{2}$$

via the usual *Ansatz* $S(t, q, Q) = Et + W(q, Q)$, we only have a 'timeless' equation of the form:

$$H\Big(q, \frac{\partial S(q, Q)}{\partial q}\Big) = 0\,, \tag{3}$$

where $S(q, Q)$ in (3) plays the role of $W(q, Q)$ in (2). This radical relationalist way of thinking about the Hamilton-Jacobi formalism leads naturally to the equations of the Wheeler-DeWitt type, such as (1), since there is a reliable heuristic, dating back to Schrödinger, that takes us from the Hamilton-Jacobi principal functional to the wavefunction (Rund, 1966, p.99-109). Radical relationalists, like Rovelli, embrace a form of timelessness even at the classical level – for them only relative variation exists, and thus neither the Hamilton-Jacobi principal functional nor the wavefunction of the universe should have any time dependance.

In a moderate relationalist approach, rather than temporal change being based merely on relative variation, such change is argued to be primitive in the sense that it is definable independently for any physical degree of freedom in isolation. This view is defended as a relationalist view on time, rather than a Newtonian absolutist view, on the grounds that although change itself is taken to be primitive, the quantification of change in terms of a temporal measure of duration is still taken to be purely relative. More formally, on the moderate temporal relationalist view defended by Gryb and Thébault, there is always assumed to exist a monotonically increasing time parametrization, but this parametrization is taken only to be defined up to diffeomorphism. This view can be reconciled with the formal arguments regarding the Hamilton-Jacobi formalism since the difference between (2) and (3) above is entirely due to an extra time boundary term, namely: the shift $S \to S + Et$, which does not affect the local equations of motion. At the classical level the two formalisms are observationally indistinguishable. See Gryb and Thébault (2016a) for extensive discussion of this point.

Given the moderate relationalist view on time, one can motivate an alternative canonical quantization procedure for the class of theories that are globally time reparametrization invariant. This is the second formal step in our argument towards timeful quantum cosmology and involves the implementation of a procedure called 'relational quantization'. The standard Dirac constraint quantization involves promoting all (first class) canonical constraint functions from the classical theory to operators that annihilate the wavefunction. The rationale behind this is a connection between canonical constraint functions and redundant 'gauge' degrees of freedom on phase space. In theories that are globally time reparametrization invariant the Hamiltonian is always a canonical constraint function. There are good

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[10] See in particular (Rovelli, 2004, §3.2).

reasons, however, to believe that this constraint function has nothing to do with redundant 'gauge' degrees of freedom. Rather, the single Hamiltonian constraint of a globally time reparametrization invariant theory results directly from the fact that the relevant Lagrange density is homogeneous of order one in the velocities. By Euler's homogeneous function theorem this implies that the Hamiltonian density must vanish (Dirac, 1964).

In fact, there is no good formal reason to believe that there are redundant degrees of freedom on phase space connected to a global Hamiltonian constraint.[11] Application of the Dirac quantization procedure to globally time reparametrization invariant theories can only be motivated, if it can be motivated at all, by adoption of radical relationalism about time. Given we adopt moderate relationalism about time we have good reason to look for an alternative quantization strategy under which time is retained. The relational quantization procedure developed by Gryb and Thébault is precisely such a strategy. It can be motivated in the context of an analysis of globally time reparametrization theories via Faddeev-Popov path integral (Gryb and Thébault, 2012), constraint quantization (Gryb and Thébault, 2014) or Hamilton-Jacobi techniques (Gryb and Thébault, 2016a). In each case, the resulting quantum formalism retains a fundamental notion of time evolution: unitary evolution of the Schrödinger-type,

$$\hat{H}\left|\Psi\right\rangle = i\hbar\frac{\partial\left|\Psi\right\rangle}{\partial t}. \tag{4}$$

This is of course in line with a classical Hamilton-Jacobi formalism understood in terms of (2) rather than (3).

The third and final step is the move that allows this Schrödinger-type evolution to be applied to the entire universe. An immediate and obvious limitation in the relational quantization procedure that we have just discussed is that it is designed for *globally* time reparametrization invariant theories with a single Hamiltonian constraint that generates *global* time evolution. General relativity is a *locally* time reparametrization invariant theory with an infinite family of Hamiltonian constraints that generate *local* 'many fingered' time evolution. Recent work, however, points towards the viability of reformulating general relativity as a globally reparametrization invariant theory.

The final move is to adopt a re-description of gravity in terms of a formalism that features a notion of preferred slicing. One attractive possibility along these lines is suggested by the *shape dynamics* formalism originally advocated by Barbour and collaborators (Barbour (2003b, 2011); Anderson *et al.* (2003, 2005)) and then brought into modern form in Gomes *et al.* (2011). Within this formalism, the principle of local (spatial) scale invariance is introduced with the consequence of favouring a particular notion of simultaneity. This selects a unique global Hamiltonian and thus allows for relational quantization to be applied. Shape dynamics is based upon a re-codification of the physical degrees of freedom of general relativ-

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[11] For extensive formal analysis of this point see Gryb and Thébault (2014, §3-4) and Gryb and Thébault (2016a, §4). Also see Barbour and Foster (2008).

ity via exploitation of a duality between two sets of symmetries. Whereas general relativity is locally time reparametrization invariant and spatially diffeomorphism invariant; shape dynamics is globally time reparametrization invariant, spatially diffeomorphism invariant, and locally scale (i.e., conformally or Weyl) invariant. In the class of spacetimes where it is possible to move from one formalism to the other (those that are 'CMC foliable') the physical degrees of freedom described by the two formalisms are provably equivalent, they are merely clothed in different descriptive redundancy.

