A new approach in classical electrodynamics to protect principle of causality

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Preprint of article published in

***Journal of Theoretical Physics and Cryptography*, vol. 5 (March, 2014)**

ABSTRACT

In classical electrodynamics, electromagnetic effects are calculated from solution of wave equation formed by combination of four Maxwell's equations. However, along with retarded solution, this wave equation admits advanced solution in which case the effect happens *before* the cause. So, to preserve causality in natural events, the retarded solution is intentionally chosen and the advance part is just ignored. But, an equation or method cannot be called fundamental if it admits a wrong result (that violates principle of causality) in addition to the correct result. Since it is the Maxwell's form of equations that gives birth to this acausal advanced potential, we rewrite these equations in a different form using the recent theory of reaction at a distance (Biswaranjan Dikshit, Physics essays, 24(1), 4-9, 2011) so that the process of calculation does not generate any advanced effects. Thus, the long-standing causality problem in electrodynamics is solved.

Key words: Advanced solution, Causality, Maxwell's equations, Wave equation, Action at a distance, Reaction at a distance

1. Introduction:

Electromagnetism is one of the oldest natural phenomena studied by modern science. Laws of electromagnetism gradually evolved from the various experimental observations. In this process, Coulomb's law was the first one which established the $1/r^2$ dependence of force between two charges. Then, Faraday discovered the induction of voltage by changing magnetic field and Ampere quantified the magnetic field generated by electric current. Finally, Maxwell introduced the concept of displacement current (i.e. time varying electric field giving rise to magnetic field) and wrote all the laws of electromagnetism in an elegant form which are commonly known Maxwell's equations. These equations in differential form are given by,

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{B} = \mu \vec{J} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t} \tag{4}$$

Eq. (1) known as Gauss law which is an alternative form of Coulombs law, Eq. (2) and Eq. (3) (Faraday's law) are all based on experimental observations. First term in the right hand side of Eq. (4) is due to Ampere's law and the second term ($\mu\varepsilon\frac{\partial \vec{E}}{\partial t}$) was added by Maxwell to save continuity equation in experiments (such as charging of a capacitor). Thus, all these four equations are just mathematical representations of experimental results.

However, a scientific law is useful if it can accurately predict a particular variable from the knowledge other variables. To achieve this objective, there was a need to devise a method to calculate electric and magnetic force (or field) in an assembly of arbitrarily moving charges. Generally in textbooks [1-3], this is done by taking differentials and forming the wave equations for scalar potential ϕ and vector potential $\vec{A}$ given by,

$$\nabla^2 \phi - \mu\varepsilon \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\varepsilon} \tag{5}$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} \qquad (6)$$

Although the wave equations Eq. (5) and (6) admit both retarded and advanced solutions for ϕ and $\vec{A}$, conventionally *onl*y the retarded solution is chosen and the advanced part is just ignored [1,3-4]. It is done because in the advanced solution, time runs backward (i.e. t is negative) and thus the effect happens before the cause. This violates principle of causality and we have never observed any acausal event in the nature till date. Although one can argue in support of advanced potential for its utility in calculation of radiation reaction force in some approaches such as action-at-a-distance [5], this radiation reaction and energy loss from an accelerating charge can also be derived without use of advanced potential by considering self force [6-9] created in the charged particle. Coulomb gauge is in no way better than Lorenz gauge as in this case also principle of causality for the potential is violated [1-2,10].

Electric and magnetic fields are calculated by following equations.

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \qquad (9)$$

$$\vec{B} = \nabla \times \vec{A} \qquad (10)$$

Although Maxwell's equations are true representations of experimental results, but an equation or law can be called fundamental only if there exists a way to predict *only the correct answer* and no wrong answers in the process. To explain it, let us take an example. Let there be a variable called x and suppose the truth is that x=5. This truth can be represented by many equations such as 2x-10=0 or x^2=25 or dx/dt=0. Among these equations, equation 2x-10=0 can be simplified to get only one value of x i.e. 5, which is the correct answer. But x^2=25 gives a value of 5 or –5 for x and dx/dt=0 predicts that x is any constant including the true value 5. Thus, although all the equations are true, equation 2x-10=0 is the fundamental equation as it predicts only the correct answer and nothing else. Arguing along these lines, as there is only one way that uses Maxwell's equations to calculate the electromagnetic effects and it generates along with the correct answer (i.e. retarded solution) a practically impossible answer (i.e. advanced solution), we cannot call the Maxwell's form of equations as fundamental. Rather this is just one of the many possible mathematical representations of the experimental observations.

