Some Consequences of Physics for the Comparative Metaphysics of Quantity

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Abstract

According to comparativist theories of quantities, their intrinsic values are not fundamental. Instead, all the quantity facts are grounded in scale-independent relations like "twice as massive as" or "more massive than." I show that this sort of scale independence is best understood as a sort of metaphysical symmetry–a principle about which transformations of the non-fundamental ontology leave the fundamental ontology unchanged. Determinism–a core scientific concept easily formulated in absolutist terms–is more difficult for the comparativist to define. After settling on the most plausible comparativist understanding of determinism, I offer some examples of physical systems that the comparativist must count as indeterministic although the relevant physical theory gives deterministic predictions. Several morals are drawn. In particular: comparativism is metaphysically contingent if true, and it is most natural for a comparativist to accept an at-at theory of motion.

1 Introduction

The notion of a physical quantity or magnitude is, so far as we know, impossible to do without. Mass is one example, and it will function as my central example here. Mass is unlike simple monadic properties in that individuals don't simply have it or lack it – it comes in degrees. These degrees, or *values* of mass, are commonly treated like monadic properties. Each individual either has or lacks a mass of one gram – but this sort of description leaves out a lot of relevant "mass facts."

One such fact is that nothing has both a mass of one gram and a mass of two grams. Also, there's a special relationship between a one-gram object and a two-gram object, since the latter is twice as massive. We need a theory of quantity that can capture all of these mass facts. Indeed, as Mundy (1987, 29) has pointed out, such a theory is implicitly presumed by any quantitative scientific theory, and thus is essential to the empirical success of science. In this sense, a successful theory of quantity is one of the most valuable contributions metaphysics can offer to the scientific project.

To this end, two general theories have been proposed: the *comparativist* theory of quantity and the *absolutist* theory. The comparativist theory holds that relations like "more massive than" and "twice as massive as" give an exhaustive list of the fundamental mass facts. The absolutist view holds that there are some extra fundamental facts as well, namely which intrinsic mass property each object has, so that without intrinsic properties like "has a mass of one gram" the list of metaphysically basic mass facts is incomplete.

If the above definition of comparativism seems imprecise, that's because no fully precise definition of comparativism has yet been proposed. Instead, most of comparativism's defenders have focused their efforts on motivating their own particular versions of the theory. But there is a common thread in the metaphysics of quantity proposed by Bigelow and Pargetter (1988), Arntzenius (2012, 49-59) and Dasgupta (forthcoming), along with that presupposed by the approach to measurement theory outlined in Suppes and Zinnes (1963) and the nominalist physics of Field (1980).

On all these comparativist accounts, the fundamental relations are independent of scale. That is to say, on these accounts, the fundamental relations holding between a one-gram massive object and a ten-gram object are exactly the same as those holding between a one-*kilogram* object and an object massing ten kilograms. This scale independence of the fundamental relations is one point of overlap between comparativist theories of quantity. The best way to understand scale independence is as a sort of *symmetry*–a class of transformations which, when applied to the ontology of a possible world, leave the comparativist's fundamental ontology unchanged. So the scale independence of the comparativist's fundamental mass facts amounts to the fact that mass doubling, mass tripling and so on do not change these facts. I explain all this in detail in §2.

The correct definition of determinism for the comparativist is difficult to formulate, but this notion of scale independence allows us to settle on what is at least a plausible necessary condition. For a comparativist interpretation of physics to count as deterministic, the scale-independent facts about the past and present must fix the scale-independent facts about the future. This will be established in §3.

Given this definition of determinism, we can formulate a fairly quick and straightforward argument that any comparativist interpretation of Newtonian gravity must be indeterministic in a wide variety of cases. But this argument's soundness depends in an interesting way on a metaphysical premise: it only works if velocity is taken to be a truly instantaneous quantity, and not a property of infinitesimal temporal neighborhoods. In §4 I show that this means the comparativist should accept a theory of motion, like the popular "at-at" theory, according to which there are no truly instantaneous velocities.

One can also construct more involved examples in which even an at-at theorist should agree that comparativist physics gives indeterministic predictions where absolutist physics does not. In some such cases, comparativist physics exhibits a peculiar sort of temporal action at a distance: the state of the entire past may be sufficient to determine the future, while the present state is insufficient. (In other cases, even the whole past history is insufficient.) These examples are explored in §5. They have the flavor of idealized toy models, so I hesitate to count them as evidence against the truth of comparativism. But it seems to me that determinism and temporal locality, in these cases, should not be ruled out as metaphysically impossible. Consequently, if comparativism is true, it is contingently true.

2 Scale independence

As I mentioned above, the precise definition of comparativism is somewhat up for grabs. Relatively few metaphysicians have gone on record as comparativists, and of those who have, most have defended specific theories of quantity rather than comparativism in general. The only exception is Dasgupta (forthcoming). Dasgupta is concerned with defending comparativism broadly speaking, rather than identifying some particular system of comparative relations as fundamental. So he characterizes comparativism in the following way:

> [T]hings with mass stand in various determinate mass relationships with one another, such as x being more massive than y or x being twice as massive as y... *[C]omparativism* is the view that the fundamental facts about mass concern how

> material bodies are related in mass, and all other facts about mass hold in virtue of them. (Dasgupta, forthcoming, 1)

Although Dasgupta gives several examples of relations that might count as fundamental for particular comparativists, he never sets down criteria for these relations to meet. In fact, he goes on to add:

> [T]he comparativist thinks that the fundamental, unexplained facts about mass are facts about the mass relationships between bodies, and all other facts about mass hold in virtue of those mass relationships. This leaves open what kinds of mass relations those fundamental facts concern: they might concern mass ratios such as an object being twice as massive as another, orderings such as an object being more massive than another, or even just linear structures such an object lying between two others in mass. But this in-house dispute will not matter for our purposes. (Dasgupta, forthcoming, 3)

This seems to indicate that for Dasgupta, any relation at all could count as fundamental as long as it expresses some comparison between different values of a quantity like mass. But when he moves on to consider arguments for and against comparativism, it becomes clear that this is not what Dasgupta has in mind.

The first of several modal arguments he considers against comparativism is "that while it is possible for everythings mass to double tonight at midnight, the comparativist cannot make sense of this since the mass relationships would be exactly the same tomorrow as they were today." (Dasgupta, forthcoming, 7)[1] But this is only true if one restricts which comparative mass relationships are allowed to count as fundamental. If relations that depend on a choice of scale are left out, Dasgupta's point is quite correct. For example, the fact that my brother's mass is greater than mine will not change if every object's mass is doubled. But on the other hand, the fact that my brother's mass is ten kilograms greater than mine will certainly change. So Dasgupta must intend to leave out relations like this family of two-place relations:

a **Kn** *b***:** *a* is *n* kilograms more massive than *b*.

[1]Note that this problem is easily solved, as Dasgupta points out, by allowing mass relations to hold between objects at different times.

It thus appears that there must be some implicit restriction on which comparative relations can count as fundamental for Dasgupta's comparativist.

I agree with this implicit commitment of Dasgupta. There must be some restriction on which relations the comparativist may count as fundamental, or else the view threatens to collapse into an uninteresting variant of absolutism. If relations like **Kn** above are allowed to count as fundamental, comparativist possible worlds will be able to contain just as much ontological structure as absolutist worlds. There is nothing incoherent about a version of comparativism that recognizes as much structure as absolutism, but I don't see any appeal in such a view. Why not just be an absolutist?

Dasgupta's main argument for comparativism is that the comparativist requires less ontological structure to "build" a possible world than the absolutist does. In effect, he means that the absolutist recognizes more differences between possibilities than the comparativist does. While the absolutist will count an otherwise empty world containing two one-kilogram brass balls as distinct from a world with a pair of five-ton balls, the comparativist identifies these two worlds as the same. But if the **Kn** relations are allowed into the comparativist's roster of fundamental relations, almost all of this ontological parsimony disappears. If a privileged "zero mass" value were also picked out, the comparativist would recognize exactly as much structure (just as many differences between possible worlds) as the absolutist. It's hard to see why anyone willing to recognize this much structure in a quantity would not just adopt absolutism about that quantity.

