What is quantum information?

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1.- Introduction

The concept of information has proved to be one of the most difficult scientific concepts to interpret (Adriaans 2013, Floridi 2015). On the one hand, the word 'information' is used with many differing meanings; on the other hand, there are several different formalisms to treat the concept quantitatively. But even when a single formalism is considered, disagreements arise when the task at issue is the interpretation of the concept (Lombardi, Fortin and Vanni 2015).

During the last decades, new interpretive problems have arisen with the advent of quantum information; those problems combine the difficulties in the understanding of the concept of information with the well-known foundational puzzles derived from quantum mechanics itself. This situation contrasts with the huge development of the research field named 'quantum information theory', where new formal results multiply rapidly. In this context, the question 'What is quantum information?' is still far from having an answer on which the whole quantum information community agrees. In fact, the positions about the matter range from those who seem to deny the existence of quantum information (Duwell 2003), those who consider that it refers to information when it is encoded in quantum systems (Caves and Fuchs 1996, Dieks 2016), and those who conceive it as a new kind of information absolutely different from classical information (Jozsa 1998, Brukner and Zeilinger 2001).

In the present article we will address the question 'What is quantum information?' from a conceptual viewpoint. In particular, we will argue that there seems to be no sufficiently good reasons to accept that quantum information is qualitatively different from classical information. The view that, in the communicational context, there is only one kind of information, physically neutral, which can be encoded by means of classical or quantum states has, in turn, interesting conceptual advantages. First, it dissolves the widely discussed puzzles of teleportation without the need to assume a particular interpretation of information. Second, and from a more general viewpoint, it frees the attempts to reconstruct quantum mechanics on the basis of informational constraints from any risk of circularity; furthermore, it endows them with a strong conceptual appealing and,

derivatively, opens the way to the possibility of a non-reductive unification of physics. Finally, in the light of the idea of the physical neutrality of information, the wide field of research about classical models for quantum information acquires a particular conceptual and philosophical interest.

For these purposes, the article is organized as follows. In Section 2 we begin by disentangling the different senses of the general notion of information in order to clarify the specific concept at issue in our discussion. In Section 3 Schumacher's formalism is introduced by contrast with Shannon's theory. Section 4 is devoted to critically asses the most common arguments for conceiving quantum information as qualitatively different from classical information. In Section 5 the relation between quantum information theory and quantum mechanics is considered, in order to make sense to the question about what peculiarities of quantum mechanics are really necessary to implement quantum protocols. Finally, in Section 6 we summarize our arguments and stress that calling into question the concept of quantum information does not imply, in any sense, downplaying the relevance of the widely developed field of quantum information theory.

2.- Which notion of information?

Since information is a polysemantic concept that can be associated with different phenomena, the first distinction to be introduced is that between a semantic and a non-semantic view of information. According to the first view, information is something that carries semantic content (Bar-Hillel and Carnap 1953; Bar-Hillel 1964; Floridi 2011); it is therefore strongly related with semantic notions such as reference, meaning and representation. In general, semantic information is carried by propositions that intend to represent states of affairs; so, it has intentionality, "aboutness", that is, it is directed to other things. Non-semantic information, also called 'mathematical', is concerned with the compressibility properties of sequences of states of a system and/or the correlations between the states of two systems, independently of the meanings of those states.

However, this distinction is not yet sufficiently specific, since in the domain of mathematical information there are at least two different contexts in which the concept of information is essential. In the *computational context*, information is something that has to be computed and stored in an efficient way; in this context, the algorithmic complexity measures the minimum resources needed to effectively reconstruct an individual message (Solomonoff 1964, Kolmogorov 1965, 1968, Chaitin 1966). By contrast, in the traditional *communicational context*, whose classical *locus* is Claude Shannon's formalism (Shannon 1948, Shannon and Weaver 1949), information is primarily something that has to be transmitted between two points for communication purposes. Shannon's theory is purely quantitative, it ignores any issue related to informational content: "[the] *semantic*

aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one selected from a set of possible messages." (Shannon 1948, p. 379). In this paper we will focus on the concept of information in the communicational context.

In spite of the formal precision supplied by mathematics, the interpretation of the concept of information in a communicational context is still a matter of debate (see Lombardi, Holik and Vanni 2016). Nevertheless, there are certain minimum elements that can be abstracted to characterize a communicational context. In fact, from a very abstract perspective, communication requires a source and a destination, both systems with a range of possible states: the sequences of the states of the source are the messages to be transmitted. As stressed above, Shannon (1948, p. 379) explicitly states that the only significant aspect of information is that a certain message is selected from a set of possible messages. Therefore, the goal of communication is to identify what message was produced at the source by means of the states occurred at the destination.

A view about information that has become very popular in the philosophical community is based on the traditional distinction between types and tokens. According to this view, given the sequence of states produced by the source, what it is intended to transmit is not the sequence of states itself, but another token of the same type. Therefore, the goal of communication is to reproduce at the destination another token of the same type as that produced at the source (Timpson 2004, 2013, Duwell 2008): this is the *type-information* (Duwell 2008, p. 201) or *pieces* of information (Timpson 2013, p. 24) to be transmitted, contrasted with the *quantity-information* or *bits* of information, that is, the measure of *how much* information the source produced (Timpson 2008).

Although very convincing at first sight, that position is contradicted by the engineering practice in communication. Since the goal of communication consists in identifying at the destination the message produced at the source, the success criterion is given by a one-to-one or one-to-many (noisy channel, see next section) mapping from the set of states of the source to the set of states of the destination. Since this mapping is completely arbitrary, the states of the source and the states of the destination may be of a completely different nature: for instance, the source may be a dice and the destination a dash of lights; or the source may be a device that produces words in English and the destination a device that operates a machine. A face of a dice and a light in a dash are not tokens of a same type in any philosophically meaningful sense of the type-token distinction (see Wetzel 2014). In other words, "*a type needs to have some content to be able to identify its tokens: the distinction between types and tokens is not merely formal or syntactic; being tokens or a same type is not an arbitrary relation.*" (Lombardi, Fortin and López 2016, p. 222).

