PBR theorem and sub-ensemble of quantum state

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Abstract

Pusey-Barrett-Rudolph theorem claims that ψ-epistemic understanding of quantum mechanics is in trouble. Not considering whether the theorem only applies for realist understanding of quantum theory, this paper instead shows that the actual issue the theorem exposes is whether every quantum state should be interpreted as representing all sub-ensemble possibilities. For example, if $|+\rangle$ was "measured" at time $t = 0$ where $|+\rangle = (|0\rangle+|1\rangle)/\sqrt{2}$, should we consider this quantum state as being solely $|+\rangle$, or representing all possible sub-ensembles such as $(+, 0), (+, 1)$? This question suggests that PBR theorem does not rule out realist/non-realist ψ-epistemic theory.

1 Analysis of PBR theorem

PBR theorem [2] starts from the idea that for a pair of non-orthogonal quantum states, $|0\rangle$ and $|+\rangle$, each has μ_0 and μ_+ as probability distribution over some "physical state" λ. Assume that the support of μ_0 and μ_+ overlap, and that the overlap area Δ has probability of q for both μ_0 and μ_+. Assuming independence of two probability distributions, one would not be able to distinguish $|0\rangle$ and $|+\rangle$ q^2 of the time from the physical state one faces.

The use of λ has been criticized [1] as being realist, but this issue is not important in this paper. Indeed, if we can find a reasonable mapping from quantum states to physical states, then the consideration of PBR theorem would remain valid.

But let us consider the following case: suppose a pair of quantum systems is prepared each with state $|0\rangle$, where $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$ and $|0\rangle$ and $|1\rangle$ are orthogonal. Instead of projecting onto the four orthogonal states used in the PBR theorem, use instead for measurement:

$$\xi_1 = |0\rangle \otimes |0\rangle$$

$$\xi_2 = |0\rangle \otimes |1\rangle$$

$$\xi_3 = |1\rangle \otimes |0\rangle$$

$$\xi_4 = |1\rangle \otimes |1\rangle$$

Then in every measurement, $|0\rangle \otimes |0\rangle$ would be measured for two quantum systems. But by the PBR assumption, for q^2 of time one would not be able to distinguish $|0\rangle$ and $|+\rangle$.

If we assume that the measurement implies that $|0\rangle \otimes |0\rangle$ represents itself, not just possibility of all sub-ensembles such as $(0, +)$, it becomes clear that the PBR assumption that at first seemed reasonable cannot be maintained. It is true that we cannot directly rule out any quantum state out of a set of orthogonal quantum states under some projection, and this certainly inspires the PBR assumption, but if measurement does "change" the quantum state we get (for example, by choosing the right projection, under some quantum theory interpretation, $|+\rangle, |-\rangle$ can only be obtained as the measurement result, not $|0\rangle, |1\rangle$), then the PBR assumption is no longer obviously reasonable even when taking realist interpretation of quantum theory. Effectively, PBR is arguing from the idea that measurement alone cannot change invariant physical reality, which just resurrects old quantum theory interpretation issues. Furthermore, even on the realist side, epistemic theory can simply argue that two quantum systems combined with appropriate measurement apparatus avoid physical states of both (two) quantum systems in the overlapping region Δ.

Thus, the below discussion is about what can be done to save PBR theorem, and whether that understanding is reasonable.

It is possible to assume instead that each quantum state represents all possible sub-ensembles. From the above example one gets $|0\rangle \otimes |0\rangle$ every time, but in this interpretation, this only means that "Out of all possibilities, $|0\rangle$ is the only definite outcome one can assume for each quantum state" and thus this quantum state only shows that indeterminacy between $|+\rangle$ and $|-\rangle$ for each quantum system, where two possibilities $+, -$ are orthogonal. Then it is certainly possible to save PBR theorem, because now we can assume that it is indeed possible to assume "invariant" physical state regardless of how measurement is done. Some realists can argue that this is how science should be done - measurement process alone should not directly affect the underlying physical "reality," but this is beyond the scope of this writing.

While the critique presented in this paper is not explored in Hofmann 2012 [1], Hofmann did explore the possibility of all possible sub-ensembles represented by a "definite" outcome, which is a quantum state. But Hofmann 2012 shows that negative probabilities must be allowed for each possible sub-ensemble. Accounting for negative probabilities, one indeed is able to replicate standard quantum theory results.

Hofmann 2012 is not troubling for those not in realist camp, but for some realists who accept assumptions and results of PBR theorem, Hofmann 2012 indeed should be troubling. If the critique presented in this writing is accurate and Hofmann 2012 is correct, then one should assign negative probability for each sub-ensemble possibility, and for realists that believe in physical reality of each sub-ensemble, only one of some sub-ensemble must be our physical reality (let us only consider the world we are located at, if one takes the many-worlds viewpoints). But one must assign negative probability for some sub-ensembles, and conventional understanding of probability must be abandoned.

2 Conclusion

It is true that our ordinary intuition is often violated in physics. And it may turn out that negative probabilities are observable and prove to have important physics consequences. But so far there is no evidence to recommend the switch. Though the author must admit that the focus made in this paper is very narrow and there are many realist quantum interpretations.

To summarize, this paper presented a critique of PBR theorem that while PBR assumes for q^2 of time $|0\rangle$ and $|+\rangle$ are not distinguishable, conventional understanding of quantum state shows that under some projection only one of them occurs 100% of the time. The paper then explored how conventional understanding may be changed to save PBR theorem and related considerations.

References

[1] Hofmann, H. (2012). The Quantum State Should Be Interpreted Statistically. Preprint. arXiv:1112.2446 v3.

[2] Pusey, M., Barrett, J. and Rudolph, T. (2012). On the Reality of the Quantum State. Nature Physics Vol. 8, pp. 475–478