Comparativism (at least when not used as a tool for implementing nominalism) is motivated by the thought that the scale-dependent features of a quantity are of no fundamental metaphysical importance. Part of what is distinctive and attractive about the comparativist picture is that these scale-dependent features are grounded in relations that are scale-independent (see Dasgupta, forthcoming, 14-16). But the **Kn** relations seem to have a scale built into them.

How should we define comparativism so as to rule out these problematic relations? I suggest the following modification (and generalization) of Dasgupta's definition: comparativism about some quantity – or family of quantities – is the view that the fundamental facts about those quantities are given by the *scale-independent* relations comparing different objects' values of the quantities. We may then define global comparativism to be comparativism about every quantity.[2]

This definition depends, of course, on a prior notion of what it is for a relation to be scale-independent. We could attempt to define this the way a physicist intuitively might, positing that the scale-independent relations are the ones that don't depend on our choice of units, or something like that. But (as Ted Sider has pointed out to me) such an attempt would be doomed. Regardless of how we normally express differences in mass, there is nothing to stop us from adopting a unit-independent language in which the relation of being five kilograms more massive is expressed without mentioning kilograms. Calling that particular relation K, we might just say that aKb whenever a is five kilograms more massive than b. And of course, this merely underscores that, while we normally represent the fact that a is five kilograms more massive than b in a way that varies depending on our choice of units, the *fact* that a is five kilograms more massive does not itself depend on our choice of units in any interesting way. Neither do the **Kn** relations.

Yet there remains a sense in which the **Kn** relations depend, not on our choice of units, but on the scale of the mass quantity. The best way to express this dependence seems to be via Dasgupta's observation that the comparative relations he regards as fundamental would not be changed by an operation like doubling the mass of every object in the universe. The **Kn** relations, on the other hand, would change. Dasgupta's comparativism (as well as Field's, and Bigelow and Pargetter's) differs from absolutism, not merely in that the fundamental facts about quantities are given by relations rather than absolute values, but also in the following way: there are transformations we can perform on the numerical values of quantities that alter which absolutist possible world the values represent, but not which comparativist world they represent. When we double the values of mass, we have changed something fundamental about the world if the absolutist is right, but not if the comparativist is right. We may therefore say there is a sort of *symmetry* to the comparativist theory of quantity that the absolutist theory lacks: transformations multiplying every value of a quantity by some constant leave the comparativist's fundamental ontology invariant, but not the absolutist's fundamental ontology.

[2] I will mostly focus on global comparativism, which I think is the most interesting and unified thesis, and which comparativists like Dasgupta, Bigelow and Pargetter endorse. But even on an absolutist approach, there remains the question, for any given quantity we use in physics, of whether that quantity's absolute values are physically meaningful (see e.g. Skow, 2011). So something akin to comparativism about particular quantities may be warranted even if global absolutism is preferable to global comparativism.

We may thus adopt the following as a definition of scale independence: A comparative relation for a quantity like mass is *scale-independent* iff, when the quantity is represented numerically, multiplying its values by a constant cannot change whether the relation holds.[3] Comparativism, in its most interesting form (and the form that has appeared in the metaphysics literature to date) is best understood as the claim that the fundamental facts about quantity are given by scale-independent relations.

Besides fitting with Dasgupta's theory, this is very much in keeping with other authors identified as comparativist. When characterizing "the many relations which are associated with a quantity like mass" in their "relational" theory of quantity, Bigelow and Pargetter (1988, 298) include "relations like 'more massive than,' 'half as massive as,' and so forth," but no scale-dependent relations. This variety of comparativism is obviously unsatisfactory for Field's purposes, since describing it requires reference to numbers. Field wants to pursue a nominalist interpretation of Newtonian physics which can be formulated without referring to any mathematical objects. Thus he defines a scalar physical quantity Q via the following system of primitive predicates:

x Q-**Bet** yz**:** x's value of Q is between y's and z's.

xy Q-**Cong** zw The (absolute value of the) difference between x's value of Q and y's is the same as the difference between z's value of Q and w's.

x Q-**Less** y x's value of Q is less than y's value of Q. (Field, 1980, 55-60)

The important thing for present purposes is that all of these relations are scale-independent. Doubling the value of Q for every object in a world will lead to no change in any of these relations.[4]

The scale independence of the comparativist's fundamental ontology will be crucial in examining the question of determinism. In particular, it will allow us to evaluate whether a deterministic comparativist version of a given absolutist scientific theory is possible. In some cases, this can be done even in the absence of a thorough definition of determinism for the comparativist–a definition which may be quite difficult to arrive at.

[3]The reader may be concerned that representing a quantity's values numerically is itself incompatible with comparativism, but the representation theorems discussed in the next section establish that this is indeed possible for the comparativist.

[4]As Field notes, further primitives will be needed to define mass density.

3 Determinism and laws for the comparativist

There is a venerable definition of what it takes for a world to be deterministic, due to Laplace. This requires that the state at a time determine the entire history:

Laplacean Determinism. A world w is deterministic iff, for any time t, there is only one physically possible world whose state at t is identical to w's.

The state of w at t corresponds to what is normally called the *initial conditions* of a physical system. Although there are other definitions of determinism (Earman, 1986, 6-22), this one is probably the most often-used in physics, and it has obvious advantages. For example, by treating initial conditions as the state at a time (instead of, for example, the state of the whole past), Laplacean determinism doesn't beg the question against the view that there is no fundamental arrow of time.

This definition, however, is potentially mysterious if we assume comparativism. The comparativist's fundamental ontology consists of comparative relations between objects at different times, as well as the present. It is this whole web of relations which grounds the (metaphysically non-fundamental) values physical quantities take on, according to the comparativist. But a physical theory is normally taken to be deterministic if the values of these quantities at a time (plus the laws) are sufficient to determine their values at all times. In other words, determinism in physics is normally defined in terms of entities that are fundamental for the absolutist, but not the comparativist. Moreover, the comparativist's fundamental ontology is extremely spare when restricted to a single instant of time. For as we saw before (fn 1), fundamental relations between objects at different times are crucial if the comparativist is to allow for metaphysical possibilities like the doubling of all objects' masses.

Nonetheless, on the most obvious picture of laws for the comparativist, it is natural to treat only the relations between objects at t as a world w's initial conditions when we ask whether w is deterministic. For example, in his discussion of laws, Dasgupta (forthcoming) re-interprets the absolutist laws featured in physics textbooks in terms of fundamental comparative relations. In particular, he offers a comparativist version of Newton's Second Law, $F = ma$. He begins by identifying those parts of the law's content that can be understood in the comparativist's fundamental terms. For example, the Second Law entails that, since my brother is 1.16 times as massive as I am, whenever we're both accelerating at the same rate, the force acting on him will be 1.16 times as great as the force on me. Generalizing from examples of this sort, Dasgupta captures the entire comparativist-friendly content of the Second Law as follows:

(L2) For any material things x and y,

(a) For any reals r_1 and r_2, if x is r_1 times as massive as y and is accelerating r_2 times the rate of y, then x has $r_1 r_2$ times as much force acting on it than y.

(b) For any real r_3, if x has r_3 times as much force acting on it than y, then there are reals r_4 and r_5 such that $r_4 r_5 = r_3$, and such that x is r_4 times as massive as y and is accelerating r_5 times the rate of y. (Dasgupta, forthcoming, 18-19)[5]

Dasgupta then offers an argument that his comparativist law (L2) reproduces all of the measurable predictions made by the Second Law–a claim we'll return to in §5. But assuming he's correct about this–and his argument generalizes to more complicated physics–the seeming absolutist character of the laws we find in our physics texts is evidently not necessary for their empirical success.

