Our brief span of life is accomplished in a world characterized more by the imperfections of its qualities than by the qualities themselves. All quantity is limited, no resource inexhaustible. Substance, whether material, radiative or field-like, is confined within compact domains of space and time. Perceptions are overburdened with error, misinterpretation and illusion; the theory of measurement is but the science of approximations, and no compounded thing escapes its law of decay.

In today’s physics, only the elementary particles are granted immortality. Even their durability has, at the level of theory, been questioned. Experimentally the proton appears in excellent shape, at least for the time being: despite the mobilizing of monumental earthworks worthy of any Hollywood science fiction superproduction, we have found no evidence that the proton is disposed to fade away, at least until, using the fashionable rhetoric of modern Eschatology, such time as Time itself will end.

No-one has ever observed the presence, temporal or permanent, of {infinite magnitudes}, my bracketing of this phrase signifying it's internal self-dissension. Nor is it anticipated that at any time soon someone in the physics community will experience confrontation with an infinite physical magnitude. The expression itself is furthermore ambiguous, since two meanings of the word ‘infinity’ are involved. Since the origins of science and philosophy with Anaximander, Anaxagorus, Empedocles and others, a distinction has been made
between actually infinite entities, such as number, spatial extension or past time, and entities merely capable of manifesting themselves in any finite quantity, however large, or the potentially infinite.

Physicists have, by and large dismissed the possibility, either in theory or practice, of infinite magnitudes in our universe. Such entities usually involves the violation of some conservation law. For example, an infinite velocity for would imply that, at least for an instant, material objects could be in two places at the same time. An infinite energy source violates all the laws of Thermodynamics. Whenever a magnitude is infinite, there is always a possibility that the part may be equal to the whole. If this part be translated or transported elsewhere, something is created out of nothing. If at the heart of an electron the electromagnetic potential be truly infinite would it not be possible, by concentrating this infinite potential, to double the electron’s charge? Or, through moving part of it elsewhere, form two electrons out of one? Rather than 'infinite potential', the term appropriate to this situation ought to be ‘potentially infinite potential’! This infinite electromagnetic potential cannot be seen, and is even thrown away in all real calculations by renormalization. What is really meant by the infinite electromagnetic, or gravitational potential at the core of a charged particle, are quantities which, when measured at a certain distance R from their center, can be increased by reducing the value of R, although beyond a certain point there is no practical way of making this reduction. This is the meaning of the term “potential infinity”. The belief that infinity is potential only has the endorsement of such names as Aristotle, Kant, Gauss and Hilbert.

1Black Holes being a notable exception which is why so many people, myself included, distrust them
Before Special Relativity, the theoretical possibility of a particle moving with infinite speed between points A and B separated in space was allowed, although, as we have mentioned, there were the obvious epistemological objections. Clearly any object moving with an infinite velocity between two points will arrive simultaneously at every point on the line connecting them.

\[ A \bullet \quad \longrightarrow \quad \bullet \quad B \]

Let the particle be cylindrical, with length \( l \), and base \( \sigma \). If the density of this particle is \( \delta \), its length is \( l \) and the length of the distance \( AB \) is \( L \), then the mass of the particle, during the magic instant of motion increases from \( \delta \sigma l \) to \( \delta \sigma L \). Infinite velocities in classical physics violate the requirement that something cannot be in two places at the same time, while infinite densities (like the ones presumed to exist in Black Holes), are inconsistent with the requirement that two things not be in the same place at the same time.

These objections persist even in the light of Bell's Theorems and their confirmations by Aspect, Grangier, etc., which exhibit a correlation that appears to be instantly propagated between two isolated points. This occurs, we are to understand, in the absence of all influence or interaction anywhere in the surrounding space! Instantaneous \textit{jumps} are not much easier to deal with than instantaneous \textit{propagations}! It isn't clear to me that the classical \textit{Weltbild} would have been any more comfortable with the notion that "a probability can be in two places at once," than with the corresponding statement involving matter. Our perplexity awaits some radically re-interpretation of locality, some New Order in the Cosmopolitics of Space - Time - Matter.
Scientists of the pre-relativistic era despite their objections to actually infinite velocities allowed for the potential infinity of velocity. The idea that there could be some universal upper limit on the speed of any material object would have been dismissed as far-fetched by most of the people who thought seriously about these problems. Einstein's postulate of Special Relativity places a barrier on material velocities, an upper bound which, paradoxically, is actually attained by a non-material energy form, the propagation of electro-magnetic radiation.

This was not in fact the first, though probably the most dramatic, announcement of an intrinsic barrier on magnitudes. 19th century Thermodynamics is based on the assertion, expressed in the form of two laws, of the non-constructibility of perpetual motion machines. Its third law, asserting that no material system in isolation can be frozen to absolute zero received its epistemological justification only after the development of Quantum Theory. The corresponding barrier is known as zero-point energy, and although it is a potential, rather than an actual barrier, it is actualized at specific values for specific entities.

The Uncertainty Principle can also be interpreted as a barrier, defining the lower quantitative bound of 'certainty'. It has never been denied that physical knowledge was necessarily uncertain. From Antiquity “physics” by definition has been restricted to the realm of the mutable, uncertain and transient, while it is “metaphysics” (Ontology, Dialectic or Logic, Theology, Ethics and so on), that took on the immutable and eternal.