Our adoption of the shape dynamics formalism at this stage is not a necessary move – any formalism for gravity with a preferred notion of simultaneity could be adopted. The unique and attractive feature of shape dynamics is that a preferred slicing is argued for on the basis of a symmetry principle rather than the introduction of preferred observers or other absolute structures. Nevertheless, there is still the worry that, in applying relational quantization to shape dynamics, or any other theory of gravity with a preferred time slicing, we might break the duality between the two sets of symmetries. In particular, it might be worried that the Schrödinger-type evolution of the wavefunction of the universe with respect to a time parameter will break the general covariance of the classical cosmological models. It is precisely in response to this concern that the notion of a Ψ-epistemic quantum cosmology becomes particularly appealing. But before we can assess this option we must consider the fortunes of ψ-epistemic approaches to quantum theory in general.

3 Local Hidden Variable Quantum Theory

3.1 The Einstein-Bell Conditions

Consider the following conditions for an interpretation of quantum cosmology, which we call the *Einstein-Bell conditions*:

1. All universal probabilities are epistemic [*God does not play dice*];
2. The universe is local;
3. Quantum cosmology is consistent with the no-go theorems.

The first condition encodes the basic assumption of our research: that, at the level of the whole universe, any probabilistic concepts must be given an epistemic interpretation. We find the concept of universal chances obscure and thus propose to investigate its alternative. It should be noted, though, that in our rejection of universal chances we take the aphorism "God does not play dice" to imply a rejection of objective indeterminacy: the ontic state of some system is sufficient to characterise uniquely a set of values for all its measurable properties. Given our schema below, however, we explicitly deny that this results in a ratification of determinism (although we note this possibility), in the sense that any instantaneous state of a system plus the associated laws are sufficient to describe the complete past and future behaviour of the system (see §3.2).

The second assumption is a consistency requirement between the quantum cosmological formalism and the general theory of relativity from which we derive our *empirically well-confirmed* classical cosmological models. We take it that, in the context of cosmology, the 'block universe' model of time is not just plausible but almost unavoidable.[12] As such, we take the statement that the universe is local to mean that the ontology of the universe is such that only arrangements of matter and interactions consistent with the causal (e.g., light-cone) structure of Lorentzian spacetimes are permitted. That is, physical influences and bodies (including observers) can only follow timelike or null spacetime trajectories.

The third requirement derives from the expectation that any quantum theory of cosmology will display correlations that violate the Bell inequalities. For this reason, we insist that the three basic 'no-go' results of contemporary quantum theory – Bell, Kochen-Specker, and Pusey-Barrett-Rudolph (PBR) – will all apply in quantum cosmology also. In this section, we will argue that there is a unique realist interpretational stance that satisfies the Einstein-Bell conditions: the causally symmetric hidden variable approach. In order to motivate this conclusion we must first define a taxonomy for interpreting the quantum mechanical wavefunction.

Following Harrigan and Spekkens (2010), we can distinguish between ψ-ontic and ψ-epistemic interpretations of the wavefunction. Referring to the complete physical state of some quantum system at some specified time as the 'ontic state' of that system, we call the wavefunction description of that system ψ-ontic if every distinct quantum state is consistent with a single ontic state. We can further distinguish ψ-ontic interpretations into ψ-complete and ψ-incomplete interpretations. A ψ-complete interpretation takes the quantum state to provide a complete description of 'reality' (there is a one-to-one correspondence between ontic states and distinct quantum states), while a ψ-incomplete interpretation requires that the quantum state be supplemented with additional ontic degrees of freedom. Many of the more well-known interpretations of quantum mechanics are ψ-ontic interpretations: many worlds and dynamical collapse interpretations are typically ψ-complete, and pilot-wave interpretations are typically ψ-incomplete, as the 'corpuscles' (a.k.a beables) provide additional ontic degrees of freedom over and above the ontic state.

The role of probabilities in ψ-ontic interpretations ranges from the straightforward to the obscure.[13] In the context of dynamical collapse, probability is an inherently ontic concept: the probabilities are objective chances primitively posited in the theory. Contrastingly, within the pilot-wave interpretation, although the wavefunction is ontic, the probabilities are essentially epistemic: they arise on the basis of ψ-incompleteness and reflect our ignorance of the full ontic degrees of freedom. The question of how we are to understand probability in the context of many worlds has been a topic of much vigorous debate that we will not attempt to review in detail here. Some critics of the approach argue that the concept 'probability' is not even an appropriate concept in the context of a quantum formalism where all

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[12] Although, see Petkov (2007), especially Ellis (2007), and also Earman (2008), for a discussion of the plausibility of a rival dynamic, or 'growing', block universe.