Setting aside for a moment the empirical predictions of (L2), it's pretty clear how we should apply the Laplacean definition of determinism to such a law. The state of the world at t is given by all the fundamental comparative relations that hold between objects at t (in Dasgupta's example, all the ratios of forces to forces, masses to masses, distances to distances, etc.). A world is then deterministic iff the fundamental relations between objects at t, plus the laws, determine all the other fundamental relations (including the relations between objects at different times).[6]

The same general approach to laws is adopted by Field (1980). Although Field does not commit himself to a particular metaphysics of laws, he seems to recognize an imperative to state the laws of his nominalist physics in terms of his primitive comparative relations. For example, he formulates Poisson's equation (for non-zero mass) as follows: "[A]t any two points where the mass density is not zero, the ratio of the Laplaceans of the gravitational

[5]Note that additional direction relations between forces and accelerations will be needed in more than one dimension; Dasgupta ignores these for brevity.

[6]As we will see in §4, the initial conditions for instant-determinism may also have to be extended to include relations between objects at times falling in an infinitesimal neighborhood of t.

potential is equal to the ratio of the mass densities." (Field, 1980, 79) Although ratios (and Laplaceans, for that matter) are not among Field's primitive relations, he shows how to formulate this sort of statement purely in terms of the congruence relations that he does count as primitive (Field, 1980, 68-70). So Field's laws could be written out in full detail using only his primitive (comparative) relations. The corresponding initial data for a possible world on Field's theory would then be the comparative relations between spacetime points lying on one time-slice.

It is also possible for comparativist laws to make use of absolutist-style values, however, using some of the resources of measurement theory. Given a sufficiently rich and constrained system of comparativist relations for a quantity like mass, a *representation theorem* can be proven. Such a theorem establishes the existence, for every possible arrangement of fundamental comparativist relations between physical objects, a mapping or *homomorphism* from the objects into some mathematical structure–for example, the real numbers or a vector space. We call this mapping a "homomorphism" because it doesn't just map the physical objects to the mathematical objects; it does so in such a way that the structure of the mathematical set parallels the structure of the fundamental relations between the physical objects. As a simple example, on one possibility according to Dasgupta's theory of mass, there are three objects A, B, C, with B's mass being twice A's and C's mass being three times as great as B's. One homomorphism from these objects into the real numbers would take A to 1, B to 2 and C to 6. Another would take A to 2, B to 4 and C to 12. A representation theorem for mass on Dasgupta's theory would establish the existence of such homomorphisms for any possible set of massive objects and fundamental mass relations.

A couple points are worth noting immediately. First, a homomorphism of this sort is not unique, and the resulting absolute values are unique only up to some factor, such as multiplication by a constant–an expression of the fact that comparativist relations are scale-independent. Second, the domain of a homomorphism includes all of the objects in a comparativist world, at all times. Thus the absolute value an object is mapped to will in general represent that object's relations to past and future objects as well as present ones.

That said, given a representation theorem for a particular comparativist theory, it is possible to characterize laws for that theory in a way that mimics absolutist laws as closely as possible. We can formulate a comparativist version of an absolutist physical law expressed by a mathematical equation, like $F = ma$, by positing the existence of a homomorphism such that the absolute values the objects are mapped to obey that equation. In the particular example of $F = ma$, we might state the comparativist law as follows:

> (L2*) For any physically possible world w, for any homomorphisms $F(x), m(x), a(x)$ where F is a homomorphism for the force relations, m for the mass relations and a for the acceleration relations, $F(x) = m(x)a(x)$ for all objects $x \in w$.[7]

The natural definition of determinism–and in particular, of initial conditions–looks quite different on this picture of laws. For it seems natural to define the initial conditions at t as the values assigned to each quantity by the homomorphisms. (Note that this is not the same as the intrinsic state of the world at t, which includes only the comparative relations between objects at t. Rather, it includes information about relations with objects at other times as well.) On this approach, a world is deterministic iff the values assigned by homomorphisms to past and future objects are fixed by the laws (which will take the form of (L2*)) plus the values at t.

There is no guarantee that these two definitions will agree about which worlds are deterministic. Which definition is superior, from the perspective of comparativism? In other words, should the comparativist believe in laws like (L2) which govern the fundamental relations, or laws like (L2*) which govern the (non-fundamental) values assigned to each quantity by homomorphisms?[8]

Laws like (L2*) will no doubt be useful for the comparativist in extracting predictions from physics, and in relating a comparativist reformulation of physics to existing absolutist theories. However, (L2*) seems to me a very poor candidate for a fundamental law of nature, and its corresponding definition of determinism seems inappropriate given comparativism. It is a familiar platitude that, while fundamentality may be a brute concept with no definition, the fundamental properties and relations "are the properties and relations that occur in the fundamental laws of physics." (Arntzenius, 2012, 41) On Lewis's popular Humean account of laws, for example, the fundamental laws are regularities in the instantiation of fundamental properties (Lewis, 1983, 368). And altering this feature of Lewis's system would rob it of much of its interest. Our best theories of physics have a particular mission: to describe the universe at its most fundamental level. Insofar as they fail to do so, either through

[7]This form for comparativist laws was suggested to me by Ted Sider.

[8]It may be more accurate to say that laws like (L2*) govern the fundamental relations indirectly, via posits concerning the existence of homomorphisms.

inaccuracy or by failing to describe reality in fundamental terms, we should take that as a sign that the true fundamental laws have not yet been discovered.[9]

Moreover, although determinism in non-fundamental laws is a topic of great foundational interest, when we ask whether our world is deterministic, for purposes of metaphysics we are most interested in whether things are fundamentally deterministic. For example, if the fundamental facts about the present did not determine the fundamental facts about the future, one can certainly imagine how an incompatibilist would see a ray of hope for free will–even if some description of the present in non-fundamental terms did determine the corresponding non-fundamental facts about the future. Especially if these non-fundamental facts about the present were not, strictly speaking, intrinsic to the present, but included lots of information about the past and future as well. The facts expressed by an assignment of values to quantities via a comparativist representation theorem are exactly like that. To ask whether our world is deterministic when described in terms of those facts would be to change the subject, quite radically, away from the concept of determinism that matters for metaphysics.

To cast the problem in more scientific terms, although determinism is a metaphysical thesis, it is one with epistemological implications. Determinism helps us derive predictions from a theory, and affects how we confirm the theory using these predictions (for example, the Principal Principle connecting chance with subjective credence is trivial for deterministic theories). It is a truism that we lack direct epistemic access to the fundamental facts about the future (unless something like time travel is allowed by the laws).[10] A notion of determinism in which the "initial conditions" include information about the future would thus appear remarkably ill-suited to the epistemic role determinism normally plays in science.

That said, representation theorems are a useful tool for studying the relationships between comparativist and absolutist laws of nature. Most physical theories do not have extant comparativist formulations (although in many cases there are obvious ones in the offing), and none has a single canonical comparativist version. Yet we would like to answer ques-

[9] I take it that Field's nominalist project, for one, arises from similar motives. If there is nothing wrong with formulating the fundamental laws in non-fundamental terms, it's hard to see why the nominalist should be unhappy with laws of physics involving Platonist assumptions.

[10] Since we also have no direct access to many facts about the present, at least according to relativity, the Laplacean definition of determinism may not be the ideal one for epistemic purposes. But my point that the ideal definition will not include facts about the future in the initial conditions surely stands.

tions of the following sort: does a given absolutist scientific theory (e.g., special relativistic electromagnetism) admit a comparativist formulation which is also deterministic?

Obviously this is not the sort of question that could be answered by examining comparativist versions of the given theory one by one, even if the theory possesses known comparativist versions. But there is a way to answer this general question in at least some cases.

As we saw in the previous section, the interesting varieties of comparativism are the ones that replace absolutist ontology with an ontology of scale-independent relations. This places a limit on the amount of ontological structure a comparativist can ascribe to a world. A system of comparativist relations can only distinguish between two worlds–or (which is equally interesting for our purposes) two time-slices of worlds–if there is some scale-independent difference between those worlds. And this is a question we can ask about absolutist worlds as well. Thus, if there is no scale-independent difference between two physical possibilities of a theory written in absolutist terms, any comparativist version of that theory must identify those two possibilities as the same. And if two sets of initial conditions–two time-slices of worlds–are indistinguishable in scale-independent terms, no (interesting) comparativist theory can treat those time slices as distinct.

