Malament-Hogarth Machines

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Abstract

We show a clear sense in which general relativity allows for a type of "machine" which can bring about a spacetime structure suitable for the implementation of "supertasks."

1 Introduction

In what follows, the intersection of two concepts in the foundations of general relativity are investigated: (1) Malament-Hogarth spacetimes which allow for a type of "supertask" in which a future infinite timelike curve is contained in the past of a spacetime event and (2) "machine" spacetimes which bring about various properties from initial conditions (e.g. "time machines" are spacetimes which bring about a particular type of unusual causal structure). After introducing a quite general characterization of machine spacetimes, we consider various definitions of "Malament-Hogarth machines" and show their existence. The upshot of our work is this: there a clear sense in which general relativity allows for a type of machine which can bring about a spacetime structure suitable for the implementation of supertasks. We close by outlining a program for future work on the subject.

2 Preliminaries

We begin with a few preliminaries concerning the relevant background formalism of general relativity.[1] An n-dimensional, relativistic *spacetime* (for $n \geq 2$) is a pair of mathematical objects (M, g_{ab}): M is a smooth (connected) n-dimensional manifold and g_{ab} is a smooth metric on M of Lorentz signature $(+, -, ..., -)$. Note that M is assumed to be *Hausdorff*; for any distinct $p, q \in M$, one can find disjoint open sets O_p and O_q containing p and q respectively.

For each point $p \in M$, the metric assigns a cone structure to the tangent space M_p. Any tangent vector ξ^a in M_p will be *timelike* if $g_{ab}\xi^a\xi^b > 0$, *null* if $g_{ab}\xi^a\xi^b = 0$, or *spacelike* if $g_{ab}\xi^a\xi^b < 0$. Null vectors create the cone structure;

[1] The reader is encouraged to consult Hawking and Ellis (1973), Wald (1984), and Malament (2012) for details. An outstanding (and less technical) survey of the global structure of spacetime is given by Geroch and Horowitz (1979).

timelike vectors are inside the cone while spacelike vectors are outside. A *time orientable* spacetime is one that has a continuous timelike vector field on M. A time orientable spacetime allows one to distinguish between the future and past lobes of the light cone. In what follows, it is assumed that spacetimes are time orientable and that an orientation has been chosen.

For some connected interval $I \subseteq \mathbb{R}$, a smooth curve $\gamma : I \to M$ is *timelike* if the tangent vector ξ^a at each point in $\gamma[I]$ is timelike. Similarly, a curve is *null* (respectively, *spacelike*) if its tangent vector at each point is null (respectively, spacelike). A curve is *causal* if its tangent vector at each point is either null or timelike. A causal curve is *future-directed* if its tangent vector at each point falls in or on the future lobe of the light cone. For any smooth curve $\gamma : I \to M$ with tangent field ξ^a, the *length* $\|\gamma\|$ is given by $\int_\gamma (\xi^a \xi_a)^{\frac{1}{2}} ds$. When γ is timelike, its length represents the elapsed proper time along the curve.

We say a curve $\gamma : I \to M$ is not *maximal* if there is another curve $\gamma' : I' \to M$ such that I is a proper subset of I' and $\gamma(s) = \gamma'(s)$ for all $s \in I$. A curve $\gamma : I \to M$ in a spacetime (M, g_{ab}) a *geodesic* if $\xi^a \nabla_a \xi^b = \mathbf{0}$ where ξ^a is the tangent vector and ∇_a is the unique derivative operator compatible with g_{ab}. A spacetime (M, g_{ab}) is *geodesically complete* if every maximal geodesic $\gamma : I \to M$ is such that $I = \mathbb{R}$. If an incomplete geodesic is timelike or null, there is a useful distinction one can introduce. We say that a future-directed timelike or null geodesic $\gamma : I \to M$ without future endpoint is *future incomplete* if there is an $r \in \mathbb{R}$ such that $s < r$ for all $s \in I$. A *past incomplete* timelike or null geodesic is defined analogously.