[13] See Timpson (2011) for a review.

possibilities are realised (Kent, 2010). Some advocates, on the other hand, claim not only to be able to give a decision theoretic derivation of the Born rule in a many worlds context (Deutsch, 1999; Saunders, 2004; Wallace, 2007), but also to be able to establish these probabilities as ontic (Wallace, 2012). The basis for Wallace's argument is the close connection between the weights that feature within the Everettian branching structure and the objects that play the functional role of probability for agents.[14] According to Wallace, since branch weights are part of the bare structure of many words quantum theory, the probabilistic concepts to which they are connected should be taken to be ontic (in our terminology).

In more general terms, since many worlds quantum theory is a ψ-complete interpretation we take it as *prima facie* reasonable to assume that *if* there is a probabilistic concept at play within the theory, *then* this concept will be an ontic one. Thus, should one be looking for an epistemic understanding of probabilities in the context of ψ-ontic interpretations, we take it that the most plausible option is a pilot-wave type approach.

We call the wavefunction description of some system ψ-epistemic when multiple distinct quantum states are consistent with a single ontic state, warranting an interpretation of the wavefunction as a representation of an observer's knowledge, rather than a representation of an objective reality. Since specifying the wavefunction does not completely specify the ontic state, ψ-epistemic interpretations are naturally ψ-incomplete. We can further distinguish ψ-epistemic interpretations into realist interpretations, wherein there exists an underlying ontic state, and anti-realist or operationalist interpretations that make no such claim for a deeper underlying 'reality'. Anti-realist interpretations include (arguably) orthodox Copenhagen interpretations as well as quantum Bayesianism[15] and other quantum informational approaches. Realist interpretations are not well explored. This is despite the fact that, as Harrigan and Spekkens (2010) argue, a realist ψ-epistemic interpretation is precisely what Einstein was advocating in his more sophisticated arguments for the incompleteness of quantum mechanics. According to the realist ψ-epistemic view, quantum mechanics is a statistical theory over the ontic states which are 'hidden' from the observer; the complete theory is then a hidden variable theory. The basic probabilistic concepts that occur in both realist and anti-realist ψ-epistemic interpretations are clearly themselves epistemic.[16] Thus, as could be expected, ψ-epistemic interpretations are natural bedfellows for epistemic notions of probability.

There is perhaps good reason for the lack of exploration of realist ψ-epistemic interpretations of quantum mechanics. The development of quantum mechanics

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[14] In this sense we can see his analysis as at least partially in the same spirit as that of Vaidman (2012). We should note, however, that Vaidman emphasises the epistemic rather the ontic aspects of his treatment of probability.

[15] Although, Fuchs (2016) argues that this is a common misconception of quantum Bayesianism. Interestingly, the 'participatory realism' detailed in (Fuchs, 2016) has notable metaphysical similarities to the causally symmetric local hidden variable theories discussed in this work.

[16] N.b. there is nothing in principle stopping a realist ψ-epistemic interpretation having supplementary stochastic structure within the dynamics of the hidden variables and so having further ontic probabilities.

was followed from the outset by a series of no-go theorems that seemingly ruled out a range of ψ-epistemic hidden variable approaches. The first of these was von Neumann's (1932) theorem that the quantum statistics could not arise from an underlying set of determined hidden variables, apparently ruling out hidden variable approaches all together. However, Bohm's (1952) (albeit ψ-ontic) model of quantum mechanics is just such a description of hidden variables that reproduces the quantum statistics, only it is explicitly nonlocal. As a consequence of Bohm's counterexample, a second no-go theorem arises, Bell's theorem (Bell, 1964), which states that there can be no hidden variable model of quantum mechanics that obeys Bell's notion of local causality, whereby spacelike separated events must be independent conditioned on a past common cause. A further no-go theorem, the Kochen-Specker theorem (Kochen and Specker, 1967), states that a hidden variable model must be contextual, whereby two operationally equivalent experimental preparation procedures may correspond to inequivalent ontic state representations (the state additionally depends on the context of measurement).

A more recent no-go theorem, the PBR theorem (Pusey *et al.*, 2012), states that the ontic states of any interpretation of quantum mechanics that fits within the Bell framework and reproduces the Born rule must be in one-to-one correspondence with the quantum states; that is, the interpretation must be ψ-ontic. Given this series of no-go theorems for ψ-epistemic hidden variable approaches, it is little wonder they remain underexplored. There is, however, one such approach that evades these no-go theorems: causally symmetric local hidden variable approaches to quantum mechanics.[17]

Causally symmetric local hidden variable (CSLHV) approaches to quantum mechanics take advantage of a 'loophole' in the assumptions that underlie the no-go theorems, assumed most explicitly in Bell's theorem. Not only does Bell assume local causality, he also assumes what could be called the 'free variables' assumption (Norsen, 2011) or, equivalently, measurement independence, whereby any hidden variables must remain independent of the choice of measurement settings to which the system is subject as part of the experimental procedure. Relaxing this assumption amounts to allowing the ontic state underlying the quantum description of a system to be directly dependent upon the measurement settings to which it will be subject in the future. But disavowing this assumption can be interpreted ambiguously: a statistical dependence between the ontic state and the measurement settings superficially appears to be suggesting that experimenters are no longer free to choose the measurement settings arbitrarily, resulting in what Bell (1990, p.244) called 'superdeterminism'. Such an interpretation, however, blindly adheres to the implicit assumption of strictly forwards-in-time causality. Another way to view the relaxation of the assumption of independence between the ontic state and the measurement settings – the direct inverse of superdeterminism – is explicitly to preserve the free choice of experimenters over the experimental settings but reject the assumption of strictly forwards-in-time causality, allowing a causal influence from

