Conceptual problems in quantum electrodynamics: a contemporary historical-philosophical approach

(Redux version)

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Conceptual problems in quantum electrodynamics: a contemporary historical-philosophical approach

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Introduction

In this work I address what can be called conceptual-mathematical anomalies in quantum electrodynamics. By this I mean conceptual and mathematical problems of the theory that do not affect 'saving the phenomena'. A well-known example is the divergent expressions that appear in the applications of the theory, which can be renormalized without implying any kind of problem in what regards the predictions of the theory.

This work can be seen as following the line of philosophy of physics studies of quantum field theory that started to emerge in a systematic way in the early eighties of last century. One example is Teller's (1995) work on standard quantum electrodynamics.[1] More recently the field has become dominated by scholars that tend to prefer more formal approaches, relying not on the set of theories of the so-called standard model but on tentative formal approaches that promise to give to quantum field theory the solid mathematical foundations that it does not have (see e.g. Fraser 2009). The particular characteristic of these approaches is that they do not deliver testable predictions.

In this work, by following a historical approach, I will return to the standard version of quantum electrodynamics (which is the only one available when we want to get numbers out to compare with experimental results). In this way I will be considering the contributions and discussions by physicists like Einstein, Bohr, Jordan, Pauli, Heisenberg, Fermi, Dirac, Feynman, and others. This does not mean that I will not take into account 'formal' results. That is not the case. Simply, I consider more interesting understanding the physical theories we really have and trying to see how they work so well in the middle of a sea of anomalies. A historical approach enables us to return to the original moments when the concepts were being developed and the problems faced for the first time; it also enables to take advantage of the insights of the physicists that created the theory. However I must call attention to the fact that I am not doing history. What I am doing is using history as a guide to a tentative clarification of some unclear aspects of the theory.

Since I am not taking into account more recent contributions, a work that goes back to the early fifties of last century and before might seem dated. Here I must distinguish between the above mentioned 'formal' approaches and technical developments made in quantum electrodynamics. Two good examples of these are the use of renormalization group technics and lattice regularization. To the best of my knowledge these more recent developments do not affect the views being presented here. They might complement them, but it was never my intention to present a full study of all the facets of quantum electrodynamics. My objective is less ambitious; it is to show that a historical approach can deliver interesting and 'new' insights regarding current philosophical issues related to quantum field theory in general and quantum electrodynamics in particular.

[1] What makes Teeler's work to be not simply a work on foundations of physics but a philosophical account of quantum field theory is, in particular, his exploration of an interpretation of quantum fields in terms of propensity (instead of substance). This has implicit worries of an ontological nature; in simple terms it relates to the philosophical question of what is the ontological implication of a physical description in terms of quantum fields.

This work spins around two main vectors. One is the divergence of the S-matrix series expansion; the other is the spatio-temporal description of physical processes in the theory. Regarding the first vector, I will be presenting an interpretation that for some will seem a bit strange (my interpretation resembles views by Bohr from the early thirties of last century); also (independently of my particular interpretation) I will explore the consequences of having just an asymptotic series to describe the interaction of radiation and matter. In a nutshell I defend that having an asymptotic series implies that the theory is intrinsically approximate, i.e. it can only describe the interaction of radiation and matter in an approximate way with just a few terms of a series expansion and not give an exact solution corresponding to treating radiation and matter as one closed system.[2] Here I am not simply accepting pragmatically a fact. The use of only a few terms of an infinite series expansion must be philosophically made acceptable by clarifying the concepts of radiation and matter and their interaction as implemented in the mathematical structure of the theory; that is, I want to provide a 'philosophical' justification for disregarding the large-order terms of the series expansion (by addressing 'gently' the question of the relation of the mathematical structure to the physical concepts this structure gives 'flesh' to).

Philosophically the typical justification of saying that the computational time would make impossible, in practice, to calculate large-order terms is not enough; neither saying that the possible contribution of these terms is irrelevant since at a high-energy new physics is coming in. This is the usual position of the believers in string theory or whatever theory of everything that might be 'underneath' the standard model. For these, quantum electrodynamics is just an effective field theory that works well in a particular energy range, being only a 'valid' approximation (even if just delivering asymptotic results) to an underlining level of description of reality. On this view the divergence of the S-matrix series expansion is considered unproblematic. I have no reason to believe in this traditional Nagel type of intertheoretical reduction. In fact the second vector of my work leads me to consider that quantum electrodynamics cannot be seen as more fundamental than classical electrodynamics, i.e. the relation of classical and quantum electrodynamics is not one of theory reduction but more complex.

The study of the spatio-temporal description of physical processes in quantum electrodynamics is the other main vector of my work. Again I present a controversial view. Quantum electrodynamics is not able to describe physical processes in time in a way similar to classical theory. In fact it *relies on* the classical temporality (as time goes by…) to construct an asymptotic temporal description, in the sense of going from $-\infty$ to $+\infty$, of physical processes (we will see for example that it is this characteristic that enables the charge renormalization procedure). This, in Feynman's words, global space-time approach has severe limitations in what regards the possibility of describing such a simple thing as a delayed interaction between charged particles, and I do not see how we can from the quantum electrodynamical level of description arrive at the temporal description of classical electrodynamics.

Here is how I delelop my views. To warm up for the discussion of the Dirac equation and its interpretation being given in chapter 3, I will consider in chapter 2 the simpler case of the Schrödinger equation and (part of) its interpretations. In chapter 3,

[2] The readers even if not agreeing with my view that quantum electrodynamics consists in an intrinsically perturbative approach should at least not too easily rely on so-called non-perturbative 'results' and take the time for a critical analysis of these. For example it is usually considered that the lattice regularization is non-perturbative because from the start the space-time lattice implies an energy-momentum cutoff to all orders of the perturbative calculation. However in lattice quantum electrodynamics we still have a divergent S-matrix, and it is this that makes the theory intrinsically approximate.

by trying to fit together the different interpretations of the Dirac equation, analyzing in particular the two-body problem, I will arrive at the well-known description of interactions in terms of quanta exchange. In chapter 4 I will consider the other cornerstone of quantum electrodynamics, the quantized electromagnetic field, and try a clarification of the concept (or better, notion) of quantum vacuum. The description of interactions in quantum electrodynamics is addressed in chapter 5. Here I will consider the problem of the divergence of the series expansion of the S-matrix and the relevance or not of the Haag theorem to the consistency of the theory. Chapter 6 is dedicated to an excursion into the history of renormalization and to recover views by Bohr and Dirac that I consider to present renormalization in a 'new' light. In chapter 7 I analyze the spatio-temporal description of physical processes in quantum electrodynamics and the status of the so-called virtual quanta (that are a crucial element in the description of interactions in terms of quanta exchange). Finally the results of chapter 7 are used in chapter 8 to defend the idea that quantum electrodynamics is an upgrade of classical electrodynamics and the theory of relativity (i.e. that classical electrodynamics does not reduces to quantum electrodynamics). In the appendix I make a digression and present an analysis of Bohr's views on space and time in quantum mechanics in relation to his quantum postulate (this will enable to address the Bohrian interpretation of the wave function followed in this work).

CHAPTER 3

THE DIRAC EQUATION AND ITS INTERPRETATION

1 Introduction

Once upon a time, Richard P. Feynman wrote that "we know so very much and then subsume it into so very few equations that we can say we know very little (except these equations – Eg. Dirac, Maxwell, Schrod.). Then we think we have the physical picture with which to interpret the equations. But these are so very few equations that I have found that many physical pictures can give the same equations" (quoted in Schweber, 1994, p. 407). He wrote this having in mind, in particular, the Dirac equation: $i\partial\!\!\!/\psi = m\psi$ (Feynman, 1961, p. 57).

In this chapter, the Dirac equation will be used as a guideline to reveal the importance of the concept of quanta in the description of interactions in quantum electrodynamics. To this end the historical evolution and interpretation of the Dirac equation is considered. In sections 2, 3, 4, and 5, I present the evolution of the Dirac equation from its first formulation as a relativistic wave equation for an electron, to a classical field equation from which an electron-positron quantum field is obtained. In this transition, the Dirac equation went from being a relativistic 'update' of the Schrödinger equation in the calculation of energy levels in atoms (basically of hydrogen) to becoming one of the cornerstones of the most successful quantum field theory: quantum electrodynamics. In section 6, I will try to clarify the relation between the different interpretations of the Dirac equation. In this way the results provided by Dirac's equation as a relativistic one-electron equation are reinterpreted from the perspective of the quantized Dirac field. Doing this, the importance of the concept of quanta in the description of bound states becomes clear. By contrast, bound states are usually only described at the level of the one-electron interpretation of the Dirac equation, which gives a distorted idea of the physical description of bound states that should be described from the perspective of quantum fields. In particular, an analysis of a two-body description of the hydrogen atom reveals a distinctive feature of quantum electrodynamics: the interaction between fermions described as an exchange of photons.

2 Before the Dirac equation: some historical remarks

Schrödinger's first attempt at a wave equation was the development of a relativistic wave equation for the hydrogen atom. As in the case of the non-relativistic wave equation, Schrödinger considered the problem of determining the eigenvibrations of the hydrogen atom. As we have seen in the previous chapter, Schrödinger took the de

Broglie relations generalizing them to the case of an electron in a central Coulomb potential and by inserting them in a second-order differential equation obtained the relativistic wave equation

$$\Delta\psi = -\frac{4\pi^2 m^2 c^2}{h^2}\left[\left(\frac{h\nu}{mc^2}+\frac{e^2}{mc^2 r}\right)-1\right]\psi = 0$$

Using this wave equation Schrödinger determined the energy levels for the hydrogen atom, and arrived at a result that was not in agreement with Sommerfeld's result for the hydrogen spectrum obtained, within the so-called old quantum theory, through the quantization of the relativistic Bohr atom. Sommerfeld's result was

$$E_{n,k} = \frac{mc^2}{\sqrt{1+\frac{\alpha^2}{\left(n-k-\sqrt{k^2-\alpha^2}\right)^2}}} - mc^2,$$

where α is the fine structure constant, n the principal quantum number, and k the azimuthal quantum number. Schrödinger obtained an expression that did not depend on n – k and k but on n – k + 1/2 and k – 1/2 (Kragh, 1981, p. 33). This meant the failure of Schrödinger's relativistic wave mechanics. Schrödinger set aside his attempt at a relativistic wavefunction and developed a non-relativistic wave equation (Schrödinger 1926a). The relativistic wave equation, later known by the name of Klein-Gordon equation, was presented by several physicists during 1926, but since it did not give the fine structure of the hydrogen spectrum it was not accepted as the correct relativistic equation for an electron (Kragh, 1984).

Another factor that would complicate matters in what regards the wave mechanics description of the electrons was the discovery of spin. In Bohr's theory each spectral term of the hydrogen atom is labeled by three quantum numbers n, k, and m. Due to an external magnetic field a spectral term labeled by n and k splits into 2k + 1 levels, the new sub-levels being distinguished by the quantum number m. This is called the Zeeman effect (Tomonaga, 1997, pp. 1-2; Sánchez Ron, 2001, pp. 336-341). Before 1900 it was already known that the spectral lines, latter described by n and k, where not unique and in reality, when an external magnetic field was applied, consisted in closely spaced lines: they showed a multiplet structure. This was called the anomalous Zeeman effect (Jammer, 1966, p. 122). In 1920 Sommerfeld introduced a new quantum number j that enabled the classification of the different energy levels within one multiplet term (characterized by n and k). Under this new classification m is still related to the specification of sublevels but now of a level specified by n, k and j. Also m must satisfy the inequality $-j \leq m \leq j$. To explain the spectroscopic evidence available Alfred Landé (and also Sommerfeld) set forward a tentative model in which it was assumed that the core of the atom had an angular momentum. There would then be a magnetic moment μ_K related to the orbital angular momentum **K** of the electron in the outermost orbit (the valence electron), given by $\mu_K = -\mathbf{K}$ (in units of the Bohr magneton). Also there would be a magnetic moment μ_R associated to the core angular momentum **R** (corresponding to a quantum number related not to the electron but to the core). The

relation between μ_R and **R** is given by $\mu_R = -g_0\mathbf{R}$, where g_0 has to be determined by fitting the model to experimental results. F. Pachen and E. Back's study of the Zeeman effect in the case of a strong magnetic field, enabled to set the value of g_0 as 2. The interaction between the two magnetic moments μ_K and μ_R of the atom leads to a slight energy change in the atomic energy levels, which results in the multiplet structure of the spectral lines (Tomonaga, 1997, pp. 11-20).

In 1924 Wolfgang Pauli showed that the association of μ_R with the atom's core was inconsistent, and considered it to be associated with the valence electron. In this way the four quantum numbers are all related to the electrons. Upon reading Pauli's ideas G. E. Uhlenbeck and S. Goudsmit proposed in 1925 to reinterpret the core angular momentum **R** as an intrinsic angular momentum of the electron. Uhlenbeck and Goudsmit had the idea that to each quantum number should correspond a degree of freedom of the electron. This led them to the idea of an intrinsic rotation of the electron, the spin, as a fourth degree of freedom to which a quantum number would be associated. As in the case of Landé's model, Uhlenbeck and Goudsmit got a discrepancy, by a factor of 2, between the theoretical and the experimental results in the case of doublet levels of alkali atoms. In 1926 L. H. Thomas presented a relativistic calculation where the missing 1/2 factor, later called the Thomas factor, was obtained (Jammer, 1966, pp. 149-152). So by 1926 there was no relativistic wave equation for the electron and there was the further complication of having to account also for the electron's spin.

In 1927 Pauli attempted to incorporate spin into wave mechanics by considering a Schrödinger wave function depending also on a degree of freedom related to spin (Kragh, 1981, pp. 45-46). The intrinsic (spin) angular momentum in any direction takes only the two values $\pm \hbar/2$. This made Pauli consider a two-component wave function, with one component corresponding to an electron's spin up $\psi(\mathbf{x}, +1/2)$ and another to an electron's spin down $\psi(\mathbf{x}, -1/2)$. This two-component wave function must be solution of two coupled equations with the form

$$H\left(\frac{\hbar}{i}\frac{\partial}{\partial q}, s\right)\psi = E\psi$$

The question was how to define in the wave equation the spin operators **s**. Pauli defined the spin operators as $s_x = 1/2\ \sigma_x$, $s_y = 1/2\ \sigma_y$, $s_z = 1/2\ \sigma_z$, where σ_x, σ_y, σ_z are the so-called Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

In the Hamiltonian for his equation Pauli considered two new terms, besides the term corresponding to an electron (without spin) in a central potential, which was already present in Schrödinger's equation. One of these terms resulted from the interaction of an external magnetic field and the valence electron, which, as we have seen, possesses besides an orbital angular momentum an intrinsic spin momentum. The other resulted from the interaction between the spin magnetic moment and the central potential due to the orbital motion of the electron: the spin-orbit coupling. In this approach Pauli considered only first-order relativistic corrections, and the way the spin operators (and g_0 and the Thomas factor) where put in the Hamiltonian was arbitrary. Most importantly Pauli was unable to extend this approach into a fully relativistic form.

3 The Dirac equation as a one-electron equation

Things changed by the end of 1927, when Paul Dirac was able to formulate a relativistic wave equation. In his first attempts towards a relativistic theory, Dirac consider a Klein-Gordon type equation written in terms of a relativistic Hamiltonian (Dirac, 1926):

$$\{p_x^2 + p_y^2 + p_z^2 - E/c^2 + m^2c^2\}\psi = 0$$

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2c^2}{h^2}\right\}\psi = 0\,.$$

Upon reading Dirac's articles using this equation, Ehrenfest asked Dirac in a letter on the motive for using a particular form for the Hamiltonian:

Why do you write the Hamilton equation in the form:

$$E^2/c^2 - p_1^2 - p_2^2 - p_3^2 = m^2c^2$$

and not:

$$mc^2\sqrt{1-(p_1^2+p_2^2+p_3^2)/m^2c^2} = E$$

Does it make a difference? (quoted in Kragh, 1990, p. 53)

By that time also Pauli was proposing to adopt instead of a second order equation a first order equation involving a square-root (Mehra & Rechenberg, 2000, p. 293).

Dirac felt that neither form of the relativistic Hamiltonian was appropriate for the development of a relativistic wave equation. Dirac considered that this equation should maintain the formal structure of the Schrödinger equation (Kragh, 1990, p. 54). On one side Dirac knew he needed an equation that was linear in the time derivative so that he could maintain in the relativistic case the statistical interpretation of the wave function adopted in the non-relativistic case (Mehra & Rechenberg, 2000, p 294, Kragh, 1990, p. 64). On the other side, this meant, due to relativistic considerations, that the equation should be linear also in the spatial derivatives. According to Dirac "an appropriate formulation of quantum mechanics will only be possible when we succeed in treating space and time as equal to one another" (quoted in Kragh, 1990, p. 54). This means that space and time must appear in the equation as the coordinates of a Minkowski space-time.

It seems that what resulted fundamental to Dirac's development of his relativistic equation was Dirac's realization of the identity:

$$|\vec{p}| = \sqrt{p_1^2 + p_2^2 + p_3^2} = \sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 p_3\,,$$

where σ_1, σ_2, σ_3 are the Pauli matrices. According to Dirac:

I was playing around with the three components σ_1, σ_2, σ_3, which I had used to describe the spin of an electron, and I noticed that if you formed the expression $\sigma_1 p_1 + \sigma_2 p_2 + \sigma_3 p_3$ and squared it, p_1, p_2 and p_3 being the three components of momentum, you got $p_1^2 + p_2^2 + p_3^2$, the square of the momentum. This was a pretty mathematical result. I was quite excited over it. It seemed that it must be of some importance. (Quoted in Mehra & Rechenberg , 2000, p. 295)

This mathematical identity was the insight that made Dirac search for a relativistic counterpart involving a term corresponding to the electron's rest mass. The problem facing Dirac was that with the Pauli matrices it was not possible to write down an expression with four squares:

$$|\vec{p}| = \sqrt{p_1^2 + p_2^2 + p_3^2 + m^2c^2} = \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \alpha_4 mc\,.$$

Dirac considered that from the wave equation developed using the previous expression $\{p_0 - (m^2c^2 + p_1^2 + p_2^2 + p_3^2)^{1/2}\}\psi = 0$ one should be able to recover the equation $\{p_0^2 - m^2c^2 - p_1^2 - p_2^2 - p_3^2\}\psi = 0$, "which is of a relativistically invariant form" (Dirac, 1958, p. 255). That is, Dirac expected his relativistic equation to contain the Klein-Gordon equation as its square, since this equation involves the relativistic Hamiltonian in its normal invariant form. This implied a set of relations for the unknown coefficients:

$$\alpha_\mu\alpha_\nu + \alpha_\nu\alpha_\mu = 0 \ (\mu \neq \nu);\ \mu, \nu = 1, 2, 3, 4,$$

$$\alpha_\mu^2 = 1.$$

There is no set of four 2×2 matrices that satisfies the previous conditions. According to his recollections, at some point Dirac "realized that there was no need to stick to quantities, which can be represented by matrices with just two rows and columns. Why not go to four rows and columns? Mathematically there was no objection to this at all. Replacing the σ-matrices by four-row and column matrices, one could easily take the square root of the sum of four squares, or even five squares if one wanted to" (quoted in Mehra & Rechenberg, 2000, p. 295). With this insight Dirac arrived at his relativistic wave equation. Dirac choose a representation where the coefficients are given by:

$$\sigma_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\rho_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \rho_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \qquad \rho_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},$$

and $\alpha_1 = \rho_1\sigma_1$, $\alpha_2 = \rho_1\sigma_2$, $\alpha_3 = \rho_1\sigma_3$, $\alpha_4 = \sigma_3$.

In this way, Dirac had immediately the relativistic wave equation for a free electron:

$$[p_0 - \rho_1(\boldsymbol{\sigma}, \mathbf{p}) - \rho_3 mc]\psi = 0,$$

where $p_0 = i\hbar\,\partial/(c\partial t)$ and $\mathbf{p} = (p_1, p_2, p_3)$, where $p_r = -i\hbar\,\partial/(c\partial x_r)$ with r = 1, 2, 3; $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is a vector formed with the above 4×4 matrices.

Dirac generalized his equation to the case of an electron in an external electromagnetic field. Dirac followed the rule of replacing p_0 by $p_0 + e/c.A_0$ and $\mathbf{p}$ by $\mathbf{p} + e/c.\mathbf{A}$ (where A_0 and $\mathbf{A}$ are the scalar and vector potentials). This gives us the equation

$$[p_0 + \frac{e}{c}A_0 - \rho_1(\boldsymbol{\sigma}, \mathbf{p} + \frac{e}{c}\mathbf{A}) - \rho_3 mc]\psi = 0,$$

which is the "fundamental wave equation of the relativistic theory of the electron" (Dirac, 1958, p. 257).

Dirac had developed his equation by considering the relativistic Hamiltonian of a free point particle, that is, Dirac did not take into account in his Hamiltonian (as for example Pauli did in the derivation of his equation) any term related to the spin of the electron. It was a surprise to Dirac that "the simplest possible case did involve the spin" (quoted in Kragh, 1981, p. 55).