A point $p \in M$ is a *future endpoint* of a future-directed causal curve $\gamma : I \to M$ if, for every neighborhood O of p, there exists a point $t_0 \in I$ such that $\gamma(t) \in O$ for all $t > t_0$. A *past endpoint* is defined similarly. A causal curve is *future inextendible* (respectively, *past inextendible*) if it has no future (respectively, past) endpoint. A spacetime (M, g_{ab}) is *effectively complete* if, for every future or past incomplete timelike geodesic $\gamma : I \to M$, and every open set O containing γ, there is no isometric embedding $\varphi : O \to M'$ into some other spacetime (M', g'_{ab}) such that $\varphi \circ \gamma$ has future and past endpoints.

For any two points $p, q \in M$, we write $p << q$ if there exists a future-directed timelike curve from p to q. We write $p < q$ if there exists a future-directed causal curve from p to q. These relations allow us to define the *timelike and causal pasts and futures* of a point p: $I^-(p) = \{q : q << p\}$, $I^+(p) = \{q : p << q\}$, $J^-(p) = \{q : q < p\}$, and $J^+(p) = \{q : p < q\}$. We say a spacetime is *J-closed* if, for all $p \in M$, the sets $J^-(p)$ and $J^+(p)$ are topologically closed. A spacetime satisfies *chronology* if there is no $p \in M$ such that $p \in I^+(p)$. A spacetime which violates chronology has timelike curves $\gamma : [s_0, s_1] \to M$ such that $\gamma(s_0) = \gamma(s_1)$; such timelike curves are called *closed*. We say a spacetime satisfies *strong causality* if, for all points $p \in M$ and every open set O containing p, there is an open set $V \subset O$ also containing p such that no causal curve intersects V more than once. A spacetime satisfies *stable causality* if there is a smooth function $t : M \to \mathbb{R}$ such that for any distinct points $p, q \in M$, if $q \in J^+(p)$, then $t(p) < t(q)$.

A set $S \subset M$ is *achronal* if no two points in S can be connected by a timelike curve. The *edge* of a closed, achronal set $S \subset M$ is the set of points $p \in S$ such that every open neighborhood O of p contains a point $q \in I^+(p)$, a point $r \in I^-(p)$, and a timelike curve from r to q which does not intersect S. A set $S \subset M$ is a *slice* if it is closed, achronal, and without edge. A spacetime (M, g_{ab}) which contains a slice S such that $D(S) = M$ is said to be *globally hyperbolic*. In such a spacetime, we say S is a *Cauchy surface*.

Two spacetimes (M, g_{ab}) and (M', g'_{ab}) are *isometric* if there is a diffeomorphism $\psi : M \to M'$ such that $\psi_*(g_{ab}) = g'_{ab}$. We say a spacetime (M', g'_{ab}) is an *extension* of (M, g_{ab}) if there is a proper subset N of M' such that (M, g_{ab}) and (N, g'_{ab}) are isometric. We say a spacetime is *maximal* if it has no extension. A spacetime (M, g_{ab}) is *past-maximal* if, for each of its maximal extensions (M', g'_{ab}) with isometric embedding $\psi : M \to M'$, we have $I^-(\psi(M)) = \psi(M)$. A *future-maximal* spacetime is defined analogously.

3 Malament-Hogarth Spacetimes

It has been argued that some models of general relativity have a spacetime structure suitable for the implementation of "supertasks" of a certain kind.[2] The idea is beautifully simple. Consider the following definition.

Definition. A spacetime (M, g_{ab}) is *Malament-Hogarth* if there is a past-extendible timelike curve $\gamma : I \to M$ and a point $p \in M$ such that (i) $\|\gamma\| = \infty$ and (ii) $\gamma[I] \subset I^-(p)$.

Consider a Malament-Hogarth spacetime (M, g_{ab}) and let $p \in M$ and $\gamma : I \to M$ be as in the definition. Let q be the past endpoint of γ. The points q and p represent, respectively, the beginning and the end of the supertask. At q, consider two observers (call them A and B) who decide to take very different future paths through spacetime. Observer A follows the path along γ. Observer B takes any path from q to p. (It follows from condition (ii) that a timelike curve exists with past endpoint q and future endpoint p.) Condition (ii) ensures that a signal can be sent from any point along γ to the point p. Condition (i) ensures that observer A has an infinite amount of future time along γ. Thus, there is a sense in which observer B can, from the point p, "view an eternity in a finite time" (Hogarth 1992).