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[17] For discussion of the relation between the PBR theorem and causally symmetric approaches see Leifer (2011); Wharton (2014).

future to past to accompany the usual causal influences from past to future. The resulting 'causally symmetric' approach circumvents the results of Bell's theorem, and so can be a local hidden variable theory (and in doing so solves any apparent tension with relativity), and thus also circumvents the PBR theorem, and so can be a ψ-epistemic interpretation of quantum mechanics. A CSLHV approach also contains an explicit contextuality of the ontic state on the experimental procedure, so fits within the bounds given by the Kochen-Specker theorem.

The loophole in Bell's theorem originates in a suggestion in the 1950s from Costa de Beauregard (1953) (a student of de Broglie) in response to Einstein *et al.*'s (1935) argument that quantum mechanics is incomplete. According to the suggestion, causal influences could propagate as both retarded and advanced waves, in a kind of 'zigzag', to avoid the problems posed by apparently nonlocal correlations. Two causally symmetric approaches to quantum mechanics that are more well-known today are the two-state vector formalism developed by Aharonov *et al.* (1964, 2014, 2015), in which forward evolving and backward evolving state vectors combine to produce the intervening quantum state, and the transactional interpretation developed by Cramer (1986), in which quantum particle trajectories emerge from a cycle of retarded and advanced waves (see also Kastner (2012)). A third approach arises from Price's (1996) foundational philosophical work on causally symmetric quantum theory supplemented by Wharton's (2010b) more recent formal extension of those foundations to develop an approach to quantum mechanics as a two-time boundary problem.

Causally symmetric local hidden variable interpretations are the most plausible option for a realist interpretation of quantum cosmology satisfying the Einstein-Bell conditions. Epistemic probabilities are not appropriate for a Ψ-complete interpretation of the universal wavefunction: any probabilities must be ontic probabilities on such a view. The first Einstein-Bell Condition thus rules out Ψ-complete interpretations and restricts us to Ψ-incomplete interpretations.[18] The combination of the second and third conditions then rules out Ψ-ontic interpretations altogether since these cannot be both local and avoid the no-go theorems. This then leaves us with either anti-realist Ψ-epistemic interpretations, for example quantum Bayesianism, or local-realist Ψ-epistemic interpretations, the only examples of which are causally symmetric local hidden variable approaches (in which the ontic state is taken to be comprised of spatiotemporally local classical variables, in accord with the Einstein-Bell conditions). If one wants to be an Einstein-Bell realist about quantum cosmology, then on our view a CSLHV approach is the most natural way to go.

3.2 Perspectivalism and the Past

In the previous section, we motivated the CSLHV approach based upon the combination of realism with the Einstein-Bell conditions for quantum cosmology. In the

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[18] We note again the subtleties regarding what, if any, interpretation of probability is appropriate in the context of many worlds theory. See Vaidman (2012); Wallace (2012).

present section, we will discuss in detail a particular variant of the CSLHV family: the 'Price-Wharton' picture.[19] The formal motivation of the Price-Wharton picture is based on Hamilton's principle with emphasis on the constraint of both initial and final boundary conditions to construct equations of motion from a Lagrangian. If we treat external measurements as physical constraints imposed on a system in the same way that boundary constraints are imposed on the action integral of Hamilton's principle, we can imagine the dynamics of a system subject to preparation and measurement procedures to emerge *en bloc* as the solution to a two-time boundary problem. Focussing solely on classical fields, Wharton (2010b) argues that constraining such fields (which characterise the ontic state) at both an initial and a final temporal boundary (or a closed hypersurface in spacetime) generates two strikingly quantum features: quantization of certain field properties and contextuality of the unknown parameters characterising the field between the boundaries. Thus, a classical field constrained at both an initial and a final temporal boundary permits, by construction, ontic variables that are correlated with the future measurement of the system. The final measurement does not simply reveal preexisting values of the parameters, but *constrains* those values (just as the initial boundary condition would) – thus, had the final measurement been different, the ontic state would have been different, rendering the picture 'causally symmetric'.

Within the Price-Wharton picture, an invariant joint probability distribution associated with each possible pair of initial and final conditions can be constructed (Wharton, 2010a, p.318), and the usual conditional probabilities can be formed by conditioning on any chosen portion of the boundary (Wharton, 2010b, p.280).[20] As a result, probability is interpreted as a manifestation of our ignorance: if we knew only the initial boundary, we would only be able to describe the subsequent ontic state probabilistically (since we lack knowledge of the final constraint). We thus interpret the solution to the Schrödinger equation ψ-epistemically as just such a description: it is an ignorance function over the unknown ontic state based on our knowledge of the initial boundary (and lack of knowledge of the final boundary). Once we obtain knowledge of the final boundary, our knowledge of the ontic state undergoes discontinuous Bayesian updating and we can then retrodict the field values between the two boundaries. There is, however, no such discontinuous evolution of the underlying ontic state. Moreover, we could equally conditionalise on the final boundary to generate a probabilistic description propagating backwards in time, but this is rarely useful in practice on account of the (assumed) forwards-in-time-facing agential perspective.