A possible move is the attempt to generalize the traditional Peircean difference between sentence-type and sentence-token in terms of *sameness of pattern or structure* (Timpson 2013, p. 18): "*the success criterion is given by an arbitrary one-to-one mapping from the set of the letters of the source to the set of the letters of the destination*" (Duwell 2008, p. 200). But this view faces two difficulties, one philosophical and the other technical (for a full development of these criticisms, see Lombardi, Fortin and López 2016). On the philosophical side, admitting arbitrary one-to-one mappings as defining the relation "*x* is a token of the same type as the token *y*" leads to admit that any two things arbitrarily chosen can always be conceived as tokens of the same type. But this trivializes the distinction type-token and deprives it of conceptual usefulness. From a technical viewpoint, the appeal to the generalization of the type-token difference in terms of sameness of structure or one-to-one mappings forgets the possibility of noisy situations, in which one-to-many mappings link the states of the source and the states of the destination (see next section). Furthermore, these noisy situations are the cases of real interest in the practice of communication engineering. Summing up, despite of the wide dissemination of the ideas that link the transmission of information with the philosophical distinction between types and tokens, it is not necessary to reproduce at the destination what happened at the source for successful communication.

In general, the messages produced at the source are encoded before entering the channel that will transmit them, and decoded after leaving the channel and before being received at the destination. Claude Shannon (1948) and Benjamin Schumacher (1995) demonstrated theorems that supply the optimal coding in the so-called classical and quantum cases, respectively. The original articles of Shannon and Schumacher were followed by an immense amount of work, both theoretical and technological. Nevertheless, those foundational articles are always consulted to track the origin of the concepts and to discuss their content. For this reason, we will begin by recalling and comparing those formalisms.

3.- Shannon and Schumacher

Shannon's theory is presented in the already classical paper "The Mathematical Theory of Communication" (1948, see also Shannon and Weaver 1949), where a general communication system consists of five parts:

- A *message source A*, which produces the message to be received at the destination.
- A *transmitter T*, which turns the message produced at the source into a signal to be transmitted. In the cases in which the information is coded, coding is also implemented by this system.

- A *channel C*, that is, the medium used to transmit the signal from the transmitter to the receiver.

- A *receiver R*, which reconstructs the message from the signal.

- A *message destination B*, which receives the message.

$$A \xrightarrow{\text{message}} T \xrightarrow{\text{signal}} C \xrightarrow{\text{signal}} R \xrightarrow{\text{message}} B$$
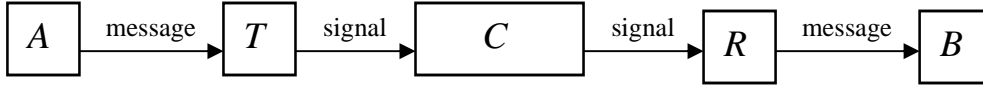
The message source *A* is a system of *n* states a_i, which can be thought as the *letters* of an alphabet $A_A = \{a_1, ..., a_n\}$, each with its own probability $p(a_i)$; the sequences of *N* states-letters are called *messages*. Analogously, the message destination *B* is a system of *m* states b_j, *letters* of an alphabet $A_B = \{b_1, ..., b_m\}$, each with its own probability. On the basis of these elements, the entropies of the source $H(A)$ and of the destination $H(B)$ can be computed as:

$$H(A) = -\sum_{i=1}^{n} p(a_i) \log p(a_i) \qquad\qquad H(B) = -\sum_{j=1}^{m} p(b_j) \log p(b_j) \qquad\qquad (1)$$

and are measured in *bits* when the logarithm to base 2 is used. When $-\log p(a_i)$ is interpreted as a measure of the information generated at the source *A* by the occurrence of a_i, $H(A)$ turns out to be the average amount of information generated at the source *A*. The aim of communication is to identify the message produced at the source *A* by means of the message received at the destination *B*.

The entropies $H(A)$ and $H(B)$ are related through the *mutual information* $H(A;B)$, that is, the information generated at *A* and received at *B*, which can be computed as:

$$H(A;B) = -\sum_{i=1}^{n}\sum_{j=1}^{m} p(a_i, b_j) \log \frac{p(a_i) p(b_j)}{p(a_i, b_j)} = H(A) - E = H(B) - N \qquad\qquad (2)$$

where the *equivocity E* is the information generated at *A* but not received at *B*, and the *noise N* is the information received at *B* but not generated at *A*. In turn, the correlations between source and destination are represented by the matrix $\left[p(b_j / a_i) \right]$, where $p(b_j / a_i)$ is the conditional probability of the occurrence of b_j at *B* given that a_i occurred at *A*, and the elements in any row add up to 1. The largest amount of information that can be *transmitted* over the channel *C* is measured by the *channel capacity CC*, defined as:

$$CC = \max_{p(a_i)} H(A;B) \qquad\qquad (3)$$

where the maximum is taken over all the possible distributions $p(a_i)$ at *A*.

The transmitter *T* encodes the messages produced by the message source: coding is a mapping from the source alphabet $A_A = \{a_1, ..., a_n\}$ to the set of finite length strings of *symbols* from the code alphabet $A_C = \{c_1, ..., c_q\}$, also called *code-words*. Whereas the number *n* of the letters of A_A is usually any number, the code alphabet A_C is more often binary: $q = 2$. In this case, the symbols are *binary digits* (binary alphabet symbols). On the other hand, the code alphabet A_C can be physically implemented by means of systems of *q* states.