Of course, your theology may not be my theology, which did not prevent speculation about the nature of God to circulate freely between the systems of Aristotle, Averroes, Maimonides and Thomas Aquinas. Conditional however upon certain unquestioned premises.
grouped together under the rubric of Faith, “theology” lays claim to being certain knowledge to the extent that, via “dialectic”, another branch of metaphysics, it demonstrates necessary conclusions from these premises.

Uncertainty underlies all perceptions, experiences and events - the last person to deny this would be the experimental physicist - once again, there was no reason to believe that there was any natural limit to its reduction. It must be admitted that there always was something dissatisfying about this belief, for if physical measurements could, even theoretically, be made certain, then there might exist a road whereby physics would, in thought if nowhere else, blend into metaphysics. Researchers might agree that error could never be eliminated entirely, yet there was no public outcry against the “potentially infinitesimal”.

Crediting the insight of the inventors of Quantum Theory, it is uncertainty rather than certainty whose attributes resemble those of traditional physical magnitudes, matter, energy, momentum, etc. “Uncertainty”, defined as $U = \Delta x \Delta p$ has a definite ‘extension’ (in a domain of phase space given by the region above the hyperbola $(x - x_0)(p - p_0) = \hbar / 2\pi$). It has a range of values, in discrete units of $\hbar$ up to $\infty$. It is homogeneous, indecomposable, otherwise obscure and impenetrable, opaque to the illumination of reason, much like Leibniz’ classical definition of matter.

Here again a barrier was discovered, embodied in a quantal unit $\hbar$ interpretable as the rate of energy transformation across some time interval. Quantum Theory exhibits a dual nature at the meta-level. In one of its aspects it presents itself as a theory of indeterminism: since position and momentum are conjugate, predictive causation’s
requirement that both loci and momenta be specified in the neighborhood of an instant cannot be satisfied.

The other face of this dualism is present in exact determinations, or eigenvalues, that can be calculated directly from the theory. These include energy levels, spins, etc., the so-called quantum numbers. Their exactness depends only on the exactness with which \( h \) can be measured, which quantity, once again, is assumed to be measurable to any degree of accuracy.\(^2\)

Quantum Theory also introduced a concept unique to the entire history of science: exact statistics. These are distributions which, being designed to handle the computational aspects of going from a wave to a particle description, have an exact character. One cites the exact ‘half-lives’ of radioactive decay, the exact distributions of spin measurements, etc. \(^3\)

Thus, barriers, even as they set up limits to size, motion and certainty, also provide us with exact parameters, the constants of nature defining them: \( c, h, \) the energy levels, Boltzmann’s constant, \( k \), etc. If a finitist universe has been ordained as our dwelling place, we have been at least entitled to intimate familiarity with its constraints.

Given the universality, flexibility and predictive success of Relativity and Quantum Theory, we judge the scientific expedition remiss in its obligations that it has not energetically embarked on a search for barriers to other discernibles of common experience. Might there not be:

(a) An upper bound to acceleration?

\(^2\) A belief that runs the risk of making \( h \) itself an object of metaphysics.

\(^3\) It is because of spin’s anomalous status between energy and momentum, between theoretical precision and theoretical uncertainty, that so many of its properties are deemed paradoxical.
(b) A durational quantum for spontaneous decay, explosion or transformation?

(c) An upper bound to matter density? Radiation density?

(d) An upper bound on gravitational and electromagnetic potentials within massive or charged particles?

(e) A discrete quantum of velocity for any object moving away from rest?

(f) A discrete time quantum?

(g) A mass quantum, closely allied with an upper bound on frequency or a lower bound on wave-length?

(h) A length quantum, causing objects to "jump' from location to location in inertial motion?

(I) Might not continuity itself, the presence of an actual infinitesimal in nature, be everywhere an illusion? Might nature not be everywhere atomic in all its manifestations?

We feel that there exists, in addition to c, h, k, etc., a complete collection of universal barriers to potential infinity for many other quantities. Putting this in the form of a pre-postulate: In any situation in which the actually infinite is proscribed by some law of nature, one will find the potentially infinite proscribed by a barrier. We dub this epistemological position: Physical Finitism.

Furthermore, the actual or potential infinitesimal can have no greater physical viability than the actual or potential infinite, all of these being essentially metaphysical categories. We will avoid all discussions of such topics as: did a “Deity” put these barriers into our world? Does the structure of some synthetic apriori inherent in thought impose the barriers as prerequisites for apprehension and understanding? Can the necessity for such barriers be proven from pure logic? From the dialectic process? From some Anthropic Principle? Etc.
The epistemological grounding for this position is rather to be found in the requirement that there be a clear line of demarcation in both perception and thought, between the physical and the metaphysical.

**On the Quantum Theory of Relativity**

We are baffled that, we might even go so far as to state that we are deeply disturbed, by the fact that Relativity and Quantum Theory, both so internally coherent, prove to be fundamentally irreconcilable in combination. Sometimes it appears as if one is speaking of distinct universes, the quantum universe and the relativity universe, inhabited by different beings, subject to different laws, with limited visitation rights between them and insurmountable obstacles to communication. Their discord is even more jarring than the 19th century’s equivalent dissonance: the conflict between Newton’s mechanics and Maxwell’s fields, exemplified in the invention of an ethereal medium with impossible properties. Both Relativity and Quantum Theory were developed to deal with this conflict. Each succeeds brilliantly on its own territory - the incredibly fast and the incredibly small - yet their mutual alienation has merely advanced a classical contradiction inherent in physical theory to a higher level of sophistication, without changing its fundamental character.