It is worth noting at this point (we will return to this issue in more depth later) that the Price-Wharton picture forces us to draw a sharp distinction between the determination of behaviour of the quantum state – the epistemic quantum wave-

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[19] This picture has been more recently developed in the jointly-authored works Wharton *et al.* (2011); Evans *et al.* (2013); Price and Wharton (2013, 2015a,b).

[20] This interpretation of probabilities maps nicely to the Feynman path integral representation of joint probabilities, wherein the joint probability of particular initial and final state pairs naturally incorporates two temporal boundary conditions, and is given by an integral over the classical action.

function description – and the underlying ontic state. When we consider determination of the behaviour of the quantum wavefunction description, then since the Schrödinger equation is parabolic, specifying the wavefunction solution on an initial boundary is sufficient to specify completely the behaviour of the wavefunction description thereafter. Thus, the Schrödinger equation and knowledge of a wavefunction description on a Cauchy surface amount to a well-posed Cauchy problem (and thus the wavefunction description renders quantum mechanics Markovian). One of the lessons of Bell's theorem is that it is not possible according to such a well-posed Cauchy problem for the wavefunction description to be comprised of classical, spatiotemporally located variables – initial data of this form cannot account for the complete observed quantum behaviour thereafter of any purported classical variables. According to the Price-Wharton picture, however, we take the wavefunction description to be ψ-epistemic and, thus, a representation of our knowledge of an underlying ontic state. If we are to think of this ontic state along the lines of Einstein-Bell realism, then we take there to be two options available for understanding the nature of the evolution of the ontic state from the initial to the final boundary. The first option, which we will focus on predominantly in this work, is that the specification of the ontic state completely on a Cauchy surface is insufficient for determining the subsequent behaviour of the state; we would additionally require information on a future boundary to obtain complete determination. Thus, complete specification of the ontic state on a Cauchy surface combined with whatever dynamical laws govern the ontic state variables cannot amount to a well-posed Cauchy problem. In other words, the laws governing the ontic state variables cannot be parabolic or hyperbolic PDEs – if they were, this would lead to an overdetermination of the ontic state at the final boundary. This issue would prove a significant challenge to the construction of a coherent Ψ-epistemic quantum cosmology and we will return to it in §4.

There is a second option for understanding the evolution of the ontic state from initial to the final boundary within a causally symmetric ψ-epistemic approach that *does* allow local hidden variables that solve a Cauchy problem. The idea would be to meet the problem of overdetermination directly. In particular, the tension that would need to be resolved is between: i) the solution of a Cauchy problem from freely, arbitrarily and (ideally) completely specifiable initial data; and ii) the symmetric expectation that the final boundary be equally freely, arbitrarily and completely specifiable. One way to escape this tension would be to remove the freedom to completely specify data on the final boundary: an agent controlling the final boundary would just happen to 'choose' a measurement that concords with the deterministic evolution of the ontic state. However, this would break the symmetry between the final and initial boundaries and would also remove the element of control that renders the Price-Wharton picture casually symmetric. To retain the symmetry would thus require some as-yet-unspecified principled constraint that limits an agent's ability to freely, arbitrarily and completely specify both initial and final boundary data. This constraint must be such that the aspects of the ontic state on the *initial* Cauchy surface that are a consequence of the choices specified at the *final* boundary are not epistemically accessible before the final boundary is specified – and vice versa. While we concede that this second option for understanding the evolution of the ontic state in the Price-Wharton picture is in principle consistent, we do not find it particularly physically enlightening. For this reason, in the remainder of the paper we will focus our attention on the first option, whereby the ontic state is characterised in terms of variables that do not solve a well-posed Cauchy problem.

It is a fundamental assumption of the Price-Wharton picture that we are in-principle ignorant of the future but we are not likewise ignorant of the past.[21] The wavefunction is ψ-epistemic because it is an ignorance function over the unknown ontic state based on our knowledge of data on some Cauchy surface *and lack of knowledge of data on some future boundary.* The supposed explanation for this asymmetry is grounded in a form of perspectivalism about temporal asymmetry. Perspectivalism about temporal asymmetry is based upon a particular way of combining a 'block universe' model of time with an 'interventionist' account of causation (Price, 2007a; Evans, 2015; Ismael, 2015). According to the former, all past, present and future events are equally real and we imagine time as ontologically on a par with a fourth dimension of space. According to the latter, we say that some event is a cause of some other event when, given an appropriate set of independence conditions, an intervention to manipulate the first event is an effective means of manipulating the second event. This provides the justification for characterising the Price-Wharton picture as causally symmetric. More precisely, X is a cause of Y just in case there is some possible (or hypothetical) intervention I that can be carried out on X that will change the probability distribution over the outcomes at Y, so long as I excludes all other possible causes of X, I is correlated with Y only through X, and I is independent of any other cause of Y.