The code-words do not have the same length: each code word w_i, corresponding to the letter a_i, has a length l_i. Therefore, coding is a fixed- to variable-length mapping. The *average code-word length* can be defined as:

$$\langle l \rangle = \sum_{i=1}^{n} p(a_i)\, l_i \tag{4}$$

$\langle l \rangle$ indicates the compactness of the code: the lower the value of $\langle l \rangle$, the greater the efficiency of the coding, that is, fewer resources $L = N\langle l \rangle$ are needed to encode the messages of length *N*. The *Noiseless-Channel Coding Theorem* (First Shannon Theorem) proves that, for sufficiently long messages ($N \rightarrow \infty$), there is an optimal coding process such that the average length *L* of the encoded message is as close as desired to a lower bound $L_{\min}$ computed as

$$L_{\min} = \frac{NH(A)}{\log q} \tag{5}$$

When the code alphabet has two symbols, then $L_{\min} = NH(A)$. The proof of the theorem is based on the fact that the messages of *N* letters produced by the message source *A* fall into two classes: one of them consisting of $2^{NH(A)}$ typical messages, and the other composed of the atypical messages. When $N \rightarrow \infty$, the probability of an atypical message becomes negligible; so, the source can be conceived as producing only $2^{NH(A)}$ possible messages. This suggests a natural strategy for coding: each typical message is encoded by a binary sequence of length $NH(A)$, in general shorter than the length N of the original message.

This formalism has received and still receives different interpretations. Some authors conceive Shannon information as a physical magnitude, whereas others consider that the primary meaning of the concept of information is always linked with the notion of knowledge (see discussion in Lombardi, Fortin and Vani 2015). In this section we do not dwell on this issue, but will only focus on the similarities and the differences between Shannon's formalism and Schumacher's formalism.

Although there were many works on the matter before the article of Benjamin Schumacher (1995) "Quantum Coding" (see, for instance, Ingarden 1976), this work is usually considered the

first precise formalization of the quantum information theory. The main aim of the article is to prove a theorem for quantum coding analogous to the noiseless coding theorem of Shannon's theory. With this purpose, Schumacher conceives the message source A as a system of n states-letters a_i, each with its own probability $p(a_i)$; then, A has a Shannon entropy $H(A)$ computed as in eq. (1). In turn, the transmitter T maps the set of the states-letters a_i of the source A onto a set of n states $|a_i\rangle$ of a quantum system M. The states $|a_i\rangle$ belong to a Hilbert space $\mathcal{H}_M$ of dimension $\dim(\mathcal{H}_M) = d$ and may be non-orthogonal. The mixture of states of the signal source M can be represented by a density operator:

$$\rho = \sum_{i=1}^{n} p(a_i)|a_i\rangle\langle a_i| \tag{6}$$

whose von Neumann entropy is:

$$S(\rho) = Tr(\rho \log \rho) \tag{7}$$

In the case that the $|a_i\rangle$ are mutually orthogonal, the von Neumann entropy is equal to the Shannon entropy: $S(\rho) = H(A)$. In the general case, $S(\rho) \leq H(A)$.

Given the above mapping, the messages $(a_{i1}, a_{i2}, ..., a_{iN})$ of N letters produced by the message source A are encoded by means of sequences of N quantum states $(|a_{i1}\rangle, |a_{i2}\rangle, ..., |a_{iN}\rangle)$, with $i \in \{1, 2, ..., n\}$. This sequence can be represented by the state $|\alpha\rangle = |a_{i1}, a_{i2}, ..., a_{iN}\rangle$ of a system M^N, belonging to a Hilbert space $\mathcal{H}_{M^N} = \mathcal{H}_M \otimes \mathcal{H}_M \otimes ... \otimes \mathcal{H}_M$ (N times), of dimension d^N. This state is transmitted through a channel C composed of L two-state systems Q called *qubits*, each represented in a Hilbert space $\mathcal{H}_Q$ of dimension 2. Therefore, the Hilbert space of the channel will be $\mathcal{H}_C = \mathcal{H}_Q \otimes \mathcal{H}_Q \otimes ... \otimes \mathcal{H}_Q$ (L times), of dimension 2^L. Analogously to the Shannon case, L indicates the compactness of the code: the lower the value of L, the greater the efficiency of the coding, that is, fewer qubits are needed to encode the messages. The *Quantum Noiseless-Channel Coding Theorem* proves that, for sufficiently long messages, the optimal number $L_{\min}$ of qubits necessary to transmit the messages generated by the source with vanishing error is given by $NS(\rho)$.

Schumacher designs the proof of the theorem by close analogy with the corresponding Shannon's theorem. Again, the idea is that all the possible states $|\alpha\rangle$ (representing the messages of N letters produced by the message source A), belonging to $\mathcal{H}_{M^N}$ of dimension $d^N = 2^{N \log d}$, fall into two classes: one of typical states belonging to a subspace of $\mathcal{H}_{M^N}$ of dimension $2^{NS(\rho)}$, and the other of atypical messages. When $N \to \infty$, the probability of an atypical state becomes negligible; so, the source can be conceived as producing only messages encoded by states belonging to a subspace of $2^{NS(\rho)}$ dimensions. Therefore, the channel can be designed to be represented in a Hilbert

space $\mathcal{H}_C$ such that $\dim(\mathcal{H}_C) = 2^L = 2^{NS(\rho)}$, and this means that the minimum number $L_{\min}$ of qubits necessary to transmit the messages of the source is $L_{\min} = NS(\rho)$.

From the above presentation it is clear that, both in Shannon's and in Schumacher's works, the stage of generating information and the stage of coding information are distinguished. It is also clear that in the generation stage there was no appeal to a particular physical theory: the physical system that plays the role of message source may be classical-mechanical, electromagnetic, thermodynamical, and even quantum-mechanical. In other words, the task of the message source may be performed for any kind of physical system producing distinguishable states that will be identified at the destination end in a successful communication. In turn, nothing is said about how the probabilities of the message source are determined or about their interpretation: they may be conceived as propensities theoretically computed, or as frequencies previously measured. It is in this sense that it can be said that the generation stage is independent of its physical substratum: the states-letters of the message source are not physical states but are *implemented* by physical states. Physical matters become relevant only when the coding stage is considered: when the transmitter encodes the output of the message source, the code symbols can be implemented by means of classical states or of quantum states. In turn, the kind of systems used for coding determines how to compute the efficiency of information transmission (nevertheless for a discussion about the quantum resources necessary to implement the protocols of quantum information theory, see Section 5).