Determinism is a thesis about which time-slices can fit into which overall histories for physically possible worlds. A world is deterministic when no time-slice of that world can fit into any other, different possible world while still obeying the laws. The comparativist can "tell the difference" between two worlds (or time-slices) just in case the differences between those worlds (or slices) are scale-independent.[11] Put together, these facts give us a necessary condition for determinism under comparativism:

> A world w (described in absolutist terms) may be deterministic under comparativism only if, for any time t, the scale-independent facts at t physically necessitate all other scale-independent facts about w.[12]

[11] Here I assume that the comparativist will want a system of fundamental relations that doesn't leave out any scale-independent differences between worlds. For example, a comparativist theory of mass on which the only fundamental relation is "more massive than" would be unsatisfactory. There may be more debatable cases, but none of the cases arising in my examples will be debatable ones, I think.

[12] To clarify: in the comparativist version of w, time itself will not be an absolutist quantity, so in a sense it will be illegitimate to talk about an absolute value like t. But it will still be possible to define a time-slice of a comparativist world as a maximally large set of (temporal stages of) objects in w, all of which are related by the "equal time" relation.

What we have, then, is a formula for looking at a possible world described in terms of ordinary (absolutist-looking) physics and determining whether the scale-independent facts a comparativist would count as fundamental (or fixed by the fundamental) could evolve deterministically under a comparativist version of the relevant laws. With this formula in hand, let's look at some cases where determinism comes under threat.

4 Escape velocity and at-at motion

I'd like to start out by looking at an instructive example from Newton's theory of gravity–a case in which determinism is threatened but can be restored by adopting further metaphysical commitments. This example will serve to illustrate why the comparativist should adopt the at-at theory of motion, as well as underscoring some relevant complexities that apply in other cases.

The informed reader may be puzzled that the question of determinism in Newtonian gravity is even on the table. After all, it is well known that Newtonian gravity is an indeterministic theory, in which (for example) swarms of massive objects may swoop in at any moment from infinitely far away (Earman, 1986, 23-37), balls may–or may not–spontaneously slide down a hill if it's shaped just right (Norton, 2003), and so on (Earman, 1986, 37-53). Why should it be surprising or interesting if the comparativist version of an indeterministic theory itself exhibits indeterminism?

What is interesting, to me at least, is the possibility that indeterminism might be more widespread in comparativist physics than it is in absolutist physics. Indeed, this possibility takes on pivotal importance when viewed in light of the fact that, whatever the *de jure* status of determinism in Newtonian gravity, the theory is *de facto* deterministic as it is ordinarily applied. In other words, there is some vaguely-defined set of implicit posits made by working physicists which serve to rule out the indeterminism in Newtonian gravity for purposes of deriving predictions from the theory. Moreover, there is some hope that these implicit restrictions could be made explicit and rigorous by means of imposing boundary conditions and restrictions on physically admissible initial conditions (Earman, 1986, 37-39, 52-53).[13] Given

[13] Another example of a restriction on initial conditions is the popular "past hypothesis" sometimes posited to explain the time-asymmetry of thermodynamics. So classical indeterminism is not the only foundational problem that seems to call for this sort of solution.

all this, it seems we should be very interested in examples where ordinary absolutist Newtonian gravity, plus the necessary extra posits, is deterministic while comparativist versions exhibit indeterminism.[14]

If there are such examples, presumably they will appear in processes where the scale of some quantity helps determine scale-independent features of the outcome. So let's look at the theory and see if we can find any. The force law for Newtonian gravity is, of course,

$$F = G\frac{m_1 m_2}{r^2} \tag{1}$$

where F is the attractive force between objects with masses m_1 and m_2, r is the distance between them and $G = 6.67428 * 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg} * \mathrm{s}^2}$ is the gravitational constant. Since none of the terms in this equation are scale-independent, this isn't immediately helpful in our search for potential comparativist indeterminism. But when combined with $F = ma$, we can derive a law in which scale-independent features of an experiment's outcome appear to depend on scale-*dependent* features of the initial conditions. This is the law governing *escape velocity.*

Like many concepts from physics, escape velocity admits both an intuitive and a technical definition. In this case the two are pretty close together. The intuitive idea behind a planet's (or other object's) escape velocity is: the velocity needed to "escape" from its surface past its orbit.

The technical version is as follows: An object's escape velocity v_e is the magnitude of the velocity directed away from their common center of mass that a projectile[15] initially located at its surface would need to asymptotically approach an infinite distance from the object in the limit of infinite time, if the two were alone in an otherwise empty universe.[16] In other words, a projectile initially located at the Earth's surface will continue to move farther from the Earth forever, without any limit to its eventual distance, if and only if its initial velocity[17] exceeds the Earth's escape velocity. Clearly–and crucially–whether a projectile

[14]If the reader remains nagged by the thought that indeterminism in Newtonian gravity is nothing new, §5 will provide an example of comparativist indeterminism in classical electromagnetism, a theory whose deterministic credentials are hard to dispute.

[15]By "projectile" I mean an object with some initial velocity which is never subject to any external force aside from gravity.

[16]This is the definition of *barycentric* escape velocity (escape velocity relative to the common center of mass of two objects). A *surface* escape velocity (the velocity relative to the planet's surface needed to escape its orbit) does depend in part on the mass of the projectile.

[17]That is, its initial velocity relative to the center of mass of the combined Earth-projectile system.

escapes is a scale-independent fact.

The important thing about escape velocity for our purposes here is that it is a function of the planet's mass, but not the mass of the projectile – which implies that its value will change if the mass of both objects is doubled. In effect, the force law (1) implies a physically necessary relationship between the planet's mass and radius and its escape velocity. The projectile's mass does not matter to this relationship. This means the theory can "tell the difference" between different initial conditions that the comparativist will identify as the same instantaneous state, since the differences between them are not scale-independent.

4.1 Earth and Pandora

Let's put a detailed example on the table. The escape velocity for a planet of mass M and radius r is given by

$$v_e = \sqrt{\frac{2GM}{r}}. \tag{2}$$

From this equation it follows that if we double the planet's mass (transform M to $2M$), its escape velocity increases from v_e to $\sqrt{2}v_e$.[18]

As an intermediary example, consider a universe containing two planets, Earth and Pandora, located far enough apart that for all practical purposes we can ignore their gravitational interaction. These two planets are identical in all their physical properties except that Earth's mass (M_E) is half of Pandora's mass (M_P). Then, as noted above, if v_E is Earth's escape velocity and v_P is Pandora's, $v_P = \sqrt{2}v_E$.[19]

Suppose that initially each planet has a projectile located at its surface with some initial outward velocity of magnitude v. We may stipulate that the projectile on Earth has mass m and the one at Pandora has mass $2m$, so that the system of Pandora and its projectile is a mass-doubled duplicate of the system of Earth and its projectile. Now suppose that $v_E < v < v_P$, that is, the projectiles both have a velocity greater than Earth's escape velocity but less than Pandora's. Then the system of Earth and its projectile will behave

[18] For a derivation of the escape velocity law, see Halliday *et al.* (1997, 331).

[19] Again, note that these escape velocities are reckoned relative to the center of mass of the entire system (planet and projectile), not the planet's center of mass. (This allows us to redescribe the example as one in which the planet reaches the projectile's escape velocity.) Since the velocity of the center of mass is determined by the planet's velocity, the projectile's velocity, and the ratio of the planet's mass to the projectile's, this quantity will also be left unchanged by mass doubling.

quite differently from its mass-doubled counterpart Pandora. The distance between Earth and its projectile will keep growing forever as the projectile shoots off "to infinity." Pandora's projectile, on the other hand, will only travel finitely far away, remaining confined by the more massive planet's gravity.

The laws of gravity seem capable of telling the difference between Earth and Pandora, despite the fact that they are indiscernible except for the difference in their masses. And since we stipulated that there was no measurable interaction between them, it should be obvious that the laws will recognize the same difference between the initial state of a universe containing only Earth and its projectile, and an initial state containing only Pandora and its projectile.[20]In the Earth universe, the projectile will fly off to infinity. In the Pandora universe, on the other hand, the projectile will go only finitely far before stopping.