Dirac set out to explore the relation between his wave equation with external potentials and the Klein-Gordon equation (based on a classical relativistic Hamiltonian), which was according to Dirac "the wave equation to be expected from analogy with the classical theory" (Dirac, 1958, p. 264):

$$\left\{\left(p_0 + \frac{e}{c}A_0\right)^2 - \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 - m^2c^2\right\}\psi = 0.$$

By squaring his equation, Dirac obtained a differential equation that included the operator of the Klein-Gordon equation and two additional terms:

$$\left\{\left(p_0 + \frac{e}{c}A_0\right)^2 - \left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 - m^2c^2 - \frac{\hbar e}{c}(\boldsymbol{\sigma}, \mathbf{H}) + i\rho_1\frac{\hbar e}{c}(\boldsymbol{\sigma}, \mathbf{E})\right\}\psi = 0.$$

Dirac concluded that

> the electron will therefore behave as though it has a magnetic moment $eh/4\pi mc \cdot \boldsymbol{\sigma}$ and an electric moment $ieh/4\pi mc \cdot \rho_1\boldsymbol{\sigma}$. This magnetic moment is just that assumed by the spinning electron model. The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether the electric moment has any physical meaning, since the Hamiltonian … that we started from is real, and the imaginary part only appeared when we multiplied it up in an artificial way to make it resemble the Hamiltonian of previous theories. (Dirac, 1928, p. 619)

Dirac showed how this internal magnetic moment resulted from the electron having a spin angular momentum. According to Dirac

> the spin angular momentum does not give rise to any potential energy and therefore does not appear in the result of the preceding calculation. The simplest way of showing the existence of the spin angular

momentum is to take the case of the motion of a free electron or an electron in a central field of force and determine the angular momentum integrals [of the motion]. (Dirac, 1958, p. 266)

By setting $\mathbf{A} = 0$ and A_0 a function of the radius r, the Hamiltonian in Dirac's equation takes the form $H = - eA_0(r) + c\rho_1(\boldsymbol{\sigma}, \mathbf{p}) + \rho_3 mc^2$. Considering, for example, the equation of motion of the x_1-component of the orbital angular momentum $m_1 = x_2p_3 - x_3p_2$, we have $i\hbar \dot{m}_1 = i\hbar c\rho_1\{\sigma_2p_3 - \sigma_3p_2\}$. We see that m_1 is not a constant of the motion. By considering also the equation $i\hbar \dot{\sigma}_1 = 2ic\rho_1\{\sigma_3p_2 - \sigma_2p_3\}$, Dirac found that $\dot{m}_1 + \hbar/2 \cdot \dot{\sigma}_1 = 0$. This means that the vector $\mathbf{m} + \hbar/2 \cdot \boldsymbol{\sigma}$ is a constant of the motion. According to Dirac "we can interpret this result by saying that the electron has a spin angular momentum of $\hbar/2 \cdot \boldsymbol{\sigma}$, which, added to the orbital angular momentum **m**, gives the total angular momentum **M**, which is a constant of the motion" (Dirac, 1928, p. 620). In this way Dirac found that his relativistic wave equation described an electron with a spin angular momentum (and corresponding magnetic moment).

Dirac did not provide an exact solution of his equation for an electron in a central potential. He only made a first order calculation. In this treatment of the hydrogen atom, Dirac was able to obtain Pauli's results on the energy levels, but without using any arbitrary assumptions: the spin angular momentum, gyromagnetic ratio (g_0) and Thomas factor all came out right. Just a few weeks after the publication of Dirac's paper on the relativistic wave equation, it was shown independently by C. G. Darwin and W. Gordon that an exact solution of Dirac's equation gave an expression for the discrete energy levels of the hydrogen spectrum which was identical to Sommerfeld's original formula derived in 1915, which was in good agreement with experimental results.

4 The problem with the negative energy solutions

Since Dirac was using 4×4 matrices in his equation, the wave function had four components (recall that Schrödinger's original equation had one component, and Pauli's equation had two components due to the spin degree of freedom). Initially Dirac thought that he could simply drop two of the components, since "half of the solutions must be rejected as referring to the charge +e on the electron" (Dirac, 1928, p. 618). This was possible in a first-order approximation. Looking at the exact solution we can see that the situation is far from that simple. Dirac's equation can be written as a set of two coupled differential equations for a pair of two-component wave functions ψ_A and ψ_B, where Dirac's wave function is given by

$$\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}.$$

These two wave functions are essential in the solution of Dirac's equation and cannot be disregarded in the calculation of the energy levels. What happens is that the lower two-components (ψ_B) are smaller that the upper two-components (ψ_A), roughly by a ratio of $\upsilon/2c$, where υ is the 'velocity' of the orbiting electron in Bohr's theory. Now, when considering the solution of Dirac's equation for a free electron with momentum **p**, we see that there are two solutions corresponding to electron states with momentum **p** and energy E_p, and two solutions corresponding to states with momentum $-\mathbf{p}$ and energy $-E_p$, or as Dirac mentioned to an electron with charge +e (and positive energy E_p). In the case of the exact solution for an electron in an external electromagnetic field, as we

have seen, we have a four-component wave function, which we can say, by resort to the free electron case, to 'have' positive and negative energy components, or as Dirac mentions, solutions referring to a charge –e and +e.[3] In this way we must take, as Dirac did, the reference to positive and negative energy components as a "rough one, applying to the case when such a separation is approximately possible" (Dirac, 1958, p. 274). When taking the non-relativistic limit of Dirac's equation, the equation for the upper two-components, takes the form of the Schrödinger-Pauli two-component wave equation (corresponding to a negative energy state), and as mentioned, the lower components are smaller than the upper components and can be disregarded. This is the procedure taken by Darwin to 'derive' Schrödinger's equation from Dirac's equation (Mehra & Rechenberg, 2000, p. 302). That is we can only disregard two of the components of the four-component wave function in the non-relativistic limit. Now, Dirac's results are not non-relativistic. The whole point of Dirac's approach was to develop and apply a relativistic wave equation.

Dirac soon recognized that there was a fundamental difficulty with his equation. Already in his early work with the Klein-Gordon equation Dirac had noticed the possibility of solutions corresponding to a charge +e (this is a general characteristic of any relativistic equation due to the relativistic formula for the Hamiltonian involving E^2). The problem is that when considering any small external electromagnetic field, "in general a perturbation will cause transitions from states with E positive to states with E negative" (quoted in Mehra & Rechenberg, 2000, 306). Accordingly Dirac considered that

> such a transition would appear experimentally as the electron suddenly changing its charge from –e to +e, a phenomenon which has not been observed. The true relativity wave equation should thus be such that its solutions split up into two non-combining sets, referring respectively to the charge –e and the charge +e. (Dirac, 1928, p. 612)

As we have seen, also in the case of an electron in a central potential as described by Dirac's equation, that is not the case. Dirac knew that; he recognized in the beginning of his paper on the relativistic wave equation that he was unable to remove this difficulty and considered that his theory "is therefore still only an approximation" (Dirac, 1928, p. 612). However Dirac expected "the probability of these transitions [to be] extremely small" (quoted in Mehra & Rechenberg, 2000, p. 306). That was not the case. Soon afterwards Werner Heisenberg showed that the probability for transitions in which an electron in a state corresponding to a charge –e goes into a state corresponding to a charge +e (a negative energy state) was much larger than Dirac's estimation. Also, Heisenberg showed that the negative-energy states where necessary to obtain the correct dispersion formulae (Mehra & Rechenberg 2000, pp. 306-307). The problem with the negative-energy solutions was highlighted by O. Klein, when, in the end of 1928, he showed that the simple case of a positive-energy wave incident on a potential barrier could give rise to a transmitted negative-energy wave (Mehra & Rechenberg, 2000, pp. 309-311).

By the end of 1929 Dirac had found a way to solve the '± difficulty' of his electron theory. In late March 1929 Heisenberg wrote to Dirac mentioning that H. Weyl thought he had a solution to the ± difficulty and asked Dirac if he knew and could give him any

[3] According to Dirac, "it is not possible, of course, with an arbitrary electromagnetic field, to separate the solutions of [the relativistic wave equation] definitely into those referring to positive and those referring to negative values [of energy], as such a separation would imply that transitions from one kind to the other do not occur". (Dirac, 1958, p. 274)

details about it. Weyl's idea consisted in suggesting that the two extra components might be ascribed not to the electron but to the proton (Dirac's equation would be in this way describing simultaneously electrons and protons). Dirac worked on Weyl's ideas and set forward a new interpretation of his equation that might provide a solution to the problem of the negative energy solutions.

Dirac's first written accounts of his new views were made in an exchange of letters with Bohr. In a letter to Dirac from November 24, Bohr had put forward the idea that there might not be a strict conservation of momentum and energy in some nuclear processes and that this might lead to a solution of the negative energy problem, resulting from the fact that, according to Klein, the potential that confines the electron in the nucleus induces transitions to negative energy states (by that time it was believed that there were electrons in the nucleus of the atoms). Dirac answered, on November 26, that "I should prefer to keep rigorous conservation of energy at all costs", and that "there is a simple way of avoiding the difficulty of electrons having negative kinetic energy" (quoted in Kragh, 1990, p. 90). Dirac then elaborate on his views:

Let us suppose the wave equation $[w/c + e/c\cdot A_0(r) + \rho_1(\boldsymbol{\sigma}\cdot\boldsymbol{\gamma} + e/c\cdot\mathbf{A}) + \rho_0 me)]\,\psi = 0$ does accurately describe the motion of a single electron. This means that if the electron is started off with a +ve energy, there will be a finite probability of its suddenly changing into a state of negative energy and emitting the surplus energy in the form of high-frequency radiation. It cannot then very well change back into a state of +ve energy, since to do so it would have to absorb high-frequency radiation and there is not very much of this radiation actually existing in nature. It would still be possible, however, for the electron to increase its velocity (provided it can get the momentum from somewhere) as by so doing its energy would be still further reduced and it would emit more radiation. Thus the most stable states for the electron are those of negative energy with very high velocity.

Let us now suppose there are so many electrons in the world that all these most stable states are occupied. The Pauli principle will then compel some electrons to remain in less stable states. For example if all the states of –ve energy are occupied and also few of +ve energy, those electrons with +ve energy will be unable to make transitions to states of –ve energy and will therefore have to behave quite properly. The distribution of –ve electrons, will, of course, be of infinite density, but it will be quite uniform so that it will not produce any electromagnetic field and one would not expect to be able to observe it.

It seems reasonable to assume that not all the states of negative energy are occupied, but that there are a few vacancies or "holes." Such a hole which can be described by a wave function like an X-ray orbit would appear experimentally as a thing with +ve energy, since to make the hole disappear (i.e. to fill it up,) one would have to put –ve energy into it. Further, one can easily see that such a hole would move in an electromagnetic field as though it had a +ve charge. These holes I believe to be the protons. When an electron of +ve energy drops into a hole and fills it up, we have an electron and proton disappearing simultaneously and emitting their energy in the form of radiation.

I think one can understand in this way why all the things one actually observes in nature have positive energy. One might also hope to be able to account for the dissymmetry between electrons and protons; one could regard the protons as the real particles and the electrons as the holes in the distributions of protons of –ve energy. However, when the interaction between the electrons is taken into account this symmetry is spoilt. I have not yet worked out mathematically the consequences of the interaction. It is the "Austausch" effect that is important and I have not yet been able to get a relativistic formulation of this. One can hope, however, that a proper theory of this will enable one to calculate the ratio of the masses of proton and electron. (Quoted in Kragh, 1990, pp. 90-91)

Bohr answered a few days later (in December 5), after discussing Dirac's letter with Klein:

We do not understand, how it works out in detail. Before all we do not understand, how you avoid the effect of the infinite electric density in space. According to the principles of electrostatics it would seem that even a finite uniform electrification should give rise to a considerable, if not infinite, field of force. *In the difficulties of your old theory I still feel inclined to see a limit of the fundamental concepts on which atomic theory hitherto rests rather than a problem of interpreting the experimental evidence in a proper way by means of these concepts. Indeed according to my view the fatal transition from positive to*

negative energy should not be regarded as an indication of what may happen under certain conditions but rather as a limitation in the applicability of the energy concept.

In the case of electrons impinging on a potential barrier examined by Klein we have, on the one hand, a striking example of the difficulties involved in an unlimited use of the concept of potentials in relativistic quantum mechanics. On the other hand, we have just in this case an example of the actual limit of applying the idea of potentials in connection with possible experimental arrangements. In fact, due to the existence of an elementary unit of electrical charge we cannot build up a potential barrier of any height and steepness desired without facing a definite atomic problem. In Klein's example the critical height of the barrier is of order mc^2, and the rise of potential shall take place within a distance of the order h/mc which is the order of magnitude of the wavelength of the electrons concerned. But if the dimensions of the barrier perpendicular to the electric force shall be large compared with this wavelength λ_0, it claims the presence of a double layer of electricity of such a strength that a surface element of size λ_0^2 of the negative layer contains al least hc/e^2 electrons. It is therefore clear that the problem in question cannot legitimately be treated as that of one electron moving in a given potential field, but is essentially a many electron problem which falls outside the range of present quantum mechanics.

On the whole it appears that the circumstance that hc/e^2 is large compared with unity does not only indicate the actual limit of the applicability of the quantum theory in its present form, but at the same time ensures its consistency within these limits. In fact the radius r_0 of the electron estimated on classical theory is $e^2/mc^2 = (h/me)(e^2/hc)$, and we can therefore never determine the position of an electron within an accuracy comparable with r_0 without allowing an uncertainty in its momentum larger than mc, thus entailing an uncertainty of energy surpassing the critical value mc^2. The idea that the reach of quantum mechanics is bound up with the actual existence of the electron would also seem to be in harmony with the fact that the symbols e and m appear in the fundamental equations of the present theory … As regards the transitions from positive to negative energy accompanied by radiation I am not sure that they present as serious a difficulty for your wave equation as it might appear. The question is, how much those features of the theory which claim the transitions in question are involved in the problems, where your theory has been found in so wonderful agreement with experiments. In this connection I must correct the statement in my former letter regarding the probability of these transitions which is not nearly so large as I believed. In discussing the problem more closely with Klein we convinced ourselves that the estimation of this probability did not take sufficient regard to the smallness of the wavelength of the radiation concerned compared with atomic dimensions. We have not made an actual calculation of any such probability, and if you have considered the problem in detail I should be very thankful for any information regarding this point. My hope is that it should be possible to defend all the successful applications of your wave equation, but I suspect that the natural limitation of these applications prevents an extrapolation of the kind you describe in your letter. (Quoted in Moyer, 1981, pp. 1057-1058 [my emphases])

It is important to notice that Bohr started seeing the Klein paradox as resulting from an unlimited mathematical application of the concept of potential (or more generally of field) in relativistic quantum mechanics. Bohr called attention to the necessity of taking into account the elementary unit of electrical charge in the determination of the actual potential barrier. Calculations made not taking into account this fact and the limits in the determination of the electron's position (and its associated uncertainty in momentum, and according to Bohr also in energy) would be beyond the 'actual limit of applicability' of the theory and any 'possible experimental arrangement'. Accordingly, Bohr considered that the elimination of Klein's paradox passed through an essential limitation of the mathematical use of the concept of field (see also Darrigol, 1991, pp. 154-155). However Bohr considered that within the domain of applicability of the theory, the concepts (being used in a restricted context) and the results obtained were consistent (in chapter 5 I will return to Bohr's view on relativistic quantum theory). Due to this, the problem of the transitions to negative-energy states would not occur in 'all the successful applications of the theory'. In this way there would be no need for the hole theory.

Dirac did not agree with Bohr's views and in a letter sent to Bohr in December 9, stressed his differences with Bohr and gave a further elaboration of his views:

I do not completely agree with your views. Although I believe that quantum mechanics has its limitations and will ultimately be replaced by something better, (and this applies to all physical theories,) I cannot see any reason for thinking that quantum mechanics has already reached the limit of its development. I think it will undergo a number of small changes, mainly with regard to its method of application, and by these means most of the difficulties now confronting the theory will be removed. If any of the concepts now used (e.g. potentials at a point) are found to be incapable of having an exact meaning, one will have to replace them by something a little more general, rather than make some drastic alteration in the whole theory … There is one case where transitions of electrons from positive to negative energy levels does give rise to serious practical difficulties, as has been pointed out to me by Waller. This is the case of the scattering of radiation by an electron, free or bound. A scattering process is really a double transition, consisting of first an absorption of a photon with the electron jumping to any state and then an emission with the electron jumping to its final state (as in Raman effect) (or also of first the emission and then the absorption). The initial and final states of the whole system have the same energy, but not the intermediate state, which lasts only a very short time. One now finds, for radiation whose frequency is small compared with mc/h, that practically the whole of the scattering comes from double transitions in which the intermediate state is of negative energy for the electron. Detailed calculations of this have been made by Waller. If one says the states of negative energy have no physical meaning, then one cannot see how the scattering can occur.

On my new theory the state of negative energy has a physical meaning, but the electron cannot jump down into it because it is already occupied. There is, however, a new kind of double transition now taking place, in which first one of the negative-energy electrons jumps up to the proper final state with emission (or absorption) of a photon, and secondly the original positive-energy electron jumps down and fills the hole, with absorption (or emission) of a photon. This new kind of process just makes up for those excluded and restores the validity of the scattering formulas derived on the assumption of the possibility of intermediate states of negative energy.

I do not think the infinite distribution of negative-energy electrons need cause any difficulty. One can assume that in Maxwell's equation div $E = -4\pi\rho$, the ρ means the difference in the electric density from its value when the world is in its normal state (i.e when every state of negative energy and none of positive energy is occupied.) Thus ρ consists of a contribution $-e$ from each occupied state of positive energy and a contribution $+e$ from each unoccupied state of negative energy.

I have not made any actual calculation of the transition probabilities from +ve to –ve, but I think they are fairly small. (Quoted in Kragh, 1990, pp. 92-93)

A paper containing Dirac's hole interpretation of his equation was published in early 1930. In this work Dirac first made clear his departure from Weyl's original idea of associating the negative energy solutions directly to the protons. According to Dirac:

One cannot, however, simply assert that a negative-energy electron is a proton, as that would lead to the following paradoxes:

(i) A transition of an electron from a state of positive to one of negative energy would be interpreted as a transition of an electron into a proton, which would violate the law of conservation of electric charge.

(ii) Although a negative-energy electron moves in an external field as though it has a positive energy, yet, as one can easily see from a consideration of conservation of momentum, the field it produces must correspond to its having a negative charge, e.g. the negative-energy electron will repel an ordinary positive-energy electron although it is itself attracted by the positive-energy electron.

(iii) A negative-energy electron will have less energy the faster it moves and will have to absorb energy in order to be brought to rest. No particles of this nature have ever been observed. (Dirac, 1930, p. 362)

Dirac then presented in more details his hole theory as described in the letters to Bohr. Dirac gave in particular an account of the scattering of radiation by an electron according to his hole theory. Dirac mentions that in a scattering process two types of intermediate states can occur. In one case we have a

transition process, consisting of first an absorption of a photon with the electron simultaneously jumping to any state, and then an emission with the electron jumping into its final state, or else of first the emission and then the absorption. (Dirac, 1930, p. 364)

In the other case

first one of the distribution of negative-energy electrons jumps up into the required final state for the electron with absorption (or emission) of a photon, and then the original positive-energy electron drops into the hole formed by the first transition with emission (or absorption) of a photon. Such processes result in a final state of the whole system indistinguishable from the final state with the more direct processes, in which the same electron makes two successive jumps. (Dirac, 1930, p. 365)

In this second case we have an intermediate state with two electrons (or one electron and the hole). It is clear that the description of the light scattering by an electron involves more than just the original electron: we are faced with a many-body theory. Dirac with his hole theory changed the character of his equation from a single-electron equation into an equation describing a many-body (in reality infinite) electron system.

Even before the publication of his paper on the hole theory, several physicists, knowing the general lines of Dirac's ideas, had a sceptical reaction to it. In particular Heisenberg made a rough calculation of the electron-proton interaction in the new theory. Heisenberg concluded that the electron and the proton had to have the same mass (Kragh 1990, p. 94). Dirac had already recognized this problem in one of his letters to Bohr, but expected that a future detailed theory of the interaction between electrons and protons (holes) might solve this difficulty. A further objection was given a few months latter, in a note by J. R. Oppenheimer (1930a) in which the author calculated the transition probability for the annihilation of an electron and a proton that corresponds to the filling of a hole in the sea. The result was not very promising. Oppenheimer obtained a mean lifetime of a free electron in matter that was too low, and totally inconsistent with the observed stability of matter (Kragh, 1990, pp. 101-102). Oppenheimer's proposition was to "return to the assumption of two independent elementary particles of opposite charge" (quoted in Kragh, 1990, p. 102). That is, to consider the electron and the proton as dissociated particles, each one being related to its 'Dirac sea' of negative-energy particles. In this way there would not be a problem of a possible proton-electron annihilation. More importantly, in 1931 Weyl published a paper in which he proved by symmetry properties of Dirac's equation that the negative-energy electrons must have the same mass as the positive-energy electrons. In this same year Dirac rethought his hole theory in face of the objections being made and presented a new view on the problem:

It thus appears that we must abandon the identification of the holes with protons and must find some other interpretation for them. Following Oppenheimer, we can assume that in the world as we know it, all, and not merely nearly all, of the negative-energy states for electrons are occupied. A hole, if there were one, would be a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron. We may call such a particle an anti-electron. We should not expect to find any of them in nature, on account of their rapid rate of recombination with electrons, but if they could be produced experimentally in high vacuum they would be quite stable and amenable to observation. (Dirac 1931, p. 61)

In 1932 a brief article by C. D. Anderson was published presenting experimental evidence for a new kind of positively charged particle with a mass much smaller than that of the proton. These experimental findings were presented without taking into account Dirac's theory. By that time P. Blackett and G. Occhialini had independent evidence for the positrons and previous to publication they discussed their findings with

Dirac. They published their results after Anderson's publication, mentioning Dirac's theory and presenting some ideas regarding the origin of the positive particle. According to Blackett and Occhialini "one can imagine that negative and positive electrons may be born in pairs during the disintegration of light nuclei" (quoted in Schweber, 1994, p. 69). However they were cautious regarding their proposed mechanism:

> when the behavior of the positive electrons has been investigated in more detail, it will be possible to test these predictions of Dirac's theory. There appears to be no evidence as yet against its validity, and in its favour is the fact that it predicts a time of life for the positive electron that is long enough for it to be observed in the cloud chamber but short enough to explain why it had not been discovered by other methods. (quoted in Schweber, 1994, p. 69)

After a thorough revision of the experimental evidence for positrons Blackett and Occhialini published another article where their support to Dirac's theory was stronger:

> these conclusions as to the existence and the properties of positive electrons have been derived from the experimental data by the use of simple physical principles. That Dirac's theory of the electron predicts the existence of particles with just these properties, gives strong reason to believe in the essential correctness of his theory. (quoted in Schweber, 1994, p. 69)

Even with the experimental evidence for the positron there was resistance to Dirac's hole theory. In relation to this discovery, Bohr considered that "even if all this turns out to be true, of one thing I am certain: that it has nothing to do with Dirac's theory of holes!" (quoted in Kragh, 1990, p. 112). And Pauli wrote to Dirac saying: "I do not believe on your perception of 'holes', even if the existence of the 'antielectron' is proved" (quoted in Kragh, 1990, p. 112). It turns out they were right and a different – field theoretical – approach was possible without any need for an infinite sea of negative-energy particles with some holes in it.