So, it will be better if we can find an alternative way to correctly predict electromagnetic effect on any charge without simultaneously creating any unusual result (like advanced solution) during the process of calculation. Thus, the causality in natural events can be protected. To achieve this objective, Jefimenko [11] has recently assumed the final forms of equations for the electric and magnetic fields as the fundamental equations and then he has derived the Maxwell equations from these equations. But, as his method does not give any explanation or reason for that particular complex form of expression for electric and magnetic fields, it looks like that we are being forced to consider a complex equation as fundamental just to avoid violation of causality. But in this paper, we propose a different form for laws of electromagnetism which we can call fundamental as they are shown to predict experimentally observed effects without simultaneously creating any unusual result like advanced solution during the process of calculation. Thus, the long-standing causality problem in electrodynamics is solved.

2. Formulation of laws of electromagnetism in a different form:

In our formulation, we will use only the established experimental results given by Coulomb, Ampere, Faraday, Maxwell and add no additional hypothesis during formulation. Our aim is to correctly calculate the *force* on a test charge Q, which is placed in any electromagnetic environment. We are not concerned about whether or not the electric field or magnetic field exists as a physical entity at particular point in space. In other words, our analysis will be a field-less theory directly calculating the force on a test charge due to position and state of other charges distributed in the universe. This is in agreement with the theory of action-at-a-distance [5, 12] proposed by Feynman and theory of reaction-at-a-distance [13-14] published by us earlier, which rule out the existence of electric field. Although we will use the symbol $\vec{E}$ for convenience, we emphasize that the electric field $\vec{E}$ just represents the force experienced by a unit test charge at a particular point of space. But if test charge is not there, $\vec{E}$ does not exist at that point. So, the proposed basic laws of electromagnetism corresponding to each experimental observation are given below.

a) As the Coulomb's law states that force between two charges is directly proportional to the $1/r^2$ and the electric field is gradient of potential (for electrostatic case), we will take the potential to vary as, $\phi \propto q/r$. We also know that Maxwell added the term $\mu\varepsilon\frac{\partial\vec{E}}{\partial t}$ in the Amperes law to save continuity equation in experimental situations (such as charging of a capacitor). The $\mu\varepsilon$ ultimately manifested as $1/c^2$ in wave equation where c is the speed of electromagnetic influence. But, in stead of bringing the Maxwell's term $\mu\varepsilon$ (or $1/c^2$) into laws of electromagnetism, we can simply state that, "A*ny charge can be electromagnetically influenced by another charge only if* ***r = ct*** *is satisfied, where* ***r*** *is the distance of the acting charge from affected charge and* ***t*** *is the time elapsed.* This is exactly same as the principle of reaction-at-a-distance proposed in our earlier publication [13-14] for solving the problems of invariance of speed of light and wave-particle duality. In theory of reaction-at-a-distance, the observer is affected by all the past and present events of the universe on its own (no carrier required), which is a fundamental property of matter. Even if some matter is created just now (say by electron–positron pair production), it gains the knowledge about all the events of the universe as soon as it is born. However, one can react exactly after a time delay, $t=r/c$, which is the inherent character of all matter. Thus, nothing (such as electromagnetic information) comes from the source; observer only reacts to the source based on its own absolute knowledge about the events that have occurred in space-time. So, using this law, potential at any point r_1 will depend upon distribution of charges in space at retarded time and mathematical expression for scalar potential ϕ will be,

$$\phi(r_1,t) = \frac{1}{4\pi\varepsilon}\int_{vol}\frac{\rho(r_2, t - r_{12}/c)}{r_{12}}dV(r_2) \qquad (11)$$

Where, r_{12} =Distance of the volume element dV at r_2 from field point r_1

ρ=Charge density in the elemental volume $dV(r_2)$at retarded time "$t- r_{12}/c$"

c = Constant of proportionality relating distance and reaction time delay (or Speed of electromagnetic wave in conventional theory)

Above Eq. (11) for scalar potential is same as the retarded generalized solution for ϕ derived by Feynman [3] from wave equation. Then using Eq. (11) and the method of "sweeping integral" as given in [3] or method of reaction line [15], we can also calculate the electric potential at any point due to a moving point charge which is commonly known as Lienard-Wiechert potential.