This example threatens indeterminism for any comparativist interpretation of Newtonian gravity. We saw in §3 that determinism only holds for the comparativist when the scale-independent facts about the world's initial state determine the scale-independent facts about other times. But the (instantaneous) initial states of the Earth and Pandora worlds agree about all scale-independent facts. The two initial states are related by mass doubling, so by our definition of scale independence they must agree on all scale-independent comparative relations. For the comparativist, the initial state of the Earth universe and the initial state of the Pandora universe are really the same initial conditions.

But these two worlds' futures differ in physically significant ways that are clearly scale independent. The differences between the future evolution of Earth's universe and Pandora's

[20]I say this should be obvious because, for the practicing physicist, approximately isolated systems can generally be treated as if they were entirely isolated (where "approximately isolated" means they are subject to almost no external forces, or other non-force external influences like quantum entanglement). This principle is what allows us to idealize an approximately isolated system, treating the balls on our pool table as if they were colliding in a vacuum instead of surrounded by a bunch of other objects (for example).

This is not to say that there couldn't be a good scientific theory in which even isolated systems behave differently in the absence of their environment. But there's an obvious scientific advantage to theories that do have this feature (they make it possible to idealize in useful ways that actually work in practice), so there is (and should be) a strong preference in favor of theories which predict that approximately isolated systems behave almost exactly the same as completely isolated systems.

Now as a metaphysical matter, under comparativism Pandora and Earth are far from "isolated," insofar as each planet's fundamental nature is partly constituted by its relations to the other planet. But as a matter of physics, if comparativism entails that approximately isolated systems (in the sense stipulated above) cannot be idealized as fully isolated, this saddles comparativism with a serious disadvantage relative to absolutism. In my discussion of this example, I will assume that the comparativist is successful in avoiding this disadvantageous commitment. (Thanks to Shamik Dasgupta for pressing me to articulate this premise.)

show up in the comparativist's fundamental relations as well as the absolutist's intrinsic values. Earth's projectile will continue to move away from the planet forever without limit; Pandora's will not. So if ratios or orderings of distances and times (for example) are among the fundamental relations, these will be quite different in the two cases. The distance between Earth and its projectile at some time will always be greater than the distance at an earlier time. The distance between Pandora and its projectile, on the other hand, will eventually begin to decrease.[21]

Since there are scale-independent differences over time between these worlds, the comparativist can certainly recognize that both Pandora's universe and Earth's are physically possible. But only at the expense of denying determinism. There can be no fundamental difference, for the comparativist, between the initial state of Earth's universe and the initial state of Pandora's. Since there is a difference between the futures of these two initial states, it must be that the same initial state can evolve into two or more distinct future states while obeying the comparativist's laws. This is the very definition of indeterminism.[22]

Or so it seems. But as Kenny Easwaran has pointed out to me, the argument of this section smuggles in a hidden assumption: that initial conditions are to be understood as truly instantaneous time-slices. The comparativist can maintain determinism in the face of my example by denying this assumption–a plausible move on its own merits, as we shall see.

4.2 A hidden premise: instantaneous initial conditions

The nature of the hidden premise will become apparent after a quick look at a puzzle surrounding the nature of velocity. In mechanics, the state of the universe at a time is normally thought to include the velocity of every material object at that time. But an object's velocity is ordinarily defined as the derivative of its position with respect to time,

[21] As noted above in fn 11, the comparativist could in principle save determinism by positing a system of fundamental relations so austere that possibilities in which a projectile escapes are treated as equivalent to possibilities in which it does not. But this would require leaving out even relations as weak as "d is a greater distance than d'," thus leaving it unclear how the resulting ontology could even represent our ordinary acquaintance with the quantity of distance. This option is obviously unsatisfactory.

[22] Although the escape velocity example is particularly straightforward, a more general argument for indeterminism is also possible. If all masses in the universe are doubled, this will double the acceleration of all massive objects, which will have scale-independent effects on the ratios of present and future velocities. It is less clear, however, that the comparativist can't get rid of this indeterminism by positing fewer fundamental relations, per fn 11.

$v = dx/dt$. And this quantity's definition involves more than just the way the world is at the exact moment t. The derivative of an object's position x at t is the limit:

$$v(t) = \frac{dx}{dt}(t) = \lim_{t' \to t} \frac{x(t') - x(t)}{t' - t} \tag{3}$$

This limit is not a property intrinsic to t; it depends on the infinitesimal neighborhood of points in time before and after t.[23] So the notion of instantaneous velocity as a component of a scientific theory's initial conditions would appear to be confused.

There are a few options for making sense of this puzzle (Arntzenius, 2000). One is to posit that velocity is a truly instantaneous, intrinsic property of the world at t. In that case, velocity is not identical with the time-derivative of position, and Eq. (3) cannot be the definition of the velocity at t. Rather, it must express either a physical or metaphysical necessity. This view has a revisionary flavor that may seem troubling, since it leaves open at least the conceptual possibility that velocity and the derivative of position could disagree.

An attractive alternative is the *at-at theory.* On this theory of motion, Eq (3) is the definition of velocity, which is not truly an instantaneous quantity. Rather, an object's velocity at t is a property extrinsic to t but intrinsic to t's infinitesimal neighborhood. And since velocity is a part of the initial conditions for classical theories of mechanics, it must be that these initial conditions don't correspond to a truly instantaneous state at a time. Rather, the "state at t" mentioned in our definition of determinism really refers to the state over a vanishingly small temporal neighborhood of t. While there are no true instantaneous velocities on this view, the limit of the velocities over smaller and smaller intervals around t serves the same purpose while maintaining the ordinary calculus definition of $v(t)$.

My argument from the Earth/Pandora examples to indeterminism covertly assumed the first, truly instantaneous picture of initial conditions. The argument proceeds by identifying the initial conditions at t with the scale-independent facts about position and velocity intrinsic to t's instantaneous time-slice. But according to the at-at picture, and its associated picture of initial conditions, none of the velocity facts are intrinsic to this time-slice, not even $v(t)$. And velocity is a necessary component of the initial conditions for any theory of mechanics–specifying just the positions at a single time while leaving out the velocities is not

[23] When I say the limit depends on t's "infinitesimal neighborhood," I do not mean to imply that there is any unique such neighborhood. Rather, I mean that the limit is not determined merely by the position at t, but is determined by the positions in any interval $(t - \delta t, t + \delta t)$.

sufficient to determine the future evolution of the state, even granting ordinary absolutist assumptions. So by identifying the initial conditions with the facts intrinsic to t, I have covertly assumed the instantaneous velocity view.

What happens to the argument if we instead assume the at-at view? On that view there is really no fundamental quantity of velocity–position is the only fundamental quantity, and velocity is defined reductively from position via Eq. (3). But the initial conditions at t consist of any facts that obtain in t's infinitesimal neighborhood–that is, facts that hold true of any finite interval $(t - \delta t, t + \delta t)$ before and after t, no matter how small. Suppose now that t is the initial stage of a universe like Earth's or Pandora's, with a projectile initially moving away from a planet of mass M. Assuming absolutism, of course, the facts about t's neighborhood together with the gravitational force law allow us to predict whether the projectile will escape. But what about if we assume comparativism? In other words, do the scale-independent facts about t's infinitesimal neighborhood determine whether the projectile will escape?

From the gravitational force law (1) and $F = ma$, we know that the acceleration of the projectile at t is GM/r^2. (The acceleration a is dv/dt, which is also determined by the projectile's position $r(t)$ in t's infinitesimal neighborhood just like v is.) Comparing this with the escape velocity equation (2), we see that $v_e = \sqrt{2ar}$, so the projectile will escape if $v^2 > 2ar$. Is it a scale-independent fact whether this inequality holds? Multiplying mass by a scalar doesn't change it, of course, since mass doesn't appear. What about if we change the scale of other quantities? In terms of the relevant fundamental quantity, the position r, the inequality is

$$(dr/dt)^2 > 2(d^2r/dt^2)r. \tag{4}$$

But if we multiply r by a scalar c, this will just multiply both sides of the inequality by c^2, which won't change whether it holds. Whether the inequality holds or not is a scale-independent fact about t's neighborhood. Since the truth or falsehood of the inequality determines whether $v > v_e$, it determines whether the projectile will escape. Therefore the scale-independent facts about the initial conditions determine whether the projectile will escape, if we understand initial conditions in the at-at theorist's way. (The argument here is due to Easwaran.)