Schumacher's formalism had a great impact on the physicist community: it is very elegant, and its analogy with Shannon's classical work is clear. Nevertheless, these facts do not supply yet an answer about the concept of quantum information.

4.- Two kinds of information?

In the literature on the matter one can find a number of implicit or explicit arguments for which quantum information is something qualitatively different from classical information. In this section we will critically analyze the most widely used arguments.

4.1.- Two kinds of source, two kinds of information?

A usual claim is that quantum information is what is produced by a quantum information source, that is, a device that generates different quantum states with their corresponding probabilities (see, e.g., Timpson 2004, 2008, 2013, Duwell 2008). Those who adopt this characterization of quantum

information in general stress the elegant parallelism between Shannon's and Schumacher's proposals.

A first difficulty of this characterization is that this is not what Schumacher says. On the contrary, following closely the terminology introduced by Shannon (which distinguishes between *message* and *signal*, and between *source* and *transmitter*, see previous section), Schumacher begins by defining the *message source A* that produces each a_i with probability $p(a_i)$, and only in the stage of coding he introduces the *quantum signal source*, which "*is a device that codes each message a_M from the source A into a "signal state" $|a_M\rangle$ of a quantum system M*." (Schumacher 1995, p. 2738). This means that the quantum states involved in the process described by Schumacher do not come from a message source, but from a quantum system *M* that is part of the device that encodes the *messages* produced by the message source and turns them into *signals* to be transmitted through the channel. In other words, the quantum system *M* is part of the device called '*transmitter*'. Schumacher calls the process developed between transmitter and receiver '*transposition*', and describes it in the following terms: "*We can therefore imagine a communication scheme based upon transposition. At the coding end, the signal of a source system M is transposed via the unitary evolution U into the coding system X. The system X is conveyed from the transmitter to the receiver. At the decoding end, the unitary evolution U' is employed to recover the signal state from X into M', an identical copy of system M* " (Schumacher 1995, p. 2741). Here it is clear that the system *X* "*is conveyed from the transmitter to the receiver*", not from the message source *A* to the message destination *B*. Moreover, the system *M* is placed at the *coding end* and the system *M'* is placed at the *decoding end*; so, *M* is not the message source *A*.

The terminology used by Schumacher along the entire paper is very coherent. In fact, even in the last section before the closing remarks, where he considers the situation in which the quantum states arise as part of a larger system that is in an entangled state (the quantum states are improper mixtures), he clearly talks about the stage of coding-transmitting-decoding: the quantum states is still characterized as "*the signal states of M*" (p. 2745), and he is still interested in the "*approximate transposition from M to M'* " (p. 2746). In other words, the focus of the paper is on the stage of coding in the transmitter, transmitting through the channel, and decoding at the receiver: there is no quantum source of quantum information that produces quantum states as messages; the quantum states involved in the processes, whether pure, proper mixtures or improper mixtures, are not the messages to be communicated but the signals to be transposed. This remark is in agreement with what is suggested by the title itself of Schumacher's article: "Quantum Coding" and not "Quantum Information".

Nevertheless, somebody might retort that, although Schumacher is clear in his paper, not even the position of a founding father of a discipline should replace a good argumentation. What prevents us from considering M a quantum source and from defining quantum information as what is generated by a quantum source? From this perspective, M and M' would be the source and the destination of the messages, and the goal of communication would be to reproduce at the destination M' the same (type) state as that produced at the source M. Besides the fact that this is not the goal of communication in the practice of science and engineering (recall Section 2), further arguments can be given against this position.

First, this view implies to confuse the effectiveness of communication, measured by the *mutual information* $H(A;B)$, with the effectiveness of transposition, measured by the *fidelity F* of the process, defined as (Schumacher 1995, p. 2742):

$$F = \sum_{i=1}^{n} p(a_i) Tr |a_i\rangle\langle a_i| \omega_i \qquad (8)$$

where the $|a_i\rangle\langle a_i|$ correspond to the signal states produced at M, and the ω_i represent the signal states obtained at M' as the result of the transposition, which do not need to be pure (here we consider pure signal states produced at M, but the definition can be generalized to mixed signal states). Since fidelity measures the effectiveness of the stage of transmission through the channel, it is a property of the channel: the fidelity of a transmission is less than unity when the channel is limited in the sense that $\dim(\mathcal{H}_C) < \dim(\mathcal{H}_{M^N})$ (although it is indefinitely close to unity when $\dim(\mathcal{H}_C) = 2^{NS(\rho)}$, as proved by the quantum coding theorem). By contrast, communication is maximally effective when $H(A;B)$ is maximum, that is, when the equivocity E is zero (see eq. (2)), and this, in turn, means that there is no loss of information between the message source A and the message destination B. In other words, all the information generated at A is recovered at B and, therefore, the states produced at the source A can be identified by means of the states occurred at the destination B. Of course, the success of a certain situation of communication based on quantum transposition will be a function of the fidelity of the transposition, but also of the reliability of the operations of coding and decoding, which correlate the states a_i of the message source A with the quantum states $|a_i\rangle$ of M, and the quantum states ω_i of M' with the states b_i of the message destination B, respectively. In other words, the closeness to success in a particular situation of communication depends on the whole communication arrangement, and not only on the transmission stage.