The interventionist account is thus a counterfactual account of causation and is not explicitly reliant on a particular temporal direction to define causation. The direction of causation is dictated by the nature of the functional dependences between the relevant variables describing a system and the nature of the relevant intervention. This permits us to understand causation as a 'perspectival' notion, wherein we have a spatiotemporally constrained perspective within the block universe: we have limited epistemic access to other spatiotemporal regions, especially future regions, such that when we act as agents there are specific natural constraints on which parts of our environment we take to be fixed and which parts we take to be controllable. It is the epistemic relation that we hold with respect to the different variables involved in the intervention that align the direction of causation with the future temporal direction. We control the intervention and thus usually know its significant preconditions. However we do not have epistemic access to the effect of the intervention in the future independently of this control.

According to (Price, 2007a, p.278), the asymmetry in our epistemic access to the future compared to the past is a feature of *creatures like us* living in a universe

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[21] Although, the mechanics of the causally symmetric schema of Price and Wharton requires that there be at least some parts of the past of which we are in-principle ignorant.

with a particular entropy gradient:

> We regard the past as fixed because we regard it as knowable, at least in principle. This is clearly an idealisation, but one with some basis in our physical constitution. As information-gathering systems, we have epistemic access to things in (what we call) the past; but not, or at least not directly, to things in (what we call) the future.
>
> Plausibly, this fact about our constitution is intimately related to the thermodynamic asymmetry, at least in the sense that such information-gathering structures could not exist at all, in the absence of an entropy gradient. Although the details remain obscure, I think we can be confident that the folk physics reflected in the temporal asymmetry of our epistemic and deliberative templates does originate in *de facto* asymmetries in our own temporal orientation, as physical structures embedded in time.

Price's concern in this matter is the asymmetry of causation: why should causes typically precede their effects? His answer is that the asymmetry of causation is deeply rooted in the inherent asymmetry of deliberation, whereby an agent can only deliberate about a desired outcome of some set of possible actions when the actual outcome is unknown to the agent (deliberation is useless where an agent knows the actual outcome in advance). The significant feature of this 'architecture of deliberation' is that we regard the past as knowable, and so we typically deliberate towards the unknown future.[22]

All together, the Price-Wharton picture provides an attractive interpretational package. It allows us to combine an epistemic interpretation of the wavefunction with a local realist ontology without contravening the no-go theorems. Whilst some (e.g., Maudlin, 2002) have characterised the resulting 'retrocausality' as a high ideological cost, and therefore wholly unappealing, Evans (2015) points out that in fact there is no ideological cost at all to the Price-Wharton picture of retrocausality (in particular, the view does not countenance 'spooky' backwards-in-time effects); rather, this view is simply a natural consequence of our limited epistemic viewpoint within a metaphysically acausal block universe. Again, in the context of cosmology, the block universe view is not just plausible but almost unavoidable.

4 Ψ-Epistemic Quantum Cosmology?

In the preceding sections, we have developed and defended a novel and, we hope, plausible proposal for Ψ-epistemic quantum cosmology. In this section, we will isolate and assess a number of important challenges to our package of ideas.

The first issue is, in a sense, the most obvious one. In §2, we argued in favour of an approach to quantum cosmology in which there is fundamental time evolution

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[22] For more on Price's views on this issue, see Price (1994, p.321-5), Price (2007b, p.7-8), Price (2007a), Price (2013, p.199) and Price and Weslake (2009, p.436-40).

in the wavefunction of the universe. Our current best theory of classical cosmology is general relativity within which the fundamental symmetry of general covariance implies that time is 'many fingered' – in particular, the local time reparametrization invariance of the theory implies that a global time evolution parameter will not in general be well defined. Does it make sense to have a symmetry at the level of the classical theory that is broken in this sense within the quantum formalism? We can now offer a good response to this worry in light of the context of the Price-Wharton picture of non-relativistic ψ-epistemic quantum theory.

In the Price-Wharton approach to quantum theory, the wavefunction may evolve non-unitarily upon measurement. Formally speaking, the unitarity of the quantum evolution has deep (and rather complicated) connection to the conditions on the flow of the Hamiltonian vector fields that guarantee consistent classical dynamics (Landsman, 2007, §5). Thus, there might seem to be a tension between a classical local hidden variable model with consistent dynamics and a quantum formalism that includes non-unitary evolution. From a ψ-epistemic perspective, such a combination is, however, not as problematic as it may seem. If the wavefunction is not something in the world then clearly conditions on the consistency of classical evolution need not be reflected in symmetries of the wavefunction. Non-unitarity relates to discrete changes in an agent's state of knowledge and should not be taken as having implications for the underlying classical dynamics. The Price-Wharton picture allows us to understand the wavefunction as evolving non-unitarily without there being any corresponding incompleteness or inconsistency in the corresponding hidden variable dynamics.[23] In a similar vein, in Ψ-epistemic quantum cosmology, it is entirely consistent to insist that the local hidden variables are generally covariant although the wavefunction of the universe picks out a preferred cosmological time. Just as the non-unitarity of the evolution of the wavefunction is a function of agential perspective so is, we argue, the existence of a preferred evolution parameter.[24] It might even be taken as a necessary precondition of our states of knowledge that their evolution is defined relative to a simultaneity class. Thus, our proposal for Ψ-epistemic quantum cosmology offers a promise of what might be a full resolution of the problem of time in quantum gravity: a coherent conceptual framework for reconciling quantum evolution with a generally covariant classical formalism.