Anti-de Sitter spacetime is the paradigm Malament-Hogarth example (see Earman and Norton 1993). In two dimensions, the spacetime is (M, g_{ab}) where $M = \mathbb{R}^2$ and $g_{ab} = \cosh^2 x \nabla_a t \nabla_b t - \nabla_a x \nabla_b x$. By inspection we see that the light cones widen rapidly as $|x| \to \infty$. It turns out that because of this fact, there exist past-extendible timelike curves $\gamma : I \to M^-$ such that $\|\gamma\| = \infty$ where $M^- = \{(t, x) \in M : t < 0\}$. Moreover, there are points p such that

[2] See, for example, the following: Pitowski 1990; Hogarth 1992, 1994; Earman and Norton 1993, 1996; Etesi, G., and I. Németi 2002; Manchak 2010; Manchak and Roberts 2016; Andréka et al. 2017.

$M^- \subset I^-(p)$. It follows that anti-de Sitter spacetime is Malament-Hogarth. (See figure 1.)

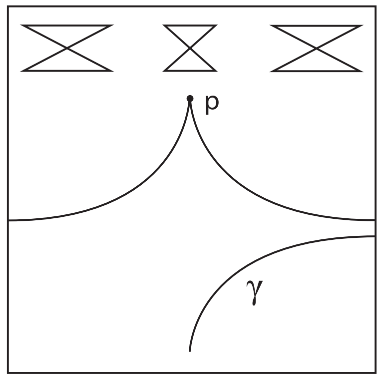

Figure 1: Anti-de Sitter spacetime is Malament-Hogarth since the past-extendible timelike curve γ is contained in $I^-(p)$ and is such that $\|\gamma\| = \infty$.

Anti-de Sitter spacetime fails to be globally hyperbolic and this turns out to be a general feature shared by all Malament-Hogarth spacetimes. We have the following (Hogarth 1992).

Proposition. All Malament-Hogarth spacetimes fail to be globally hyperbolic.

The cosmic censorship hypothesis championed by Penrose (1979, 1999) can be stated as: "All physically reasonable spacetimes are globally hyperbolic" (Wald 1984, 304). Suppose for a moment that this statement is true. Then, a corollary to the above proposition is the statement that no Malament-Hogarth spacetime is physically reasonable. Now since Penrose's version of the cosmic censorship is quite controversial (see Earman 1995), there is still room to argue that there that non globally hyperbolic spacetimes can be physically reasonable in some sense. But there are other potential problems for Malament-Hogarth spacetimes (see Earman and Norton 1993). In the anti-de Sitter example, the curve γ has an infinite total (integrated) acceleration. This means that observer A would need an infinite amount of fuel to take such a path through spacetime. Another complication is the "divergent blueshift" problem. The worry is this: the frequency of signals sent from observer A to observer B is amplified more and more as time goes on. Eventually, "even the slightest thermal noise will be amplified to such an extent that communication is all but impossible" (Manchak and Roberts 2016). Both the infinite acceleration and divergent blueshift problems can be easily avoided if chronology is violated. But we wish to emphasize here that even if one restricts attention to stably causal (and hence chronological) spacetimes, one can still find Malament-Hogarth spacetimes which avoid all of the problems mentioned above and more (see Manchak 2010; Andréka et al. 2017). In other words, it remains an open question whether there exist physically reasonable Malament-Hogarth spacetimes.

4 Machines

Malament-Hogarth spacetimes are certainly fascinating. But we are ultimately interested in the possibility of "bringing about" their properties in our own universe. We know that whatever else is the case, one cannot make sense of this "bringing about" by employing the usual notion of causal determinism present in general relativity; as mentioned above, a globally hyperbolic spacetime can never be Malament-Hogarth. The literature on "machines" – especially "time machines" and "hole machines" (Earman, Wüthrich, and Manchak 2016) – provides a framework to understand the "bringing about" notion in a quite general way.[3] Consider the following.

