**The Relativistic Einstein-Podolsky-Rosen Argument**

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**Abstract**

 We present the possibility of a relativistic formulation of the Einstein-Podolsky-Rosen argument. We pay particular attention to the need for a reformulation of the so-called reality criterion. We introduce such a reformulation for the reality criterion due to Ghirardi and Grassi and show how it applies to the nonrelativistic EPR argument. We elaborate on Ghiradi and Grassi’s proof and explain why it cannot be circumvented. Finally, we review and summarise our own views. This is a continuation of the paper by myself and Patrick La Riviere whose defects are exposed, and corrected, in the present work.

**1. Preliminaries**

At first glance, the realist interpretations of quantum mechanics such as Bohm's offer many advantages over standard interpretations of the theory. In particular, they give a clear, intuitive picture of many potentially paradoxical, physical situations, such as the two-slit experiment and the phenomenon of barrier penetration. At the same time, their chief drawback- a form of nonlocality that seems, as we have claimed, to conflict with the constraints of relativity theory- is apparently shared by the standard 'anti-realist' interpretations that reject hidden variables and assume completeness, as was demonstrated by the original Einstein-Podolsky-Rosen (EPR) argument.

However, while the Bell argument that establishes nonlocality for realistic interpretations such as Bohm's has been formulated in a relativistic context, there is no well-established relativistic formulation of the EPR argument. In the absence of such a formulation, it seems hasty to conclude that the tension between the standard interpretations and relativity theory is just as great as that between Bohmian interpretations and relativity. Clearly, if a relativistic formulation of EPR could be given that did not entail nonlocality, anti­ realist interpretations would have an advantage over the Bohmian interpretation.

Let us now investigate the possibility of a relativistic formulation of the EPR argument. First, the standard nonrelativistic version of the EPR argument is reviewed and the problematics of translating it into a relativistic context are considered, paying particular attention to the need for a reformulation of the so-called reality criterion. Then, we introduce one such reformulated reality criterion, due to GianCarlo Ghirardi and Renata Grassi, and show how it is applied to the nonrelativistic EPR argument. Next, we discuss the application of the new reality criterion in a relativistic context.

A relativistic version of the EPR argument must differ from the nonrelativistic version in two principal ways. First, the particle states must be described by a relativistic wave-function. The details don't concern us here; we need only require that the wave-function preserve the maximal, mirror-image correlations of the nonrelativistic singlet state. And indeed, the existence of maximal correlations in the vacuum state of relativistic algebraic quantum field theory has recently been demonstrated (see Redhead 1995). Second, the argument must not depend on the existence of absolute time ordering between the measurement events on the left and right wings of the system, for in the relativistic argument these may be spacelike separated. As it turns out, the nonrelativistic version of the argument does invoke absolute time ordering. To see how to get around this problem, we must briefly review the standard formulation of the incompleteness argument.

For EPR, a necessary condition for the completeness of a theory is that every element of physical reality must have a counterpart in the theory. To demonstrate that quantum mechanics is incomplete, EPR need simply point to an element of physical reality that does not have a counterpart in the theory. In this vein, they consider measurements on a pair of scattered particles with correlated position and momentum, but the formulation of the argument in Bohm, in terms of a pair of oppositely moving, singlet-state, spin-1/2 decay products of a spin-0 particle, is conceptually simpler. In this case, the formalism of quantum mechanics demands a strict correlation between the spin components of the two spatially separated particles, such that a measurement of, say, the z-component of spin of one particle allows one to predict with certainty the outcome of the same measurement on the distant particle . This ability to predict with certainty, or at least probability one, the outcome of a measurement is precisely the EPR criterion for the existence of an element of reality at the as-yet-unmeasured particle. By invoking one final assumption, a locality assumption stating that elements of reality pertaining to one system cannot be affected by measurements performed 'at a distance' on another system, EPR can establish that the element of reality at the unmeasured particle must have existed even before the measurement was performed at the distant particle . But the quantum formalism describes the particles at this point with the singlet state, and thus has no counterpart for the element of reality at the unmeasured particle. It follows that the quantum description was incomplete. Schematically,

Quantum Formalism ^ Locality  ~ (1)

 Completeness.

Alternatively, if one assumes completeness, the argument may be rearranged as a proof of nonlocality:

Quantum Formalism ^Completeness  ~ (2)

Locality.