In the second place, when working with non-orthogonal states, the state at the supposed destination M' cannot be distinguished from other states by measurement, so it cannot be used to identify the state occurred at the supposed source M. So, if M were the quantum source that

generates quantum information, quantum information would be something that, in principle, that is, on the basis of the theory itself, cannot be communicated. However, the protocols of quantum information do not abandon the goal of communication: they only intend to make communication secure or to improve its efficiency. Then, strategies to recover the information of the source even in these cases can be designed. As Dennis Dieks clearly explains: "*This* [the generic non-orthogonality of quantum states] *does not mean that messages sent via quantum coding will always remain partly illegible: one can devise strategies that make the probability of error as small as one wishes in the long run. A basic strategy here is to introduce redundancy by sending the same information multiple times: comparison of the measurement outcomes on repeated encoded words will make it possible to reconstruct the original message with an increasing level of reliability*." (Dieks 2016, p. 1). But, at the end of the day, the goal is always communication in the traditional sense, which, as noticed in Section 2, requires the identification of the state occurred at the source by means of the state occurred at the destination.

Thirdly, the very idea of a quantum source of information leads to conceptual perplexity. If the quantum states to be transmitted were the elements of the message produced by the quantum source of messages, where would the coding process be located? In fact, what is produced by the message source would be the same as what is transmitted, and the term 'coding' would turn out to be vacuous.

Finally, if quantum information were fully identified with the quantum states produced by a quantum message source, the transmission of information would be reduced to the transposition of quantum states. Indeed, if the fact that transposition is only a part of the communication process were forgotten and the roles played by the message source and the message destination were disregarded, nothing would change in the discourse about quantum information if the term 'quantum information' were replaced by the term 'quantum state.' The argument can be posed in other terms: since quantum information is what is communicated and a quantum state is what is transposed, the identification between communication and transposition amounts to the identification between quantum information and quantum state. As Armond Duwell clearly states, although it can be argued that there are specific properties that motivate a new concept of information, different from Shannon's, when those properties are revised, "[i]*t is obvious that there is already a concept that covers all of these properties: the quantum state. The term 'quantum information' is then just a synonym for an old concept*" (Duwell 2003, p. 498). In other words, 'quantum information' turns out to mean quantum state, and the whole meaningful reference to communication gets lost.

4.2.- Two kinds of coding, two kinds of information?

Another strategy to conceive quantum information as a different and peculiar kind of information is to link the very meaning of the concept of information with the coding theorems: if the theorems are different in the classical and the quantum case, the corresponding concepts of information are also different. For instance, Christopher Timpson *defines* the concepts of information in terms of the noiseless coding theorems: "*the coding theorems that introduced the classical (Shannon, 1948) and quantum (Schumacher, 1995) concepts of information$_t$* [the technical concept of information] *do not* merely *define measures of these quantities. They also introduce the concept of* what it is *that is transmitted,* what it is *that is measured.*" (Timpson 2008, p. 23, emphasis in the original). But this definitional strategy also has a number of conflicting consequences (see detailed discussion in Lombardi, Holik and Vanni 2016).

The first point to notice here is that, as explained in Section 3, the coding theorems are proved for the case of very long messages, strictly speaking, for messages of length $N \to \infty$. Therefore, if the noiseless coding theorems embodied the nature of classical and quantum information, one wonders whether short messages can be conceived as conveying information, to the extent that they are not covered by those theorems. Moreover, if the theorems defined the very concepts of information, they would allow us to conceive as information only the Shannon and the von Neumann entropies. As a consequence, it would make no sense to talk about the individual amount of information conveyed by a single state of the message source. Or it should be accepted that the information per letter associated with a message is derivatively defined in terms of the entropy $H(A)$ of the message source (see Timpson 2013, p. 22). In either case, against the usual understanding, the entropy $H(A)$ of the message source can no longer be interpreted as an average amount, since only in terms of previously defined individual amounts an average can be defined as such. The distinction between conceiving the entropies of the source and the destination as measuring amounts of information or average amounts of information might seem an irrelevant detail; however, this is not the case when we are interested in elucidating the very notion of information.

Secondly, let us recall that, when explaining the elements of the general communication system, Shannon (1948, p. 381) characterizes the transmitter as a system that operates on the message coming from the source in some way to produce a signal suitable for transmission over the channel. And he adds that, in many cases, such as in telegraphy, the transmitter is also responsible for encoding the source messages. However, as any communication engineer knows, in certain cases the message is not encoded; for instance, in traditional telephony the transmitter's operation "*consists merely of changing sound pressure into a proportional electrical current.*" (Shannon 1948,

p. 381). If information is defined in terms of the noiseless coding theorem, how to talk about information in those situations that do not involve coding? If one insisted on this definitional strategy, the entropy of the source would turn out to be defined in terms of something that is not essential to the source itself −coding.

In the third place, the strategy of defining the concept of information in terms of the coding theorems leads to some conceptual puzzles. In fact, the message source A would generate different kinds of information with no change in its own nature: the kind of information generated would depend not on itself, but on how the messages will be encoded later. Moreover, if the kind of coding to be used at the coding stage were not decided yet, the very nature −classical or quantum− of the information generated by the message source A would be indefinite, and would remain as such up to the moment at which the decision were made.

All these difficulties immediately disappear when two concepts involved in communication are carefully distinguished: the information generated at the message source, which depends on the probability distribution over the source's states and is independent of coding −even independent of the very fact that the messages are encoded or not−, and the resources necessary to encode those states, which depend not only on that probability distribution, but also on the particular coding selected, classical or quantum.

4.3.- The peculiarity of teleportation

Teleportation is one of the most discussed issues in the field of quantum information. Although a direct result of quantum mechanics, it appears as a weird phenomenon when described as a process of transmission of information. Broadly speaking, an unknown quantum state $|\chi\rangle$ is transferred from Alice to Bob with the assistance of a shared pair of particles prepared in an entangled state and of two classical bits sent from Alice to Bob (the description of the protocol can be found in any textbook on the matter; see, e.g., Nielsen and Chuang 2010). In his detailed analysis of teleportation, Timpson poses the two central questions of the debate: "*First, how is so much information transported? And second, most pressingly, just how does the information get from Alice to Bob?*" (Timpson 2006, p. 596). Each question leads to its own specific difficulty.

