Entanglement, Modality, and Indeterminacy

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Abstract

This paper aims to contribute to the metaphysical foundations of the Everett or 'many-worlds' interpretation of quantum mechanics (cf. Everett, 1957; Wallace, 2012). I focus on the nature of the indeterminacy countenanced by states of entanglement, and argue that an account which clarifies the nature of the possible worlds at issue might serve to elucidate both the notion of metaphysical indeterminacy as well as the status of probability in the interpretation. I endeavor to elucidate the claim that the compossible states exhibited by entangled superpositions are real. I advance, then, three interpretations of the reality of the worlds at issue, and examine their interaction with the actuality operator. Finally, I examine which combinations of the approaches are consistent, and I argue in favor of a property-based approach to possible worlds.

1 Introduction

This paper aims to contribute to the metaphysical foundations of the Everett or 'many-worlds' interpretation of quantum mechanics (cf. Everett, 1957; Wallace, 2012). I focus, in particular, on the nature of the indeterminacy countenanced by states of entanglement, and argue that an account which clarifies the nature of the possible worlds at issue may serve to elucidate both the notion of metaphysical indeterminacy as well as the status of probability in the interpretation.

A separable Hilbert space is a space of countable complex-valued vectors representing physical magnitudes (A,B, ...). The inner product of a Hilbert space is a linear functional on vectors which specifies their expectation values. Density operators, f, on Hilbert space map real-valued vectors to complex numbers. The operators are self-adjoint, such that for all one-object categories with

a homomorphism, a, from a target category to its underlying set, there is a unique homomorphism, b, such that a is isomorphic to b (cf. Awodey, 2006: 10, 180-181); linear [f(A + B) = f(A) + f(B)]; non-negative [f(A) $\geq$ 0]; normed [f(T) = 1]; and countably additive [if A and B are orthogonal, then f(A $\cup$ B) = f(A) + f(B)] (cf. Ruetsche, 2011: 21-23). A spin-state vector is the sum of the probabilities that a particle is spinning either upward or downward. If there are two particles which can be spinning either upward or downward, then both particles can be spinning upward; both particles can be spinning downward; particle-1 can be spinning upward while particle-2 spins downward; and particle-1 can be spinning downard while particle-2 spins upward. The state vector, V – which records the superposition of the foregoing possibilities – will be equal to the product of the spin-state of particle-1 and the spin-state of particle-2. However, the superposition of the two states, V, is unable to witness the discernibility of the particles when both are spinning upward or downward, such that two of the above four possibilities will be indiscernible. Relative to the value of each particle vector, i.e., its eigenvalue, the probability that particle-1 will be spinning upward is .5 and the probability that particle-2 will be spinning downward is .5, such that the probability that both will be spinning upward or downward = .5 x .5 = .25. If, however, the particles are both spinning upward or both spinning downward, then the value of the superposition V will be .5. Thus, the indiscernibility of the spin-state vectors when both are spinning upward or downward will be inconsistent with the product of the eigvenvalues of the spin-state vectors. When the superposed vector is thus unequal to the product of the foregoing eigenvalues, the superposition is said to be entangled.

According to the Everett interpretation, both possibilities concerning the value of the entangled state are not only compossible, but are also real (cf.

Wallace, op. cit.). Thus, no indeterminacy is purported to obtain with regard to the value of the superposed vector (Wallace op. cit; Hawthorne, 2010). Rather, many-worlds are instantiated at once, such that both possibilities are claimed to be in some sense real, rather than e.g. intra-world representational counterparts.

In this note, I endeavor to elucidate the notion, in the Everett interpretation, according to which the compossible states exhibited by entangled superpositions are real. I advance three interpretations of the reality of the worlds at issue, and examine their interaction with the actuality operator. I examine, then, which combinations of the approaches are consistent, and I argue in favor of a property-based approach to possible worlds. Section **3** provides concluding remarks.

2 Modality and the Everett Interpretation

According to the Everett interpretation, both possibilities concerning the value of the entangled state are not only compossible, but are also real (cf. Deutsch, 2010; Wilson, 2011). Thus, no indeterminacy is purported to obtain with regard to the value of the superposed vector (Hawthorne, 2010). Rather, many-worlds are instantiated at once, such that both possibilities are claimed to be in some sense real.

In the remainder of this paper, I endeavor to elucidate the claim that the compossible states exhibited by entangled superpositions are real. I focus, in particular, on the interpretation of the worlds at issue, as well as the interaction of the latter with the notion of actuality.