 The problematic assumption of absolute time ordering entered the argument in the reality criterion, which turns on the possibility of predicting with certainty the outcome of a measurement along one wing *subsequent* to having obtained the result of a measurement along the other. Of course, for spacelike separated events, notions like precedence and subsequence are reference-frame dependent, not absolute. So to translate the EPR argument to a relativistic context requires a modified criterion for the attribution of elements of reality that is not contingent on the time ordering of the measurement events. Ghirardi and Grassi have undertaken to formulate just such a criterion. For the sake of clarity, we shall first describe how this criterion applies to the nonrelativistic version of the argument.

Ghirardi and Grassi's criterion rests on the truth of certain classes of counterfactual statements

-statements of the form 'if φ were true, then ψ would be true' where the antecedent φ is in general known to be false. In particular, they wish to link the attribution at a time *t* of the property corresponding to [observable **a**having value a] to the truth of the counterfactual assertion: if measurement of **a** were performed at time *t,* then the outcome would be a ( they use David Lewis’s analysis of counterfactuals here).

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With this criterion in hand, Ghirardi and Grassi can now run the *nonrelativistic* EPR argument essentially as before. They assume a measurement of property **a**is performed on the right-hand particle at time *t*R, yielding a specific result a. To ascertain whether an element of reality corresponding to property a = a' exists at the left-hand particle, they must assess the truth of the counterfactual assertion: 'if I were to perform a measurement of property **a** at the left-hand particle at time *t*L, I would obtain the result a'.' In the nonrelativistic case, the truth of this counterfactual assertion follows naturally from the presence of absolute time ordering. For if *t*R < *t*L, then the outcome of the right-hand measurement can be assumed to be the same in all of the 'accessible' (most similar) worlds used to evaluate the counterfactual, because it is strictly in the past of the counterfactual's antecedent. The strict correlation laws of quantum mechanics, also assumed to hold in all accessible worlds, then demand that the result of a measurement on the left wing also be fixed in all possible worlds (specifically, the laws require that a' = -a). Thus the counterfactual is true, and an element of reality can be said to exist at the left-hand particle. From here, the argument unrolls in the usual way, and by supplementing this reality criterion with a locality assumption (they call it G-Loc, after Galileo), Ghirardi and Grassi can deduce that quantum mechanics is incomplete. Once again, we can represent their argument schematically by

Quantum Formalism ^ G-Loc → ~ (3)

Completeness or

Quantum Formalism ^ Completeness → ~ (4)

G-Loc.

**2. The crux of the argument**

'A system cannot be affected by actions on a system from which it is isolated. In particular, elements of physical reality of a system cannot be influenced by actions on systems from which it is isolated.' An examination of the structure of Ghirardi and Grassi's argument reveals that they make use not of the general principle stated but of a special case of this general principle, namely that elements of reality cannot be brought into existence 'at a distance.' It is this special case of G-Loc, call it ER-Loc (for elements of reality) that enters toward the end of the argument to establish that the measurement at the right wing could not have created an element of reality at the left wing and thus it must have existed prior to the measurement at the right wing, when the quantum formalism said the particles were in the singlet state. Thus they conclude that quantum mechanics is incomplete. All is well so far, but when one turns the argument around, assuming completeness and dispensing with locality, one must ask, can one be more precise as to which locality principle should be given up: the principle they label G-Loc, or the special case ER-Loc? Indeed it is the latter, for only it entered into the argument. As it turns out the distinction between G-Loc and ER-Loc does not affect their conclusions in the nonrelativistic case, because the conclusion they choose to highlight- the creation of elements of reality at a distance- is precisely one that does follow from dispensing only with ER-Loc.

In the relativistic case they want to claim peaceful coexistence between nonlocality and special relativity. However, greater care must be taken with the statement of the locality principle, this time called L-Loc (after Lorentz by Ghirardi and Grassi), because a locality principle must enter at the very beginning of the argument as well as in the usual way at the end. The argument begins in the same way as in the nonrelativistic case, with the occurrence of a measurement on the right-hand side, but now the absence of absolute time ordering means the result of this measurement can no longer tacitly be assumed to be the same in all the accessible worlds used to evaluate the element-of-reality counterfactual at the left­hand side. Locality must be invoked to establish the independence of the outcome of the right­hand measurement from the occurrence of the measurement at the left. This done, Ghirardi and Grassi then demonstrate the existence of an element of reality at the left-hand side following the same reasoning as above. From here, the argument unrolls once again in the usual way and locality makes a second appearance in its familiar place at the end of the argument. In this way, Ghirardi and Grassi can again prove that standard quantum mechanics plus 'locality' implies incompleteness.