This discussion leads us to a second potential worry; this one much more difficult to deal with. In taking Ψ-epistemic quantum cosmology seriously we must reconsider exactly what quantum-classical limiting procedures mean. One of the requirements of the Price-Wharton picture was that the local hidden variables do not obey dynamical equations with a well-posed Cauchy problem.[25] Such a require-

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[23] See Aharonov *et al.* (2014) for consideration of this issue within the perspective of the two-state vector formalism.

[24] We should note that the issue at hand is a subtle one. In particular, whilst non-unitary quantum evolution does not directly contradict a classical symmetry principle, a preferred time in the context of quantum cosmology certainly would. Thus, the coherence of our timeful Ψ-epistemic quantum cosmology rests on a subtle reinterpretation of symmetries in the context of classical-quantum limits.

[25] Given, of course, the considerations noted in §3.2.

ment is clearly also necessary in Ψ-epistemic quantum cosmology: without it our approach would fall foul of the relevant no-go theorems (i.e., Bell, Kochen-Specker, PBR) that must reasonably be assumed to apply in the cosmological context. However, one would also expect that any sensible hidden variable theory of quantum cosmology must still contain a limit where it is approximately reproducing classical Lorentzian field theory – for example, electromagnetism. The problem here is that, in this limit, the two-time boundary problem for Lorentzian field theories is not well defined since the field equations are typically hyperbolic due to the Lorentzian signature of the spacetime. There is a worrying tension between the demands that a Ψ-epistemic cosmology must *both* describe an underlying dynamics of hidden variables that *do not* solve a Cauchy problem *and* contain a limit where it recovers classical field theories that *do* solve a Cauchy problem. There is, however, a viable route of escape from this seemingly fatal impasse. There are at least two formal resources found within modern physics that one can draw upon to obtain a Lorentzian field theory described by hyperbolic equations from a Euclidean field theory described by elliptic equations. Such resources give us a means to 'square the circle' and describe a system as both solving a Cauchy problem (in some limit) but not-solving a Cauchy problem (in the fundamental dynamics). What we have in mind here is using techniques developed in the context of 'emergent gravity' and 'Wick rotation'. We will spend some time explaining the potential applicability of each approach below.

Within the 'emergent gravity' approach it is argued that classical field theories, including electromagnetism and general relativity, can be understood as low energy limits of a field theory of fundamentally different character. Particularly prominent implementations of such an idea include various forms of the 'entropic gravity' proposal (Jacobson, 1995; Verlinde, 2011). More straightforwardly, but in the same spirit, one can make the simple observation that if our classical field theories are emergent, in the sense of resulting from structurally different underlying dynamics, then the character of the partial differential equations of Maxwell's theory (for example) might also be an emergent feature. That is, one might imagine that a dynamics described by hyperbolic PDEs might emerge from a more fundamental theory that features PDEs which are elliptic. Such an idea is explicitly examined by Barceló (2007), who points out that there is a very large set of systems which can be appropriately described in an averaged fashion by a hyperbolic system of PDEs, even though the fundamental equations are elliptic. The essence of the idea comes from techniques used in the context of analogue gravity,[26] wherein linearised fluctuations in a medium can, under certain conditions, obey effective equations with Lorentzian signature, while the bulk medium is governed by the (Euclidean) equations of non-relativistic continuum mechanics. In such a context, it is quite plausible for the universe to be described by local hidden variables that do not solve a Cauchy problem, whilst simultaneously there is an emergent classical dynamics that does.

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[26] The original proposal of analogue gravity was by Unruh (1981). Reviews are Barceló *et al.* (2005); Visser and Weinfurtner (2007). See Dardashti *et al.* (2015) for a philosophical discussion.

In analogue gravity, a Lorentzian field theory with hyperbolic equations may be understood as *emergent* from an underlying Euclidean field theory with elliptic equations. A much more standard technique for moving between two such systems of equations is 'Wick rotation'. Wick rotation is used in quantum field theory to convert a complex Lorentzian path integral to a real Euclidean partition function. In that context, it is a technique used to prove convergence and to control certain divergences of the Lorentzian path integral – see, for example, Ticciati (1999). Our claim is that, on top of being an important tool for rigorously analysing Lorentzian quantum field theories, Wick rotation may also provide a second approach for resolving the apparent tension within our proposal. This is because a Euclidean partition function: i) can be well defined as a two-'time' boundary problem; and ii) can, under certain conditions, be analytically continued to an equivalent Lorentzian path integral.[27] Moreover, the manner in which the analytic continuation and subsequent complex rotation is performed is intimately connected to the form of the propagators one uses for quantization. Since these, in turn, directly determine the form of microcausality implemented in the Lorentzian field theory, Wick rotation allows us to get direct access to influences in the Euclidean field theory that could be analytically continued back into causally symmetric influences in the Lorentzian framework. Euclidean field theory has, therefore, features that mark it out as a good starting point to construct a causally symmetric local hidden variable theory.[28] The fundamental equations of such a theory would be taken to fail to solve a Cauchy problem and yet, after some coarse-graining, lead to a partition function that could be suitably Wick rotated to a Lorentzian path integral.