They disagree over many key issues, among them:

(a) Relativity requires stable reference frames, established and maintained through optical surveys. Quantum Theory admits of no well-defined locations, and builds theoretical uncertainties into all surveying.

(b) Relativity equates matter with energy. Quantum Theory calls matter a parameter and energy an operator.

(c) Momentum is a primitive term of the Quantum Theory. There is no equivalent to Newton’s Second Law, since the time derivative of
momentum has no meaning in this theory. General Relativity is a brilliant extension of Newton’s Second Law.

(d) Quantum Theory deals with simultaneous measurements of complementary or conjugate observables. Relativity abolishes simultaneity.  
(e) Quantum Theory treats time as a parameter and length as an operator. Relativity puts time on an equal footing with length, both spatial dimensions subject to a pseudo-Euclidean metric geometry.
(f) Quantum Theory eliminates expressions like ‘the path of the electron’. Relativity depends on the existence of inertial paths.

The list goes on indefinitely (or until such time as Time itself shall end!) We are suggesting here that a way of reconciling these incompatible cosmologies may be through the promotion of other barrier theories, the properties of which could establish a bridge between these two splendid yet isolated summits.

Extra-Physical Axioms

Definitions:

A ‘discernible’ refers to some homogeneous, autonomous, objective, measurable and conserved entity, invariant under the full space-time isometry group, (translations, rotations, reflections, transposition to past and future, and time reversal).

A discernible possesses locality if it cannot assume two value at the same time and place. Arguments have been presented on previous pages to show that discernibles possessing locality cannot be present in infinite quantities in a compact domain.

4Bell’s Theorem embodies a simultaneity paradox that highlights the contrary requirements of the two theories.
The following set of 3 axioms appear to us to be necessary consequences of the philosophical requirement that there be a line of demarcation between physics and metaphysics:

**Barrier Axiom I**: Any discernible which possesses locality will also be bounded away from potential infinity by an upper or lower numerical limit, called a barrier.

**Barrier Axiom II**: Upper barriers are always realized in some natural phenomenon. These phenomena have properties which are sufficiently distinct from those phenomena whose magnitudes lie below the barrier, that it is impossible, (save by spontaneous transformation), to force them into the state of those objects at the barrier limit.

Here we use the Postulate of Special Relativity as a model.

**Barrier Axiom III**: The lower barriers will be present in the form of a quantum, a minimum discriminatory unit upon which the magnitude is built. Continuity is thus always absent at the level of the microcosm, but may be obtained in the limit as the magnitude moves to the upper limit.

**Summarizing our intentions**: The universe is both potentially and actually finite. Finitude also excludes the domain of the infinitely small. Continuity is only an important mathematical abstraction and does not exist anywhere in nature. Not only is it the case that perfectly continuous substances or transformations are never encountered, but there is always a lower barrier, a minimum discernible quantum, for every objective magnitude.

About the meaning of ‘autonomous’ and ‘objective’ discernibles: Experience has shown a discernible known as a ‘griegle’, an atom with 6 legs and a persecution complex, does not exist in this

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5 Question: Is God continuous or discrete? That would have kept the Medieval scholastics hopping!
universe. Let \( g \) = the number of griegles. Then the number, \( l = 1/g \) is infinite: We have apparently discovered an actually infinite magnitude in our midst! Discernibles must be defined in such a fashion that arguments like this one are excluded.

By an *autonomous discernible*, we will mean such things as matter, radiation, space, time, gravity, force or combinations of these. Einstein did not make things any easier for us by showing that matter and energy are not autonomous, but rather two faces of the same discernible. This of course is only true relativistically: in the Quantum Theory, energy is an operator and matter a parameter. Furthermore, time is a parameter and length is an operator, which is contrary even to the character of the basic conservation laws, which ally energy conservation with time invariance, and momentum with space invariance!  

By an *objective discernible* we mean one that is independent of our imaginations, cognizable, unlike unicorns and griegles, through the evidence of our senses. We know that this raises more problems than it solves, but we are interested in only so much philosophy as we need to justify the search for upper or lower barriers on the magnitudes of discernibles. There are as many imaginations in this world as there are minds, human, dog, cat, insect, etc. Perhaps I am the only person in the world who sees griegles through my electron microscope. Perhaps all men but only a few women see griegles. I bet we all see griegles, but because each person believes that he alone can see them, he dares not confess his knowledge to others. All of these, and related scenarios of social psychology, are excluded from the definition of an objective discernible. Objectivity means, therefore, independence from

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6What I’m really saying in all this is that modern physics is horribly confused in this matter of the proper designation of the autonomous discernibles of Nature.
observers, individual or collective. This is a paradox, because it applies only to observed phenomena, but we are not going to get into this problem here.

We postulate, and it seems simple common sense to do so, that all the autonomous, objective, homogeneous and measurable discernibles of our universe manifest themselves in finite amounts only, as invariants, (in a fixed reference frame), under the full isometry group of space and time translations, and locally co-variant in moving frames. Radiation is, has been, always will be, (until such time as Time...etc!). It can and is found everywhere. It is always present in finite quantities, large and small. No observable amount of radiation can have infinite magnitude, because one can isolate portions of it, and show that the part is less than the whole.