But there are two quite distinct cases of L-Loc that are actually being employed, one used in getting the argument started and the other appearing in the conclusion. Ghirardi and Grassi define L-Loc as the following: 'An event cannot be influenced by events in spacelike separated regions. In particular, the outcome obtained in a measurement cannot be influenced by measurements performed in spacelike separated regions; and analogously, possessed elements of physical reality referring to a system cannot be changed by actions taking place in spacelike separated regions.' As in the nonrelativistic case, it is not the general principle but rather the two special cases, call them OM-Loc (for outcome of measurement) and ER-Loc (again for element of reality), that are doing the logical work in their argument. OM-Loc affirms that the outcome of a measurement cannot be influenced by performing another measurement at a spacelike separation, while ER-Loc affirms that elements of reality cannot be created by performing a measurement at spacelike separation. Ghirardi and Grassi invoke OM­ Lac at the beginning of the argument while applying the counterfactual reality criterion, as discussed above, and they invoke ER-Loc at the end of the argument, as they did in the nonrelativistic case. So if we write L-Loc = OM-Loc ^ ER-Loc, then, schematically, their argument looks like this:

Quantum Formalism ^ OM-Loc ^ ER-Loc →

~Completeness (5)

or

Quantum Formalism ^ Completeness → ~OM-

Loc v ~ER-Loc. (6)

Ghirardi and Grassi now argue, in effect, as follows. Assuming OM-Loc we can again demonstrate from Completeness a violation of ER-Loc, i.e. Einstein's *spooky* action-at­a-distance *creating* elements of reality at a distance. But if we don't assume OM-Loc, then we cannot deduce a violation of ER-Loc. All this is quite correct, but the price we have to pay for *not* being able to demonstrate a violation of ER-Loc is precisely that we have to accept a violation of OM-Loc!

In other words, the relativistic formulation of the EPR argument does not help with the thesis of peaceful coexistence between quantum mechanics and special relativity, unless one argues that violating ER-Loc is more serious than violating OM-Loc from a relativistic point of view. This is hard to maintain since violating OM-Loc involves a case-by-case version of what Shimony refers to as violating parameter independence, i.e. the independence of the probability of the outcome on one wing of the EPR experiment with respect to the parameters controlling the type of experiment being performed on the other wing. By analogy, violating ER-Loc is also a form of parameter dependence. So Ghirardi and Grassi’s claim of peaceful coexistence is already justified , with a novel idea of how it works out, a violation of OM-Loc.

**3. The thesis of myself and La Riviere’s paper**

However they say they need to identify an additional assumption omitted from (5) and (6) , which, if challenged, could undermine the inference. Recall that to run the argument in either the nonrelativistic or relativistic case, Ghirardi and Grassi must establish that the outcome of, say, the right-hand measurement is the same in all accessible worlds. With this established, the correlation laws of quantum mechanics imply that the outcome of the left-hand measurement is the same in all accessible worlds, and hence establish the truth of the counterfactual assertion about the left-hand measurement result that permits the attribution of an element of reality to the left­ hand particle. In the nonrelativistic case, the constancy of the right-hand result is a natural consequence of the absolute time ordering as discussed above; in the relativistic case. A premise akin to one that Michael Redhead, following Stapp, labels the Principle of Local Counterfactual Definiteness (PLCD) is needed to do this sort of work.

In the present case, PLCD may be taken to assert that the result of an experiment that *could* be performed on a microscopic system has a definite value that does not depend on the occurrence of a measurement at a distant apparatus. Ghirardi and Grassi implicitly assume that PLCD is licensed by their locality principle, for they invoke only OM-Loc to establish the constancy of the right-hand outcome in all accessible worlds. But we argue that PLCD does *not* follow directly from any typical locality principle, certainly not from one like OM-Loc, which asserts that the outcome obtained in a measurement cannot be influenced by measurements in spacelike separated regions. The reason is quite simple: while invoking locality may prevent measurements on the left­ hand particle from influencing the result at the right and from breaking the constancy of the accessible worlds as far as the right-hand result is concerned, it does not prevent *indeterminism* from wreaking that sort of havoc. Intuitively, we can imagine that we run the world over again, this time performing the measurement on the left-hand particle. If we consider this left-hand measurement schematically as a point event with a backward light cone identical to that in the actual world, we are concerned with what will happen in the complement of the forward and backward light cones. Under indeterminism the events in this complement (the absolute elsewhere) simply cannot be assumed to remain the same.