5 The field theoretical interpretation of Dirac's equation

To understand how this field theoretical interpretation of Dirac's equation came to be, we need (at least) to go back to a work by Dirac published in 1927. In this work, Dirac presented a non-relativistic treatment of the interaction of electromagnetic radiation and atoms, which enabled him to give a dynamical derivation of Einstein's laws for the emission and absorption of radiation, which Einstein had obtained by statistical considerations (Darrigol, 1986, p. 226). Dirac followed initially an approach in which he considered an assembly of classical particles (that would interact with an atom), which were described by the Schrödinger equation. For the case of light quanta, Dirac knew that he could not use all the available wave functions, but had to select only symmetrical wave functions, corresponding to Bose-Einstein statistics: "The solution with symmetrical eigenfunctions must be the correct one when applied to light quanta, since it is known that the Einstein-Bose statistical mechanics leads to Planck's law of black-body radiation" (Dirac, 1926, p. 672).

In his 1927 work, Dirac did not follow the simpler procedure of imposing symmetrical wave functions (Dirac, 1958, p. 225), but a procedure more complex and physically unclear. This so-called 'second quantization' turns out to be simply a method that guaranteed that the (quantized) particles, that is particles described by a Schrödinger equation, satisfied Bose-Einstein statistics (Schweber, 1994, p. 28). Dirac did not arrive at the method by some physical insight. According to his words:

I remember the origin of that work was just playing about with equations. I was intending to get a theory of radiation at the time. I was just playing about with the Schrödinger equation. I got the idea of applying the quantization to it and worked out what it gave and found out it just gave the Bose statistics. (Quoted in Darrigol, 1984, p. 461)

In this work Dirac provided a different approach to the interaction of radiation and atoms, by considering the electromagnetic radiation as a classical wave (and not as constituted by particles), which after quantization satisfies Bose-Einstein statistics. Dirac made the bold move of quantizing not directly the electric and magnetic field but the vector potential (Dirac, 1927a, p. 262; Miller, 1994, p. 22; Kragh, 1990, p. 126), which is resolved into its Fourier components. Using a non-relativistic approximation consistent with the one adopted for the light quanta case, Dirac found it possible – by making a necessary reinterpretation of the state corresponding to zero light quanta as a state with an infinite number of unobservable light quanta with zero energy and momentum – to find a Hamiltonian for the system constituted by a (quantized) electromagnetic field interacting with an atom, that "takes the same form as in the light-quantum treatment" (Dirac, 1927, p. 265). This result led Dirac to consider that:

Instead of working with a picture of the photons [light quanta] as particles, one can use instead the components of the electromagnetic field. One thus gets a complete harmonizing of the wave and corpuscular theories of light. One can treat light as composed of electromagnetic waves, each wave to be treated like an oscillator; alternatively, one can treat light as composed of photons, the photons being bosons and each photon state corresponding to one of the oscillators of the electromagnetic field. One then has the reconciliation of the wave and corpuscular theories of light. They are just two mathematical descriptions of the same physical reality. (Quoted in Schweber, 1994, p. 31)

The fact that Dirac considers that there is "a complete harmony between the wave and light-quantum description" (Dirac, 1927a, p. 245), does not mean that he takes over this equivalence to the case of the electrons. In fact, as Dirac clearly states in this article, he makes a sharp distinction between the case of electromagnetic radiation and matter. For Dirac there simply is no real de Broglie wave that, after quantization, permits the description of the electrons (Dirac, 1927a, p. 247).

Jordan's reading of Dirac's work was quite different. Going back to the cumbersome method of second quantization that for Dirac was "nothing but a convenient way to take Bose statistics into account" (Darrigol, 1986, p. 229), Jordan interpreted the scheme of second quantization as the quantization of a classical wave described by a classical wave equation that could be the Maxwell-Lorentz equations for the case of the electromagnetic field or a Schrödinger equation for the case of electrons seen not as particles but as de Broglie waves (Darrigol, 1986, pp. 229-230). This view of Jordan had the advantage to make it possible to treat the quantized waves in a three-dimensional space (or four-dimensional Minkowski space-time) instead of using a 3n (multidimensional) space in the case of an n-particle system (Schweber, 1994, p. 36). In a paper sent to publication in July 1927, Jordan made a conceptual turnaround on Dirac's approach, and applied his method to the quantization of quanta obeying Pauli's exclusion principle, [4] that is to electrons. Contrary to the case of photons (light quanta), in which there is no limit to the number of particles that may occupy the same state, the electrons (according to Pauli's exclusion principle) cannot be in the same state, that is, the occupation number for each possible state can only be 0 or 1 (another way of making this statement is to say that the electrons are fermions, that is, they obey Fermi-

[4] Regarding Pauli's exclusion principle see e.g. Sánchez Ron (2001, pp. 348-350).

Dirac statistics). Jordan's approach was to take the method used by Dirac with the light quanta, and to apply it to the electron as a de Broglie wave – for which there was experimental evidence (Darrigol, 1986, p. 219) – that was a solution of a (classical) Schrödinger equation. In this way, Dirac's 'second quantization' was from Jordan's perspective a 'first quantization' of a classical wave. This was simply implemented by taking the coefficients of each normal mode to be matrices satisfying certain commutation relations. In this work, Jordan did not take correctly into account the phase factors in the matrices "necessary to guarantee that the creation operators for different energy states anticommute with one another" (Schweber, 1994, p. 37). This point was settled by the end of the year in a collaboration with E. Wigner, where the correct anticommutation relations were given. In any case, already in his first incomplete work Jordan was able, by his quantization of the wave, to obtain "an evolution identical to the one given by anti-symmetrical wave functions in configuration space" (Darrigol, 1986, p. 231). That is, Jordan showed the equivalence, for the electrons, of adopting as a classical starting point, not the electrons as particles but electrons as classical de Broglie waves. With this procedure there was no need for a multidimensional abstract configuration space, since it was possible with the wave approach to maintain the description solely in terms of a quantized wave described in a three-dimensional space. In this way, already in this first incomplete work, Jordan was able to conclude that "a quantum-mechanical wave theory of matter can be developed that represents electrons by quantum waves in the usual three-dimensional space" (quoted in Darrigol, 1986, p. 232). This conclusion made possible for Jordan to make the 'programmatic' assertion: "The natural formulation of the quantum theory of electrons will be attained by conceiving light and matter as interacting waves in three-dimensional space" (quoted in Darrigol, 1986, p. 232). In the abstract of the work done with Wigner similar considerations were made:

> The problem at hand is to describe an ideal or nonideal gas that satisfies the Pauli exclusion principle with the idea of not using any relation in the abstract (3N-dimentional) configuration space of the atoms of the gas, but using only three-dimensional space. This is made possible by representing the gas by a three-dimensional quantized wave, for which the particular non-commutative properties for multiplying wave amplitudes are simultaneously responsible for the existence of corpuscular atoms of the gas and for the validity of Pauli's exclusion principle. (Quoted in Schweber, 1994, p. 38)

Heisenberg and Pauli adopted Jordan's approach in the development of a quantum field approach to the description of the interaction of radiation and matter. In their case they took Dirac's equation as a classical wave equation. The 'de Broglie' solution of Dirac's equation is then quantized according to the procedure set forward by Jordan. The elementary excitations (quanta) of the field resulting from the quantization are the particles. In a sense this field theoretical approach as applied by Heisenberg and Pauli is incomplete. The point is that the energy of the field can be negative. This is so due to the presence of the negative energy quanta. This means it was still necessary to make use of Dirac's hole theory to make sense of the field quantization. The negative energy states where taken to be full and a positron was identified with an empty negative-energy state. That is, a positron in this field theoretical view was taken to be the lack of an elementary excitation in an infinite sea of negative-energy elementary excitations.

In November 1933, V. Fock published a paper where he made a symmetrical treatment of free electrons and positrons without using negative-energy particles, following a procedure by Heisenberg from 1931 in which he explored "a far-reaching analogy between the terms of an atomic system with n electrons and those of a system in which n electrons in a closed-shell are lacking" (quoted in Pais, 1986, p. 379). In a

letter to Pauli from July 1933 Heisenberg had presented that same approach, and used it in a paper published in 1934, considering the case where an external field was present (Darrigol, 1984, p. 479; Miller, 1994, p. 63). In this paper he required that "the symmetry of nature in the positive and negative charges should from the very beginning be expressed in the basic equations of [the] theory" (Heisenberg 1934, p. 169). Taking the Dirac equation and its adjoint equation as classical field equations derived from a classical Lagrangian, an arbitrary field can be expanded in terms of the complete set of free-particle solutions (Schweber, 1961, pp. 222-223):

$$\psi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{m}{E_{\mathbf{p}}}\right)^{1/2} \left\{ \sum_{r=1}^{2} b_r(\mathbf{p}) w^r(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{x}} + \sum_{r=3}^{4} b_r(-\mathbf{p}) w^r(\mathbf{p}) e^{-i\mathbf{p}\cdot\mathbf{x}} \right\}.$$

The quantization scheme consists in replacing the expansion coefficients by operators satisfying the anticommutation relations $[b_n, b_m]_+ = [b_n^*, b_m^*]_+ = 0$ and $[b_n, b_m^*]_+ = \delta_{nm}$. With this procedure $\psi(\mathbf{x})$ and the adjoint spinor field $\psi^*(\mathbf{x})$ become operators that act on state vectors of a Fock space;[5] and $b_r(\mathbf{p})$ and $b_r^*(\mathbf{p})$ are interpreted as the annihilation and creation operators of an electron in the state ($\mathbf{p}$, r). Redefining the operators for the negative-energy states as $b_{r+2}(-\mathbf{p}) = d_r^*(\mathbf{p})$ and $b_{r+2}^*(-\mathbf{p}) = d_r(\mathbf{p})$ with r = 1, 2, these operators can be interpreted as the creation and annihilation operators for a positive-energy positron (Schweber, 1961, p. 223; Miller, 1994, p. 56), and the expansion of the $\psi(\mathbf{x})$ operator is now given by

$$\psi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{m}{E_{\mathbf{p}}}\right)^{1/2} \sum_{r=1}^{2} \left\{ b_r(\mathbf{p}) w^r(\mathbf{p}) e^{i\mathbf{p}\cdot\mathbf{x}} + d_r^*(\mathbf{p}) \upsilon^r(\mathbf{p}) e^{-i\mathbf{p}\cdot\mathbf{x}} \right\}.$$

With this formulation there are no negative-energy states (identified as the positive-energy positrons), and so no need for the infinite sea of negative-energy electrons. Also in the field operators $\psi(\mathbf{x})$ and $\psi^*(\mathbf{x})$ we have simultaneously components related to electrons and positrons. Let us consider the total charge operator

$$Q = \sum_r \int d^3p [-a_r^*(\mathbf{p}) a_r(\mathbf{p}) + b_r^*(\mathbf{p}) b_r(\mathbf{p})] = \int d^3p [n^+(\mathbf{p}) - n^-(\mathbf{p})],$$

where $n^-(p)$ is the number of the quanta identified as electrons and $n^+(p)$ is the number of quanta identified as positrons (Jauch & Rohrlich, 1976, p. 64). As we can see from this expression, as Jordan proposed, the quantization of charge and subsequent emergence of a particle-like concept of an electron can be seen as a result of the quantization of the classical field.

6 Combining results from the different views on Dirac's equation

[5] Considering the vacuum state, which is the state with no quanta, an n-quanta Hilbert space can be defined by n applications of creation operators. The Fock space is the (infinite) product of the n-quanta Hilbert spaces: $H^{(0)} \oplus H^{(1)} \oplus H^{(2)} \ldots$ (Schweber, 1961, pp. 134-137; Gross, Runge & Heinonen, 1991, p. 21).

With this field theoretical reinterpretation of Dirac's equation we are still facing a problem. We know that the original one-electron interpretation is not consistent: how then can we relate the solutions of Dirac's equation as a relativistic wave equation for an electron to the view imposed by the quantum field perspective?[6]

I will look again into the case of the hydrogen atom. As Dirac mentioned "in the general case of an arbitrary varying electromagnetic field we can make no hard-and-fast separation of the solutions of the wave equation into those referring to positive and those to negative kinetic energy" (Dirac, 1930, p. 361). L. L. Foldy and S. A. Wouthuysen gave a more detailed account of the situation:

> If we regard the electric field as a perturbation, then one can say that the electric field induces transitions of the particle between the positive- and negative-energy states of a free particle. This is one way of viewing the physical situation.
>
> On the other hand, one knows that for sufficiently weak fields the Hamiltonian above possesses a complete set of eigenfunctions with energy eigenvalues which may be classified according to whether they are positive or negative. There exists for these weak fields a clear-cut distinction between these two sets of stationary states since they are separated by a relatively large energy gap of order 2m. Furthermore, the wave functions corresponding to positive energies show a behavior of the particle appropriate to a particle of positive mass, in that the particle tends to be localized in regions of low potential energy; while the negative-energy solutions show a behavior of the particle appropriate to a particle of negative mass, in that the particle tends to be localized in regions of high potential energy.
>
> Either of the two descriptions of the behavior of the particle in a weak field given above is of course correct, although the distinction between what are called the positive- and negative-energy states is different in the two descriptions. However, the question of terminology for positive- and negative-energy states being left to our own choice, we are free to choose our definitions in such a way as to give the more graphic (and perhaps more intuitively satisfying) description of the actual physical events which are being described. In this spirit we feel that the second description is to be preferred since it has a perfectly reasonable classical limit. It would be difficult indeed to picture classically the motion of a particle in a weak field in terms of transitions between free-particle motions with positive and negative mass.
>
> Consider now what happens when the particle interacts with strong rather than weak fields. Under such circumstances, the division of states into those of positive and negative mass is no longer clear-cut, since the energy separation of the two sets of states is reduced to a relatively small amount. Furthermore, the wave functions describing these states no longer appropriately describe the motion of a particle of fixed sign of mass according to our customary notions. In fact, if we try to interpret the wave function in these terms, we encounter certain well-known paradoxes – the Klein paradox, for example. While the energy of any stationary state will still have a definite sign, the statement that the particle is in a state of positive energy will no longer carry with it the validity of any intuitive conceptions as to the behavior of a classical particle with positive energy, and there will be little qualitative difference between certain states of positive energy and certain states of negative energy. Hence, in the presence of strong fields, the usefulness of a description in terms of positive and negative-energy states will be lost. (Foldy & Wouthuysen, 1949, pp. 33-34)

When considering the exact solution of the one-electron Dirac equation in a central potential (the hydrogen atom) we have a four-component wave function. In simple terms we can say that in the four-component wave function we have components that, from a quantized field perspective, are related to both electrons and positrons. As we have seen, the quantized field perspective relates the (free) positive-energy solutions to

[6] A simple answer in what regards the equation itself is that we can see the Dirac one-electron equation as a 'semi-classical' equation resulting from using the so-called external field approximation (Jauch & Rohrlich, 1976, p. 303), where there appears to be a classical potential within the quantum formalism, but that really is due to a quantum field theoretical description of the interaction with a very heavy charged particle (described by a quantum field) when its recoil is neglected (Schweber, 1961, p. 535). It is within the external field approximation that a Dirac field operator equation with an 'external' field appears, and from which the relativistic one-electron equation with a 'classical' potential can be seen to emerge from the full quantum electrodynamics (Jauch & Rohrlich, 1976, pp. 307 & 313).

the electrons and the (free) negative-energy solutions to the positrons. If we want, taking into account this perspective, a simple model for the hydrogen atom with only one electron, while using the Dirac equation as a relativistic one-electron equation, we must develop a model that uses only two-component positive-energy wave functions to describe the electron. This approach is also important if we want to make a clear connection between the relativistic and non-relativistic equations, that is, between the Dirac and the Schrödinger equations.

Considering the four-component solutions of the Dirac equation in the presence of electromagnetic coupling $\psi = (\psi_A\ \psi_B)$, in the non-relativistic limit the lower two components ψ_B are smaller than the upper two ψ_A. When calculating matrix elements like $(\psi, \gamma_4\psi) = \psi_A^*\psi_A - \psi_B^*\psi_B$, neglecting terms of order $(v/c)^2$, we obtain an expression only in terms of the large components $\psi_A^*\psi_A$, reducing the matrix element to its non-relativistic form in terms of two-component wave functions (Mandl, 1957, pp. 214-215). In the non-relativistic limit the large components can be seen as the solution of the Schrödinger-Pauli two-component wave equation. Concerning this approach to the problem of the non-relativistic limit of the Dirac equation, Foldy and Wouthuysen argued that "the above method of demonstrating the equivalence of the Dirac and Pauli theories encounters difficulties […] when one wishes to go beyond the lowest order approximation" (Foldy & Wouthuysen, 1949, p. 29). Foldy and Wouthuysen proposed a new method (using a different representation than Dirac's original one) which would not only provide better results for higher-order approximations but also the definition of new operators for position and spin "which pass over into the position and spin operators in the Pauli theory in the non-relativistic limit" (Foldy & Wouthuysen, 1949, p. 29).

In the case of the Dirac equation for a free electron it is possible to perform a canonical transformation on the Hamiltonian that enables the decoupling of the positive- and negative-energy solutions of Dirac's equation, each one becoming associated to a two-component wave equation. This means we get two independent equations for two-component wave functions, and that we can identify the equation with positive-energy solution as the Schrödinger-Pauli equation. The case of an electron interacting with an external electromagnetic field is more involved. The trick is to consider the electromagnetic field as a perturbation and to make a sequence of transformations to obtain the separation of negative- and positive-energy solutions. In the non-relativistic limit, like in the previous method, the Schrödinger-Pauli equation is obtained.

It is then possible to rework the relativistic Dirac one-electron equation in a way in which only positive-energy solutions are considered. Foldy and Wouthuysen applied their method to the case where a Dirac electron interacts with an external electromagnetic field. By making three canonical transformations and using only terms of order $(1/m)^2$ they obtained a Hamiltonian (incorporating relativistic correction to this order) that enabled a clear separation of positive- and negative-energy solutions. With this method the non-relativistic limit of Dirac's equation results in two uncoupled equations one with positive-energy solutions and the other with negative-energy solutions. With their three canonical transformations Foldy and Wouthuysen were able to obtain the same wave equation as in the Pauli theory. However it is important to notice that in the previous method we are not taking into account directly the quantized Dirac field, and we are basically maintaining the inconsistent one-electron interpretation of Dirac's equation. Also, Foldy and Wouthuysen's definition of the positive-energy solutions was made by taking into account the classical limit, and as they mentioned it is not unique. As we have seen the quantization of the (free) Dirac field leads to an association of the quanta to individual terms of a plane-wave expansion of the field

corresponding to either positive or negative energy eigenvalues, which implies choosing another definition of positive-energy solutions.

This leads us to the necessity of taking a quantum field approach to the case of the hydrogen atom. One possibility is to use the so-called Furry or bound interaction representation within the external field approximation. This gives a method for calculating corrections (due to a quantized electromagnetic field) to the energy levels of a bound electron (due to a static external potential) determined by the Dirac equation as a relativistic one-electron equation (Berestetskii, Lifshitz & Pitaevskii, 1982, p. 487). But the starting point is the field operator defined by

$$\psi(x) = \sum_{r} u_{r}(\mathbf{x}) a_{r} e^{-iE_{r}t} + v_{r}(\mathbf{x}) b_{r} * e^{iE_{r}t},$$

where $u_r(\mathbf{x})$ and $v_r(\mathbf{x})$ are obtained by solving the Dirac equation for a positive-energy particle representing the electron, and a negative-energy particle representing de positron: $Hu_a(\mathbf{x}) = E_a u_a(\mathbf{x})$ and $Hv_b(\mathbf{x}) = -E_b v_b(\mathbf{x})$, where $H = i\gamma^0\boldsymbol{\gamma}.\nabla - e\gamma^0\gamma_\mu\varphi^\mu + i\gamma^0 m$, with φ^μ a static external field (Jauch & Rohrlich, 1976, p. 313). The first equation is exactly the one solved in the case of the one-electron interpretation of the Dirac equation. This means that E_a gives the positive energy levels obtained by this method, and that in spite of identifying $u_a(\mathbf{x})$ as the electron's positive-energy wave function, it contains what in the limit of a free-particle solution are positive- and negative-energy components (Schweber, 1961, p. 566). Now, what is needed is a method in which the free particle positive-energy characteristic of the electron is maintained during the interaction with no mixing of positive- and negative-energy components.