With respect to the proper identification and description of autonomous discernibles, we will only be interested for the purposes of this paper in density, velocity, acceleration, time, and energy, (in the form of photon frequency).

We will say that acceleration is distinguished from velocity, even though velocity is measured in units $\text{LT}^{-1}$, and acceleration is measured in units $\text{LT}^{-2}$, because velocity is associated with inertial paths and acceleration is not. On the other hand, all higher levels of acceleration, of the form $\text{LT}^{-n}$, are not necessarily distinguished physically. Most significantly, the two basic kinds of acceleration, continuous growth and spontaneous change, are not treated as autonomous.

We will consider density as an autonomous discernible, distinct from spatial volume and mass, because a volume can, in theory, be reduced to a point, (topological reducibility in a finite open domain, side-stepping the issue of an open or closed universe), which mass
cannot,(Aristotle: no matter without shape); while a quantity of mass can, in theory, be sprinkled over an infinitely large space, (even though in reality, space may be of finite volume). However, density cannot be reduced to a point (we can’t give any meaning to the ‘density’ of an isolated point), nor spread over infinite space (a finite density at every point of an infinite volume implies an infinite quantity of matter, which is contrary to our assumptions). Briefly, density is not topologically reducible.

From the dawn of time, mankind has been preoccupied by the problem of the correct designation of the primary qualities of Nature,(what we are calling the autonomous, objective, homogeneous, measurable discernibles). In the Ancient World, earth(-iness), fire(-iness), air(-iness) and water(-iness) were thought to be the primary qualities. The problem was given an exotic twist by the scholastics of the late Middle Ages, desperate for some scientific justification of the dogma of the transubstantiation of the Host: the Inquisition was not adverse to burning up people who did not understand how “Christ-iness” could, following certain magic incantations, become the autonomous, objective, homogeneous, measurable discernible of wafer and wine.

Win some, lose some: the 19th century relegated the 4 primary elements of the ancients to secondary "states of matter", but also revitalized the programme of Democritus and Lucretius. Now the visible world was analysed in terms of elements, molecules, atoms and, eventually, elementary particles. The 20th century saw the introduction of scalar, tensor and vector force fields, the wave-particle ambiguity and the equivalence of matter and energy, followed by strings, superstrings, chromodynamics, Witten instantons, and so forth. The correct identification of the primary qualities is once again a topic
of intense controversy: it may be that we were never meant to know if the chicken or the egg came first.

In the remainder of this article, we will be examining the elementary consequences of postulating upper barriers on density, acceleration, spontaneous energy transformation and photon frequency, and lower quantum barriers for time, velocity and photon wave length. We recognize that our treatment of the density barrier is controversial, using some arguments that many will rightly consider dubious. Density right from the beginning is a slippery concept.

On the other hand, we are unequivocal in feeling that a time quantum is built into Nature. The evidence for it comes to us from a great many directions. We apologize in advance if this section of the article will strike some as proselytizing. Let's put it this way: we believe that physicists will eventually accord the same degree of reality (in the sense of Einstein, Podolsky and Rosen) to the time quantum that they give now to the Postulate of Relativity and the Second Law of Thermodynamics.

THE DENSITY BARRIER

What are the immediate consequences of postulating that matter cannot be compressed beyond a universal density limit, $\Delta$? By the equivalence of matter and energy, this implies also that a photon cannot have an infinite wave-length, or zero frequency, although this state is assumed to exist in Black Holes.

By Quantum Theory, if $E$ is the kinetic energy of a light-beam, $\nu$ its frequency, then $\hbar = E/\nu$. If $\nu$ cannot be reduced to 0, even by gravity, then neither can $E$. Black Holes are inconsistent with a
universal density barrier. One might however be able to combine the density limit $\Delta$, with the concept of a Black Hole by coupling the longest wavelength $\lambda$, to the Hubble length, $H_0$. A light ray with a wavelength that is so long that it unfolds over the entire span of the universe at a given moment in time, can be considered essentially undetectable. Rather than Black Holes one ought then speak of “cold” or “frozen” stars, within which all light quanta are reduced to an ultimate minimal energy state, $\omega$ the weakest possible quantum. $\Delta$ and $\omega$ are equivalent barriers, except for the fact that we have not yet shown how to make $\Delta$ Lorentz-invariant. Such undetectable radiation could be added to the ledger of the hypothesized missing matter in the universe.

Interpreting $\Delta$ in terms of pressure: An object is at the barrier density $\Delta$, if further increase in density necessitates an infinite increase in pressure. The upper density limit is $\Delta$. Let density $= \rho$, pressure $= p$, $dp/d\rho = g(\rho)$. $g(\rho) = \infty$ at $\rho = \Delta$, or

$$\lim_{\rho \to \Delta} g(\rho) = \infty$$

The concept of density in physics is more difficult to define rigorously than it in mathematics, in which “substance” is just a numerical parameter associated with a Lesbesgue measure over a region with compact support. Density is usually thought of in a very naive way as the intrinsic ratio of matter to the volume of its shape, from which in some mysterious way all empty space has been extracted. More precisely, one forms the ratio $d = \text{matter}/\text{volume}$, in a sphere of radius $r$ centered on a specific point, $q$, takes the limit of this ratio as $r$ goes to zero, then says that one has a positive mass density at that point, $q$, if this limit exists and is $> 0$. This is an indirect way of permitting a density at an isolated point, something essentially non-material.
Democritus’ solution of this difficulty was the atom. Dirac’s was the $\delta$-function. Our proposed solution is $\Lambda$.