Regarding to the first question, it is usually said that the amount of *classical* information generated at the source is, in principle, infinite, because two real numbers are necessary to specify the state $|\chi\rangle$ among the infinite states of the Hilbert space. It is also claimed that, even in the case that a coarse-graining is introduced in the Hilbert space, the amount of information is immensely greater than the two bits sent through the classical channel, and this great amount of information

cannot be transported by the two classical bits that Alice sends to Bob. However: how is classical information computed to support these claims? In order to compute the Shannon entropy $H(A)$, it is necessary to know which the possible states of the source A are and to count with the distribution of probability over those states: a source might have immensely many states such that only one of them has a probability almost equal to one; in this case, $H(A)$ would be close to zero. This means that describing a phenomenon as teleportation in informational terms makes no sense if the message source, with its possible states and their probabilities, is not precisely characterized.

However, if the qualitative difference between classical and quantum information is accepted, what about *quantum* information? How much quantum information is transferred? The usual answer is: one qubit per successful run of the teleportation protocol. But at this point it is necessary to recall that the term 'qubit' is endowed with a dual meaning: a qubit is primarily conceived as a two-state quantum system used to encode the messages produced by a source; but it is also understood as a unit of measurement of quantum information, which is quantified by the von Neumann entropy $S(\rho)$. If 'qubit' refers to a two-state quantum system, we cannot say that a qubit was transferred in teleportation: there is no quantum system that Alice sends to Bob. But if 'qubit' is interpreted as the unit of measurement of the quantum information carried by $|\chi\rangle$, difficulties do no disappear: the von Neumann entropy $S(\rho)$ corresponding to the state $|\chi\rangle$ is zero, because $|\chi\rangle$ is a pure state.

The perplexities related with Timpson's first question vanish when the role played by teleportation in communication is clearly understood. In fact, teleportation is not a process of communication, but of transposition: "*"quantum teleportation"* […] *is a rather exotic example of a transposition process*" (Schumacher 1995, p. 2741). In other words, teleportation is a physical process that allows a quantum state to be transferred between two spatially separated quantum systems without leaving a copy behind, and this process does not need to be conceptualized in informational terms to be understood: it can be better explained with no reference to information.

Let us now consider the second question: how does the information get from Alice to Bob? In traditional communication, the information is always transferred from the transmitter to the receiver by means of some physical signal. But in teleportation there is no physical carrier other than that represented by the two classical bits that Alice sends to Bob. Might it not be this feature what makes quantum information qualitative different from classical information? Whereas classical information always requires a physical carrier that travels through space in a finite amount of time, quantum information would not need a physical carrier but could be transferred by means of entanglement, which does not involve a physical signal traveling through space. This view, suggested as a possibility by Jeffrey Bub in a personal communication, would justify talking about quantum information in teleportation. Nevertheless, it has to be considered with care.

First, although teleportation is a way of taking advantage of entanglement to implement transposition, this does not mean that any transposition process needs to be implemented by entanglement. Transposition needs the signal to be conveyed from the transmitted to the receiver: "*We can therefore imagine a communication scheme based upon transposition. At the coding end, the signal of a source system M is transposed via the unitary evolution U into the coding system X. The system X is conveyed from the transmitter to the receiver. At the decoding end, the unitary evolution U^{-1} is employed to recover the signal state from X into M', an identical copy of system M [...] The system X is the quantum channel in this communication scheme, and supports the transposition of the state of M into M'.*" (Schumacher 1995, p. 2741). It is clear that this process can be carried out by means of entanglement, in particular, of the "rather exotic" case of teleportation. But transposition can also be met by sending a quantum physical system *X* from *M* to *M'* through space and time, and the whole formalism of quantum information theory still applies. This means that quantum information cannot be defined by the fact that it is transmitted without a physical carrier traveling through space and time. Eventually, the essential feature of quantum information would be, as Bub suggests, that *it is possible* to transmit it without a physical carrier.

However, the idea that the hallmark of quantum information is that it does not need a physical carrier to be transferred faces the same conceptual puzzle as that already pointed out in the previous subsection. Again, the message source *A* would generate different kinds of information, quantum or classical, with no change in its own nature, but depending on a feature of the stage of transmission, in this case, whether the information may be transmitted without a physical carrier or not.

Timpson (2004, 2013) is right in finding the origin of the puzzles usually attached to teleportation in a particular physical interpretation of information, which assumes that the transmission of information between two points of the physical space necessarily requires an information-bearing signal, that is, a physical process propagating from one point to the other. He cuts the Gordian knot of teleportation by adopting a deflationary view of information, according to which "*there is not a question of information being a substance or entity that is transported, nor of 'the information' being a referring term.*" (2006, p. 599). The moral of the present subsection is that, when teleportation is understood as a kind of transposition process and not as a whole process of communication, the difficulties vanish without the commitment to a particular interpretation of information. Therefore, if there is a puzzle in teleportation, it is the old quantum puzzle embodied in non-locality, and not a new mystery about a new kind of information

5.- *Quantum information and quantum mechanics*

According to several authors (Timpson 2003; Duwell 2003; Lombardi 2004, 2005; Lombardi, Fortin and Vanni 2015), the information described by Shannon's theory and measured by the Shannon entropy is not classical, but is neutral with respect to the physical theory that describes the systems used for its implementation. Armond Duwell expresses this idea very clearly: "*The Shannon theory is a theory about the statistical properties of a communication system. Once the statistical properties of a communication system are specified, all information-theoretic properties of the communication system are fixed.* [...] *Hence, the Shannon theory can be applied to any communication system regardless whether its parts are best described by classical mechanics, classical electrodynamics, quantum theory, or any other physical theory.*" (Duwell 2003, p. 480). By contrast, quantum information is usually conceived as inextricably linked to quantum mechanics. The idea that quantum mechanics dictates the need of a new kind of information is very widespread in the physicist community (Jozsa 1998, Brukner and Zeilinger 2001). It is interesting to notice that this view breaks the usually stressed parallelism between the classical and the quantum case: whereas Shannon information is physically neutral, quantum information would be essentially tied to quantum mechanics.