As we will see in the following chapters, the main working tool in quantum electrodynamics, the S-matrix, was designed for scattering problems where we have free particles in the beginning and free particles in the end of an interaction (scattering). This means that the S-matrix is not very appropriate to deal with the case of a bound particle, at least not in a direct way. Moreover, one of the most important characteristics of quantum field methods is that the interaction between fermions is represented by the exchange of photons: quanta of the electromagnetic field (e. g. Carson, 1996, pp. 127-129). If we make a model of the atom in which a classical Coulomb field gives the effect of the nucleus, this quanta view is lost (as in the external field method previously discussed). A way to overcome these difficulties is to address directly the two-body problem using the Bethe-Salpeter equation. In this method, the two-body problem is addressed by considering directly the two-particle propagator for an electron and a proton (which in the calculations is taken to be a 'big' positron with the same mass as the proton).[7] Considering a power series expansion of the two-particle propagator, the binding energy between a proton and an electron is basically calculated using what is known as the ladder approximation (Schweber, 1961, p. 713). Concerning this approximation, H. A. Bethe and E. E. Salpeter remarked that "although the probability for the exchange of a quantum during a small time interval is fairly small, during the infinite time of existence of the bound state an indefinite number of quanta may be exchanged *successively*. It is just such processes that the ladder-type graphs deal with" (Salpeter & Bethe, 1951, p. 1234). Thus, in the quantum field theory approach, the binding of the electron in the atom is achieved by an exchange of photons with the

[7] There is an indirect method to calculate the energy levels of bound states from the S-matrix, which consists in determining the poles of the exact scattering amplitude. But in practice this approach leads to a summation of an infinite series of diagrams that corresponds to solving the Bethe-Salpeter equation (Berestetskii, Lifshitz & Pitaevskii, 1982, p. 553-556).

proton. We see that from a quantum field theory perspective, the description of the hydrogen atom (as a two-body problem) leads to a physical picture of the process going on in the atom quite different from the one obtained when using inconsistently the Dirac equation as a one-electron equation. That is, we see, when going from a central potential approach to a quantum field two-body treatment of the hydrogen atom, the importance of the quanta concept in the description of interactions in quantum electrodynamics. The exchange of quanta is a basic characteristic of the description of physical processes in quantum electrodynamics. We will look at this in detail in later chapters.

CHAPTER 5

THE INTERACTION OF RADIATION AND MATTER

1 introducion

The more basic and fundamental elements of quantum electrodynamics are already present in Dirac's 1927 work. In it, the electromagnetic field and matter are described by classical Hamiltonians; a further term gives the interaction between the field and matter (Jordan's reinterpretation of matter as waves or Pauli and Heisenberg's Lagrangian formalism does not change the mathematical core of the theory and procedures used in its applications). All this can be developed within a correspondence approach with classical mechanics and field theory, that is, this type of Hamiltonian can be put to use in the Maxwell-Lorentz classical electrodynamics or a classical theory of fields in interaction (Barut, 1964, p. 138; Bogoliubov & Shirkov, 1959, p. 84). Then a second 'layer' is put on top of the classical description (in the fully developed theory, matter is described by the Dirac equation) through which the quantization of the individual fields is achieved (the so-called 'second quantization'). That is, the generalized coordinates (and conjugate momenta) of each field are submitted to commutation or anticommutation relations, and the terms in the Hamiltonian for each field become operators, as is also the case for the term describing the interaction between the fields. But it is important to notice that the fields are quantized as free non-interacting fields, each by itself. Then we are into the game. For practical purposes Dirac makes use of perturbation theory to treat the interaction of radiation and matter.[8] So it was then, and it still is now.

In section 2 we will look into the details of setting quantum electrodynamics (as the theory that describes the interaction of matter and radiation) into 'motion'. It turns out that quantum electrodynamics *is* a perturbative approach. Also quantum electrodynamics relies on the doubtful method of adiabatically switching on/off the interaction between radiation and matter. However, not looking too closely into the mathematical structure of the theory and considering only a few order terms perturbative calculations, quantum electrodynamics presents an astonishing agreement with experimental results.

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[8] The use of perturbative methods has a long history in celestial mechanics. One example is the development of an analytical perturbation theory for the three-body problem: the Sun-Earth-Moon system (Hoskin & Taton, 1995, pp. 89-107). From the planets, perturbative methods went to the planetary models of atoms, being a calculational tool present in the so-called old quantum theory (Darrigol, 1992, pp. 129 & 171). Also it became fundamental in the creation of matrix mechanics, as it was from the perturbative study of the anharmonic oscillator that Heisenberg developed his quantum-theoretical approach (Darrigol, 1992, pp. 266-267; Paul, 2007, pp. 4-5). Soon after, Heisenberg and Max Born put together a perturbation theory within the formalism of quantum mechanics recently developed (van der Waerden, 1967, pp. 43-50; see also Lacki, 1998).

Unfortunately there appear to be severe problems with the previous view. These will be addressed in section 3. According to John Earman and Doreen Fraser, the Haag theorem seems to imply the mathematical inconsistency of the usual treatment of interactions in quantum electrodynamics using perturbative methods (Earman & Fraser, 2006), in which the concept of quanta is central. Fraser presents the idea that "once infinite renormalization counterterms are introduced … it is no longer possible to prove Haag's theorem" (Fraser, 2006, p. 2). I shall argue that simply calling attention to the fact that renormalization "renders the theory mathematically not well-defined" (Fraser, 2006, p. 171), does not provide any answer to the question: "why perturbation theory works as well as it does" (Earman & Fraser, 2006, p. 307). This is not an unimportant question, since we are facing the puzzling situation that the predictions of quantum electrodynamics are so accurate, while the theory from which they are derived appears to be mathematically inconsistent. In this section I will try to provide an answer to this question. In the process I will try to show that Earman and Fraser's conclusion that "Fock representations are generally inappropriate for interacting fields" (Earman & Fraser, 2006, p. 330) or Fraser's related conclusion that "an interacting system cannot be described in terms of quanta" (Fraser, 2008, p. 842), are not valid in quantum electrodynamics: we do not have an interacting system but two weakly interacting fields (systems), and their interaction is described in terms of quanta.

In a nutshell my argument will be as follows: it turns out that quantum electrodynamics, because the series expansion of the S-matrix is divergent, is unable to treat radiation and matter as one closed system. Rather the theory can only give an approximate description (using a perturbative approach) of the interaction between radiation and matter as distinct systems. If we try to close the gap and treat radiation and matter as one closed system, corresponding to an exact solution of the coupled non-linear Maxwell-Lorentz and Dirac equations, our perturbative approach fails. My view is that there is a one-to-one correspondence between having meaningful mathematical results and being clear about the physical assumptions used to set up the theory (in quantum electrodynamics I take part of the input physical concepts to be radiation and matter taken to be independent systems that are independently quantized due to the weak interaction between then). This is so, because the physical concepts are defined in connection with a specific mathematical 'support', not beyond or independently of maths. In a situation where we have an infinity popping up we cannot have a well-defined use of the physical concepts involved. In the case of quantum electrodynamics this situation occurs when trying to give a full description of the (weak) interaction between matter and radiation, which would correspond to treating them as one closed system. I see the divergence of the S-matrix series expansion as a fingerprint of a tentative application of the mathematics of the theory beyond its physical content.

By focusing on the basic physical concepts used to set up the theory, and not solely on its (ill-defined) mathematical structure, we can make sense of the success of quantum electrodynamics (independently of the Haag theorem). This provides a different perspective than the one adopted by Earman & Fraser that not only contradicts several of their conclusions, but also, in my view, makes quantum electrodynamics more intelligible.

2. Quantum electrodynamics as a perturbative approach

In his 1927 paper Dirac dealt with a system consisting of an atom interacting with electromagnetic radiation. Dirac used a non-relativistic Hamiltonian and made his

calculations in what would be called the Coulomb gauge. As already mentioned, in this article Dirac was able to obtain a dynamical derivation of Einstein's coefficients for the spontaneous and stimulated emission of radiation by an atomic system. In a subsequent paper from April 1927 Dirac applied his techniques to the development of a quantum theory of the dispersion of radiation by an atom. The starting point was the classical Hamiltonian for an atom (an electron in a central potential ϕ) interacting with an electromagnetic field (a transverse radiation field with two polarization components) described by a vector potential **A**:

$$H = c\sqrt{m_0^2c^2 + (\vec{p} + \frac{e}{c}\vec{A})^2} - e\phi .$$

In the non-relativistic approximation we have

$$H = H_0 + \frac{e}{c}\vec{v}\cdot\vec{A} + \frac{e^2}{2m_0c}\vec{A}^2 .$$

Each Fourier component of the vector potential is written as

$$k_r = 2\left(\frac{h\nu_r}{c\sigma_r}\right)^{1/2} N_r^{1/2}\cos\theta_r / h ,$$

where in particular ν_r is the frequency of the radiation in the mode k_r. Then Dirac takes the field to be a quantum operator, by taking the canonically conjugated variables N_r and θ_r to be quantum operators (q-numbers in Dirac's approach). N_r is now the number of light-quanta of the component r, and θ_r is its corresponding phase (operator). They satisfy the commutation relation

$$[\theta_r, N_r] = ih.$$

As in his previous 1927 work Dirac is describing the radiation field by using the vector potential **A**, and making a Fourier expansion of **A**. It is the vector potential that is treated as a quantum operator by applying the quantum rules to each of its components.

To treat the dispersion of radiation Dirac found it necessary to use perturbation theory. According to Dirac:

In applying the theory to the practical working out of radiation problems one must use a perturbation method, as one cannot solve the Schrödinger equation directly. One can assume that the term (V say) in the Hamiltonian due to the interaction of the radiation and the atom is small compared with that representing their proper energy, and then use V as the perturbing energy … In the present paper we shall apply the theory to determine the radiation scattered by an atom … If V_{mn} are the matrix elements of the perturbing energy V, where each suffix m and n refers to a stationary state of the whole system of atom plus field (the stationary state of the atom being specified by its action variables, J say, and that of the field by a given distribution of energy among its harmonic components, or by a given distribution of light-quanta), then each V_{mn} gives rise to transitions from state n to state m; more accurately, it causes the eigenfunction representing state m to grow if that representing state n is already excited, the general formula for the rate of change of the amplitude a_m of an eigenfunction being

$$ih/2\pi\,\dot{a}_m = \sum_n V_{mn} a_n = \sum_n \upsilon_{mn} a_n e^{2\pi i(W_m - W_n)t/h} \quad (1)$$

where υ_{mn} is the constant amplitude of the matrix element V_{mn}, and W_m is the total proper energy of the state m. To solve these equations one obtains a first approximation by substituting for the a's on the right-hand side their initial values, a second approximation by substituting for the a's their values given by the first approximation, and so on. (Dirac, 1927b; pp. 711-712)

Up to second order Dirac found that the Hamiltonian operator gave rise to two processes of scattering of radiation by the electron in the atom. In one case, which Dirac called direct or true scattering processes, we have a transition "in which a light-quantum jumps directly from a state s to a state r" (Dirac, 1927b, p. 717). The other case, which shows up only when making an approximate second order calculation, Dirac named 'double scattering processes'. When considering two states m and m' without an appreciably difference in energy, there can be a scattering of radiation which "appears as the result of the two processes m' → n and n → m, one of which must be an absorption and the other an emission" (Dirac, 1927b, p. 712), where n is a third state different from m and m'. According to Dirac in neither of the two processes "is the total proper energy even approximately conserved" (Dirac, 1927b, p. 712).[9]

As we have already seen, Dirac's approach was improved by Pauli, Heisenberg, and Fermi, in the development of a relativistic theory of the interaction of quantized radiation and matter. Looking now at quantum electrodynamics from the established Lagrangian approach, we have two classical fields described by the Maxwell-Lorentz equations and the Dirac equation. As we have already seen, the Dirac equation can be taken to be a classical equation of a spinor field (and its adjoint field). Using the usual procedure of Fourier expansion of a wave function this field can be resolved into its Fourier components, whose amplitude coefficients become operators after the quantization and satisfy anticommutation relations according to Fermi-Dirac statistics. An equivalent procedure is taken for the quantization of the electromagnetic field (following Bose-Einstein statistics). Up to this point we are dealing with two independently quantized fields. Quantum electrodynamics is about the description of the interaction between radiation and matter as described by these quantum fields. In classical electrodynamics, the relativistic equation of motion of a charged particle in a given external field is

$$K = e\left(\vec{E} + \frac{1}{c}\left[\vec{v} \times \vec{H}\right]\right).$$

This equation can be derived from the Hamiltonian representing the total energy of the particle expressed as a function of the canonical coordinates and momenta

$$H = e\phi + \sqrt{m_0^2 c^4 + (\vec{p} - e\vec{A})^2}\,,$$

where the total energy, which can be given by $T + e\phi$, is the fourth component of a four-vector p_μ given by

$$p_\mu = u_\mu + eA_\mu$$

[9] In chapter 7 I will look at what might be the physical meaning of these transitory states (the virtual states) that appear in the perturbative treatment of the interaction of radiation a matter.

(u_μ is the kinetic momentum given by the relativistic expression for the momentum-energy four-vector for a free particle).

From the Hamiltonian it is a simple task to derive the equation of motion for the charged particle in an external field:

$$\frac{1}{c}\frac{d}{dt}(p_x - eA_x) = e\left(E_x + \frac{1}{c}[vH]_\xi\right).$$

In this way the Hamiltonian for the particle contains also the term describing the interaction of the particle with the field.

In the application of Dirac's equation as a one-electron equation to the case of electrons in an external field (e.g. the hydrogen atom) we use the prescription of going from the 'kinetic momentum' to the 'total momentum' by making the replacement

$$p_\mu \rightarrow p_\mu - \frac{e}{c}A_\mu .$$

In this way, Dirac's equation in the presence of an external (classical) field is given by

$$\gamma^\mu\left(i\hbar\partial_\mu - \frac{e}{c}A_\mu(x)\right)\psi(x) = mc\,\psi(x),^{10}$$

instead of simply

$$i\hbar\gamma^\mu\partial_\mu\psi(x) = mc\,\psi(x),$$

which is the case for a free field. As we have seen it is this last equation that is used when making the quantization of the Dirac field. The case of the electromagnetic field is similar. We quantize the free field, but the (operator) equation for the (quantized) electromagnetic field in the presence of (quantized) charges is

$$\left(\frac{\partial^2}{c\partial t^2} - \nabla^2\right)A_\mu = -\frac{\partial L}{\partial A^\mu} = -e\overline{\psi}\gamma_\mu\psi .$$

This set of coupled equations, for the Dirac and electromagnetic field, can be derived from a Lagrangian representing the Dirac field interacting with the electromagnetic field

$$L = -\frac{1}{2}\frac{\partial A_\mu}{\partial x_\nu}\cdot\frac{\partial A^\mu}{\partial x^\nu} - \frac{1}{2i}\left[\overline{\psi}\gamma^\mu\cdot\left(\frac{\partial}{\partial x^\mu} - ieA_\mu\right)\psi + im\overline{\psi}\cdot\psi\right]$$

$$+\frac{1}{2i}\left[\left(\frac{\partial\overline{\psi}}{\partial x^\mu} + ieA_\mu\overline{\psi}\right)\gamma^\mu\cdot\psi - im\overline{\psi}\cdot\psi\right].$$

[10] After quantization this equation becomes an operator equation for a quantized Dirac field in interaction with a quantized electromagnetic field.

This Lagrangian can be written in terms of the Lagrangians for the free Dirac field, the free electromagnetic field and a term representing the interaction between the two fields

$$L = L_m + L_{em} + e\overline{\psi}\gamma^{\mu} \cdot \psi A_{\mu} .$$

Since the interaction term in the Lagrangian does not contain any time derivatives of the field operators "the canonical momenta are therefore the same functions of the dynamical variables as [in the free field case], and we can immediately write down the equal time canonical commutation relations" (Källén, 1972, p.76). That is we could apparently start from the Lagrangian for the Dirac and electromagnetic fields in interaction to develop the quantization procedure. But contrary to the case of free field quantization it is not possible now to obtain commutation relations valid for all times since this implies solving first the coupled equations for the fields (Schweber, 1961, p. 276; Källén, 1972, p.76). The problem is that, as Freeman Dyson mentioned,

these equations are non-linear. And so there is no possibility of finding the general commutation rules of the field operators in closed form. We cannot find any solution of the field equations, except for the solutions which are obtained as formal power series expansions in the coefficient e which multiplies the non-linear interaction terms. It is thus a basic limitation of the theory, that it is in its nature a perturbation theory starting from the non-interacting fields as an unperturbed system. Even to write down the general commutation laws of the fields, it is necessary to use perturbation theory of this kind. (Dyson, 1952a, p. 79)

This implies that to treat the interaction of radiation with matter we must start from the quantized free fields and then by using perturbation theory treat the interaction between the two fields. According to Dyson

Since the perturbation theory treatment is forced on us from the beginning, it is convenient not to set up the theory in the Heisenberg representation but to use the interaction representation. The IR is just designed for a perturbation theory in which the radiation interaction is treated as small. In the IR the commutation rules can be obtained simply in closed form, and so the theory can be set up with a minimum of trouble. (Dyson , 1952a, p. 79)

I will consider for the time being that we can (apparently) describe the interaction of matter and radiation in terms of a wave function describing the fields as one closed system. We take, as Dirac did in 1927, this joint system to be described by a Hamiltonian $H = H_0 + V$, where H_0 describes the free non-interacting fields and V describes the interaction between them. In the Schrödinger picture (representation) the equation of motion of the system is the time-dependent Schrödinger equation

$$i\hbar\partial_t \left|\phi_S(t)\right\rangle = H_S \left|\phi_S(t)\right\rangle = (H_{0S} + V)\left|\phi_S(t)\right\rangle .$$

In the interaction representation (or Dirac picture) the state vector describing the system is defined in terms of the state vector in the Schrödinger representation as

$$\left|\phi_D(t)\right\rangle = e^{iH_{0S}t\hbar} \left|\phi_S(t)\right\rangle .$$

This state vector satisfies the equation

$$i\hbar\partial_t\left|\phi_D(t)\right\rangle = e^{iH_{0S}t\hbar}Ve^{-iH_{0S}t\hbar}\left|\phi_D(t)\right\rangle = V_D(t)\left|\phi_D(t)\right\rangle;$$

also, an operator in the interaction representation is related to the corresponding Schrödinger representation operator by

$$Q_D(t) = e^{iH_{0S}t\hbar}Q_S e^{-iH_{0S}t\hbar},$$

whose time dependency is determined in terms of the unperturbed Hamiltonian

$$i\hbar\partial_t Q_D(t) = \left[Q_D(t), H_0\right].$$

In this way we see that in the interaction representation the time evolution of the state vector (describing the two fields in interaction) is dependent only on the interaction term, while the time evolution of the operators depends on the free-field Hamiltonian.