It is not only the elliptic form of the equations that make the Euclidean formalism attractive from a Ψ-epistemic perspective. Unlike its Lorentzian cousin, the Euclidean path integral can be interpreted as a genuine statistical mechanical partition function. That is, one can interpret each path in the sum over histories as a genuine element of a statistical mechanical ensemble since each term of the sum is real and, therefore, there is no interference between individual paths. Additionally, a coarse-graining (or some limiting procedure that would effectively integrate out the retrocausal modes) would be required to transform a non-Cauchy theory, in the Euclidean setting, to one that can be analytically continued to a Lorentzian theory. This means that there is a natural way to analyse our ignorance in terms of the coarse-grained degrees of freedom. In this context, a statistical interpretation of the resulting coarse-grained theory is perfectly natural. Thus, Euclidean field theory naturally complements a Ψ-epistemic interpretation of a local hidden variables theory. Whether motivated by emergent gravity or Wick rotation, in the context

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[27] More precisely, there are a set of necessary and sufficient conditions under which Euclidean Green's functions are guaranteed to define a unique Wightman quantum field theory. See Osterwalder and Schrader (1973).

[28] Of course, our Euclidean theory should not be fully equivalent to the Lorentzian field theory, else we would end up running into the no-go theorems again. Rather, the idea would be to formulate a theory that, although it strictly violates the Osterwalder-Schrader conditions, admits a coarse-grained description in which the partition function can be Wick rotated to a valid Lorentzian field theory.

of Euclidean field theory the hope of reconciling our set of seemingly irreconcilable desiderata should no longer be taken to be an entirely vain one.

The final issues facing Ψ-epistemic quantum cosmology relates to the arrow of time. As the quote at the end of Section §3.2 suggests, that we know about the past and not the future is, according to Price, a function of the thermodynamic asymmetry and the entropy gradient; thus, the asymmetry of causation is grounded, via our perspective as temporally embedded agents, in the thermodynamic asymmetry. However, the precise nature of the connection between the thermodynamic asymmetry and our asymmetric epistemic relationship to the past and future is a matter that remains largely unexamined in the literature on causally symmetric quantum theory. This, unfortunately, is particularly problematic for the case of quantum cosmology where an explanation in terms of local thermodynamic arrow of time is inadequate. The fact that the universe is clearly not in thermodynamic equilibrium means that it is not possible to employ a thermodynamic notion of entropy in the cosmological setting. This immediately implies that Price's account simply cannot be applied directly to cosmological observers (or any observers outside of thermodynamic equilibrium) and, thus, cannot be applicable, for instance, to measurements of the primordial fluctuations in the power spectrum of the CMB.[29]

On a more fundamental level, unlike a quantum formalism for sub-systems of the universe, any complete construction of a causally symmetric local hidden variable theory of quantum cosmology must provide an explanation for our asymmetric epistemic access to the past and future that is applicable to the universe treated as a closed system. If this explanation is to rely on the emergence of a thermodynamic arrow of time for local observers, then this explanation must necessarily explain how that local thermodynamic arrow of time is to emerge in the first place, otherwise it cannot be a complete cosmological theory. One common line of approach is to explain the local arrow by employing a 'past hypothesis' (Albert, 2001). The past hypothesis, however, is more of an aspiration for a solution to the problem than an actual solution itself because it relies on the assumption that it is possible to define the entropy of the universe as a whole without specifying which particular notion of entropy one should use, how to calculate the entropy of an infinite closed system (and, in particular, how to regularize it in an unambiguous way) or why the entropy should not either be completely arbitrary or simply zero. Thus, without a concrete implementation of the past hypothesis (which is currently well beyond reach) it is not possible to rely on it to provide a concrete proposal for a causally symmetric quantum formalism for the cosmology.[30] What we require instead is a concrete mechanism that can be realized in an explicit model illustrating how the local thermodynamic arrow of time emerges, and, along with it, an explanation for our asymmetric epistemic access to past and future. We regard this as a key

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[29] Although the power spectrum itself is near thermal, the combined system of the density fluctuations and us observing it now is certainly not.

[30] For various discussions of the problems and prospects of using special initial conditions to derive an arrow of time see Loschmidt (1876); Price (1996); Albert (2001); North (2002); Callender (2004); Price (2004); Earman (2006); Callender (2010); Wallace (2010).

question to address in future work.[31]

5 Final Thoughts

This paper has included a rather large number of new and controversial ideas. We feel, however, that we have presented a plausible conceptual platform upon which a Ψ-epistemic quantum cosmology might be built. The next step is the construction of a concrete cosmological model which contains all the necessary mathematical and conceptual structures. This will be the focus of future work.

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[31] One promising proposal is to follow the ideas developed by Barbour *et al.* (2014) and use models with dynamical attractors – where the attractors themselves represent systems with better and better record keeping devices – to simultaneously explain the emergence of the local arrow of time dynamically (without reference to a past hypothesis) and local observer's asymmetric epistemic access to past and future.

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