So if we leave out the hidden premise, the argument for indeterminism does not succeed. Does this give the comparativist good reason to deny the premise–to deny that initial conditions are instantaneous? I believe so. The goal of any comparativist interpretation of Newtonian gravity should be to achieve the same scientific aims as the absolutist version of the theory, without the absolutist's more elaborate ontology. If the comparativist and absolutist theories differ in their predictions–if they differ about which physical possibilities are deterministic–the comparativist has failed in this goal.

Indeterminism by itself is not so bad, of course. We're happy to accept the possibility of objective chances in quantum theory. But indeterminism in classical physics is more troubling, since the theory includes no probability measure over outcomes to interpret as chance. In the case of escape velocity, an indeterministic comparativist interpretation will simply predict that a projectile near a planet's surface will either escape its orbit or not, without offering any statistical predictions about the likelihood of these outcomes.

So the comparativist should prefer the at-at theory of motion over the alternative view that there are instantaneous velocities. A way to preserve instantaneous velocities and initial conditions may seem to suggest itself: just make acceleration an instantaneous fundamental quantity as well. Whether a projectile will escape is determined by whether inequality (4) holds. It appears that knowing the scale-independent facts about position, velocity and acceleration is sufficient to determine this fact. And the view that acceleration, as well as velocity, is an instantaneous fundamental quantity has been independently defended (Lange, 2005).

Adding instantaneous accelerations as well as velocities to the mix will not suffice to determine this, however. Not if they are fundamental quantities distinct from position, as the instantaneous view has it. For then their relationship with position is no longer definitional–velocity is not defined to be the derivative of position, but rather there must be some law that makes this the case. And this means it doesn't follow that when we change the scale of position, for example by doubling it, we must also double the value of velocity and acceleration. These are separate fundamental quantities, and thus it will make sense from the comparativist point of view to change the scale of one without changing the scale of the others. But it was the fact that changing the scale of position automatically changed the scale of velocity and acceleration by the same factor, on the at-at view, that allowed us to determine whether inequality (4) would be satisfied. That response to the Earth/Pandora argument does not succeed if we assume that position, acceleration and velocity are separate fundamental quantities.

We've seen that if the comparativist adopts the at-at picture of motion, as they should, the Earth/Pandora examples do not imply that a comparativist version of Newtonian gravity must be indeterministic. But more involved examples will bring back the specter of indeterminism.

5 Indeterministic examples and metaphysical modality

What is the factor that prevents the Earth/Pandora examples from exhibiting indeterminism, on the at-at view? The fact that certain comparative relationships between the different derivatives of position–velocity and acceleration–are scale-independent is what sinks the otherwise promising argument for indeterminism. One way to make some of these relationships either trivial or undefined is to set one or both of these derivatives to zero. So one way to start looking for indeterminism is to study systems with zero initial acceleration.

The example I have in mind is one I'll call Friction World (because the details can be filled in with an idealized Newtonian account of frictional forces). Think of the mass-m hockey puck in Figure 1 as sliding over some surface with an initial velocity v, feeling no forces at all. But upon entering the shaded region, it feels a constant force F in a direction opposite v. $F = ma$ tells us that the resultant acceleration will slow the puck (reducing the value of v). The shaded region is only L meters wide, so if the puck can make it L meters without slowing to a stop it will continue moving in the leftward direction forever afterward.

Suppose we describe these initial conditions solely in scale-independent terms. Here's a question we won't be able to answer: will the puck make it past the shaded area and continue moving left? To answer this question, there would need to be a scale-independent difference between the different possible initial conditions for Friction World. But suppose that v is just barely great enough for the puck to make it past the shaded region. Then if we double F–a transformation that makes no scale-independent difference–the puck will no longer be moving fast enough to make it. Or suppose the puck is just barely too slow to make it past. Halving F will halve the puck's acceleration as it moves over the shaded region, permitting it to make it past.[24]

As in the escape velocity case, the differences between the possible outcomes in which

[24]If we understand this example in terms of friction, these transformations correspond to doubling or halving the coefficient of friction.

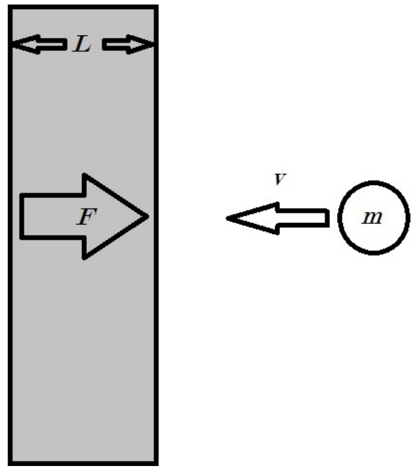

Figure 1: Friction World

the puck stops or slides past are clearly scale-independent. In some cases, the puck remains in the shaded area permanently. In others, it continues moving forever (which is to say, the scale-independent ratios between its distances from the shaded area's edge at different times will keep increasing eternally). So here we have an indisputable case of comparativist indeterminism for a physical system that is deterministic on the ordinary, absolutist understanding of classical mechanics.

Interestingly, the indeterminism in the Friction World example eventually "goes away" once the puck becomes subject to the force. There are stages of Friction World's history whose infinitesimal neighborhoods do determine the scale-independent facts about the puck's future. For the puck will immediately begin accelerating at a rate $a = F/m$ once it enters the shaded area. At that point, we may ask again whether it will stop or not, and there is more scale-independent information to work with. Assume for the moment that the puck will eventually stop. It will take an amount of time equal to $t = v/a$ to do so. Since the acceleration a will be constant, its average velocity will just be $v_{avg} = v/2$. So it will cover a total distance $v_{avg} * t = v^2/2a$ over the course of its journey. Clearly, this means it would have slid past the shaded area iff the area's length L were less than this distance. So in general, the puck will make it past iff

$$\frac{v^2}{2a} > L. \tag{5}$$

As we saw in our discussion of Earth and Pandora, assuming at-at motion, when we double the quantity of distance we must also double velocity and acceleration. So transforming the scale of distance will multiply both sides of this inequality by the same constant, leaving the same inequality. So it is a scale-independent fact whether (5) obtains. Thus, once the puck enters the shaded area and begins accelerating, the scale-independent data about Friction World's present will determine the scale-independent facts about its future. The indeterminism disappears once the puck begins to experience the force.[25]

Another peculiarity of this sort of example (understood the comparativist way): the indeterminism can actually *reappear* at later times if we change the example in the right way. For suppose we modify Friction World by adding a second, identical shaded region of force, some distance beyond the first. To make it past both regions, one after the other, will clearly require that

$$\frac{v^2}{2a} > 2L, \tag{6}$$

since the total velocity lost by the puck as it slides over two successive regions of length L will be the same as if it had to slide over one region of length $2L$. At the time when it enters the first region, the scale-independent facts will, as before, determine whether inequality (6) is satisfied and hence will determine the scale-independent facts about the future (such as whether the puck will pass the second region). But (assuming the puck gets past the first region) consider a time after it has left the first region and before it has entered the second region. At this point its acceleration will again be zero, and the scale-independent facts about the infinitesimal neighborhood surrounding this moment in time will not determine whether it will pass through the second region or be captured. The instantaneous situation will be exactly as in the original Friction World case. So the indeterminism has "gone away" while the puck was moving on the first surface, only to "come back" again between the two surfaces.

A moment's reflection will reveal another bizarre feature of this case. Suppose that, at the time it enters the first region of force, the puck does in fact satisfy inequality (6). Then it is physically impossible for the puck to later stop within the second region. Its velocity

[25] This "disappearing indeterminism" is a feature shared with one of the (debatable) cases of indeterminism in classical mechanics. In the dome example described by Norton (2003), at the initial time it is indeterministic whether and when the ball will begin to slide down the dome (and which direction it will go). But if it does begin to slide down, it will move deterministically from then on (assuming we've ruled out other potential sources of indeterminism like "space invaders" from infinity).