In quantum electrodynamics, the majority of its applications are made using the S-matrix formalism. This formalism is particularly tailor-made for the description of scattering processes but is also applicable to bound-state problems (Veltman, 1994, pp. 62-67). I follow Dyson's presentation of a typical scattering process as described within quantum electrodynamics:

> The free particles which are specified by a state A in the remote past, converge and interact, and other free particles emerge or are created in the interaction and finally constitute the state B in the remote future. (Dyson, 1952a, p. 81)

Dyson calls attention to the fact that:

> The unperturbed states A and B are supposed to be states of free particles without interaction and are therefore represented by constant state-vector ϕ_A and ϕ_B in the interaction representation. The actual initial and final states in a scattering problem will consist of particles each having a self-field with which it continues to interact even in the remote future and past, hence ϕ_A and ϕ_B do not accurately represent the initial and final states.(Dyson, 1952a, p. 81)

Dyson presents what can be considered an operational justification for using the states of free particles (usually referred to as bare states) in the calculations, by taking into account how scattering experiments are really done (see Falkenburg, 2007, pp. 129-131):

> Let $\Psi_B(t)$ be the actual time-dependent state-vector of the state B in the IR. We are not interested in the dependence of $\phi_B(t)$ on t. In an actual scattering experiment the particles in state B are observed in counters of photographic plates or cloud-chambers and the time of their arrival is not measured precisely. Therefore it is convenient to use for B not the state-function $\Psi_B(t)$ but a state function ϕ_B which is *by definition* the state-function describing a set of bare particles without radiation interaction [that is without self-interaction with its own field], the bare particles having the same momenta, and spins as the real particles in state B. (Dyson, 1952a, p. 94)

The transition amplitude of the scattering process is given by $S_{AB} = (\phi^*_B S\phi_A)$, where S is the so-called S-matrix. This scattering amplitude S_{AB} can be written as (ψ_B^-, ψ_A^+), where we have: $\lim_{t_0\to-\infty} U(0,t_0)\phi_A = \psi_A^+$ and $\lim_{t\to+\infty} U(0,t)\phi_B = \psi_B^-$, where $U(t, t_0)$ is the time displacement unitary operator defined by

$$U(t,t_0)\left|\psi(t_0)\right\rangle = \left|\psi(t)\right\rangle$$

(where we are using the state vectors and operators in the Dirac picture), which satisfies the equation

$$i\hbar\partial_t U(t,t_0) = V(t)U(t,t_0)\,.$$

In this way the S matrix is simply related to the operator $U(t, t_0)$ by the formal expression $S = U(\infty,-\infty)$. By using the boundary condition $U(t_0, t_0) = 1$ the previous equation for $U(t, t_0)$ is equivalent to the integral equation

$$U(t,t_0) = 1 - \frac{i}{\hbar}\int_{t_0}^{t} V(t')U(t',t_0)dt'.$$

This equation is solved by an iteration procedure. We have

$$\begin{aligned} U(t,t_0) = 1 &+ \left(-\frac{i}{\hbar}\right)\int_{t_0}^{t} dt_1 V(t_1) \\ &+ \left(-\frac{i}{\hbar}\right)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 V(t_1)V(t_2) \\ &+ \left(-\frac{i}{\hbar}\right)^3 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} dt_3 V(t_1)V(t_2)V(t_3) \\ &+ \ldots . \end{aligned}$$

By defining a chronological operator

$$P(V(t_1)\ldots V(t_n)) = \sum_P \theta(t_{\alpha_1} - t_{\alpha_2})\ldots\theta(t_{\alpha_{n-1}} - t_{\alpha_n})V(t_{\alpha_1})\ldots V(t_{\alpha_n})$$

(where we must sum over all permutations of $t_1, \ldots t_n$), so that we have

$$P(V(t_1)\ldots V(t_n)) = V(t_i)\ldots V(t_j)\ldots V(t_k)\,, \text{ with } t_i > \ldots t_j \ldots > t_k,$$

the expansion for $U(t, t_0)$ can be written as

$$U(t,t_0) = \sum_{n=0}^{\infty} \frac{1}{n!}\left(-\frac{i}{\hbar}\right)^n \int_{t_0}^{t} dt_1 \int_{t_0}^{t} dt_2 \ldots \int_{t_0}^{t} dt_n P(V(t_1)V(t_2)\ldots V(t_n))\,.$$

In applications to scattering problems we need to calculate the S-matrix, that is, $U(\infty,-\infty)$. This is the case because in the case of scattering processes we only have experimental access to the cross-section. In quantum electrodynamics the scattering cross-section is calculated from the transition probability per unit space-time volume, which is related to the S-matrix in a simple way (Jauch & Rohrlich, 1976, pp. 163-167). According to B. Falkenburg "the effective cross-section is the physical magnitude with which the current field theories come down to earth. As a *theoretical quantity*, the cross-section is calculated from the S-matrix of quantum mechanics … as an *empirical*

quantity, it is the measured relative frequency of scattering events of a given type" (Falkenburg, 2007, p. 107)

When calculating the S-matrix we must extend the initial time t_0 to $-\infty$ and the final time t to $+\infty$. As we have seen in Dyson's presentation the initial (and final) state of the system is taken to be an eigenstate of H_0 the Hamiltonian for the non-interacting fields. This idea can be given a more formal presentation by considering that the interaction between particles in a scattering process is adiabatically switched on in the remote past and adiabatically switched off in the remote future (Lippmann & Schwinger, 1950, p. 473; Bogoliubov & Shirkov, 1959, p. 197). What this means is that at infinity the interaction term is taken to zero, that is, there is no interaction between matter and radiation. This has important implications. One is that since there is no interaction we are really considering two uncoupled systems, the Dirac field, and the electromagnetic field. Another point is that in this case the state vectors are taken to be a product of Fock states of each field: bare states. Looking at this in terms of scattering of particles (quanta of each field), the initial and final states of the scattered particles are states of the (free-particle) Fock space.

As we have seen the S-matrix is given as a series expansion in powers of the interaction term V(t). We can look at the description of an interaction process as described by the S-matrix as a perturbative approach in which only Fock states are considered. The use of the interaction representation in the S-matrix approach can be seen then as part of a perturbative approach in which the interaction term is leading to a perturbation of the free states of the fields.

To see the importance of the adiabatic 'switching on' and 'switching off' of the interaction to this overall perturbative approach let us look into Dyson's take on this subject. As we have seen Dyson gave an operational justification to the use of a Fock state function ϕ_B to describe the initial and final state of the Dirac field, instead of $\Psi_B(t)$ the state-vector representing for example a self-interacting electron. Dyson asks the question: "what is the connection between $\Psi_B(t)$ and ϕ_B?"(Dyson, 1952a, p. 94). Let us look into his presentation of the adiabatic trick.

Suppose t_B to be a time so long in the future after the scattering process is over, that from t_B to $+\infty$ the state B consists of separated outward-travelling particles. Then the relation between $\psi_B(t)$ and ϕ_B is simple. We imagine a fictitious world in which the charge e occurring in the radiation interaction decreases infinitely slowly (adiabatically) from its actual value at time t_B to zero at time $(+\infty)$. In the fictitious world, the state $\psi_B(t_B)$ at time t_B will grow into the bare-particle state ϕ_B at time $+\infty$. Thus

$$\phi_B = \Omega_2(t_B)\, \psi_B(t_B) \tag{489}$$

where

$$\Omega_2(t_B) = \sum_{n=0}^{\infty}\left(\frac{e}{\hbar c}\right)^n \frac{1}{n!}\int_{t_B}^{\infty}\dots\int_{t_B}^{\infty} dx_1 \dots dx_n P\{\overline{\psi}\mathbb{A}\psi(x_1),\dots,\overline{\psi}\mathbb{A}\psi(x_n)\} g_B(t_1)\dots g_B(t_n) \tag{490}$$

and $g_B(t)$ is a function decreasing adiabatically from the value 1 at $t = t_B$ to zero at $t = \infty$. Similarly, when t_A is a time so far in the past that the state A consists of separated converging particles from $t = -\infty$ to $t = t_A$ we have

$$\Omega_1(t_A) = \sum_{n=0}^{\infty}\left(\frac{e}{\hbar c}\right)^n \frac{1}{n!}\int_{-\infty}^{t_A}\dots\int_{-\infty}^{t_A} dx_1 \dots dx_n P\{\overline{\psi}\mathbb{A}\psi(x_1),\dots,\overline{\psi}\mathbb{A}\psi(x_n)\} g_A(t_1)\dots g_A(t_n) \tag{492}$$

where $g_A(t)$ is a function increasing adiabatically from $t = -\infty$ to $t = t_A$.

The scattering matrix element between states A and B is given exactly by

$$M = (\psi_B^*(t_B) S_{t_A}^{t_B} \psi_A(t_A)) \tag{493}$$

$$S_{t_A}^{t_B} = \sum_{n=0}^{\infty} \left(\frac{e}{\hbar c}\right)^n \frac{1}{n!} \int_{t_B}^{t_A} \dots \int_{t_B}^{t_A} dx_1 \dots dx_n P\{\bar{\psi} \not{A} \psi(x_1), \dots, \bar{\psi} \not{A} \psi(x_n)\} g(t_1) \dots g(t_n) \tag{494}$$

Of course (493) is independent of the times t_A and t_B. When t_A and t_B are chosen so far in the past that (489) and (491) are satisfied, then (493) may be written in the form (487), where now

$$\begin{aligned} S &= \Omega_2(t_B) S_{t_A}^{t_B} \Omega_2(t_A) \\ &= \sum_{n=0}^{\infty} \left(\frac{e}{\hbar c}\right)^n \frac{1}{n!} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dx_1 \dots dx_n P\{\bar{\psi} \not{A} \psi(x_1), \dots, \bar{\psi} \not{A} \psi(x_n)\} g(t_1) \dots g(t_n) \end{aligned} \tag{495}$$

and g(t) is a function increasing adiabatically from 0 to 1 for $-\infty < t < t_A$, equal to 1 for $t_A \le t \le t_B$, and decreasing adiabatically from 1 to 0 for $t_A < t < \infty$. Thus we come to the important conclusion that [the] formula [$M = (\phi_B^* S \phi_A)$ (487)] for the matrix element is correct, using the bare particle state-functions ϕ_A and ϕ_B, provided that [the] formula

$$S = \sum_{n=0}^{\infty} \left(\frac{e}{\hbar c}\right)^n \frac{1}{n!} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} dx_1 \dots dx_n P\{\bar{\psi} \not{A} \psi(x_1), \dots, \bar{\psi} \not{A} \psi(x_n)\} \tag{488}$$

for S is interpreted by putting in the slowly varying cut-off function $g(t_i)$ to make the integrals converge at $t_i = \pm\infty$. The cut-off functions are to be put in as they appear in (495), and then S is defined as the limit to which (495) tends as the rate of variation of g(t) is made infinitely slow.

The main practical effect of this limiting process in the definition of S is to justify us in throwing away all terms in the integrals which oscillate finitely at $t_i = \pm\infty$. There are however certain cases in which the integral (488) is in a more serious way ambiguous due to bad convergence at $t_i = \pm\infty$. In these cases the cut-off functions have to be kept explicitly until a late stage of the calculations before going to the limit g(t) = 1. In all cases, if the limiting process is done in this way, the matrix element M is obtained correctly and unambiguously.

The use of bare-particle wave-functions ϕ_A and ϕ_B in (487) is thus justified. (Dyson, 1952a, pp. 94-95)

In this way, in the S-matrix approach we will be calculating transition amplitudes between free-particle states ($M = (\phi_B^* S \phi_A)$). Since the interaction term is given by $e\bar{\psi}\gamma^\mu \cdot \psi A_\mu$, we can also see the series expansion of the S-matrix as given in powers of *e*, where *e* is the electric charge (or in powers of the so-called fine structure constant $\alpha = e^2/4\pi$): $S = 1 + eS^{(1)} + e^2S^{(2)} + \dots$ (Källén, 1972, p. 88).

Let us consider, for example, the quantum electrodynamical treatment of the two-photon annihilation of an electron and a positron (Sakurai, 1967, pp. 204-208): $e^+ + e^- \rightarrow 2\gamma$. The initial state corresponds to a Dirac field with two quanta, one corresponding to the electron, and the other to the positron (the electromagnetic field is taken to be in the vacuum state). In the final state after the 'scattering', the Dirac field is in the ground-state, that is, with no quanta, and the electromagnetic field is in a state with two photons. The second-order transition or scattering amplitude is given by

$$S_{fi} = (-e)^2 \int d^4x_1 \int d^4x_2 \left\langle 2\gamma \middle| A_\mu(x_1) A_\nu(x_2) \middle| 0 \right\rangle (\gamma_\mu)_{\alpha\beta} (\gamma_\nu)_{\gamma\delta}$$

$$\times\Big[\big\langle 0\big|\Psi_{\beta}^{(+)}(x_1)\overline{\Psi}_{\gamma}^{(-)}(x_2)\overline{\Psi}_{\alpha}^{(+)}(x_1)\overline{\Psi}_{\delta}^{(+)}(x_2)\big|e^{-}e^{+}\big\rangle\theta(t_1-t_2)$$
$$-\big\langle 0\big|\Psi_{\gamma}^{(+)}(x_2)\overline{\Psi}_{\beta}^{(-)}(x_1)\overline{\Psi}_{\alpha}^{(+)}(x_1)\overline{\Psi}_{\delta}^{(+)}(x_2)\big|e^{-}e^{+}\big\rangle\theta(t_2-t_1)\Big].$$

I will not go into the details of this expression but only address some relevant features. The expression is an example of the perturbative quantum electrodynamical treatment of the interaction between the Maxwell and Dirac fields. In it we have the description of the propagation of quanta between two space-time points x_1 and x_2 (corresponding to Dirac's 'third state'), and components related to the non-interacting states of each field (Fock states): $\left|e^{-}e^{+}\right\rangle$, $\left|2\gamma\right\rangle$, and the vacuum states of each field $\left|0\right\rangle$. That is, we use only the Fock space for each field to calculate the interactions in quantum electrodynamics. Also, each quantum field operates in different Fock spaces.

3 *Possible problems for quantum electrodynamics: the Haag theorem and the divergence of the S-matrix series expansion*

It seems then that we have a simple procedure to calculate whatever situation of interaction between radiation and matter we might have by simply addressing each case as if it is a scattering problem and dealing with it using the S-matrix approach with the adiabatic switching trick, where we can use as initial and final states Fock states of each field. However the situation is not that simple. One crucial aspect of all this adiabatic switching trickery is that the following supposition (called the adiabatic theorem) is being made: "If a state is an eigenstate of the Hamiltonian and if a parameter in the Hamiltonian is adiabatically changed …, then the same state is also an eigenstate, after the Hamiltonian is changed, but with a different eigenvalue" (Källén, 1972, pp. 52-53). As we have seen in Dyson's presentation this is being taken for granted: 'the state $\psi_B(t_B)$ at time t_B will grow into the bare-particle state ϕ_B at time $+\infty$. Thus $\phi_B = \Omega_2(t_B)\,\psi_B(t_B)$'. Is this really the case?

No! From Haag's theorem (Haag, 1955) we know that we cannot have a unitary transformation that relates the field operators corresponding to the free Hamiltonian H and the interacting field Hamiltonian H_I. Considering that at t_0 the Heisenberg picture and the Dirac picture (interaction representation) coincide (Earman & Fraser, 2006, p. 320), it would seem that the state vector in the interaction representation, in the limit $t \to \pm\infty$, corresponds to free particles due to the fact that the interaction part of the Hamiltonian is negligible. But from Haag's theorem it seems that "at times $t = \pm\infty$, all the assumptions of the theorem hold for the Heisenberg representation, which represents an interaction, and for the interaction representation, which is a Fock representation for a free system" (Earman & Fraser, 2006, p. 322). In informal terms Haag's theorem implies that the state vectors in the interaction representation, that for $t \to \pm\infty$ are *supposed* to represent the free field, and the state vector in the Heisenberg representation for the interacting fields, are not in a common domain of both H and H_I (Schweber, 1961, p. 416).

From Haag's theorem we can conclude that if we have a free field at $t = -\infty$, the interaction representation describes also a free field at any time t_0. This means that we need to have a state of the full interacting Hamiltonian from the start so that we can consistently give the interaction representation its usual interpretation as giving a different time dependency to the state vector and the operators (Schweber, 1961, p. 317).

Both the Heisenberg and Dirac pictures can hypothetically be used in either free or interacting systems, if we can separate the Hamiltonian in two parts. The change of representation does not change the physical situation whether it concerns free or interacting fields. There really is no "interaction picture's assumption that there is a time at which the representation for the interaction is unitarily equivalent to the Fock representation for a free system" (Fraser, 2006, p. 54).[11] This 'assumption' has nothing to do with the representation being used. The 'assumption' is that with an adiabatic switching on of the interaction, the state vector for the interacting systems can be constructed from the state vector of free fields (Schweber, 1961, p. 320):

$$\lim_{t\to-\infty} U(t_0,t)\left|\varphi_a\right\rangle = \left|\psi_a\right\rangle .$$

This is what is supposed to be achieved in the adiabatic switching on of the potential that 'connects' a free field Hamiltonian with the interacting field Hamiltonian (Jauch & Rohrlich, 1976, p. 134; Schweber, 1961, p. 322): $\lim_{t\to-\infty} H_\varepsilon(t) = \lim_{t\to-\infty}(H_0 + e^{-\varepsilon|t|}V(t)) = H_0$, where ε is a parameter that is taken to zero in the end of the computations. The point is, as we have seen, that at infinite times before and after the adiabatic switching on/off of the interaction potential, the state vector in either the Heisenberg or the Dirac picture is *assumed* to be describing free fields.

The question here is not thus the representation being used but whether it is possible to connect the interacting state to a free field state. It seems clear from the consequences of Haag's theorem that the usual adiabatic switching on/off of the interaction will not do the trick. In this way we are in the situation of explaining how it is that with a mathematically incorrect procedure it is possible to develop applications from the theory that give so good results when compared with experiments.

It is well known that in the applications of quantum electrodynamics there are problems with divergent integrals. These problems are circumvented in practice with renormalization techniques in which (basically) all the divergent integrals appearing in the series expansion of the S-matrix are related to corrections to the mass and charge of the fermions. Since the value of the mass and the charge are not defined by the theory but result from measurements, the terms in the series expansion that are divergent (but formally should be smaller and smaller) are taken to be part of the observed mass and charge.

It might seem that by using renormalization techniques the consequences of Haag's theorem might be evaded because "once infinite renormalization counter terms are introduced, the interaction picture is not mathematically well-defined" (Fraser, 2006, p. 2). From this it might seem that "renormalization addresses this problem not by refining the assumptions, but by rendering the canonical framework mathematically ill-defined" (Fraser, 2006, p. 90). But it would be rather strange, to say the least, that by considering

[11] Earman and Fraser's take it that in the infinite past and the infinite future "particles are assumed to be infinitely far apart and therefore not interacting" (Earman & Fraser, 2006, p. 321). In this case they consider that at t=±∞ "the representation is taken to be a Fock representation" (p. 321). And this is taken to be a Dirac picture (interaction representation) assumption: "In the interaction picture ... at t = ±∞ the representation is the Fock representation for the free field" (p. 321). At this point I thus disagree with Earman and Fraser's presentation of the consequences of the Haag theorem. I simply do not think that the assumption that at infinite past we can take the state of a particular field to be a free state as described by a Fock space has to be seen as part of the assumptions of the Dirac picture. But this is a question of detail. The main aspect of their presentation is irrefutable: according to the Haag theorem the perturbative approach used in quantum electrodynamics to describe interactions is mathematically inconsistent.

an impoverished mathematical structure, suddenly, something physically equivalent to a unitary transformation connecting free and interacting field operators might emerge. In reality this argument does not provide any explanation for why the theory is effective, and this is a central question which has correctly been considered to be relevant: "There is, however, unfinished business in explaining why perturbation theory works as well as it does" (Earman & Fraser, 2006, p. 307).

If we followed Fraser's account we would be back where we started since, without taking into account the ill-defined mathematics of renormalization (and this is possible by considering only the lowest-order calculations), one could say that the adiabatic switching on/off of the interaction is also of doubtful mathematical rigor precisely due to the Haag theorem. This means that when imposing the adiabatic switching on/off we are developing an ill-defined approach. The primary question of the effectivity of the theory would remain, independently of the probable impossibility of taking into account the Haag theorem in this ill-defined mathematical context (now due to the adiabatic switching on/off of the interaction).

Another aspect of not having a rigorous basis for a procedure to adiabatically construct the interacting states from the free states is the following. In the Fock space we have a direct connection of quanta to the normal modes of classical field configurations. It is from the Fourier splitting of a classical wave into positive and negative frequency normal modes, providing the basis for the configuration space, that the concept of quanta emerges (via a quantization procedure which results in associating quanta to each normal mode). In the case of interacting fields it is not possible to make this Fourier expansion (Fraser 2006, p. 136). This implies that it is not possible to use the Fock representation for a free field to represent the interacting Dirac and Maxwell fields (Earman & Fraser, 2006, p. 322). Following this line of reasoning, this means that the quanta concept is unavailable when we consider full interacting fields (Earman & Fraser, 2006, p. 330; Fraser, 2008, pp. 2-3). It could seem that the perturbative S-matrix approach could overcome this problem by providing a rigorous procedure to describe the interacting Dirac and Maxwell fields in the scattering problem by using only the Fock states representing free quanta from each field. According to the Haag theorem this is not possible. It seems then, according to Earman and Fraser, that we cannot use the concept of quanta when dealing with interacting fields.

It does not seem that the effectiveness of the perturbative approach, in spite of Haag's theorem, is a consequence of the need for a mathematical ill-defined mass and charge renormalization. At least there is no argument that shows how the efficiency of this scheme might result from developing it from a mathematical structure that is ill-defined due to renormalization. Moreover the renormalization technique is not the only element that makes the theory mathematically ill-defined. It seems that we cannot arrive at any solid conclusions by analysing from a mathematical perspective, what appears to be an ill-defined mathematical structure.

My view is that the explanation for the good results of the perturbation theory approach to scattering (and bound state) problems in spite of the Haag theorem, will not be found in the mathematical structure of quantum electrodynamics. As will be seen below, the point about Haag's theorem is that the question of its applicability, is not even addressed in the way the physical theory is really used: *we have to consider the physical input assumptions of the theory together with its mathematical formulation and application, not the mathematical structure all by itself.*

As mentioned above, the theory is developed from a canonical quantization of two independent classical fields. The description of the interaction between the fields is given, like in the classical counterpart, by an interaction term. Formally we can adopt

whatever representation is mathematically possible. But since we are (apparently) dealing with an ill-defined mathematical structure one should not consider formal aspects of the theory, like the full formal S-matrix for a scattering process (which in the applications turns out to be an asymptotic series). We must consider the applications of the theory that provide results that we can check with experiments. In these circumstances the use of the interaction representation only attains a physical meaning at the level of applications in which we can consider different systems with an interaction that can be considered as a perturbation of their independent states. In this way the fields maintain their identity as separated physical systems even during the interactions. In this sense the use of the interaction representation is part of the applications. We simply use part of the Hamiltonian, which is possible since the theory was developed considering distinct parts in the Lagrangian, one corresponding to the free Dirac field, another to the free Maxwell field, and another to the interaction. This is the one pulled apart from the others in the interaction representation.