Here is a rough idea for the Equation of State of an object compressed onto the barrier density. Define the variable $s$ by $s = 1/p$, where $p$ is the pressure. In the neighborhood of $\Lambda$, $dV/ds$ has a critical value: the volume curve relative to $s$ flattens out. This means that near $\Lambda$, we have a parabolic relationship

$$V = As^2 + V_0.$$  

$A$ will be linearly dependent on temperature, so

$$A = aT$$

$$V_0 = M/\Lambda$$

Therefore, The equation of state at the barrier is

$$\rho = \frac{M\Delta p^2}{M^p^2 + aT\Lambda}$$

Lorentz Covariance. The desire for an absolute density barrier independent of reference frame leads to the necessity for a non-commutative velocity measurement that is a function of the density of the object being measured. Let observers be $O$ and $P$. Let $d$ be the “rest density” (a matter difficult to define, but we won’t worry about that), of a bar-shaped mass $M_P$ at $P$, with “rest volume” $V = HWL$ (height, length, width). Then

$$d = M_P/V = M_P/HWL$$

Let $P$ now be set in motion, with a velocity $v$ relative to $O$, in the direction $L$. If $P$ is being observed by $O$, then $L$ will appear to shrink in the ratio $L' = L\beta$, where $\beta = \sqrt{1 - \frac{v^2}{c^2}}$. The mass $M_P$ will appear to increase by the same amount, $M' = M_P/\beta$. Combining these, we see that the density of $P$ as seen from $O$, will appear to have increased by the amount $d^* = \frac{d}{\beta^2}$.

If there is a universal maximum density, $\Delta$, then the equation
\[ \Delta = \frac{M}{V \beta^2} \text{ determines a maximum velocity } v_d < c, \text{ which is a function of } P's \text{ rest density, } d. \]
\[ \frac{d}{\beta^2} = \Delta \]
\[ d / \Delta = 1 - \frac{v^2}{c^2} \]
\[ v_d = c \sqrt{1 - \frac{d}{\Delta}} \]

This formula suggests that we can ascribe a maximum upper velocity to moving objects, based on their intrinsic density. Since P and O may presumably have different rest densities, it follows that velocity measurement can be non-commutative. This is possible only if the density of an object influences the measurement of its distance from another object. It is distance rather than time that must change, because there is no way to understand how clocks on P can influence the time-reckoning on O!

Assume an ideal length \( X \) between two systems without density, i.e. massless points, and therefore, an ideal velocity, \( u \), of some spatial point, (or massless particle like a neutrino) moving in one’s reference frame. Let us say that O’s measurement of \( X \) is influenced by P’s density, and P’s measurement of \( X \) influenced by O’s density, as follows:

\[ X_O = X \delta_P, \quad X_P = T \delta_O, \text{ where} \]
\[ \delta_P = c \sqrt{1 - \frac{d_P}{\Delta}} \]
\[ \delta_O = c \sqrt{1 - \frac{d_O}{\Delta}} \]

Both of us measure the elapsed time as \( T \). Then P’s speed as seen by O is \( v_O = u \delta_P \), while O’s speed as seen by P is \( v_P = u \delta_O \). This result
follows inevitably from the hypothesis of an upper limit to density which shall be independent of velocity.

B. MINIMUM VELOCITY INCREMENT

Lower Relativity Postulate: We examine the consequences of postulating a minimum velocity increment \( \epsilon \). Any system moving away from rest must have a velocity greater than or equal to \( \epsilon \).

Write \( \epsilon = ctanh\eta \), where \( \eta \) is the relativistic angle. Imagine that you and everything around you is at rest. Suddenly a particle \( p \) in your rest frame begins to move with velocity \( \epsilon \): our hypothesis is that it cannot move at less than this speed. Let \( J \) be an observer moving with \( p \), and suppose that a particle \( q \) now moves in \( J \)'s rest-frame with this same velocity \( \epsilon \). You will perceive \( q \) to be moving at the velocity

\[
\epsilon_2 = \frac{(\epsilon + \epsilon)}{(1 + \frac{\epsilon^2}{c^2})} = c \tanh 2\eta.
\]

This leads to the notion of a Lorentz-invariant set \( \Pi \) of permissible velocities for any reference frame. Definition: The set \( \Pi \) of permissible velocities for a given reference frame is defined as follows:

(i) 0, that is to say, rest, is a permissible velocity

(ii) \( \epsilon \) is a permissible velocity
(iii) If \( r \) and \( t \) are permissible velocities, then \( (r+t)/(1+rt/c^2) \) is a permissible velocity.

(iv) All permissible velocities are derived from applications of (1), (2) and (3).

THEOREM I: The set \( \Pi \Pi \Pi \Pi \) of permissible velocities is given by
\[
\{ c \tanh (N\eta) \} , \quad N = 0, \pm 1, \pm 2 , \ldots \ldots
\]
Readily proven by induction. One need only observe that
\[
c \tanh(A + B) = \frac{c(tanh A + tanh B)}{(1 + tanh A tanh B)}
\]
is the relativistic velocity addition law.