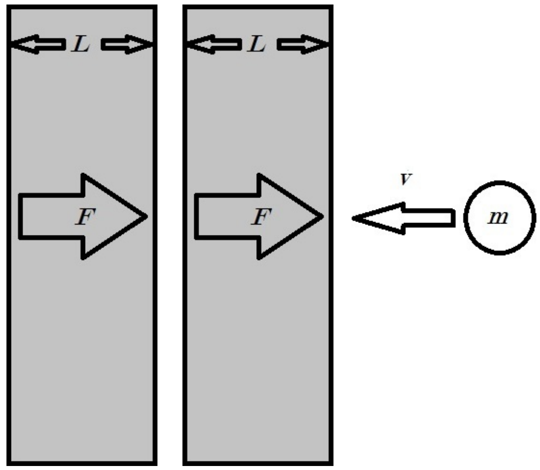

Figure 2: Double Friction World

is high enough that it must keep moving forever. But holding fixed only the instantaneous (neighborhood) state while it is somewhere between the two surfaces, it is physically possible for the puck to stop within the second region. (This is just another way of saying that the instantaneous state at the first time determines the future evolution, while the state at the second time does not.) Something very curious is happening: given the earlier state, it is physically necessary that the puck will keep moving forever, but given the later state it is physically possible for the puck to stop moving. So the Double Friction case exhibits a sort of temporal action-at-a-distance. While the puck is in between the two regions, the entire past of the system physically necessitates an outcome that is *not* physically necessitated by the state at the present time. According to the comparativist, in a case like this the past can influence the future without said influence being mediated by the state of the world in the present.

5.1 A more realistic example

These aspects of Friction World are odd and foundationally interesting. The reader may wonder whether they can be reproduced in a more realistic setting. Friction World is, after all, a toy model that bears little resemblance to any complete theory of physical forces. In fact we can develop an analogous example in Newtonian gravity by modifying the escape velocity example, although idealizations will remain.

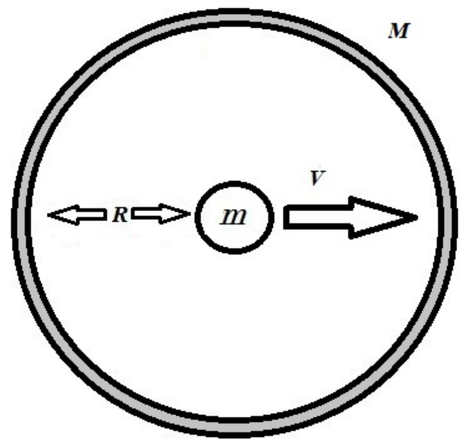

Figure 3: Shell World

The key to doing so is the gravitational Shell Theorem. This theorem has two consequences: the gravitational force outside a uniform spherical shell of mass M is the same as that from a mass-M particle located at the center of the shell; and a uniform, hollow spherical shell exerts no net gravitational force on any object inside it.

Our new example, Shell World, is formed by modifying the Earth/Pandora-type examples. Instead of a planet, the initial state consists of a uniform, rigid shell of mass M, with a projectile initially located at the center of the shell with outward velocity v. By the Shell Theorem, the projectile will initially feel no force and its acceleration will be zero. Here comes the idealization: we must suppose that the projectile will somehow pass through the shell.[26] One possibility would be to just suppose that the objects in the example are permeable and only interact via the gravitational force. Another possibility would be to assume the projectile is a point particle and there is a point-sized gap in the shell at the place where the projectile's trajectory will intersect the shell. This may be the best option, although the idea of a hole with no spatial extension may seem conceptually confused. Finally, we could make some assumptions about what will happen when the projectile collides with the shell (perhaps the shell will break apart?). I will assume one of these solutions is postulated. For our purposes it won't matter which one.

[26] A further significant idealization: we must ignore whatever forces hold the rigid shell together, preventing its gravitational collapse. This idealization strikes me as unproblematic, though, since filling in these details will not undermine the morals of the example unless they introduce some initial acceleration.

Regardless, once the projectile passes through the shell, it will immediately feel a gravitational force from the shell's mass, just as if the shell were a solid planet with total mass M (this also follows from the Shell Theorem). At that point it will begin to accelerate toward the shell. The case will then become exactly parallel to the Earth and Pandora examples: the projectile will escape if v exceeds the shell's escape velocity.[27] But think about the initial time, before the projectile exits the shell. At that point we may also ask whether the projectile will escape or not. The ordinary way to find out, of course, is to compare the projectile's velocity v with the shell's escape velocity. But because there is no initial acceleration due to gravity, we may now pull the trick that made the Earth/Pandora cases look indeterministic: we may double the mass of every object in Shell World. And since the acceleration in Shell World is initially zero, the comparativist may not respond by using inequality (4) to determine whether the projectile will escape. The scale-independent facts about Shell World are not changed when we double the mass, even if we assume the at-at theory of motion. So the type of indeterminism that at first appeared to be present in the Earth/Pandora examples is actually present in Shell World.[28]

As in Friction World, the indeterminism is temporary: once the projectile passes through the shell and feels the force, the instantaneous state will determine the future evolution. What about temporal action-at-a-distance? Can we get the indeterminism to "come back" by modifying the Shell World case, the way we did with Friction World?

This may seem challenging (what if we assume the shell breaks?), but actually it is easy to construct a case of temporal action-at-a-distance by modifying Shell World. The key ingredient is the time-reversal invariance of classical mechanics: since the history of Shell World is physically possible, so is the time-reverse of that history. Consider this time-reversed copy of Shell World. In effect, the projectile will begin at the right of the diagram in Figure 3 and move to the left, entering the shell.[29] At the end of this process, it will be at the

[27]If we postulate that the projectile collides with the shell and breaks it, the problem of whether it escapes will be more complicated to solve but not qualitatively different.

[28]An anonymous referee has raised a concern about Shell World and related examples. Note that the initial conditions of Shell World are very unusual: only a tiny class of physical states will exhibit the sort of symmetry required to guarantee zero initial acceleration. Indeed, these states make up a set of measure zero in the space of all states. Does this undermine the foundational importance of the example? I don't see why it should. The significance of some pathological or peculiar behavior of certain states is not lessened unless the states themselves can be ruled out as somehow unphysical or uninteresting. The fact that they are rare or atypical states should not by itself disqualify them in this way.

[29]Depending on how the collision is handled, the pieces of the shell may also converge to form the shell.

center of the shell, moving to the left with velocity $-v$. In effect, this temporal stage of the time-reversed Shell World will be the mirror image of Shell World's initial conditions. As before, the projectile will escape iff v is greater than the shell's escape velocity–which is not determined by the scale-independent facts about this temporal stage.

But if we rewind time to a point before the projectile entered the shell, the instantaneous state at *that* time will determine whether the projectile will eventually escape the shell's orbit. For at that earlier time, we will have scale-independent data about the acceleration the moving object undergoes due to the shell's gravity–and we know it will feel the same acceleration when it leaves the shell once more. More generally, since the state of the modified Shell World while the projectile is inside the shell is just the mirror image of the initial state of the original Shell World–and the past history of the modified Shell World is the time-reverse of the original Shell World–we know that the future of the modified Shell World will necessarily be the mirror image of the time-reverse of its past. So the past history of this modified Shell World physically necessitates its entire future, but the instantaneous state while the projectile is inside the shell does not. Just as in the Double Friction case, the comparativist must accept temporal action-at-a-distance.

It is worth mentioning that a close analog of the Shell World example also exists in classical electromagnetic theory. Although the theory is relativistic, I will describe the initial conditions for this example in one frame (the rest frame of the shell). Since there is also an electromagnetic Shell Theorem, we may construct an example in which a positively-charged projectile is initially located at the center of a negatively-charged shell, with some initial outward velocity. All that's needed is to modify the Shell World example so that the projectile has electric charge Q and the spherical shell has charge $-Q$. We can call the modified example Charge World. Once the projectile passes through the shell it will be attracted to the shell's charge by the Coulomb force:[30]

$$F = k\frac{q_1 q_2}{r^2}. \tag{7}$$

The scale-independent facts about the initial state of Charge World will not determine whether the projectile escapes this attractive force. This example is especially interesting

[30]Here, the constant $k = 8.988 * 10^{-9} N * m^2/C^2$ (in units of meters, newtons of force, and coulombs of charge).

because relativistic field theories like electromagnetism are entirely deterministic as ordinarily understood, unlike Newtonian gravity (Earman, 1986, 55-78). So here we have a case in which comparativism and absolutism disagree, with no caveats needed, about whether a certain physical theory has indeterministic solutions. And since classical electromagnetism is also time-reversal invariant, one may modify the example in the same way that we modified Shell World to introduce temporal action-at-a-distance.