The description of scattering is developed from the theory considering an initial state corresponding to a limited number of free particles (quanta), and with an adiabatic switching on of the interaction between the fields, a full interacting state ψ_b^- is *apparently* obtained. The interacting state ψ_a^+ that corresponds to a well-defined number of quanta in the final state is defined in an equivalent way. The scattering amplitude S_{ab} is given by (ψ_b^-, ψ_a^+) (Schweber, 1961, p. 323). The point is that we really do not work with these doubtful interacting states. What is going on is quite different. We are only considering a few terms of a perturbation expansion of the scattering matrix. When considering the applications we are taking advantage of the way the theory was developed. We always have clearly distinct fields. For the description of their interaction it is not necessary to have a description of both fields as a closed interacting system. On the contrary, as we will see next, if we try to make a full description of the interaction considering all the terms of the power series expansion of the S-matrix, it can "at best only be an asymptotic expansion" (Schweber, 1961, p. 644).

One of the major achievements of Dyson in the development of quantum electrodynamics was showing that the perturbative expansion of the S-matrix is renormalized to all orders. As mentioned previously, quantum electrodynamics (QED) had tremendous problems of divergent integrals that made impossible but a few lower order calculations. This problem was circumvented by the procedure of mass and charge renormalization. Dyson showed, in a paper published in 1949, that the renormalization procedure could be applied to all orders of the perturbative expansion of the S-matrix (Schweber, 1994, pp. 527-544).

Soon afterwards, in the summer of 1951, Dyson came out with a physical argument that strongly suggested that, after all, "all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge" (Dyson, 1952b, p. 631). [12] According to Dyson, the series expansion of the S-matrix is divergent, and this has nothing to do with renormalization (Dyson 1952b). That is, even if there were no divergent integrals appearing in the terms of the S-matrix, the series would still be divergent. According to Dyson, if we try to make a full description of the interaction considering all the (infinite) terms of the power series expansion of the S-matrix, it can "only be an asymptotic series" (Schweber, 1994, p. 565). That is, according to Dyson's physical argument, we can expect at some point that the term of

[12] It is important to notice that Dyson's is a heuristic physical argument not a rigorous mathematical derivation. In my view, this is not a deficiency of Dyson's argumentation since we are considering a theory with an ill-defined mathematical structure.

order n+1 will not be smaller than the term of order n and the series starts to diverge: "the terms … will decrease to a minimum and then increase again without limit, the index of the minimum term being roughly of the order of magnitude 137" (Dyson, 1952b, p. 632). In Dyson's view the "divergence will not prevent practical calculations being made with the series" (Dyson, 1952b, p. 632). But Dyson's view was that in a certain way the theory only provided a basis for developing the S-matrix series expansion (Cushing, 1986, p. 122). In Dyson's own words: "I had this rather positivistic view that all QED was the perturbative series. So if that failed you didn't really have a theory" (quoted in Schweber, 1994, p. 565). Even if strict mathematical proof of the divergence of the S-matrix does not exist, further strong evidence in favor of Dyson's claim has been given in the last decades (Aramaki 1989, 91-92; West, 2000, 180-181; Jentschura, 2004, pp. 86-112; Caliceti et al, 2007, pp. 5-6). I will now look in more details into Dyson's argument and mention some of the contemporary results that reinforce his claims.

During the summer of 1951 while visiting the ETH In Zurich Dyson found a heuristic argument that convinced him that the perturbation theory, used trough the S-matrix series expansion, diverges. According to Dyson

> All existing methods of handling problems in quantum electrodynamics give results in the form of power-series in e^2. The individual coefficients in these series are finite after mass and charge renormalization ... The purpose of this note is to present a simple argument which indicates that the power-series expansions obtained by integrating the equations of motion in quantum electrodynamics will be divergent after renormalization … The argument for divergence is as follows. According to Feynman, quantum electrodynamics is equivalent to a theory of the motion of charges acting on each other by a direct action at a distance, the interaction between two like charges being given by the formula
>
> $$e^2\delta_+(s_{12}^2), \tag{1}$$
>
> where e is the electron charge. The action-at-a-distance formulation is precisely equivalent to the usual formulation of the theory, in circumstances where all emitted radiation is ultimately absorbed. We shall suppose that conditions are such as to justify the use of the Feynman formulation of the theory. Then let
>
> $$F(e^2) = a_0 + a_2e^2 + a_4e^4 + ... \tag{2}$$
>
> be a physical quantity which is calculated as a formal power series in e^2 by integrating the equations of motion of the theory over a finite or an infinite time. Suppose, if possible, that the series (2) converges for some positive value of e^2; this implies that $F(e^2)$ is an analytic function of e at e = 0. Then for sufficiently small values of e, $F(-e^2)$ will also be a well-behaved analytic function with a convergent power-series expansion.
>
> But for $F(-e^2)$ we can also make a physical interpretation. Namely, $F(-e^2)$ is the value that would be obtained for F in a fictitious world where the interaction between like charges is $[-e^2\delta_+(s_{12}^2)]$ instead of (1). In the fictitious world, like charges attract each other. The potential between static charges, in the classical limit of large distances and large numbers of elementary charges, will be just the classical Coulomb potential with the sign reversed. But it is clear that in the fictitious world the vacuum state as ordinarily defined is not the state of lowest energy. By creating a large number N of electron-positron pairs, bringing the electrons together in one region of space and the positrons in another separate region, it is easy to construct a "pathological" state in which the negative potential energy of the Coulomb forces is much greater than the total rest energy and kinetic energy of the particles. This can be done without using particularly small regions or high charge densities, so that the validity of the classical Coulomb potential is not in doubt. Suppose that in the fictitious world the state of a system is known at a certain time to be an ordinary physical state with only a few particles present. There is a high potential barrier separating the physical state from the pathological states of equal energy: to overcome the barrier it is necessary to supply the rest-energy for the creation of many particles. Nevertheless, because of the quantum-mechanical tunnel effect, there will always be a finite probability that in any finite time-interval the system will find itself in a pathological state. Thus every physical state is unstable against the spontaneous creation of large numbers of particles. Further, a system once in a pathological state will not remain steady; there will be a rapid creation of more and more particles, an explosive disintegration of the

vacuum by spontaneous polarization. In these circumstances it is impossible that the integration of the equations of motion of the theory over any finite or infinite time interval, starting from a given state of the fictitious world, should lead to well-defined analytic functions. Therefore $F(-e^2)$ cannot be analytic and the series (2) cannot be convergent. (Dyson, 1952b, pp. 631-632)

Almost at the same time as Dyson set forward his argument, C. A. Hurst, working on the so-called φ^3 theory concluded that the perturbative approach was divergent. In particular

He enumerated the number of graphs in field theories with the interaction Hamiltonian of general type and showed that the number of graphs with n vertices increases like $n^{n/2}$ as n increases so that the perturbation expansion cannot converge unless the matrix elements decrease with correspondingly great rapidity as n increases. He studied the case of a three-scalar field interaction $\lambda\varphi^3$ and obtained the conclusion, by evaluating a lower bound for the matrix elements, that in this case the perturbation expansion with respect to λ cannot converge. Hurst remarked that the excellent agreement of QED with experiment would indicate that the series is an asymptotic expansion about the singular point = 0, as Dyson conjectured. (Aramaki, 1989, pp. 91-92)

Dyson's and Hurst's results stimulated W. Thirring to investigate the convergence in the $\lambda\varphi^3$ theory, and he got the same conclusion regarding the series divergence (Aramaki, 1989, p. 92). Also the calculation of A. Petterman and A. Jaffe with particular models supported Dyson's contention that the renormalized perturbation series diverges (Gill & Zachary, 2002, p. 29). More evidence for the perturbation series divergence was found in recent years. According to U. D. Jentschura

A priori, it may seem rather unattractive to assume that the quantum electrodynamics perturbation series may be divergent even after the regularization and the renormalization. However, as shown by explicit nontrivial 30-loop calculations of renormalization group γ functions in a six-dimensional φ^3 theory, and in a Yukawa theory …, we believe that the ultimate divergence of the perturbative expansion can be regarded as a matter-of-fact, clearly demonstrated by explicit high-order calculations. (Jentschura, 2004, pp. 108-109)

For Dyson his 1951 conclusion on the divergence of the S-matrix approach meant the end of his involvement with quantum electrodynamics. According to Dyson

All my efforts up to that point had been directed toward building a complete convergent theory. Finding out that after all the series diverged convinced me that was as far as one could go … That was of course a terrible blow to all my hopes. It really meant that this whole program made no sense. (Quoted in Schweber, 1994, p. 565)

This makes it even more imperative to justify the perturbative approach (which provides excellent agreement with experimental results), and to explain how to get rid of large-order terms of a divergent series that simply would make it impossible to use the theory.

Just from a mathematical perspective the use of only a few terms of a divergent series is difficult to defend, but by considering the input physical assumptions of the theory the justification of throwing away large-order terms will become clear. In part it is related to the weakness of the interaction between the Maxwell and Dirac fields (Mandl & Shaw, 1984, p. 95). *This by itself can explain why a few order terms in a perturbative approach can give so good results*. But it does not by itself justify throwing away terms that should be smaller and smaller but which will ultimately result in a divergent series.

As mentioned, the scattering matrix amplitude S_{AB} can be written as (ψ_B^-, ψ_A^+), where we have $\lim_{t_0 \to -\infty} U(0,t_0)\varphi_A = \psi_A^+$ and $\lim_{t \to +\infty} U(0,t)\varphi_B = \psi_B^-$. Here U is the time displacement unitary operator determined by solving the Schrödinger equation, φ_A and φ_B are the initial and final free states, and the in and out states ψ_A^+ and $\psi_B^-$are taken to be eigenstates of the full interacting Hamiltonian (Schweber, 1961, pp. 317-323). Formally these interacting states can be derived from the (complete) S-matrix. Since the S-matrix series expansion is divergent we can conclude that we cannot obtain these interacting states (Scharf, 1995, pp. 314-318), i.e. it is not possible to find solutions of the coupled non-linear Maxwell-Lorentz and Dirac equations as (infinite) power series expansions in the coefficient *e* (the electric charge). In other words, it is not possible to find a solution for a closed interacting system of an electromagnetic field and charged particles (Dirac field).

Let us recall that the theory was structured around the idea that the interaction between radiation and matter is weak (due to the small coupling constant). We do not quantize the interacting system, but each field separately; i.e. when we speak of weak interaction this has two related aspects: one is that the interaction term in the Lagrangian (or Hamiltonian) is small; the other is that from this we are justified to consider the quantization of radiation and matter *separately* as the quantization of free fields, and to take the interaction term as a perturbative one.[13] As Dyson mentioned, quantum electrodynamics "is in its nature a perturbation theory starting from the non-interacting fields as … unperturbed systems" (Dyson, 1952a, p. 79). Since we are starting from the non-interacting fields, we need to use the adiabatic switching on/off trick to 'connect' the two quantized systems and so to be able to describe their interaction. Now, as we have just seen, if we try to describe within the theory the full weak interaction between radiation and matter (corresponding to determining the eigenstates of the full interacting Hamiltonian) we get into trouble (i.e. we obtain divergent results).

We conclude then that, *in practice* (i.e. in the theory we really have and work with), the notion of weak interaction implies an intrinsic approximate approach, i.e. there is, *in practice*, an unbridgeable gap between the notion of weak interaction and the idea of a full (complete) description of the (weak) interaction (since a full description of the interaction would correspond to obtaining a complete expansion of the S-matrix, which is not possible). Thus the divergence of the S-matrix series expansion implies that we *are unable* to bridge the gap that exists between our starting physical assumption of two independent unperturbed systems and the (*ideal*) closed system of fully interacting radiation and matter; and there is a good reason for this.

As Earman & Fraser showed from a *formal* consideration of fully interacting fields (corresponding to an exact solution that we cannot obtain in the case of quantum electrodynamics), we cannot describe them in terms of the Fock representation for free fields. This means that, *formally*, for a closed system of interacting fields we cannot use all the physical input of quantum electrodynamics associated with the notion of weak interaction of radiation and matter (in particular our starting physical assumption of two non-interacting fields); i.e. the *formal* considerations imply an, *in principle*,

[13] It is important to remember that contrary to what formal presentations of the theory might lead us to think (e.g. Dyson, 1952a, pp. 58-59; Källén, 1972, pp. 75-80), we do not start with a Lagrangian for an interacting system of radiation and matter and then due to practical problems in solving a set of coupled non-linear equations we feel forced to resort to perturbative calculations. On the contrary the theory was pretty much developed along two separate lines – one of them the quantization of the free electromagnetic field (a subject not addressed here) the other the development of a relativistic equation for the electron –, from the start taking into account, implicitly, that we were dealing with two clearly distinct weakly interacting physical entities: (quantized) charged particles and (quantized) electromagnetic field.

incompatibility between the notion of *weak* interaction as implemented in the theory and the (formal) notion of a closed system of fully interacting fields. Also, as we have seen, considering the complete series expansion of the S-matrix would correspond to the description of a closed system of fully interacting fields. *Here the formal results are valid*, i.e. we would have two contradictory mathematical results. This is not the case since the series expansion of the S-matrix is divergent.

We see then that we disregard the large-order terms not simply for pragmatic reasons but for physical reasons. Including these terms would correspond to an improper use of the mathematical structure of the theory beyond its physical content (in a tentative description of a closed system of radiation and matter): in quantum electrodynamics we have the concepts of radiation and matter, and of a weak interaction between them, not of fully interacting fields.[14] At this point *I would like to establish a correspondence between getting meaningful mathematical results and the way we set up the physical concepts in the theory*. In this case, (meaningful) approximate calculations of the *weak* interaction between different physical systems (radiation and matter).

I think that the situation we are facing here can be illuminated by recalling some of Bohr's views related to the Klein paradox and the problem with infinities in the theory (later addressed by renormalization). As we have seen, in the exchange of letters with Dirac in late 1929, Bohr addressed the negative energy problem of Dirac's equation. Bohr called attention to the fact that it arises from not taking into account the elementary unit of electrical charge in the determination of the actual potential barrier, i.e. from not considering the physical concepts inscribed in the theory. According to Bohr, if calculations are made in which this fact and the limits in the determination of the electron's position are not taken into account, we would be facing the

> actual limit of applying the idea of potentials in connection with possible experimental arrangements. In fact, due to the existence of an elementary unit of electrical charge we cannot build up a potential barrier of any height and steepness desired without facing a definite atomic problem. (Quoted in Moyer, 1981, pp. 1057)

In a nutshell, to Bohr the paradox resulted from "an unlimited [mathematical] use of the concept of potentials in relativistic quantum mechanics" (quoted in Moyer, 1981, p. 1058; see also Darrigol, 1991, pp. 154-155).

An analogous situation occurs with the (renormalizable) infinities in the theory. According to Alexander Rueger's presentation of Bohr's ideas,

> only for an [atomic] electron weakly interacting with the electromagnetic field could the radiation reaction, which would render the electron's orbit unstable, be ignored; as Bohr stressed repeatedly, strong interactions would make the idea of approximately stationary states of the electron in the atom impossible. (Rueger, 1992, pp. 317-318)

In these circumstances, Bohr recalls that "the whole attack on atomic problems … is an *essentially approximate procedure*, made possible only by the smallness of [the coupling constant]" (Bohr, 1932a, p. 378). That is, to Bohr

> the attempts to treat the radiation effects on rigorous lines by considering the atoms and the electromagnetic field as a closed quantum-mechanical system led to paradoxes arising from the appearance of an infinite energy of coupling between atoms and field. (Bohr, 1932b, p. 66)

[14] With this justification for disregarding the large-order terms of the S-matrix series expansion, the excellent empirical results of the theory follow simply – in the perturbative approach – from the weakness of the interaction between the two *separately* quantized fields.

We have then, according to Bohr, that the physical conditions used to set up the theory imply an essentially approximate approach of an electron *weakly* interacting with the electromagnetic field. Like in the case of the Klein paradox if we extend the mathematical structure of the theory beyond its physical content we face a breakdown in the calculations. In my view, this is exactly the situation we are facing with the divergence of the S-matrix series expansion.

When trying to close the gap between two weakly interacting systems (described by an approximate approach) and fully interacting fields (corresponding to obtaining the exact solution for a closed quantized system of interacting fields), we face 'the actual limit of applying the idea' of non-interacting fields that are part of our physical input assumptions.

What we have then is the impossibility of defining within the theory a fully interacting state from the two fields that are defined and quantized as independent entities. Thus, in my view, we have a theory that is able, on an approximate level, to describe (using a few terms in perturbative calculations) the interaction between two separate fields, and not a theory describing as a whole a system of fully interacting fields. From a formal perspective the Haag theorem says that it is not possible to connect the separate fields with (fully) interacting fields when starting from the physical assumptions used to articulate the theory, i.e. from the notion of weak interaction as it is implemented in quantum electrodynamics.

In the theory the consequences of the Haag theorem are circumvented not because we are facing a "canonical framework mathematically ill-defined" (Fraser, 2006, p. 90) but because *we are not even trying to describe a system of (fully) interacting fields* (this eventual possibility is excluded *in practice* due to the divergence in the series expansion of the S-matrix and *in principle,* on formal grounds, by the above mentioned incompatibility between the notion of weak interaction as it is implemented in the theory and the formal results related to interacting fields). We are just trying to describe, by an 'essentially approximate procedure', the weak interaction between radiation and matter as distinct systems.[15] Thus, there is no conflict in quantum electrodynamics with the Haag theorem.

As the divergence in the series expansion of the S-matrix shows, the Lagrangian of quantum electrodynamics does not provide us with the possibility of describing a system of interacting Dirac and Maxwell fields, but with the possibility to describe in an intrinsically approximate way the interaction between the two fields. The descriptions of interactions in the theory are based on the use of the Fock space for each field and the idea of (virtual) quanta exchange. There are no alternatives in quantum electrodynamics. From the start the theory was not developed to treat the question of fully interacting fields, but to treat the question of the interaction between distinct fields that are separately quantized. To consider that "Fock representations are generally

[15] However I agree with Fraser's view that we cannot apply Haag's theorem when working with a "canonical framework mathematically ill-defined" (Fraser, 2006, p. 90). What I do not agree with is Fraser's view that renormalization is the factor that makes it possible to evade the consequences of Haag's theorem enabling the theory to be effective (also, as I mentioned, there are other factors which can be taken to render the theory mathematically ill-defined). That is because, in my view, in quantum electrodynamics, the problem of circumventing the Haag theorem is included in the broader problem of explaining how the theory can give so good results. This involves addressing the divergence of the S-matrix series expansion (necessary to justify the perturbative approach) and circumventing the Haag theorem. But these are not unrelated matters. In reality, as we have seen, when addressing the divergence of the S-matrix series expansion, the consequences of Haag's theorem become irrelevant (independently of the fact that we are considering a mathematically ill-defined approach).

inappropriate for interacting fields" (Earman & Fraser, 2006, p. 330) is, in the context of quantum electrodynamics, to turn upside down the theory as it was developed. The theory is built on top of the physical idea of independent entities whose interaction describes change in nature. When accepting this approach, and its intrinsic limitations, it is difficult to consider inappropriate, at least from an empirical point of view, the results of quantum electrodynamics; and so, contrary to Earman and Fraser's views, we can retain the concept of quanta in the description of interactions.

4 A note regarding the concept of vacuum in quantum electrodynamics

The revision of the role of the Lagrangian of quantum electrodynamics as simply giving rise to an essentially approximate approach has immediate consequences on the interpretation of the mathematical formalism of the theory. According to this view, even if from an abstract point of view we can talk about the Hilbert space of the physical states of the full Hamiltonian of the two fields and their interaction, from a physical point of view we cannot build up these formal states from the individual states corresponding to each field by itself. Since the theory gives rise only to approximate procedures we can only build a physical description of the interaction between the fields with low-order perturbative calculations using the individual states of each field.[16] This

[16] This does not mean that we do not need to take into account for example the renormalization of the electron's mass and charge to get agreement with experimental results. The point is the interpretation given to this. From an experimental point of view it might seem that the so-called physical electron with its 'cloud' of virtual photons (the renormalized quanta of the Dirac field) is the physical concept of particle we have in the theory when considering bound electrons. But recalling again Bohr's ideas we see that that is not the case when making our considerations from the perspective of the physical-mathematical structure of the theory. According to Bohr, "the classically estimated ratio between the radiative reactions on the electron and the nuclear attraction is … of the same order of magnitude as α^3 [where α is the fine structure constant]. It is just this circumstance which affords a justification for the neglect of the radiative reaction in a description of the stationary states including the fine structure" (Bohr, 1932b, p. 66). In this way, according to Bohr, "in the account of the simplest features of the radiation phenomena, we may neglect entirely the radiation reaction in the calculation of the transition probabilities" (Bohr, 1932b, p. 67). But Bohr considers possible "the treatment of such problems as the width of spectral lines and the retardation effects in the interaction of electrons bound in atoms. Still, the condition for such applications is that the effects in question can be treated as small perturbation of the phenomena to be expected if the finite propagation of forces would be neglected" (Bohr, 1932b, p. 67). That is, we *must* consider the electron's self-energy (due to the radiation reaction) as a small perturbation to the 'bare' electron in a central Coulomb potential, in order to justify for example the Lamb shift calculation regarding the energy shift of *stationary states* of the electron in the atom. As Rueger called attention to, "Bohr stressed[ed] repeatedly, [that] strong interactions would make the idea of approximately stationary states of the electron in the atom impossible" (Rueger, 1992, p. 317).