Definition. A past-maximal, globally hyperbolic spacetime is a ($\mathscr{P}$, $\mathscr{Q}$)-*machine* if (i) some extension to the spacetime has $\mathscr{P}$ and (ii) every extension which has $\mathscr{P}$ also has $\mathscr{Q}$.

A past-maximal, globally hyperbolic spacetime represents a "time" before the machine is switched on; let us call such a spacetime a *starter* in what follows. Property $\mathscr{P}$ is used to pare down the space of starter extensions to those which are "physically reasonable" in some sense.[4] Property $\mathscr{Q}$ is the one intended to be brought about by the machine. Condition (ii) captures the idea that all "physically reasonable" starter extensions have $\mathscr{Q}$. Condition (i) is added to avoid a nuisance case; we don't want to count a starter as a ($\mathscr{P}$, $\mathscr{Q}$)-machine simply because (ii) is vacuously true.

Research on ($\mathscr{P}$,$\mathscr{Q}$)-machines has (so far) focused primarily on existence results where $\mathscr{Q}$ is the failure of chronology. Consider the following "time machine" existence result, for example (Manchak 2011a).

Proposition. There exist ($\mathscr{P}$, $\mathscr{Q}$)-machines where $\mathscr{P}$ is J-closedness and $\mathscr{Q}$ is the failure of chronology.

The starter used to exhibit the above proposition is the "bottom half" of Misner spacetime (see Hawking and Ellis 1973). Consider Misner spacetime (M, g_{ab}): $M = \mathbb{R} \times S$ and $g_{ab} = 2\nabla_{(a}t\nabla_{b)}\varphi + t\nabla_a\varphi\nabla_b\varphi$ where the points (t, φ) are identified with the points $(t, \varphi + 2\pi n)$ for all integers n. The bottom half of Misner spacetime is (M^-, g_{ab}) where $M^- = \{(t, \varphi) \in M : t < 0\}$. (See figure 2.) One can verify that (M^-, g_{ab}) is a starter; it is past-maximal and globally hyperbolic. Now, some extensions to the starter have $\mathscr{P}$ and some do not. But it turns out that all extensions to the starter which have $\mathscr{P}$ also have $\mathscr{Q}$.

[3] See also: Earman 1995; Earman, Smeenk, and Wüthrich 2009; Krasnikov 2002, 2014; Manchak 2009a, 2009b 2011a, 2014a; Smeenk and Wüthrich 2010.

[4] See Manchak 2011b for a discussion of our limitations in determining the class of "physically reasonable" spacetimes.

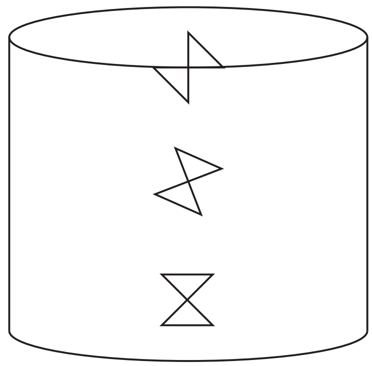

Figure 2: The "bottom half" of Misner spacetime is a ($\mathscr{P}$, $\mathscr{Q}$)-machine where $\mathscr{P}$ is J-closedness and $\mathscr{Q}$ is the failure of chronology.

We close this section with a word concerning the structure of existence results like those considered above. Suppose for some properties $\mathscr{P}$ and $\mathscr{Q}$ one finds there is a ($\mathscr{P}$, $\mathscr{Q}$)-machine. It is important to note that one's choice of $\mathscr{P}$ is, in general, crucial for the existence result to go through: If $\mathscr{P}_0 \Rightarrow \mathscr{P} \Rightarrow \mathscr{P}_1$ for some properties $\mathscr{P}_0$, $\mathscr{P}_1$, there is no guarantee that either a ($\mathscr{P}_0$, $\mathscr{Q}$)-machine or a ($\mathscr{P}_1$, $\mathscr{Q}$)-machine exists. The former is not guaranteed since $\mathscr{P}_0$ may be so strong that the starter used to exhibit the ($\mathscr{P}$,$\mathscr{Q}$)-machine may not even have a $\mathscr{P}_0$ extension. The latter is not guaranteed either; if the class of "physically reasonable" spacetimes is enlarged by $\mathscr{P}_1$, it is possible that the starter used to exhibit the ($\mathscr{P}$, $\mathscr{Q}$)-machine may be such that one of its $\mathscr{P}_1$-but-not-$\mathscr{P}$ extensions is not $\mathscr{Q}$.