COROLLARY: If \( H \) is my rest frame, and \( J \) is any frame moving relative to \( H \) at a permissible velocity, then objects moving in \( H \) at a permissible velocity, are also moving in \( J \) at a permissible velocity.

THEOREM II: It is consistent with Special Relativity to propose that we live in a world of permissible inertial paths, that is to say, of objects that, in the absence of gravity, only move at permissible velocities.

Since \( \lim_{N \to \infty} (\tanh(N + 1)\eta - \tanh N\eta) = 0 \), it follows that velocities become progressively more continuous as they approach light. Since the normal velocities of our experience are both much larger than \( \varepsilon \) and much smaller than \( c \), we can assume for most practical purposes that velocity is both continuous and additive. Lorentz contractions and time dilations also become more continuous as one approaches the speed of light. Events in a system moving rapidly away from us appear to happen more smoothly than ones in our immediate space-time neighborhood. The term, “streamlining” might describe this phenomenon.
C. UPPER BARRIERS ON ACCELERATION

Our goal is a condition on velocity that will prohibit infinite accelerations.

The word “acceleration” is customarily applied to both continuous and discontinuous changes of velocity. Physicists have no problem with this, although they continue to be unhappy about the possibility of measuring velocity through discontinuous jumps across empty space. What are the consequences of invoking a maximum acceleration barrier for discontinuous or spontaneous change of velocity?

**Time Limit Postulate for spontaneous velocity transitions:**

Any abrupt transition from velocity $v_1$ to $v_2$ requires a minimum time-interval $t(v_1, v_2)$. This time limit is determined by the upper acceleration barrier $\alpha$. Nature, therefore, is neither discontinuous nor continuous: Not discontinuous because spontaneous transitions are ruled out. Not continuous because the time interval $t$ cannot be reduced.

Assuming an ultimate limit $\alpha$ on spontaneous acceleration exists, we must have $(v_2 - v_1)/t < \alpha$. Therefore, the shortest time for any such transition is given by $t = (v_2 - v_1)/\alpha$

Likely candidates for the value of $\alpha$ can be derived through examining various natural phenomena. We defer the introduction of our candidate for the upper barrier to spontaneous acceleration until we have presented a list of 5 arguments in support of a discrete time quantum.
Argument 1: The requirement for a standard clock

Relativity does not make a distinction between time and distance. Since distance is measured by rulers, time by clocks, one is tempted to think that clocks can, in theory, be everywhere replaced by rulers. But clocks and rulers are machines, instruments constructed to perform mathematical groups of operations in parallel or series. Clocks do not function under the same operational group as rulers. The “clock group”, $G_C$, and the “ruler group”, $G_R$, are neither isomorphic nor homomorphic.

“Rulers” can move freely in 3-Dimensional space. The surveyor avails himself of the full freedom of the 3-dimensional Euclidean isometry group (rotations, translations and reflections).

“Clocks” however cannot move freely from the present to the past or future. There is simply no spatial equivalent to the existence of the present, the moment at which the time-reckoner says, “It is now”. Traditional physics has never given a warm welcome to the property of “presentness”, so contrary to the goal of a time-invariant science. It is only recently, with the emergence of Anthropic Principle models that the “now” mode of temporality is being tentatively introduced.

We can’t go back to “yesterday”, or “tomorrow”, then return to “today”, save through a axiomatic scheme of theoretical assumptions on the functioning of ideal harmonic oscillators. Rulers measure intervals, while clocks ‘tick off’ a succession of instants. Our confidence in the regularity of this metronomic march is based on the belief that our clocks return to an exactly identical state after the passage of a complete cycle. Anyone who has ever had trouble in keeping the beat when playing music will know what I am talking about.
This being the case, there exists no ready criterion for stating that two distinct intervals of time, measured at different times (say the day’s length on July 17th, 1992, and July 17, 1993), are of exactly the same length. I am obliged always to pass through the time that I am measuring. How can I be sure that yesterday’s period of the earth’s rotation was (more or less) the same as today’s? The claim is based on a consensus of opinion about the validity of certain laws governing the behavior of dynamical systems, in this case Newton’s laws, which assure us that all (recent—say over the past millennium) days have been more-or-less identical.

Time reckoning therefore depends upon the states of dynamical systems in isolation, not continuously, but only at their period endpoints— at which moments, however, one is obliged to render an exhaustive description of every parameter entering into the state function, including all time derivatives, vibratory modes, mass locations, etc.

In fact, the Earth’s rotation periods are obviously not exactly identical. How do we know this? We can compare the moment of sunrise on each day against the ticking of other, far more accurate clocks, watches, chronometers, and so on. The assertion that any system can return to an exact copy of its initial state after a finite interval of time is always an article of faith, based on observations, theoretical assumptions and physical laws. By using increasingly more accurate watches, (ultimately the vibrations of a cesium atom), we are doing little more than transmitting the self-referencing character of time reckoning to other levels. An infinite chain of descent is epistemologically unacceptable, as this provides no basis for asserting the equality, through comparison, of distinct time intervals.
In any case, this method is bound to fail as long as we are restricted to mechanical clocks since, by the 2nd Law of Thermodynamics, every mechanical clock must ultimately wear out. Unless the universe contains a clock that does not wear out, whose periodic cycles do not lengthen or shrink, there can be no meaning to time as a measurable discernible. There must be a class of phenomena that escape the 2nd Law to avoid a self-referencing absurdity. If my watch happens to be the most accurate and durable in the Cosmos, the standard by which all other watches are set, then its cycles are by default, always exactly identical. In this case, the Second Law of Thermodynamics will be violated, as it depends on time, whose measurement depends on my unique watch.