Considering the theory as giving rise only to essentially approximate procedures, *we do not really have a coherent quantum electrodynamical concept of a self-interacting electron*, which would be "always in interaction with the surrounding cloud of virtual particles" (Thirring, 1958, p. 140). Another aspect, related to this is the following: in quantum electrodynamics we describe the physical processes as resulting from the interaction of two clearly distinct fields (which are quantized as free independent fields). Due to the mass renormalization there is a mismatch between the concept of electron as quanta of the Dirac field and the applications where the electron is described by taking into account also the electromagnetic field. When considering the electron's mass, we can no longer make the simple association of the electron to quanta of the Dirac field. In the applications of the theory we must consider the electron's mass as resulting from a contribution from the two fields. It is clear that when we go beyond the lowest-order approximations, and the mass renormalization is needed to render the results finite, we cannot maintain a simple identification of the electrons with quanta of the Dirac field. The observed 'particle' – the electron – is built, in the applications of the theory, from contributions from the two fields (I will came back to this in the next chapter).

means in particular that *there is no physical meaning within quantum electrodynamics to the concept of vacuum (ground state) of the interacting fields*. It is usually thought that the coupled fields vacuum state can be "formally expanded as a superposition of ϕ_0" (Redhead, 1982, p. 86; Schweber, 1961, p. 655), where ϕ_0 are the vacuum states of the free fields. But in the theory we really have, this is a vacuous mathematical statement without any physical counterpart. This does not imply that the concept of vacuum is not relevant in the theory, as we have just seen in the previous chapter.

5 Conclusions

As we have seen, Haag's theorem implies that the perturbative approach used in quantum electrodynamics to treat the interaction between the quantized Maxwell and Dirac fields is not mathematically consistent. This brings up two big questions. How can the way the theory is implemented (giving a prominent role to the concept of quanta in the description of interactions) be justified? And even more importantly, "why perturbation theory works as well as it does" (Earman & Fraser, 2006, p. 307). Earman and Fraser do not provide any answer to this last question as regards quantum electrodynamics. And with respect to the first question, Fraser merely proposes to take shelter in the ill-defined mathematical structure of the theory to justify the inapplicability of the Haag theorem (Fraser, 2006, p. 90). However this argument has no explanatory power. It is an argument based solely on the mathematical structure of the theory (not taking into account its physical content) and it is not providing an explanation of the good results and soundness of the perturbative S-matrix approach.

According to Dyson the series expansion of the S-matrix, used in the description of scattering (and bound state) processes, is divergent. This indicates that the theory only provides a description of interactions using a few lower-order terms (which works well due to the small coupling constant between the fields). This means that the theory can only provide results if we are close to a free field situation. In other words, only when considering the interaction between two different fields as a small perturbation to their individual free states can the theory provide results in agreement with experiments.

We see that one has problems when trying to give a full description of the interaction, which corresponds to treating matter and radiation as one closed system. This would imply to go beyond the initial physical set up of the theory based on the idealization of totally non-interacting fields. In this way I think that quantum electrodynamics can be seen as providing only an approximate approach to the description of the interaction between two fields taken to be different physical systems. Only the lower-order terms of the series expansion can be kept. To take into account the large-order terms would mean to disrupt the physical input assumptions provided by the implementation of the notion of weak interaction (i.e. the possibility of quantization of free fields and the description of their interaction perturbatively, using the adiabatic switching on/off of the interaction). Due to the small coupling constant between the fields, the lower-order terms already provide good results.

From this perspective, how can Earman and Fraser's conundrum be solved while saving the use of quanta in the description of interactions? It is true that we are in an ill-defined mathematical context. However, we do not really need that to make the consequences of the theorem irrelevant in the theory. If we forget about Haag's theorem and set the machinery into motion, we face the situation that we cannot go from a free fields situation to a fully interacting fields situation (exactly as the Haag theorem says). This occurs because we are stretching the physical concepts too much and the

calculations break down, i.e. we have a theory describing the weak interaction between different fields, not a theory describing fully interacting fields. In this way we are outside the scope of Haag's theorem.

Regarding the concept of quanta – which follows naturally from the quantization procedure – this is, as we have seen, a central concept in the quantum electrodynamical description of the weak interactions between the fields (as this description involves free-particle Fock spaces). However we are left with a tension regarding the concept of quanta. The point is that we start with the idealization of non-interacting fields, and, as we have seen, we need the unphysical adiabatic switching on/off trick to set quantum electrodynamics as a perturbation theory into motion. The problem is that when addressing scattering problems, we take the particles (for example electrons) to be far apart before (and after) the scattering, and because of this not interacting (i.e. electrons as quanta of the Dirac field without self-interaction). In this way, we are describing the particles *observed* after the scattering process is over with the idealization of charged particles with a 'disconnected' charge, and this is rather unphysical – since implicit in the possibility of observation of an electron is the possibility of electromagnetic interaction with it.

CHAPTER 6

ASPECTS OF RENORMALIZATION IN QUANTUM ELECTRODYNAMICS

1 Introduction

The appearance of divergent integrals in higher-order calculations in quantum electrodynamics where the so-called radiative corrections are taken into account has been seen as, at least, indicating that the theory fails for high energies. As J. Schwinger stated, "electrodynamics unquestionably requires revision at ultra-relativistic energies" (quoted in Aramaki, 1989, p. 93). Even considering the accuracy of the theory at lower energies, Schwinger considered that the renormalization procedure, that permits avoiding the infinites in the results of calculations, ultimately has to be excluded from physics (Cao & Schweber, 1993, p. 50). Regarding this problem the position of P. Dirac was even less sympathetic: "I am very dissatisfied with the situation, because this so-called "good theory" does involve neglecting infinities which appear in its equations" (quoted in Kragh, 1990, p. 184). In general the position of leading physicists was very critical regarding quantum electrodynamics, and some pinpointed structural problems that go beyond the high-energy behaviour of quantum electrodynamics. For example N. Bohr considered that the whole program only made sense taking into account the weakness of the coupling constant, which means applying the theory only in situations where the electron interacts weakly with the electromagnetic field (Rueger, 1992, p. 317).

In this chapter I will offer a historical account of the renormalization program and recover the views of several physicists that I think permits a more enlightening account of the meaning of renormalization than more recent accounts. In sections 2 and 3 the historical emergence of the problem of infinites in quantum electrodynamics is considered, as well as the 'provisional' solution attained in the late forties with the completion of a renormalization program. The historical approach will enable to recover forgotten aspects of Dirac's subtraction physics and relate them to some of Bohr's views. This will enable a different view regarding renormalization than the one that has become 'standard'. The conceptual motivation behind Dirac's subtraction physics is to be contrasted with the post-war attitude of the physicists that completed a working renormalization approach. We will look in particular into the contributions of Schwinger and Feynman. Feynman's regularization approach is to be contrasted with Dirac's subtraction physics (which technically is similar) in its lack of any conceptual justification for regularization. Looking in detail into Feynman's work will also enable us to follow his first-hand account of his overall space-time approach to the description of interactions. This is an important subject whose conceptual implications will be developed in the next chapters.

Some more technical details of the renormalization program are considered in sections 3 and 4: the calculation of the self-energy of the electron and the photon, and

the role of the cut-off procedure that provides a 'regularization' of the theory, previous to the renormalization proper. We will see that mass renormalization brings with it a mismatch with the presupposition in the theory of two different fields independently quantized. The electrons (and positrons) are taken to be quanta of the Dirac field, characterized in part by what are called the bare mass and bare charge. According to the applications of the theory the bare mass of the electrons has no observational meaning; the observed mass results from a simultaneous contribution from the Dirac and Maxwell fields. In this way we cannot associate the electron exclusively to quanta of the Dirac field. It will also be addressed not only the dubious mathematical procedure related with the calculation of the photon's self-energy, but also the even more dubious – from a physical point of view – procedure of attaching the infinite constant that pops out in the photon self-energy calculation to the charges of the electrons 'connected' by the photon in an interaction process. In particular the charge renormalization procedure is considered in a second-order radiative correction to the Møller scattering amplitude. We will in this case notice limitations regarding the temporal description of physical processes, which are related with the charge renormalization procedure.

2 The emergence of infinites in quantum electrodynamics

When in 1929-30, Heisenberg and Pauli presented in two papers a relativistic quantum theory of the interaction between the quantized Maxwell and Dirac fields, they moved from Heisenberg's first view that the self-energy of the electron did not constitute a problem and the infinite Coulomb self-energy could be neglected, to a more circumspect position recognizing that this problem might even render the theory inapplicable (Darrigol, 1984, pp. 484-486). In the first paper, published in 1929, Heisenberg and Pauli discarded the infinite Coulomb self-energy of the electrons as they did with the zero-point energy of the vacuum, because they considered these divergences to be irrelevant infinite constants that disappear as soon as one evaluates quantities that are observable like the difference between two energy eigenvalues. Also according to Pauli,

> the theory can be called a correspondence theory, insofar as all expressions for the Lagrangian of the field are indeed taken over directly or indirectly from the classical theory … I believe that we have now arrived at the natural limit of range of the correspondence idea on the basis of wave mechanics. Our theory naturally fails at all places where the classical picture fails. (Quoted in Mehra & Rechenberg, 2000, p. 316)

It is well know that in the classical theory of a point-like electron we already have a problem with the electron's self-energy: it is infinite. Even so Pauli had aesthetic reservations regarding the self-energy problem in quantum electrodynamics, and even though he considered that the infinite constants might be removed in practical calculations they represented a 'defect of beauty in principle' (quoted in Mehra & Rechenberg, 2000, p. 316). According to J. Mehra and H. Rechenberg, in the section 8 of their paper, Pauli and Heisenberg

> progressed to a perturbation scheme … under the assumption that the interaction terms could be expanded in a series of small perturbations … The Heisenberg-Pauli solution, however, also contained divergent terms of the form $1/r_{PP}$ (the subscript P referring to the position of the particle), which corresponds to the self-interaction of the charged particle, say, the electron. This additive infinite term occurring in the energy of the total system may simply be neglected (subtracted), so long as the number of electrons does not change. (Mehra & Rechenberg, 2000, p. 325)

By the middle of 1929 Pauli (with whom J. R. Oppenheimer had began to collaborate) was seeking to improve the theory he had developed with Heisenberg. A three-man paper was being planned, where in particular Jordan's criticism regarding gauge invariance would be addressed (Mehra & Rechenberg, 2000, p. 327). By July 1929 Heisenberg still felt that "the catastrophic self-interaction of the electron does not disturb me too much" (Mehra & Rechenberg, 2000, p. 328). Finally Heisenberg and Pauli published a second part of their quantum electrodynamics in 1930, and Oppenheimer published a separate note regarding specifically the self-energy problem. In their paper Heisenberg and Pauli obtained Oppenheimer 's result for the Coulomb self-energy of the electron. Now however they recognized that the infinite self-energy, "in many cases will make application of the theory impossible" (quoted Miller, 1994, p.34). Also, the fact that the self-energy problem could not simply be traced back to a similar situation occurring already in classical electrodynamics was soon revealed by Oppenheimer (1930b) who found out a new (infinite) contribution to the self-energy without any classical counterpart. Using Dirac's second-order perturbation formula

$$H_{fi} = \sum_j \frac{H^1_{fi} H^1_{ji}}{E_i - E_j},$$

and adopting

$$H^1 = -\int \vec{j}\vec{A}^{tr} d\vec{x}, \quad \vec{j} = ie\overline{\psi}\vec{\gamma}\psi$$

for the perturbation term in the Hamiltonian H^1, we can consider the particular case where i = j. This situation represents the perturbation to the energy of the electron's state i arising from the self-interaction of the electron. Considering for simplicity a single free electron (with momentum **p** and energy $E(p) = c(p^2 + m^2c^2)^{1/2}$), the sequence of transitions i → j → i is

$$e \rightarrow e' + \text{photon} \rightarrow e.$$

According to A. Pais the "virtual states [e' + photon] correspond to all momentum-conserving partitions of **p** between e' and the photon. There are infinitely many such states" (Pais, 1986, p. 373). The self-energy of the electron is found to be

$$W(\vec{p}) \sim \frac{e^2\hbar^2c^2}{\hbar cE(p)} \int k\, dk,$$

in addition to smaller terms including the electrostatic (Coulomb) self-energy. What Oppenheimer obtained with this result was that while the classical self-energy diverges linearly (~ 1/r) as we take the electron radius to approach the point-like limit (r → 0), the quantum electrodynamical calculation predicted also a quadratic divergence of the term W(**p**). Also, $W(\mathbf{p}_1) - W(\mathbf{p}_2)$ is not finite (as Heisenberg and Pauli initially expected) but also infinite, which means, "as Oppenheimer stressed, [that] self-energy effects causes infinite displacements of spectral lines" (Pais, 1986, p. 373). A disastrous result for quantum electrodynamics.

The situation of quantum electrodynamics during the thirties did not improve, on the contrary. As we have seen, to solve inconsistencies of his electron theory related to the existence of negative-energy solutions, Dirac proposed his hole theory. From this, a new infinity problem popped out. In Dirac's hole theory we have an infinite sea filled with negative-energy electrons. This made Dirac consider that the electromagnetic field is generated by "the difference in the electric density from its value when the world is in its normal state (i.e. when every state of negative energy and none of positive energy is occupied)" (quoted in Pais, 1986, p. 378). That is, the Maxwell-Lorentz equation for the electric field is given, in Dirac's hole theory, by Div $\mathbf{E} = -4\pi(\rho - \langle\rho\rangle_{vaccum})$.

Going back to chapter 3, we can recall that in his 1930 paper on the hole theory Dirac remarked that "in the general case of an arbitrary varying electromagnetic field we can make no hard-and-fast separation of the solutions of the wave equation into those referring to positive and those to negative kinetic energy" (Dirac, 1930, p. 361). The knowledge of this situation led Dirac to consider in more detail the effect of an 'external' electromagnetic field (that could simply result from the presence of a sole electron above the negative-energy sea) on the definition of the 'normal' state (vacuum state) of the negative-energy sea. According to Dirac

> when applied to space in which there is an electromagnetic field, … one must specify just which distribution of electrons is assumed to produce no field and one must also give some rule for subtracting this distribution from the actually occurring distribution in any particular problem. (Quoted Schweber, 1994, p. 114)

In a letter to Bohr from September 10, 1933, Dirac summarized his findings:

> Peierls and I have been looking into the question of the change in the distribution of negative-energy electrons produced by a static electric field. We find that this changed distribution causes a partial neutralization of the charge producing the field. If it is assumed that the relativistic wave equation is exact, for all energies of the electron, then the neutralisation would be complete and electric charges would never be observable. A more reasonable assumption to make is that the relativistic wave equation fails for energies of the order $137mc^2$. If we neglect altogether the disturbance that the field produces in negative-energy electrons with energies less than $-137mc^2$, then the neutralization of charge produced by the other negative-energy electrons is small and of order 1/137. We then have a picture in which all the charged particles of physics electrons, atomic nuclei, etc. have effective charges slightly less than their real charges, the ratio being about 136/137. The effective charges are what one measures in all low energy experiments, and the experimentally determined value for e must be the effective charge on an electron, the real value being slightly bigger. (Quoted in Schweber, 1994, p. 116)

Dirac presented his results at the seventh Solvay Congress held in October 1933. Due to the fact that in relativistic mechanics the energy is given by $W^2 = m^2c^4 + c^2p^2$, it can take positive and negative values. According to Dirac, "it has not been possible to develop a relativistic quantum theory of the electron in which the transitions from a positive to a negative value of the energy should be excluded" (Dirac, 1934a, p. 136). In particular transitions between the positive and negative energy states are "predicted in general for all processes putting into play exchanges of energy of the order mc^2" (p. 136). Dirac considered that

> it seems there are no reasons of principle against the applicability of the quantum mechanics to similar exchanges of energy. It is true that quantum mechanics does not seem applicable to phenomena which involve distances of the order of the classical radius e^2/mc^2, since the present theory can in no manner discuss the structure of the electron. But such distances, considered as electron wavelengths, correspond to energies of the order $(\hbar c/e^2)(mc^2)$, which are much greater than the changes in question. It seems that the most reasonable solution is to search for a physical meaning for the negative energy states. (pp. 136-137)

Dirac went on to present again his hole theory which gave a physical interpretation for the negative-energy states:

Let us admit that in the universe such as we know it, the states of negative energy are nearly all occupied by electrons, and that the distribution thus obtained is not accessible to our observation on account of its uniformity throughout space. Under these conditions every unoccupied negative energy state represents a break in that uniformity, and must reveal itself as a kind of hole. It is possible to admit that these holes constitute positrons.

This hypothesis resolves the principal difficulties of the interpretation of the states of negative energy … the hole takes exactly the aspect of an ordinary particle, positively electrified. (p. 137)

Dirac then addresses the problem facing his interpretation when an external field is present. As already mentioned when there is no external field it is simple to take into account the infinite negative-energy sea in the Maxwell-Lorentz equation by considering that "the distribution of electrons produces no field in which no state of positive energy is occupied. And that it is the deviations from that distribution which determine the fields" (p. 138). However as Dirac acknowledges, this hypothesis

is completely satisfactory when it is a question of a region of space where there exists no field, and where the distinction between the positive energy states and those of negative energy is cleanly defined; but one must specify when it is a question of a region of space where the electromagnetic field is not zero in order to be able to lead to results free of all ambiguity. We must specify mathematically which distribution of electrons is supposed to produce no field, and also give a rule for subtracting that distribution from the one which exists effectively in each particular problem, in such a way as to obtain a finite difference that can figure into equation [div $\mathbf{E} = -4\pi\rho$], since, in general, the mathematical operation of subtraction between two infinities is ambiguous. (p. 138)

Dirac set out to consider the case of a weak, time-independent electrostatic field using the Hartree-Fock approximation. Dirac defined the density matrix R as

$$(q'|R|q'') = \sum_r \overline{\psi}_r(q')\psi_r(q''),$$

where the ψ's are four-component wave functions that are solutions of Dirac's equation for each individual electron, and the summation runs over all occupied negative energy states. The wave functions are determined in the Hartree-Fock approximation, where each electron is taken to move in an effective field that is the same for all electrons. The equation of motion for R is

$$i\hbar\dot{R} = HR - RH,$$

where $H = c\rho_1(\boldsymbol{\sigma}, \mathbf{p}) + \rho_3 mc^2 - eV$ is the Hamiltonian for an electron moving in the electric field $\mathbf{E} = -\nabla V$; also due to the exclusion principle we must have $R^2 = R$. Dirac assumes that the distribution R_0 that produces no field is given by

$$R_0 = \frac{1}{2}\left(1 - \frac{W}{|W|}\right),$$

where $W = c\rho_1(\boldsymbol{\sigma}, \mathbf{p}) + \rho_3 mc^2$ is the kinetic energy of an electron. Dirac then looks for a "permanent state for which the equation of motion $i\hbar\dot{R} = HR - RH$ reduces to $HR - RH$

= 0" (p. 139). In particular Dirac looks for a solution of the form $R = R_0 + R_1$ where R_1 is a quantity of first order in V. Dirac considers that

the quantity that interests us is the electric density corresponding to the distribution R_1. In order to obtain it we must form the diagonal sum of R_1, with respect to the spin variables, and then take the general diagonal element, multiplied by –e, of the resultant matrix with respect to the position variables x. (p. 140)

Dirac denotes this quantity by $D(R_1)$. Dirac found that when doing the integration of the expression for $D(R_1)$, "the result contains an infinite logarithm" (p. 141). Dirac's reaction was to use a cut off which rendered the result finite. But, according to Dirac there is a physical justification in the use of a cut off:

We could believe, at first sight, that the presence of that infinity renders the theory unacceptable. However, we cannot assume that the theory applies when it is a question of energies greater than the order of $137mc^2$, and the most reasonable way to proceed seems to be to limit arbitrarily the domain of integration to a value of the momentum … corresponding to electron energies of the order indicated. (p. 141)

This is an important point. As we have seen, Dirac had mentioned that

quantum mechanics does not seem applicable to phenomena which involve distances of the order of the classical radius e^2/mc^2, since the present theory can in no manner discuss the structure of the electron … such distances, considered as electron wavelengths, correspond to energies of the order $(\hbar c/e^2)(mc^2)$ [$\cong 137mc^2$]. (pp. 136-137)

This goes along the lines of Bohr's views on quantum electrodynamics. In Bohr's terms an unbound limit of integration would mean to apply the theory not taking into account the physical assumptions used to set up the theory, since we would be disregarding that the theory treats the electron as a point-charge; in it we are always considering distances larger than the electron's 'diameter' (Bohr, 1932b, pp. 63-64). In his Faraday lecture Bohr mentioned that

The scope of the quantum mechanical symbolism is essentially confined, however, to problems where the intrinsic stability of the elementary electrical particles can be left out of consideration in a similar way as in the classical electron theory. In this connexion, *it must not be forgotten that the existence of the electron even in classical theory imposes an essential limitation on the applicability of the mechanical and electromagnetic concepts. Indeed, the finite propagation of electromagnetic forces brings with it the existence of a fundamental length, the so-called "electron diameter" defining a lower limit for the extension of the region where the idealization according to which the electron is considered as a charged material point is justifiable*. Not only would a concentration of the charge of the electron within a smaller space result in an essential modification of its mass, but we even meet here with a limitation of the unambiguous use of the idea of inertial mass. In fact, we lose any simple basis for a sharp separation between ponderomotoric forces and radiative reactions when we consider processes in which the electron undergoes a velocity change of the same order as the velocity of light within a length of path equal to the electron diameter. It is true that such considerations lose their significance to a large extent on account of the existence of the quantum of action which imposes an essential limit to the analysis of motion. *The fertility of quantum mechanics as applied to the problem of atomic stability lies just in the fact that the linear dimensions of the regions ascribed to even the firmest electron-bindings outside the nucleus are still very large compared with the classical electron diameter*. (Bohr, 1932a, pp. 377-378 [my emphases])

Returning to Dirac's take on the logarithmically divergent integral, Dirac considered that

if P is the value of the vector momentum … to which we limit the integration domain, the final result, obtained after a complicated integration, is:

$$-e\left(x\left|D(R_1)\right|x\right)=-\frac{e^2}{\hbar c}\frac{2}{3\pi}\left(\log\frac{2P}{mc}-\frac{5}{6}\right)\rho-\frac{4}{15\pi}\frac{e^2}{\hbar c}\left(\frac{\hbar}{mc}\right)^2\nabla^2\rho,$$

where ρ is the electric density producing the potential V, so that

$$\nabla^2 V = -4\pi\rho,$$

and where the terms containing the derivatives of ρ of order greater than second have been neglected. (Dirac, 1934a, p. 141)

Now, Dirac's view was that we could not apply the theory for energies greater than $137mc^2$. This means taking the cut off P to be of order 137mc. For this cut off value the first term in the expression for $-e\left(x\left|D(R_1)\right|x\right)$ is equal to $-(e^2/\hbar c)\,\rho$ (Dirac took the second term not to be an important correction in the present conditions). Dirac's interpretation of this result is as follows:

As a result of the foregoing calculation, it would seem that the electric charges which one ordinarily observes on electrons and protons and the other particles of physics are not the actual charges which these particles carry (appearing in the fundamental equations) but are all slightly smaller, in a ratio of about 136/137. (Quoted in Schweber, 1994, pp. 115-116)

Dirac tried to improve his approach by presenting a more systematic procedure, which, contrary to the previous case (Dirac 1934a), was relativistic and might be applied to the case of external time-dependent fields. In a letter to Bohr of November 10, 1933, Dirac mentioned his new approach:

I have been working at the problem of the polarization of the distribution of negative-energy electrons, from a relativistic point of view. If I have not made a mistake, then there is just one relativistically invariant, gauge invariant treatment, which gets over all the difficulties connected with the infinites, to the accuracy with which the Hartree-Fock method applies … I have not yet seen whether this relativistic treatment leads to any kind of compensation of charge arising from the vacuum polarization. (Quoted in Schweber, 1994, p. 117)

In his development of the density-matrix formalism Dirac again considers the density matrix R; and again it is considered that

each electron moves in a definite electromagnetic field, which is the same for all electrons. This field will consist of a part coming from external causes and a part coming from the electron distribution itself, the precise way in which the latter part depends on the electron distribution being one of the problems we have to consider. (Dirac, 1934b, p. 146)

Dirac's objective was to find "some natural way of removing infinities from $\sum_k \left(xt\left|R\right|xt\right)_{kk}$ and $\sum_k \left(xt\left|\alpha_s R\right|xt\right)_{kk}$ [which is the current density] so as to leave finite remainders" (p. 148).