5.2 Metaphysical modality

In light of these examples, I want to consider the proposition that comparativism about quantity is not only true, but metaphysically necessary. I'll proceed from the assumption that any reasonable system of physical laws are the laws of some metaphysically possible world(s).

The motivation for this assumption is as follows: In the past, physicists considered seriously the possibility that Newton's laws were true. Indeed, they remain interested in many counterfactuals concerning what would be true if Newton's laws were the actual laws.[31] It would be bizarre if, in doing so, past and present physicists were entertaining a hypothesis as impossible as the proposition that water is not H_2O, or the existence of an all-red, all-green object, or an object with a mass of both one gram and five grams. The view that these laws are metaphysically impossible but logically possible would entail this bizarre consequence. Moreover, the laws of our present-day best theories (general relativity and quantum field theory) are known to be false, although they approximate the truth closely in broad domains. Again, it would be bizarre to suppose that we entertain a metaphysical impossibility every time we apply these laws.[32]

By a reasonable system of laws, I mean a collection of putative laws that good physicists might hypothesize to be the fundamental laws of nature if presented with the right sort of

[31] One reason for this is that Newton's laws are very good at approximating the more accurate laws of relativity and quantum theory. Another reason is that they share many broad physical principles (which one might, following Lange (2007), call "meta-laws"), such as conservation laws, with more accurate theories.

[32] Arguments for so-called causal structuralism have been thought to imply that non-actual but reasonable-seeming laws are metaphysically impossible (Shoemaker, 1998). The thought is that, for example, it is essential to the quantity of charge that opposite charges attract, and so a universe in which opposite charges repel is a metaphysical impossibility. But as Fine (2005, §2) has shown, this implies only that non-actual laws may have to involve alien properties, or the absence of familiar fundamental properties (e.g. a world with no charge), not that they are metaphysically impossible.

experimental data. Newton's laws are like this: faced with the experimental data available at that time, the best scientists of the early modern period hypothesized that they were the fundamental laws. The laws of Friction World–Newton's laws of motion plus a toy force law–seem to me like reasonable laws in this sense. I am certain that the laws of Shell World and Charge World–the laws of Newtonian gravity or classical electromagnetism, plus perhaps some laws about collisions which I've neglected to fill in–are reasonable.

I've been a bit unspecific just now. By "the laws of Friction World (or Shell/Charge World)," do I mean the absolutist version of these laws or some comparativist version of them? This question gets to the heart of the matter. I have no argument that comparativist laws for Friction World or Shell World are unreasonable. But it seems clear to me that the absolutist version of Friction World's (or Shell/Charge World's) laws–as distinct from any possible comparativist laws–are reasonable laws that ought to be true of some metaphysically possible world. For it would be reasonable, in a case like Friction World or Charge World, to posit that the laws of nature are deterministic.

It would be entirely reasonable, after all, for physicists, faced with the sort of experiments that might lead them to accept classical electromagnetism, to suppose that the laws are deterministic under initial conditions like those of Charge World. All the observable predictions of classical electromagnetism are consistent with determinism, and so it would seem rather *ad hoc* to postulate (as the comparativist must) that the theory is indeterministic under certain narrow special conditions despite otherwise exhibiting determinism.[33] Even more plausibly, a theory that avoids temporal action-at-a-distance under any conditions might be preferred. Lange (2002, 7-17), for example, has argued for a principle of *temporal locality* according to which there must be no temporal gaps between an event and its causes. If we understand causation to involve physical necessitation, this principle is violated by comparativist physics in cases like Double Friction and the analogous modification of the Shell World example. Again, I'm not claiming that the comparativist's laws for Friction/Shell/Charge World are unreasonable laws. But I do maintain that some deterministic system of laws governing these cases must be reasonable–and since any such laws will be absolutist, I conclude that absolutism must be a metaphysically possible thesis.

It is, of course, open to the comparativist to maintain that his view is metaphysically

[33]An alternative would be to stipulate that Charge World's initial conditions are physically impossible, but this appears equally *ad hoc* given the straightforwardness of the example.

necessary by denying my principle that reasonable laws must be metaphysically possible.[34] At this point it becomes difficult to respond, beyond registering my disagreement. But readers who trust their modal intuitions about specific cases more than I do may find themselves filled with the intuition that it is metaphysically possible for Friction World, Shell World or Charge World to exhibit deterministic, temporally local laws. Such readers will then be obliged to agree with the thesis of this section, even if they disagree with my principle about the metaphysical possibility of reasonable physical laws.

6 Conclusions

I have argued that comparativism is not metaphysically necessary, and that it brings with it certain theoretical commitments. Namely, comparativists should accept the at-at picture of motion (or some close cousin of it), and should also accept that temporal action-at-a-distance is possible according to their view (unless they want to deny that Friction World, Shell World and Charge World are real possibilities). What does this imply about the truth or falsehood of comparativism? Do we have any new reasons for rejecting the view?

To begin with, some will be attracted to the view that the true theory of quantity must be metaphysically necessary. If the argument of §5.2 is correct, comparativism cannot succeed on these terms. But for my part, I don't see why a theory of quantity should have to be metaphysically necessary in order to be true of our world. It seems entirely plausible to me that the space of possibilities includes both absolutist and comparativist worlds. Kripke has taught us, of course, that certain scenarios which are logically and (in some sense) conceptually possible are nonetheless not intelligible ways a universe could be. But it seems obvious, from the existence of both comparativism and absolutism as formal theories of quantity with well-defined models, that absolutism and comparativism are both intelligible ways for a universe to contain quantities. I take this to be a strong indication that both views are metaphysically possible.

If one is persuaded by arguments like those of Lange (2005) that velocity must be an instantaneous fundamental quantity, one should probably reject comparativism, to avoid accepting indeterminism in a wide variety of physically important cases like the example of

[34] I expect this would be Dasgupta's position, since he takes seriously the proposition that only the actual world is metaphysically possible (Dasgupta, in progress).

escape velocity. But at-at motion is pretty plausible on its own merits, so I don't see this as a huge theoretical cost for comparativism.

The sort of disappearing, reappearing indeterminism exhibited by the comparativist versions of Friction World and Shell World is pretty peculiar, as is the temporal action-at-a-distance that arises in these examples. On the other hand, these two examples are rather idealized (especially Friction World) and don't much resemble any of the physical possibilities according to our actual best theories. So at most, they commit the comparativist to the metaphysical possibility of temporal action-at-a-distance. Again, this does not strike me as a huge cost–especially given that determinism in Newtonian physics is already known to be a vexed issue.

The most troubling example is Charge World. Here we have an example where the comparativist must disagree with the absolutist–and with physics as ordinarily understood–about whether a relativistic theory of force is deterministic and temporally local. Classical electromagnetism is non-fundamental, which draws some of the sting. By itself, Charge World shows at most that comparativism would lead to unwelcome pathologies if electromagnetism were the fundamental theory of our universe. But classical electromagnetism is also a limiting case of quantum electrodynamics, one of our most fundamental quantum field theories. This raises the uncomfortable possibility that our most fundamental theories might also exhibit chanceless indeterminism and temporal action-at-a-distance under comparativism.

This is a difficult question to address, given the outstanding controversy over which quantities are fundamental in our most successful quantum theories. And even if those theories were better understood, they are known not to be truly fundamental. But given comparativism's track record with determinism and temporal locality in classical theories, I would not want to bet on its success in quantum theory. Comparativism should be explored further, but metaphysicians who accept the theory run the risk that it might lead to pathologies in fundamental physics as bad as the ones it causes for classical physics. Absolutism is a safer option in this regard.

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