There are at least three interpretations of the reality of the worlds at issue in the foregoing approach to physical ontology. The first two of which have yet

to receive protracted examination in the literature.

(i) According to the first interpretation of the reality of the worlds at issue, possible worlds are counterfactual properties – e.g., alternative histories – of the actual world (cf. Adams, 1974; Stalnaker, 1976, 2012; Kripke, 1980: 18-20; Wallace, op. cit.: 287-289). This interpretation of the notion of possible worlds provides an easy answer to the inquiry into how possible worlds may be both real and compossible, while it is determinately true that only one of the possibilities actually obtains. The interpretation may be developed as follows. The counterfactual properties may, by a regularity condition, have objective probability measures defined thereon, such that the value of a spin-state vector is possible only if it has a non-zero probability. One virtue of this approach to the interpretation of probability and possibility is that it provides a means of coherently explaining how objective probabilities, i.e. chance, can be real, while yet consistent with the determinacy of the states at issue. A third virtue accruing to the foregoing interpretation is that it is consistent with the constructor theory proffered by Deutsch (2013), according to which quantum information is constitutively modal. The modality of quantum information theory is taken, in constructor theory, to consist in the counterfactual profile of the transition function taking spin-state vectors as arguments, as defined over the state space of a physical system.

(ii) According to the second interpretation of the worlds at issue, possible world-states in the Everett interpretation may all be actual, in virtue of a necessitist modal ontology. The proposal is consistent with Lewis' (1986: 1.8) modal realism, according to which the possible worlds as necessary existents are causally isolated, recombinations of regions of spacetime (cf. Deutsch, 2010: 545-546; and – for a language-relative variation of realism with regard to the

possible branches of spacetime in cases of entangled superpositions – Wilson, op. cit.). At first-order, the foregoing entails that objects have necessary being. At higher-order the above entails that properties of objects have necessary being, including haecceitistic properties according to which for all x and y there is a condition such that x's satisfaction of that condition entails that x is that y, i.e. x bears a property which is the property of possessing a unique identity. By the above, all propositions have necessary being. If the logic to which the Barcan formula and its converse are augmented is 'metaphysically universal' because canonical – i.e., the set of propositions are true on all interpretations of their atomic subformulas on the intended interpretation of the modal operator – then the truth of the universal generalizations of the propositions on their intended interpretation will be a guide to their truth simpliciter, rather than their truth-in-a-model (cf. Williamson, op. cit.). The foregoing suffices thereby to relate the truth-in-a-model with actual, metaphysical truth, and thereby account for how the set of merely possible propositions converges with the actuality thereof. If the modal propositions at issue are taken to be counterfactual properties, then necessitist modal ontology provides a second means for interpreting the contention that distinct possible worlds may be actual: Only one of the foregoing is spatio-temporally concrete, and the remaining are contingently non-concrete.

(iii) According, finally, to the third interpretation, the reality of the worlds stipulated as being compossible in the Everett interpretation is yet consistent with the notion of metaphysical indeterminacy. On this interpretation, metaphysical indeterminacy is taken to consist in the indeterminacy with regard to which world in a set of possibile worlds is actual (cf. Saunders and Wallace, 2008). Metaphysical indeterminacy is thus argued to obtain, only if it is not determinate which world in a set of worlds is actual, subsequent to operations

of precisifying the determinacy of subsets of the total set of worlds (cf. Barnes and Williams, 2011; Lewis, 2016: 277-278).

The third interpretation is inconsistent with the first and second. The third interpretation is inconsistent with the second, because while both interpretations stipulate that the sets of worlds at issue are unpointed such that no world is designated as being actual, the third interpretation takes the lack of an actual world stipulated in the model to explain in virtue of what there is metaphysical indeterminacy, whereas the second interpretation takes the unpointed set of worlds itself to be actual because metaphysically universal. The third interpretation is inconsistent with the first, because, whereas the third interpretation argues that there can be a set of possibilities according to which it is unsettled which possibility is actual, the first interpretation argues that possibilities are counterfactual properties of the actual world, such that all possibilities are actual even if only one is realized. Finally, although Stalnaker (op. cit.) argues in favor of contingentist systems of modal metaphysics such that (i) the Barcan formula and its converse are invalidated both at first- and second-order and (ii) necessarily everything could be nothing when not concrete, it would be possible for the first and second interpretations to be consistent, if the first interpretation were to be governed by a logic which is augmented with the Barcan formula and its converse at first- and higher-order. Author (ms) and Williamson (2016) provide arguments, for example, which endeavor abductively to vindicate the truth of necessitism when restricted to physical modality.