Another idea that pervades the literature on the subject is that, since for a mixture of orthogonal states $S(\rho) = H(A)$, Shannon information is a particular case of quantum information: it is the case in which the states are distinguishable. Jeffrey Bub explicitly expresses this view: "*Classical information is that sort of information represented in a set of distinguishable states −states of classical systems, or orthogonal quantum states− and so can be regarded as a subcategory of quantum information, where the states may or may not be distinguishable.*" (Bub 2007, p. 576). Or, the other way around, von Neumann entropy is conceived "*as a generalization of the notion of Shannon entropy.*" (Bub 2007, p. 576). From this viewpoint, Shannon information is classical and, as a consequence, it loses its physical neutrality. Moreover, Shannon/classical information is a particular case of quantum information. In other words, the basic or more fundamental concept would be that of quantum information, because it does not introduce constraints regarding orthogonality, whereas classical information would be a secondary concept, since restricted to the case of orthogonality. Although for different reasons, this view also breaks the parallelism between the classical and the quantum case: the notions of classical and quantum information are not at the same level from a conceptual viewpoint. What underlies it is the assumption that classical mechanics is also a kind of particular case of quantum mechanics: classical systems are quantum systems in the classical limit.

These different ways of conceiving quantum information as strongly tied to quantum mechanics have consequences on the attempts to reconstruct quantum mechanics in informational terms (Fuchs 2002, Clifton, Bub and Halvorson 2003). If the reconstruction has no other purpose than showing that it is possible to express quantum mechanics in informational terms, the link between quantum mechanics and quantum information is a mere manifestation of that possibility. But if the reconstruction is conceived as a foundational program, designed to show that the foundations of quantum mechanics are informational, the program runs the risk of becoming circular. In fact, if the quantum-informational constraints used to reconstruct quantum mechanics are due to the existence of quantum information, whose nature and features, in turn, depend on the features of quantum mechanics, something sounds odd in the whole foundational proposal. The risk of circularity is averted from an alternative conceptual position: there is no quantum information as different from classical information; there is a single kind of information, which is not tied to a particular physical theory, and that can be encoded by means of classical or quantum resources. Therefore, any attempt to reconstruct a physical theory −not only quantum mechanics− in informational terms will rely on physically neutral bases.

This neutral conception of information has an additional conceptual advantage. Either for simplicity reasons or due to the conviction that reality is a harmonious whole and not an incoherent plurality, during the history of science the unification of different theories has been widely considered a desirable goal. In turn, in most historical cases, such a goal was pursued by reductionist means. However, at present –and already since several decades ago− reductionism tends to be viewed with, at least, a grain of skepticism, both in the physical and in the philosophical communities. In the face of this situation, the physical neutrality of information allows to preserve the ideal of unification without commitment to reductionism, since it opens the way for a non-reductive unification of physics: if different physical theories can be reconstructed on the same neutral informational basis, they could be meaningfully integrated into a single theoretical network and compared to each other, with no need to search for reductive links among them.

Additionally, this physically neutral way of conceiving information paves the way to consider, from a conceptual viewpoint, a question that has attracted much theoretical attention during the last decades: how much of quantum mechanics is necessary to implement the protocols of quantum information theory?

In the context of discussions about interpretation, the philosophy of physics has provided different classical models of quantum mechanics. Examples of them are those supplied by Diederik Aerts, in particular, that of an elastic band in a sphere, which produces a quantum structure isomorphic to the structure of a two dimensional complex Hilbert space (Aerts 1986, 1998). From a

more general viewpoint, Wayne Myrvold (2010) stresses that some features traditionally considered specifically quantum can be recovered in a formalism that deals with classical mixed states defined as probability measures over a classical phase space (or in the Hilbert space formalism of classical statistical mechanics, see Koopman 1931). In particular, two classical mixtures can be defined as orthogonal if and only if their supports are disjoint; therefore, non-orthogonal states can be also defined in a classical framework.

During the last decades, much physical work has been directed in the same conceptual direction. Howard Barnum and collaborators (2007) proved a generalized version of the no-broadcasting theorem (Barnum *et al.* 1996), according to which a set of states is broadcastable if and only if it is contained in a simplex generated by states that are jointly distinguishable by measurement. On this basis, this theorem is considered as a criterion to distinguish between classical and quantum: the theories whose sets of states are represented by a simplex are usually considered "classical". However, this is a formal result: the question about whether and to what extent a non-simplex of states can be "simulated" by classical mechanics is still a matter of intensive research.

As it is well known, according to Schrödinger (1936), the essential difference between the classical and the quantum is located in entanglement. However, there are "classical" simulations of quantum mechanics that recover some features of the theory even regarding entanglement (Spreeuw 1998, Collins and Popescu 2002). Furthermore, some classical simulations of the violations of Bell inequalities have been proposed (e.g. Aerts *et al.* 2000, Frisch 2002, Mor 2006, Goldin *et al*. 2010).

Perhaps the first attempts to simulate non-locality by classical means appear precisely in the field of information theory, regarding teleportation. For instance, Sandu Popescu (1994) shows that it is possible to implement the teleportation protocol with reasonable fidelity by means of states that do not violate Bell's inequalities. This implies that teleportation can be obtained with states that can be modeled by local hidden variables. More recently, the possibility of classical implementation of teleportation has been argued for in the field of classical optics (Spreeuw 2001, Francisco and Ledesma 2008).

Still in the context of information theory, even the no-cloning theorem, originally obtained in the quantum context (Dieks 1982, Wootters and Zurek 1982; see the extension to mixtures in Barnum *et al*. 1996), can be proved in the classical statistical domain by taking overlapping probability distributions with non-trivial supports as dynamical variables (Daffertshofer *et al*. 2002; see discussion in Teh 2012).