Thus it is maintained that Ghirardi and Grassi need both OM-Loc and an assumption of determinism to get their argument off the ground.

We need, schematically, to replace (6) by

Quantum Formalism ^ Completeness ^

Determinism → ~OM-Loc v ~ER-Loc. (7).

Thus, Determinism  OM-Loc (  PLCD )

and hence by the disjunctive syllogism, we can infer ~ER-Loc,

So text of myself and La Riviere ends here.

**4. But there is a catch…..**

But there is a catch . We note first that Determinism  Bell Inequality (assuming randomness assumption is assumed to be true, for which independent engagement can be given). But this contradicts the conclusion that the Bell Inequality is false. So we cannot infer ~ER-Loc.

But, according to Lewis (private commuication) , OM-Loc  PLCD and hence allows the Bell Inequality, independently of Determinism. The argument here appears to be the following. For possible worlds we allow so-called quasi-miracles to happen, but also they might not happen. To prove the counterfactual we need to close down worlds where miracles happen, leaving the counterfactual to succeed.

We disagree with this claim since it involves discussion of possible worlds, whereas in our world the counterfactual comes out as false.

Put succinctly, the argument is the following. A counterfactual supposition about an event can, and indeed should, be taken to preserve as much as possible of the actual facts, compatible with the truth of the supposition. Since in a relativistic setting, a counterfactual supposition about an event is compatible with preserving the the actual facts that are spacelike separated from it. It seems to us , under indeterminism, one’s treatment of a counterfactual supposition should ‘respect’ the indeterminism by allowing the ways that history could play out, compatibly with the couterfactual supposition to be considered.

But if we follow Lewis we again allow the Bell Inequality so on both counts the attempt to rescue the justification of ~ER-Loc fails.

We make a remark on Ghirardi and Grassi’s proof on peaceful existence. For Ghirardi and Grassi there are two sorts of argument: 1) EPR argument assuming locality; 2) argument showing, correctly, how nonlocality works in violating Bell. I confused the two sorts of arguments, starting with 1) and trying to prove peaceful existence according to 2)! I learned from my mistakes as Popper would have said!

But notice how we have shown, using 1), that peaceful coexistence actually works out.

**5. Conclusion**

So what conclusion concerning nonlocality can we draw? Taking first the realist option, in which all observables have sharp values at all times, if we assume determinism or restrict the discussion to cases of strict correlation or anticorrelation where we can derive determinism on plausible assumptions, there seems very little scope for avoiding the conclusion of nonlocality for any 'realist' reconstruction of quantum mechanics. But experimentally we never observe absolutely *strict* correlations, so it may be argued that discussions of nonlocality should not deal with this ideal case, but should be based upon the experimentally realistic nonideal case. We are not convinced by this argument, since idealization is an essential aspect of any scientific theorizing. Be that as it may, the assumption of determinism for the nonideal case where correlations are less than perfect, may well be suspect. In such cases one is forced, as we have seen, to go to the stochastic hidden-variable framework, and much of the recent discussion has focused on the significance of violating the locality assumptions involved in this framework. Following Jon Jarrett these break down into two classes.

1. Independence of the probability for a particular outcome of measurement on one particle,

conditionalized on the collective hidden variables, from the *outcome* of measurement carried

out on a remote particle. This is what Shimony calls parameter independence, which is already referred to above.

2.Independence of the probability for a particular outcome of measurement on one particle

conditionalized on the collective hidden variables, from the type of measurement being

carried out on a remote particle. This is what Shimony calls outcome independence.