5 Malament-Hogarth Machines

Using the framework from the preceding section, one defines a "Malament-Hogarth machine" to be a ($\mathscr{P}$, $\mathscr{Q}$)-machine where $\mathscr{Q}$ is the property of being a Malament-Hogarth spacetime. What about $\mathscr{P}$? As noted above, one's choice of $\mathscr{P}$ is generally quite important; an existence result may fail if $\mathscr{P}$ is either too strong or too weak.

One often restricts attention to "physically reasonable" spacetimes by invoking a pair of global conditions – one to rule out artificial "holes" in spacetime and one to rule out "bad" causal structure (see Earman 1995). There are two useful logical hierarchies to consider. We have a "no-holes" hierarchy (Manchak 2014b): geodesic completeness (GC) $\Rightarrow$ effective completeness (EC) $\Rightarrow$ maximality (M). And we have a (simplified) hierarchy of causal conditions (Wald 1984): global hyperbolicity (GC) $\Rightarrow$ stable causality (SC) $\Rightarrow$ strong causality (Str) $\Rightarrow$ chronology (C). Using these hierarchies, one can settle a quite a few open questions concerning Malament-Hogarth machines in one fell-swoop. We have the following.

Proposition. There exist ($\mathscr{P}$, $\mathscr{Q}$)-machines where $\mathscr{Q}$ is the property of being Malament-Hogarth and $\mathscr{P}$ is any property such that ((SC) & (GC)) $\Rightarrow \mathscr{P}$.

Proof. Here we construct an example which is conformally equivalent to a portion of Minkowski spacetime. The example is two-dimensional for the sake of simplicity but can be easily generalized to any dimension $n \geq 2$.

Consider Minkowski spacetime $(\mathbb{R}^2, \eta_{ab})$ in standard (t, x) coordinates: $\eta_{ab} = \nabla_a t \nabla_b t - \nabla_a x \nabla_b x$. Let us agree that our temporal orientation is such that the vector $(\partial/\partial t)^a$ is future-directed. Let $M = \mathbb{R}^2 - J^+((0,0))$ and let $g_{ab} = \Omega^2 \eta_{ab}$ where the function $\Omega : M \to \mathbb{R}$ is defined by $\Omega(t, x) = (t^2 + x^2)^{-1}$. The spacetime (M, g_{ab}) is conformally equivalent to (and therefore has the same causal structure as) the portion (M, η_{ab}) of Minkowski spacetime. Since (M, η_{ab}) is globally hyperbolic, so is (M, g_{ab}). (The set $\{(t, x) \in M : t = -1\}$ is one Cauchy surface.) By construction, (M, g_{ab}) is past-maximal. We now show that any extension to (M, g_{ab}) is a Malament-Hogarth spacetime.

Let (M', g'_{ab}) be any extension at all of (M, g_{ab}). Consider the curve $\gamma : (0, 1) \to M$ defined by $\gamma(s) = (s - 1, 0)$. So the tangent vector ξ^a is $(\partial/\partial t)^a$ at every point along γ. Thus, γ is a future-directed timelike curve with past endpoint $(-1, 0)$. We have $dt/ds = \xi^a \nabla_a t = 1$ and so $\|\gamma\| = \int_\gamma (\xi^a \xi_a)^{\frac{1}{2}} ds = \int_\gamma (\xi^a \xi_a)^{\frac{1}{2}} dt$. Because $\xi^a \xi_a = \Omega^2$ we have $\int_\gamma (\xi^a \xi_a)^{\frac{1}{2}} dt = \int_\gamma (t^2 + x^2)^{-1} dt$. But since $x = 0$ along γ, the last quantity simplifies to $\int_\gamma t^{-2} dt = -t^{-1}\big|_{-1}^{0} = \infty$.