In the last times, the research on classical analogues of quantum features has experienced a strong development in classical wave-optics. Besides the already mentioned case of teleportation, certain effects usually associated with quantum entanglement have been obtained with classical fields (Lee and Thomas 2002, Qian and Eberly 2011), and different quantum-information processes and phenomena, such as Greenberger-Horne-Zeilinger entanglement and a quantum error correction network, have been reproduced by means of classical optical processes (Spreeuw 2001). The appeal to classical optics has permitted, during the last years, the empirical implementation of the theoretical results previously obtained: at present there is a growing field of experimental research in this field (see, e.g., Borges *et al.* 2010, Töppel *et al.* 2014, Qian *et al.* 2015).

Optical simulation has been applied particularly to the computational context (see, e.g. Man'ko *et al.* 2001; for a simulation of a Hadamard gate, see Francisco *et al.* 2006a; a simulation of a quantum walk is presented in Francisco *et al*. 2006b). For instance, certain operations of a quantum computer can be performed by means of optical elements. A particular case is the optical implementation of Grover's algorithm for efficient searching; according to certain authors, this is achieved by means of "*a physical system that relies on superposition, interference, and non-factorizable states to function. Since these may also be classical phenomena, we conclude that many ingredients of quantum algorithms are not necessarily non-classical.*" (Kwiata *et al*. 2000, p. 265; for a physical implementation of the algorithm, see Bhattacharya *et al.* 2002). This and other quantum algorithms have been simulated by using programmable liquid-crystal displays (Puentes *et al.* 2004). It has also been shown that a nontrivial quantum computing optical device can easily be constructed if the number of component qubits is not too large (Cerf *et al.* 1998). A relevant theoretical result in the computational context is given by the Gottesman-Knill theorem (Gottesman 1999), according to which quantum algorithms that only use certain specific operations (those belonging to the so-called Clifford group) can be efficiently simulated by classical operations (for an interesting analysis of the conceptual meaning of the theorem, see Cuffaro 2015).

During the last decade a new classical model of quantum phenomena has been proposed and physically implemented. Droplets bouncing on a vibrating non-coalescent liquid generate waves as they bounce, and interact with their own waves: due to this phenomenon, they mimic non-local interactions reminiscent of the particle-wave associations in quantum mechanics. The droplets also interact with the environment (and other bouncing droplets), generating a complex dynamics which resembles that of particles guided by pilot waves (Couder *et al.* 2005a, Couder *et al*. 2005b, Couder and Fort 2006). Furthermore, it has been experimentally shown that bouncing droplets can be used to reproduce single-slit and double-slit diffraction, interference phenomena, tunneling, quantized energy levels, quantum random walks, and the creation/annihilation of droplet pairs (Protière *et al.*

2006, Eddi *et al*. 2011, Oza *et al*. 2013, Moláèek and Bush 2013). And, as it might be expected, these bouncing droplets have already led to interpretive reflections (Brady and Anderson 2014).

Of course, this theoretical and experimental trend, although widespread and prolific, does not cancel the differences between classical and quantum physics. Nevertheless, on this basis we may wonder which quantum resources are needed for quantum information theory. The whole theoretical power of quantum mechanics is necessary in every case, or certain quantum information processes can be implemented by classical models? The peculiarities of quantum mechanics are required for the possibility of implementing the information protocols, or only for obtaining efficient implementations? These questions open a wide field of conceptual and philosophical research based on recent theoretical and experimental results. However, such a work would not find a comfortable place in the framework of a position that presupposes the qualitative difference between classical and quantum information from the very beginning. In fact, if classical and quantum information are two different kinds of information according to their own nature, what kind of information, classical or quantum, is transferred in the case of a quantum informational protocol implemented by physically classical means? On the contrary, the view of information as physically neutral is particularly adequate for undertaking that conceptual research: it has the sufficient flexibility to accommodate different ways of implementing informational processes, to the extent that they are not tied a priori to a particular physical theory.

6.- Concluding remarks

In the present article we have argued that there seems to be no sufficiently good reasons to accept that there is a kind of information, the quantum information, qualitatively different from classical information. In particular, we have presented several arguments directed to challenge the idea that there are two different kinds of information source, classical and quantum, and against defining information in terms of the classical and quantum coding theorems. On this basis, we have defended the view that, in the communicational context, interpreting information as physically neutral is more adequate. Many conceptual challenges simply vanish when it is assumed that the difference between the classical and the quantum case is confined to the coding stage and does not affect the very nature of information.

We have also argued that this physically neutral view of information has, in turn, interesting conceptual advantages. First, teleportation loses its puzzling features with no need of commitment with a particular interpretation of information. Second, the reconstructions of quantum mechanics on the basis of informational constraints acquire better foundations. Third, the ideal of a non-

reductive unification of physics also finds support in the physical neutrality of information. Finally, the active research about classical models for quantum information attains a particular conceptual and philosophical interest.

The fact that many conceptual challenges vanish from our neutral view does not imply that all the interpretive problems about the concept of information disappear. In fact, there are several conceptual questions that can be posed in the context of information theory even before considering the different ways in which information is encoded. For instance: Is the concept of information a formal or an empirical concept? Is it a concrete or an abstract concept? Does information have any relationship with knowledge? Is there any sense in which information might be conceived as a physical magnitude? (see detailed discussion in Lombardi, Holik and Vanni 2016). Of course, the advent of quantum information has a relevant influence on the answers to these questions. Nevertheless, they remain as questions open to debate even when it is accepted that there are not two qualitatively different kinds of information, classical and quantum.

These conclusions do not intend to underestimate the relevance of the so-called 'quantum information theory.' This is a field that has grown dramatically in recent decades, supplying many new and significant results with promising applications. Our aim here has been exclusively conceptual. As it has been claimed previously −although for different reasons− (Timpson 2013, p. 237), the theory is not "(quantum information) theory", that is, a theory of quantum information, but "quantum (information theory)", that is, a theory about quantum resources applied to information theory. In this article, our purpose has been to support this claim from a philosophical perspective based on the physical neutrality of information.

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