**Theorem:** No proper meaning can be given to the notion of time as a measurable quantity unless there is a class of perpetual motion machines, standard clocks violating the 2nd Law of Thermodynamics.

**Corollary:** No material system can be a standard clock

Einstein gave us our system of standard clocks: the frequencies of unimpeded photons. The Third Law “protects” their world-lines from Entropy. As long as the path of the photon is unobstructed it will vibrate forever at the same rate.

One can argue further that there should exist a photon, with a frequency, say, of $\psi$ Hertz, which “sets the beat for the rest of the orchestra”\(^7\). All other frequencies (whether electro-magnetic or mechanical) would then be integral multiples of $\psi$. Were this not the case, there would be no standard of comparison between natural clocks. Without the possibility of comparing clocks, there can be no time dimension. Interference patterns between two photons whose frequencies were incommensurable would produce almost-periodic

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\(^7\) Kepler’s ‘Music of the Spheres’ in an unexpected guise.
phenomena with partial periods smaller than $1/\psi$, and if there were two photons whose periods were commensurable, but not integral multiples of $1/\psi$, their destructive interference would produce a photon with a frequency higher than $\psi$. Thus, if $\psi$ is the highest frequency of the Cosmos, then $1/\psi$ must be the time quantum $\tau$.

**Argument 2: Quantum Time Derivatives**

Quantum Theory raises many obstacles to reliable time-reckoning, leading us to question if a metrizable time is even possible. Among the primitive notions of Quantum Theory one does not find a well-defined concept of time duration. The theory does admit a peculiar “Arrow of Time”, in the sense that the moment just before an observation, when the wave-packet has not yet been collapsed, differs from the moment just after the observation. There is no reactive effect of the observation back onto a system’s past.\(^8\)

Quantum Theory’s only acknowledgment of time duration, and that not very convincing, is the “Weak Uncertainty Relation”:

$$\Delta t \cdot \Delta E > h/2\pi.$$  

The time quantum $\tau$, as an entity that is neither discrete nor continuous, returns duration to Quantum Theory. It also permits us to speak of time derivatives, which are otherwise utterly meaningless:\(^9\): the process of differentiating any quantity $Q$ requires two measurements, $Q(0)$ and $Q(\delta t)$, so close to one other that the ratio $\delta Q/\delta t$ can, at least in theory, be returned to $t=0$. This is meaningless within the formalism of Quantum Theory, as the first measurement of $Q$ throws off the second measurement of $Q$ by an enormous uncertainty.

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\(^8\) Allowing for such retroactive effects might resolve some of the quantum paradoxes.

\(^9\)”Quantum Theory and Time Derivatives”, Roy Lisker, Ferment X#8
The minimal discrete time quantum $\tau$ will allow us to speak, theoretically at least, about the time derivatives of physical quantities in the microcosm.

**Argument 3: Zeno’s paradoxes.**

Continuous motion, such as that of Zeno’s arrow, implies the existence of continuous processes. But the nature of time-reckoning never gives us anything but a train of minimal discrete pulses, like the ticks of a stop-watch. Since continuous duration is unmeasurable, in fact unobservable, one can argue that continuous processes, by Occam’s Razor, are unscientific.

What is being called into question is the seemingly natural 1-1 mapping of length $R$ onto time $T$

$$\Phi: R \rightarrow T$$

Notice the many operational differences between range $R$ and domain $T$. “Lengths” can be bisected by going into 3 dimensions and using ruler and compass. The only way to bisect a time interval $[0, \sigma]$ *is to build two clocks $C_1$ and $C_2$*, the first of which pulses in the period $\sigma$, while the second pulses twice in the same period. We cannot know in advance if our second clock will work; we must use trial and error, until by sheer luck we hammer together the right clock, $C_2$. Since the midpoint of a line segment is constructible by method, while the determination of the mid-point of a time interval is dependent on luck, (back reconstruction), an appeal to intuition would strongly urge that *we are speaking of two inherently different quantities*, which cannot readily be mapped onto each other.

Indeed, consider the reverse mapping

$$\Phi^{-1}: R \rightarrow T$$

I doubt that anyone would suggest that spatial measurement ought to be done by building a special clock for every conceivable time
interval, then using the path of a light-beam to map those durations onto length!

The time quantum $\tau$ replaces continuous processes by ones that jump through extremely minute steps from one moment to the next. By setting limits on the constructibility of clocks, it supplies a “finitistic” solution to Zeno’s Paradoxes.

**Argument 4: The collapse of the wave packet**

At the root of many of the problems involved in the reconciliation of Relativity with Quantum Theory lies the following observation: A Schrödinger wave-function $\Psi(x,t)$ is at any single instant $t$ spread out over all of space. The probability, for example, of finding an electron in any small box $B$ is given by the integral of $\Psi^*\Psi$ taken over the box. However, if by chance the electron is “discovered” in the box, the wave function “collapses” to zero instantly over all of the rest of space - an egregious violation of Special Relativity.