The first and second interpretations of the modality provide, then, a means of explaining the determinacy and reality of compossible states in the many-worlds interpretation of quantum mechanics. Both possibilities concerning the values of the spin-state vectors from the vantage of the entangled, superposed

state are real. On the first interpretation, both are counterfactual properties of the actual world, which may further correspond, as in constructor theory, to counterfactual transition functions whose values are the spin-state vectors at issue. On the second interpretation, the states are possible and actual, because both states have necessary being even if only one of the states is concrete.

Eschewing the third interpretation appears at first pass to entrain the rejection of an interpretation of entangled superpositions which construe the latter as cases of metaphysical indeterminacy. It is interesting to note, however, that – even on an interpretation of entangled superpositions which vindicate the first and second proposals outlined above – the contention that the many-worlds interpretation is committed to there being fundamental indeterminacy might yet be defended. One way to render the first two interpretations consistent with fundamental indeterminacy is to countenance the latter hyperintensionally, rather than in modal terms. Thus, while the value of the entangled superposed state might both be actually .5 and actually .25, it can yet be fundamentally indeterminate which possibility is fundamentally real. The notion of fundamental reality which might need to be availed of is that of grounding, where the latter is an operator which takes scope over propositions (cf. Fine, 2012). The logic of the operator might be non-well-founded and thus irreflexive, such that sets of propositions have no least element, precluding cases in which the propositions might also reflexively ground themselves (cf. Barnes, 2015).

A second virtue accruing to the hyperintensional interpretation of indeterminacy is that – beyond being consistent with the actuality of compossible values in entangled superpositions – metaphysical indeterminacy may be avoided by reinstating a well-founded chain of grounding states. If so, then the fundamental level of grounding may vindicate a priority monistic conception of physical

ontology, according to which the parts of a physical system will hyperintensionally ontologically depend on the system as a whole (cf. Schaffer, 2010; Ismael and Schaffer, 2016). A variation on physicalist priority monism takes the fundamental level to target, e.g., the complex-valued wavefunction in configuration spacetime, consistently with the derivative reality of the particles residing in 3-dimensional spacetime. The foregoing enables a combination of views according to which the compossible values of entangled superposed states are each real and actual – either because they are modal (counterfactual) properties of the actual world (interpretation 1) or because they have actual, necessary being (interpretation 2) – consistently with a priority-monistic conception according to which it is determinate what the value of the complex-valued wavefunction in a configuration spacetime might happen to be.

3 Concluding Remarks

In this note, I have endeavored to elucidate one aspect of the metaphysics of the Everett interpretation. According to the Everett interpretation, the distinct, compossible values of entangled superposed states are each real, and entangled superpositions are thus not indeterminate. In order to clarify the sense in which two possible worlds are countenanced by a single state without there being metaphysical indeterminacy, I provided three interpretations of the notion of possible worlds at issue, as well as of the interaction between the possibilities and the notion of actuality. According to the first, possible worlds are counterfactual properties of the actual world, where the counterfactual properties might, as in constructor theory, be modal transition functions ranging over probabilistic real-valued spin-state vectors. According to the second, possible worlds are actual because of a vindication of necessitist modal ontology, such that physical states

have nececessary being even when the latter are contingently non-concrete. According to the third interpretation, the Everett interpretation is metaphysically indeterminate, when indeterminacy is interpreted so as to consist in an upointed set of worlds in which it remains indeterminate which world in the set is actual upon a supervaluational filtering of the set with precisifications. Consistently with the proposed commitments of the many-worlds approach, I argued for the plausibility of the first and second interpretations over the third. I argued however that a rejection of the third interpretation is yet consistent with a conception of fundamental indeterminacy which targets hyperintesional grounding operators, such that – while entangled possibilities might both be actual – the reality of the states would yet be consistent with there being fundamental indeterminacy in the grounds in virtue of which those possibilities obtain. A second approach to the movement to the hyperintensional setting is that it might instead vindicate the consistency of there being many actual worlds with the priority monism of a fundamental level of truths – those concerning, e.g., the values of complex-valued functions in higher-dimensional space. Thus, the status of the reality and determinacy of the compossible states proffered in the many-worlds interpretation of quantum mechanics is consistent with there yet being either hyperintensional fundamental indeterminacy, or hyperintensional monistic determinacy, in the subvening ontological grounds.

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