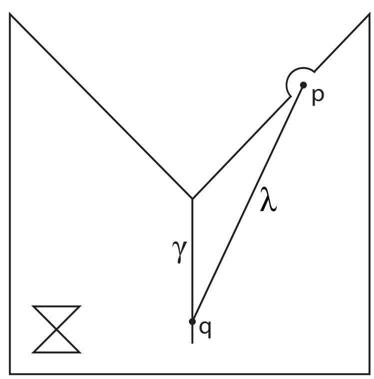

Figure 3: The region $M \cup O$ of M'. A future-directed timelike curve λ can be constructed from every point $q \in \gamma[I]$ to the point p.

Let $p \in M'$ be any point on the boundary of M. One can extended the coordinate system on M to $M \cup O$ for some neighborhood O of p. Clearly, $p = (p_t, p_x)$ is such that $p_t > 0$ and $|p_x| = p_t$. Without loss of generality, let us assume that $p_t = p_x$. (An analogous argument can be given for $p_t = -p_x$.) Let $q = (q_t, 0)$ be any point on $\gamma[I]$. Let $\lambda : [0, 1] \to M'$ be defined by $\lambda(s) =$

$((p_t - q_t)s + q_t, sp_x)$. We find that the tangent vector ζ^a at every point along λ is $(p_t - q_t)(\partial/\partial t)^a + p_x(\partial/\partial x)^a$. So $\zeta^a \zeta_a = \Omega^2[(p_t - q_t)^2 - p_x^2]$. Because $p_t = p_x > 0$ and $q_t < 0$, we see that ζ^a is a future-directed timelike vector. Since $\lambda(0) = q$ and $\lambda(1) = p$, it follows that $p \in I^+(q)$. (See Figure 3.) Since q is an arbitrary point on $\gamma[I]$, we have $\gamma[I] \subset I^-(p)$. Thus, (M', g'_{ab}) is a Malament-Hogarth spacetime.

We are done if we can find an extension to (M, g_{ab}) which is stably causal and geodesically complete. But this is easy: consider the extension (M', g'_{ab}) such that $M' = \mathbb{R}^2 - \{(0,0)\}$ and $g'_{ab} = \Omega'^2 \eta_{ab}$ where the function $\Omega' : M' \to \mathbb{R}$ is defined by $\Omega'(t,x) = (t^2 + x^2)^{-1}$. $\square$

6 Conclusion

We close with a suggestion for future work. In the preceding, no imposition has been made on the local structure of spacetime; in particular, Einstein's equation (with some reasonable matter source) did not enter into the discussion. Essentially, we have leaned heavily on the idea that "one's lack of concern with Einstein's equation in these examples is a reflection of the experience that things which can happen in the absence of this equation can usually also happen in its presence" (Geroch and Horowitz 1979, 215). That said, it might be of interest to see if an existence result can still be obtained if one restricts attention to Malament-Hogarth spacetimes whose stress energy tensor T_{ab} (given by Einstein's equation) satisfies some energy condition or other (see Earman and Norton 1993).

Let (E) be any one of several energy condition of interest (see Curiel 2017). Given the hierarchies mentioned in the previous section, sixteen open questions present themselves which (without further argument) are independent of each other. (There is plenty of work to do here!)

Question. Do there exist ($\mathscr{P}$, $\mathscr{Q}$)-machines if $\mathscr{Q}$ is the property of being Malament-Hogarth and $\mathscr{P}$ is any one of the following?

1. (E)
2. (E) & (M)
3. (E) & (EC)
4. (E) & (GC)
5. (E) & (C)
6. (E) & (C) & (M)
7. (E) & (C) & (EC)
8. (E) & (C) & (GC)
9. (E) & (Str)
10. (E) & (Str) & (M)
11. (E) & (Str) & (EC)
12. (E) & (Str) & (GC)
13. (E) & (SC)

14. (E) & (SC) & (M)
15. (E) & (SC) & (EC)
16. (E) & (SC) & (GC)

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