b) Just as the Ampere's law gives the relation between the magnetic field and current, we state that the relation between magnetic vector potential and current distribution is given by (after incorporating the concept of reaction time delay as earlier),

$$\vec{A}(r_1,t) = \frac{1}{4\pi\varepsilon c^2}\int_{vol} \frac{\vec{J}(r_2, t - r_{12}/c)}{r_{12}} dV(r_2) \qquad (12)$$

Where, r_{12} =Distance of the volume element dV at r_2 from field point r_1

$\vec{J}$ =Current density in the elemental volume $dV(r_2)$at retarded time "t- r_{12}/c"

c) Faraday's law is accounted by incorporating the term $\left(-\frac{\partial \vec{A}}{\partial t}\right)$ in the expression for electric field. So, in place of $\vec{E} = -\nabla\phi$, we take $\vec{E}$ as,

$$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \qquad (13)$$

d) Net force on Q is experimentally found out be,

$$\vec{F} = Q[\vec{E} + \vec{v}\times(\nabla\times\vec{A})] \qquad (14)$$

Substituting a new variable $\vec{B}$ for $(\nabla\times\vec{A})$, we get the conventional equations given by,

$$\vec{F} = Q[\vec{E} + \vec{v}\times\vec{B})] \text{ and } \nabla\cdot\vec{B} = 0$$

Thus, we can calculate the force on any charged particle due to other charges by using Eq. (11) to Eq.(14). We note here that the E and B above are just intermediate mathematical terms to calculate the net force on Q from knowledge of potentials and they do not exist as physical entities in space in accordance with the theory of reaction-at-a-distance [13-14].

3. Conclusion:

We have represented the Maxwell's equations expressing the experimental results in a different form so that the problem of advanced fields does not arise during the process of calculation of electromagnetic effects (or force) which otherwise happens in the wave equation approach. We have constructed this alternative form of Maxwell's equations using the theory of reaction-at-distance, which is a field-free electromagnetic theory. Thus, the long-standing causality problem in electrodynamics could be solved.

REFERENCES:

[1] David J. Griffiths, *Introduction to Electrodynamics*, (PHI Learning Private Limited, New Delhi, 2009), p 422, 425,421, 431

[2] J D Jackson, *Classical Electrodynamics* (Wiley, New York, 1975) p 221, 223

[3] Richard Philips Feynman, Robert B Leighton and Matthew Sands, *The Feynman Lectures on Physics (vol. II)*, (Narsoa Publishing House, New Delhi, 1992), p21-3, 20-14, 21-5, 21-10

[4] F Rohrlich, “Causality, the Coulomb field, and Newton’s law of gravitation”, American Journal of Physics, **70** (4), 411-414 (2002)

[5] John Archibald Wheeler and Richard Philips Feynman, “Interaction with the absorber as the mechanism of radiation”, Reviews of Modern Physics, **17** (2-3), 157-181 (1945)

[6] F. Rohrlich, “The self-force and radiation reaction”, American Journal of Physics, **68** (12), 1109-1112 (2000)

[7] David J Griffiths, Thomas C Proctor and Darrell F Schroeter, “Abraham-Lorentz versus Landau-Lifshitz”, American Journal of Physics, **78** (4), 391-402 (2010)

[8] Philip Pearle, “Classical electron models” in book *Electromagnetism Paths to Research*, (Edited by Doris Teplitz, Plenum press, New York, 1982), p 211-258

[9] Biswaranjan Dikshit, “A new approach in the derivation of the Larmor formula for radiation loss from an accelerating charged particle”, Canadian Journal of Physics, **91**(7), 501-505 (2013)

[10] C W Gardiner and P D Drummond, “Causality in the Coulomb gauge: A direct proof”, Physical review A, 38 (9), 4897-4898 (1988)

[11] Oleg D Jefimenko, ”Presenting electromagnetic theory in accordance with the principle of causality”, European Journal of Physics, **25,** 287-296 (2004)

[12] John Archibald Wheeler and Richard Philips Feynman, “Classical Electrodynamics in terms of Direct Interparticle Action”, Reviews of Modern Physics, **21** (3), 425-433 (1949)

[13] Biswaranjan Dikshit, “A new concept of light: Solving the problems of invariance of speed of light and wave-particle duality”, Physics Essays, **24**(1), 4-9 (2011)

[14] Biswaranjan Dikshit, “Derivation of Lorentz transformation equations from the new concept of reaction-at-a-distance”, Physics Essays, **24**(4), 508-511 (2011)

[15] Biswaranjan Dikshit, “Space-time diagram approach in derivation of Lienard-Wiechert potential for a moving point charge”, Canadian Journal of Physics, **91**(7), 519-521 (2013)