Can particle configurations represent measurement results in Bohm's theory?

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Abstract

It is argued that if the relative configuration of Bohmian particles represents the measurement result, then the predictions of Bohm's theory may be inconsistent with the Born rule in some situations.

The measurement problem of quantum mechanics originates from the incompatibility of the following three claims: (1). the wave function of a physical system is a complete description of the system; (2). the wave function always evolves in accord with the Schrödinger equation; and (3). each measurement has a definite result (Maudlin, 1995). One approach to solving the measurement problem is to deny the claim (1) and add some hidden variables and corresponding dynamics to explain definite measurement results. A well-known example is Bohm's theory (Bohm, 1952). A key issue in Bohm's theory is what determines the measurement result. A popular view is that the Bohmian particles themselves determine the measurement result, and in particular, the relative configuration of Bohmian particles represents the measurement result (Lewis, 2007). It has been argued that this view leads to the problem of allowing superluminal signaling (Brown and Wallace, 2005; Lewis, 2007). In this paper, I will argue that this view may lead to a more serious problem of being inconsistent with the Born rule.

Consider a simple spin measurement. Suppose a measuring device or an observer M measures the x-spin of a spin one-half system S that is in a superposition of two different x-spins, $\alpha \left|up\right\rangle_S + \beta \left|down\right\rangle_S$. According to the Schrödinger equation, the wave function of the composite system after the measurement will evolve into the superposition of M recording x-spin up and S being x-spin up and M recording x-spin down and S being x-spin down:

$$\alpha \left|up\right\rangle_S \left|up\right\rangle_M + \beta \left|down\right\rangle_S \left|down\right\rangle_M . \quad (1)$$

In Bohm's theory, although the post-measurement wave function is a superposition of two definite result branches, the configuration of the Bohmian particles of the device is definite after the measurement, being in one of the two branches with epistemic probability consistent with the Born rule. This may be enough for solving the measurement problem if assuming relative particle configurations indeed represent measurement results.[1] The question

[1] Note that the absolute configuration of Bohmian particles in an inertial frame, which

is: Can this assumption be true?

According to the Born rule, the modulus squared of the amplitude of each result branch of a post-measurement superposition gives the probability of obtaining the measurement result corresponding to the branch. For example, the modulus squared of the amplitude of the branch $|up\rangle_S |up\rangle_M$ in the above superposition, $|\alpha|^2$, gives the probability of obtaining the x-spin up result. This means that the Born rule requires that the quantities that represent the measurement results should be correlated with these result branches of the superposition. Thus, if relative particle configurations represent measurement results, then the relative configurations of the Bohmian particles that reside in different result branches (in configuration space) should be different. In other words, in order that the measurement result is represented by the relative configuration of Bohmian particles, there must exist a one-to-one correspondence from the relative configurations of the Bohmian particles to the result branches of the post-measurement superposition.

Let us see whether this requirement can always be satisfied. Suppose the spatial part of $|down\rangle_S$ is $\psi(x_0, y_0, z_0, t)$, the spatial part of $|up\rangle_S$ is $\psi(x_0 - a_0, y_0, z_0, t)$, and the spatial part of $|down\rangle_M$ is $\phi(x_1, y_1, z_1, ..., x_N, y_N, z_N, t)$, the spatial part of $|up\rangle_M$ is $\phi(x_1 - a_1, y_1, z_1, ..., x_N - a_N, y_N, z_N, t)$, where a_0, a_1,... and a_N are large enough so that the two branches of the superposition (1) are non-overlaping in configuration space and the superposition may be a valid post-measurement state. When all a_i (i=0, ... N) are different, and the difference between two of them is larger than the spreading size of the wave function $\psi(x_0, y_0, z_0, t)\phi(x_1, y_1, z_1, ..., x_N, y_N, z_N, t)$ in configuration space, then obviously there is a one-to-one correspondence from the

is not invariant in all inertial frames, cannot represent the measurement result, since the representation of a measurement result should be independent of the selection of an inertial frame.

relative configurations of the Bohmian particles to the two branches of the post-measurement superposition.

However, it can be seen that there are also situations in which the one-to-one correspondence does not exist. Here is an example (Gao, 2017). When $a_0 = a_1 = ... = a_N$, one branch of the superposition (1) is a spatial translation of the other branch. In this case, if a relative configuration of the Bohmian particles appears in the region of one branch in configuration space in some experiments, it may also appear in the region of the other branch in configuration space in other experiments. Moreover, the epistemic probability of the configuration appearing in both regions are the same. This means that the relative configurations of the Bohmian particles that reside in different branches of the superposition may be the same, and there does not exist a one-to-one correspondence from the relative configurations of the Bohmian particles to the result branches of the post-measurement superposition.