Violation of (1) has been generally held to mark probabilistic causal dependence between the outcome of a local measurement and the 'setting' of a remote piece of apparatus, in prima facie conflict with standard interpretations of special relativity as prohibiting causal links between spacelike separated events. Violation of (2) has led to more controversy. It seems to demonstrate that the precise specification of the state of the source, or more generally of the particles before the measurements are undertaken, cannot be regarded as a common cause of the measurement outcomes. Indeed a direct causal connection exists between the events consisting of the measurement results on distant particles obtaining in the way they actually do. We have challenged this view by proposing to interpret violation of (2), in the way proposed, but in terms of a noncausal direct dependence between measurement outcomes that lacks the necessary ' robustness' in respect of how the measurement results are brought about, to merit description as a causal connection. This approach gives up the stochastic hidden-variable framework in favour of trying to understand the correlations between distant events in terms of a harmony-at-a distance, or, as Shimony describes it, a passion­at-a distance.

This idea ties in with, but is really quite distinct from, the fact that violation of (2) cannot be used to transmit signals between distant locations since the necessity to recover quantum probability distributions by averaging over the hidden variables, means that we have no way of controlling the local marginal distributions for the results of measurement on one particle, by varying the type of measurement performed on distant particles. The no-signalling result is often cited to defuse the tension between nonlocality in quantum mechanics and the constraints of special relativity, but, at a deeper level, the 'nonrobustness' argument may be preferable.

Turning to the anti-realist option, we have examined Ghirardi and Grassi's attempt to reformulate the EPR argument in a relativistic context and argued that peaceful coexistence does arise.

There remains of course the question of how to interpret the violation of outcome independence. Assuming completeness, outcome dependence famously follows. In the EPR set-up this means that when measurements are performed at spacelike separation on the two wings of the experiment, the results are mirror-image correlated. As one potentiality gets actualized on the left, say, how does this happen exactly in tandem with the opposite result on the right? Are we faced with a causal effect, namely result-to-result causation, so that peaceful coexistence with relativity is still challenged? Similar answers involving no-signalling or nonrobustness may be provided as in the discussion above of the 'realist' option.

But it must be stressed that the mysterious harmony of the result-to-result correlations remains arguably 'spooky' even if it does not involve causal dependence.

Another distinguishing feature of this harmony is its symmetrical character, quite unlike the asymmetry that one would normally want to ascribe to a causal connection. Shimony's phrase 'passion-at-a -distance' seems exactly the right one to capture what is going on, even if one concedes that the mystery of the EPR correlations is not eliminated merely by introducing an apt nomenclature.

For the antirealist the role of measurement is to actualize potentialities. If there are no measurements, then there are no actualities! This observation is particularly relevant to cosmological applications of quantum mechanics, where there is nothing 'outside' the universe to serve as a measuring device! So, from the cosmological perspective, one can argue that the realist option is after all to be preferred.

Here one can argue that the nonseparability approach, as exemplified in blocking the nonlocality proof by denying the 0-Loc principle, may in the end be the best way of understanding the peculiar features of entangled states in quantum mechanics.

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Many people find it difficult to see the distinction between violating 0-Loc (nonseparability) and violating E-Loc (action­ at-distance). A simple example due to David Lewis may help here. Consider someone with what she calls a bilocal hand. There is in reality just one hand that is manifested in two different places. If you shake hands with this curiously disabled person, her other hand will move in synchrony with the one you are shaking, not because there is an interaction between the two hands, but just because, in reality, there is only one hand, bilocally located! According to Lewis, this is an example of nonseparability as distinct from action-at-distance

**Appendix**

**Realist Interpretations**

E-Loc: Local elements of reality cannot be affected by changes in a distant, spacelike separated environment.

O-Loc: Local elements of reality can be specified independently of a holistic context.

**Antirealist interpretations**

G-Loc: Events cannot be affected by a distant measurement performed simultaneously.

L-Loc: Same as for G-Loc but with ‘simultaneously’ replaced by ‘at spacelike separation’.

ER-Loc: Elements of reality cannot be created by distant measurements performed simultaneously/at spacelike separation – a special case of G-Loc/L-Loc.

OM-Loc: The outcome of a measurement at one location cannot be affected by a distant measurement performed at spacelike separation – another special case of L-Loc.

PLCD: The outcome of a measurement that could be performed has a definite value independent of whether or not another measurement takes place at a distant spacelike separated location. OM-Loc implies PLCD under an assumption of determinism, but not of indeterminism.

Parameter independence: The *statistics* of measurement results at one location is independent of the parameters defining a measurement procedure at a distant spacelike separated location.

Outcome independence: The *statistics* of measurement results at one location is independent of the outcome of measurements performed at a distant spacelike separated location.

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