The minimal time quantum, $\tau$, suggests a solution to this inconsistency: if the box $B$ is very small, the uncertainty $\Delta p$ in the electron’s momentum is very large: my discovery of the electron causes it to fly away to anywhere at any speed. The Schrödinger wave distribution of the electron over all of space is restored almost immediately after its detection.

Proclaiming that nothing can be seen or known within any sub-interval of the time quantum interval $[0, \tau]$, allows time for the wave packet to collapse and be restored without anyone being the wiser. A combination of relativistic limitations with quantum limitations on knowledge has restored our faith in Nature. Physical theory is saved by unavoidable ignorance!

**Argument 5: Placing the Uncertainty Principle in Nature.**
There are many reasons why we want Heisenberg’s Uncertainty Principle to exist in Nature independently of observation. If quantum uncertainty in phenomena depends only on the way people look at them, how account for the decay processes that have nothing to do with observation? Do we need to keep watching everything to make the Third Law of Thermodynamics come true, or can substances drop to Absolute Zero if we don’t look at them? Does a Black Hole have a temperature only when observed? Are there intermediate energy levels between the eigenstates? Must Nature be in an eigenstate to be seen, or does nothing exist outside of a eigenstate?

An argument along the following lines suggests that the time quantum situates the Uncertainty Principle in Nature itself: Observation always involves an interactive loop between the observer and the thing observed. This loop bears some relationship to the Twin’s Paradox loop of relativity, interpreted as a forward leap in time. Consider a situation involving the phenomenon P, the observer O, and a channel, C, along which some agent (quanta, electron beams, etc. - the ‘messenger’) moves. This messenger establishes contact with the phenomenon, and brings back a ‘news report’ of its attributes.

The messenger, M, abides in his proper time, s, while we, the observers at O, make our observations in a time interval r which must, by Relativity, be greater than s. Both r and s are larger than the time quantum, $\tau$, and because this is so, there is an intrinsic uncertainty, within the channel itself, that is magnified when transmitted to the observer.

This interpretation of “time quantum jumps” as “Twin’s Paradox Loops”, is similar to the interpretation of discrete energy levels as loop

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10 “I’ll believe in Black Holes when I see one!” (Phillip Morrison)
11 As biased and inaccurate as any journalist’s, but the best we have.
integrals around atomic orbits. This is encouraging, because time and energy are conjugate observables in Quantum Mechanics, and a quantization of time may be the missing step in understanding the properties of a time operator conjugate to the energy operator, or Hamiltonian.

Having presented our arguments for the necessity of a time quantum, we now assume its existence for the determination of the acceleration barrier, $\alpha$.

**Time-Energy Postulate:** No system, (or subsystem), can convert all of its potential energy to kinetic energy in a time interval less than $\tau$.

Consider an electron at rest spontaneously decaying into gamma radiation moving away at velocity $c$. By the above postulate, this cannot occur in a time interval less than $\tau$. Let $v(t)$ be the velocity of the moving object, (electron or radiation, or some intermediate substance), in the time interval $0 < t < \tau$. Assuming a linear growth of velocity in this interval, we derive:

\[
\alpha = \frac{c}{\tau}
\]

In general, if the kinetic energy of any isolated particle with velocity $v$ is given by $K = \frac{1}{2}(mv^2)$, we want to say that the rate of change of the ratio $K/m$, or

\[
\frac{d}{dt} \left( \frac{K}{m} \right) = \frac{d}{dt} \left( \frac{v^2}{2} \right)
\]

is less than some universal constant, $\mu$.

This gives

\[
\frac{d}{dt} \left( \frac{K}{m} \right) = v \frac{dv}{dt} \leq \mu.
\]

Now $(v_2 - v_1)/t \leq \alpha$, and $v \leq c$, so we also have
Equating these upper limits gives us
\[ \mu = c \alpha. \]
Hence
\[
\mu = \frac{c^2}{\tau}
\]

**Assertion:** (Not rigorous enough to call a theorem): In all transformations of a particle of mass, m, whenever the kinetic energy changes by an amount \( \Delta K \), we have \( \Delta K/m < c^2/\tau \), where \( \tau \) is the minimal time quantum.

### The Uncertainty Principle

The Uncertainty Principle connects the time quantum to a photon frequency upper barrier. The Uncertainty Principle states that the energy change across the period of an observation is in inverse relationship to the time needed to make the observation. Invoking the principle in the form \( \Delta t \cdot \Delta E > h \), we identify \( \Delta t \) with the minimal time quantum, \( \tau \), and use the above relation \( \Delta E/\tau < mc^2/\tau \), to derive the inequality:

\[
(mc^2/\tau)(h \nu/2\pi \tau) \geq \Delta E \geq h/2\pi \tau^2
\]

Replacing the particle of mass m, with the photon of highest frequency \( \psi \), we can substitute \( h\psi/2\pi \) for \( mc^2 \), and equate the outer ends of the above inequalities. This gives:

\[
h\psi/2\pi \tau = h/2\pi \tau^2, \text{ or } \tau = 1/\psi
\]
The length of the minimum time quantum is equal to the inverse of the highest possible frequency.

The same result is derived independently on page 26. The above demonstration presents one of the ways by which the time-quantum situates the Uncertainty Principle in Nature, independently of the interaction of observer and observed.