Since the Born rule requires that there should exist such a correspondence relation when assuming that the measurement result is represented by the relative configuration of Bohmian particles, the non-existence of the correspondence relation means that this assumption is wrong. This result can be seen more clearly as follows. If assuming that the relative configuration of Bohmian particles represents the measurement result, then no matter which branch of the above post-measurement superposition the Bohmian particles reside in, the measurement result will be the same. In other words, there will be only one measurement result with probability one under the assumption. This is obviously inconsistent with the Born rule.

One may object that it is misleading to describe the above superposition as a post-measurement situation. Since anything that deserves to be called a measurement is a situation in which different results are encoded in different relative configurations of things including the wave functions, the above superposition, even if it is a valid post-measurement state, also corresponds to one measurement result with probability one. Thus Bohm's theory with the above assumption is not inconsistent with the Born rule.

This is a significant objection. In my view, the objection is indeed valid. However, one may avoid this objection by somewhat changing the above superposition and also considering the properties of the Bohmian particles. Consider situations in which one branch of the superposition is formed by first spatially translating the other branch and then exchanging the coordinates of two non-identical subsystems. For example, $|down\rangle_M$ is $\phi(x_1, y_1, z_1, x_2, y_2, z_2, ..., x_N, y_N, z_N, t)$, and $|up\rangle_M$ is $\phi(x_2 - a, y_2, z_2, x_1 - a, y_1, z_1, ..., x_N - a, y_N, z_N, t)$, where subsystems 1 and 2 are not identical. In these situations, since the two branches of the superposition are non-overlaping, and one branch of the superposition is not a spatial translation of the other branch, the superposition may describe a post-measurement situation.[2] Now, if Bohmian particles have no properties other than position (as many Bohmian think), which means that exchanging the coordinates of two Bohmian particles does not change their relative configuration, then when the Bohmian particles reside in one branch of the above superposition, their relative configuration always has its translated counterpart in the other branch. Thus, similar to the previous example of spatial translation, no matter in which branch of the superposition the Bohmian particles reside, the measurement result will be the same. This is inconsistent with the Born rule.

[2]Note that if one branch of the superposition is formed by spatially translating the other branch and exchanging the coordinates of two identical subsystems, then the superposition is not a valid post-measurement state with two possible results as before.

This result also means that if assuming the relative configuration of Bohmian particles represents the measurement result, then one need to endow the Bohmian particles with more properties than position. These additional properties are intrinsic and can distinguish one Bohmian particle from the other, and thus exchanging the coordinates of two Bohmian particles will change their relative configuration. In this way, the problem of violating the Born rule may be solved in the above example. However, the measurement result will be determined not only by the position property of Bohmian particles, but also by these intrinsic properties of Bohmian particles which determine their identities.

Moreover, it can be further argued that the Bohmian particles of a quantum system must have all intrinsic properties possessed by the system such as mass, charge and spin in order to avoid the violation of the Born rule. Assume this is not the case, e.g. the Bohmian particles of a quantum system have all but one intrinsic property of the system such as spin. Consider a post-measurement superposition similar to the above superposition, in which one branch of the superposition is formed by first spatially translating the other branch and then exchanging the coordinates of two subsystems which have only different values of spin. Then, since the Bohmian particles of the two subsystems have no spin property, they are identical and exchanging their coordinates does not change their relative configuration. Then, similar to the above analysis, no matter in which branch of the superposition the Bohmian particles of the system reside, the measurement result, which is represented by the relative configuration of Bohmian particles, will be the same. Again, this is inconsistent with the Born rule.

However, it is well known that all observables other than position, including spin, are contextual properties of Bohmian particles, which means that they are not intrinsic properties of Bohmian particles which exist independently of the context of being measured. Thus it seems that we have obtained an interesting result, namely that if relative particle configurations represent measurement results in Bohm's theory, then the predictions of the theory may be inconsistent with the Born rule in some situations. This suggests that relative particle configurations may be not eligible to represent measurement results in Bohm's theory.

Here one may also object that the above superposition is not a valid post-measurement state. But the reason cannot be the same as before, since the relative configurations of the wave functions are different in different branches of the superposition. Moreover, it is worth pointing out that if using Bohm's result assumption, namely assuming that the branch of the wave function occupied by the Bohmian particles represents the measurement result, then the predictions of Bohm's theory can still be consistent with the Born rule in the above situations.[3]

Finally, I note that the above analysis also raises concern about the whole strategy of hidden-variable theories to solve the measurement problem. Why add hidden variables such as positions of Bohmian particles to quantum mechanics? It has been thought that adding these variables which have definite values at all times is enough to ensure the definiteness of measurement results and further solve the measurement problem. Indeed, the existing no-go theorems for hidden-variable theories, such as the Kochen-Specker theorem (Kochen and Specker, 1967), consider only whether observables can be assigned sharp values or whether there exist such hidden variables. However, if these hidden variables cannot determine the measurement results, then even though they have definite values at all times, their existence does not

[3]Unfortunately, it has been argued that Bohm's result assumption is problematic (Stone, 1994; Brown and Wallace, 2005; Lewis, 2007).

help solve the measurement problem.

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