In the case of no external field, Dirac found that the singularities of $\left(x't'\left|R\right|x''t''\right)_{k'k''}$ all occurred in the light cone (p. 151). In the case of an external field present, Dirac supposed "that the singularities are of the same form as in the case of no field, but have unknown coefficients" (p. 152). Dirac showed that the density matrix could be divided into two parts $R = R_a + R_b$, where R_a contains all the singularities and "the electric and current densities corresponding to R_b are those which are physically

present, arising from the distribution of electrons and positrons. (p. 156). Dirac's idea was that with this division of R in two parts "we can remove the infinities" (p. 156).

Since the singularities were located in the light cone, this means that when $x = x' - x'' \neq 0$ R_a is finite. According to Pais, "Dirac's prescription for extracting finite results was: first subtract these singular terms, then let $x \rightarrow 0$" (Pais, 1986, p. 382). Also, according to A. I. Miller, "the intent, of course, is to propose a counter-term $-R_a$ so that $R - R_a$ and, consequently, the measured charge densities are finite. This will be accomplished by Heisenberg" (Miller, 1994, p. 60).

The uniqueness of Dirac's subtraction method was immediately questioned (Miller, 1994, p. 60; Pais, 1986, p. 383). Heisenberg tried to improve Dirac's method, and in the process, due to his use of a second quantized version of Dirac's formalism in which electron and positrons were treated in a symmetrical way, came up with the existence of an "infinite self-energy of the light-quanta" (Heisenberg, 1934, p. 186). According to Heisenberg, when "compared to Dirac's treatment, [his] paper emphasizes the significance of the conservation laws … and the necessity of formulating the basic equations of the theory in a manner extending beyond the Hartree approximation" (p. 169). In the first part of his paper Heisenberg used the density matrix formalism. Following Dirac's approach Heisenberg considers that "one will have to subtract from the density matrix [R_s] another density matrix [S] which is determined uniquely by the external fields, in order to obtain the 'real' density matrix [r, i.e. a density matrix without singularities]" (p. 171). The problem with Dirac's method is according to Heisenberg that it does not provide a unique specification for S and because of this for the equation of motion of the system (p. 172). Heisenberg's idea is that "by taking into account the conservation laws of charge, energy, and momentum, the possibilities for S can be restricted insofar that a particular value can be distinguished as the simplest assumption" (p. 172). Heisenberg went on to calculate the vacuum polarization, obtaining a corrected second term for the induced charge density (Miller, 1994, p. 64). Like Dirac, Heisenberg took this term as having no physical significance. This is not the case. As shown in 1935 by E. Uehling, this term yields measurable effects. Uehling found for the hydrogen atom the level 2S to be displaced by $\Delta\nu = -27$ megacycles per second (Pais, 1986, p. 383).

In the second part of his paper Heisenberg extended the formalism, treating the Dirac wave function as a quantized Dirac field and also treating the electromagnetic field as a quantized field. In this part Heisenberg gave a symmetrical treatment to electrons and positrons being both treated on equal footing as quanta of the quantized Dirac field (Heisenberg, 1934, p. 183). Adopting Hamiltonian methods, Heisenberg noted that the subtraction of infinites could be done order by order in perturbation theory, noticing nevertheless the presence of self-energy-like terms that the subtraction procedure could not remove (Schweber, 1994, p. 118). According to Heisenberg "the perturbation method can be continued in principle, unless an infinite self-energy causes the method to diverge" (Heisenberg, 1934, p. 184).

This formalism enabled Heisenberg to treat, in particular, the creation and annihilation of electron-positron pairs. Heisenberg found that the process by which a photon creates an electron-positron pair, which subsequently annihilates creating a photon, gives rise to a logarithmically divergent term. Heisenberg interpreted this term (in analogy to the electron's self-energy) as the (infinite) self-energy of the light quanta: "we shall treat the matter density associated with a light quantum and in particular the self-energy of the light quantum derived on the basis of this matter density ... The energy of this matter field becomes infinite, in precise analogy to the infinite self-energy of the electrons" (p. 185).

Regarding the self-energy problems there was not much improvement during the thirties. The only solid result was V. F. Weisskopf's demonstration that the electron self-energy is 'only' logarithmically divergent (Pais, 1986, p. 385). Going back to Oppenheimer's calculation of the electron's self-energy (Oppenheimer, 1930b), Oppenheimer found using single-particle theory (i.e. Dirac's equation as a one-electron equation) a new quantum-mechanical term in the electron's self-energy that diverges quadratically. Weisskopf completed in early 1934 a second-order calculation of the electron's self-energy in hole theory (Weisskopf, 1934, p. 158). Weisskopf divided the electron's electromagnetic field in two parts: a rotation-free part and a divergence-free part. According to Weisskopf the electrostatic self-energy is given by

$$E^S = \frac{1}{2}\int \rho\Phi' d\vec{r}\,,$$

and the electrodynamic self-energy is given by

$$E^D = -\frac{1}{2}\int (\vec{i}_{tr}\vec{A}_{tr})d\vec{r}$$

where $\mathbf{i}_{tr}$ is the divergent-free part of the current density. Expanding the self-energy operators by powers of the electric charge, in the second-order calculation only terms proportional to e^2 are kept. The electrostatic self-energy calculated in the Dirac single-electron theory diverges linearly. Weisskopf made his calculation in hole theory, i.e. considering a multi-electron system where all the states with negative energy are occupied. According to Weisskopf, "to calculate the self-energy of a multi-electron system, it is advantageous to use the method of quantized waves, in which the charge and current densities act as operators on the eigenfunctions, whose variables are the occupation numbers $N^k(\mathbf{p})$ of the stationary states $\mathbf{p}^k$, k=1, …, 4 of the free electron" (p. 160). He then calculated the diagonal element of the self-energy operator E^S for a particular occupation of states. In this way, in the expression for the electrostatic self-energy there is a summation over all the negative-energy states plus the occupied positive-energy state: the electron whose self-energy we want to calculate. Weisskopf found that "the electrostatic self-energy diverges *logarithmically* in the 'hole' theory" (p. 163). In this work Weisskopf made a mistake in the calculation of the electrodynamic part of the self-energy, obtaining initially a quadratic divergence (like in Oppenheimer's calculation). Soon after W. Furry called Weisskopf's attention to his error. In a correction to his first paper, also published in 1934, Weisskopf presented the corrected result. He found like in the case of the electrostatic part of the self-energy that in Dirac's hole theory E^D also had a logarithmic divergence (Miller, 1994, p. 61).[17] Writing to Weisskopf in February 1935, Heisenberg called attention to a shortcoming in Weisskopf's calculation. Heisenberg repeated Weisskopf's calculation and found that

$$E^S = \frac{e^2}{h\sqrt{m^2c^2 + p^2}}(2m^2c^2 + p^2)\int \frac{dk}{k}$$

[17] In 1939 Weisskopf showed that the self-energy of the electron was logarithmically divergent to every approximation in an expansion of the self-energy in powers of the fine structure constant α (Weisskopf, 1939).

$$E^D = \frac{mc}{h\sqrt{m^2c^2 + p^2}}(m^2c^2 - 4/3)p^2 \int \frac{dk}{k}.$$

Heisenberg found these results suspicious because

one must expect on relativistic grounds that

$$E^S + E^D = \text{constant}\frac{e^2}{h}\sqrt{m^2c^2 + p^2}\int \frac{dk}{k}. \text{ (Quoted in Schweber, 1994, p. 125)}$$

According to Schweber, "the lack of proper relativistic covariance was to plague all self-energy calculations in the prewar period" (Schweber, 1994, p. 125).

3 The submergence of infinites in quantum electrodynamics

Things changed drastically in 1947, with W. Lamb's experimental results on the shift of the $2^2S_{1/2}$ state relative to the $2^2P_{1/2}$ states in the hydrogen atom. H. A. Bethe coming from a conference in Shelter Island, held on 2-4 June 1947, did some calculations on a train going from New York to Schenectady. In the conference W. Lamb presented his recent results on the level shift. Bethe made a nonrelativistic calculation of the Lamb shift, taking into account the suggestion by J. Schwinger, Weisskopf, and Oppenheimer that the self-energy of the electron was responsible for the shift in the energy levels, and Kramers's idea of mass renormalization (Pais, 1986, pp. 455-456; Schweber, 1994, p. 228-231). According to Bethe: "Kramers suggested that what one really ought to do was to renormalize the mass of the electron, taking into account its interaction with its own electromagnetic field. Then only those parts of the self-energy which are not contained in the mass of the particle would be observable" (quoted in Mehra & Rechenberg, 2001, p. 1039). Bethe calculated the self-energy W of a bound electron and subtracted to it the self-energy W_0 of a free electron (with the same average kinetic energy). This corresponds to Kramers's idea of mass renormalization. The difference $W–W_0$ was according to Bethe "the true shift of the levels due to interactions" (quoted in Schweber, 1994, p. 231). This expression is logarithmically divergent. Bethe considered that there should be (in the relativistic theory) a natural cut-off at energies around mc^2 (which is not the case). By taking into account this *ad hoc* cut-off, Bethe was able to obtain a result in good agreement with the observed value.

As seen above, the idea of renormalization, in the case of the electron's charge, was basically present in Dirac's report to the Solvay conference of 1933 (Dirac, 1934a). His ideas are stated more clearly in the letter to N. Bohr written after the preparation of the report:

We then have a picture in which all the charged particles of physics, electrons, atomic nuclei, etc. have effective charges slightly less than their real charges, the ratio being about 136/137. The effective charges are what one measures in *all* low energy experiments, and the experimentally determined value of *e* must be the effective charge of an electron, the real value being slightly bigger. (Quoted in Schweber, 1994, p. 116)[18]

[18] Another statement of the idea of charge renormalization due to the effect of the vacuum polarization can be seen in Weisskopf's paper on vacuum polarization from 1936. According to Weisskopf "the polarizability could in no way be observed, but would only multiply all charges and field strengths by a constant factor" (Weisskopf , 1936, p. 208).

A similar approach regarding the electron self-energy started to emerge in the end of the thirties, in Dirac's own work, and in H. A. Kramers's investigation of the renormalization of the electromagnetic mass at the classical level (as a first step for dealing with the problem at the quantum level). Kramers's intention was to sidestep the problem by obtaining a consistent model for a finite size electron – that avoided the classical self-energy divergence –, considering from the start the experimental mass of the electron (that contained the mechanical mass and the electromagnetic mass). In this way Kramers "tried to present the theory in such a fashion that the questions of the structure and the finite extension of the particles are not explicitly involved and that the quantity that is introduced as the 'particle mass' is from the very beginning the experimental mass" (Kramers, 1938, p. 254). But mass renormalization was only put to use in quantum electrodynamics in 1947, in the quantum-mechanical (non-relativistic) train-ride calculation of Bethe.

A few months after the conference, Schwinger worked on a non-covariant relativistic calculation of the Lamb shift using the mass and charge renormalization recipe, and obtained finite results to order $e^2/\hbar c$. Knowing of G. Breit's suggestion that the electron might have an intrinsic magnetic moment different from the one predicted by the Dirac equation – that explained the discrepancy with the experimental results regarding the hyperfine structure of the hydrogen atom –, Schwinger calculated the so-called anomalous magnetic moment for an electron in an externally applied homogeneous magnetic field, which accounted for the previous hyperfine discrepancies between theory and experiment (Mehra & Rechenberg, 2001, p. 1045). Schwinger published his results in a short note from late December 1947. This note did not include the precise results of his Lamb shift calculations due to discrepancies between the Coulomb (Lamb shift) and the magnetic field (anomalous magnetic moment) calculations, because there existed a difference in the result for the magnetic moment in the case of the electron in an atom and a free electron (Schweber, 1994, p. 319; Mehra, 1994, pp. 238-239). In this brief note Schwinger presented his view on the need for renormalization in quantum electrodynamics. According to Schwinger

> Attempts to evaluate radiative corrections to electron phenomena have heretofore been beset by divergent difficulties, attributable to self-energy and vacuum polarization effects. Electrodynamics unquestionably requires revision at ultra-relativistic energies, but is presumably accurate at moderate relativistic energies. It would be desirable, therefore, to isolate those aspects of the current theory that essentially involve high energies, and are subject to modification by a more satisfactory theory, from aspects that involve only moderate energies and are thus relatively trustworthy. This goal has been achieved by transforming the Hamiltonian of current hole theory electrodynamics to exhibit explicitly the logarithmically divergent self-energy of a free electron, which arises from the virtual emission and absorption of light quanta. The electromagnetic self-energy of a free electron can be ascribed to an electromagnetic mass, which must be added to the mechanical mass of the electron. Indeed, the only meaningful statements of the theory involve this combination of masses, which is the experimental mass of a free electron. It might appear, from this point of view, that the divergence of the electromagnetic mass is unobjectionable, since the individual contributions to the experimental mass are unobservable. However, the transformation of the Hamiltonian is based on the assumption of a weak interaction between matter and radiation, which requires that the electromagnetic mass be a small correction ($\sim(e^2/\hbar c)m_0$) to the mechanical mass m_0 …It is important to notice that the inclusion of the electromagnetic mass with the mechanical mass does not avoid all divergences; the polarization of the vacuum produces a logarithmically divergent term proportional to the interaction energy of the electron in an external field. However, it has long been recognized that such a term is equivalent to altering the value of the electron charge by a constant factor, only the final value being properly identified with the experimental charge. Thus the interaction between matter and radiation produces a renormalization of the electron charge and mass, all divergences being contained in the renormalization factors. (Schwinger, 1948a, p. 416)

The calculations mentioned in the note were done using non-covariant hole-theoretic methods. This is the motive for the discrepancy in the calculations. Afterwards, Schwinger developed a covariant formulation of the Heisenberg and Pauli quantum electrodynamics. Nevertheless, "there were a great many ambiguities in the procedure" (Schweber, 1994, p. 333) of identifying the divergent contributions. Also the calculation method was terribly complicated and threatened to become insurmountable in higher-order calculations.

A key aspect of Schwinger's covariant formulation was the deduction of an equation (later known as Tomonaga-Schwinger equation) that was a manifestly Lorentz covariant generalization of the Schrödinger equation. This was a functional derivative equation that describes the state function ψ as a functional $\psi[\,\sigma\,]$ of a general three-dimensional surface σ in space-time:

$$i\hbar c\,\frac{\delta\psi[\sigma]}{\delta\sigma(x)} = H(x,t)\,\psi[\sigma].$$

According to J. A. Wheeler's notes on Schwinger's presentation of his covariant formulation at the Pocono conference (held from March 30 to April 2, 1948), "these equations contain nothing more than Heisenberg-Pauli formalism and would not be required if one knew how to carry out Heisenberg-Pauli calculations consistently" (quoted in Schweber, 1994, p. 324). Schwinger applied his formalism in the determination of the radiative corrections to the motion of an electron in an external electromagnetic field, i.e. the calculation of the anomalous magnetic moment and the Lamb shift.

At the Ann Arbor summer school (from July 19 to August 7, 1948), Schwinger gave a more detailed presentation of his formalism including an improved treatment of vacuum polarization. The value for the Lamb shift given by Schwinger was

$$\frac{8\alpha^3}{3\pi}\frac{Z^4}{n^3}\left(\ln\frac{mc^2}{\Delta E} - \ln 2 + \frac{3}{8} - \frac{1}{5} + \frac{1}{2}\right) = 1040\,\mathrm{Mc}\,,$$

which includes the 1/5 Uehling term, and where the 1/2-term is the magnetic moment effect (which were not included in the expression Schwinger presented at the Pocono conference).

Schwinger's lectures were based on his paper 'quantum electrodynamics I' (Schwinger 1948b) and a first draft of a sequel where Schwinger applied the formalism to determine the vacuum polarization and the self-energy (Schwinger 1948c).

In the introduction to the first paper Schwinger gave further insights on his views regarding quantum electrodynamics. According to Schwinger

> The unqualified success of quantum electrodynamics in applications involving the lowest order of perturbation theory indicates its essential validity for moderately relativistic particle energies. The objectionable aspects of quantum electrodynamics are encountered in virtual processes involving particles with ultra-relativistic energies. The two basic phenomena of this type are the polarization of the vacuum and the self-energy of the electron.
>
> The phrase "polarization of the vacuum" describes the modification of the properties of an electromagnetic field produced by its interaction with the charge fluctuations of the vacuum. In the language of perturbation theory, the phenomenon considered is the generation of charge and current in the vacuum through the virtual creation and annihilation of electron-positron pairs by the electromagnetic field. If the electromagnetic field is that of a light quantum, the vacuum polarization effects are equivalent to ascribing a proper mass to the photon. Previous calculations have yielded non-vanishing, divergent

expressions for the light quantum proper mass. However, the latter quantity must be zero in a proper gauge invariant theory. The failure to obtain this result from a gauge invariant formulation can be ascribed only to a faulty application of the theory, rather than to an essential deficiency thereof. When the electromagnetic field is that of a given current distribution, one obtains a logarithmically divergent contribution to the vacuum polarization current which is everywhere proportional to the given distribution. This divergent result expresses the possibility, according to the present theory, of creating electron-positron pairs with unlimited energy, a situation that presumably will be corrected in a more satisfactory theory. Thus the physically significant divergence arising from the vacuum polarization phenomenon occurs in a factor that alters the strength of all charges, a uniform renormalization that has no observable consequences other than the conflict with empirical finiteness of charge.

The interaction between the electromagnetic field vacuum fluctuations and an electron, or more exactly, the electron-positron matter field, modifies the properties of the matter field and produces the self-energy of an electron. The mechanism here under discussion is commonly described as the virtual emission and absorption of a light quantum by an otherwise free electron … in a Lorentz invariant theory, self-energy effects for a free electron can only result in the addition of an electromagnetic proper mass to the electron's mechanical proper mass. Calculations performed for a stationary electron have yielded a logarithmically divergent electromagnetic proper mass, a divergence that results from the possibility of emitting light quanta with unlimited energy … the electromagnetic proper mass merely produces a renormalization of the electron mass that has no observable consequences, other than the conflict with the empirical finiteness of mass.

It is evident that these two phenomena are quite analogous and essentially describe the interaction of each field with the vacuum fluctuations of the other field. The effect of these fluctuation interactions is simply to alter the fundamental constants e and m, although by logarithmically divergent factors. However, it may be argued that a future modification of the theory, inhibiting the virtual creation of particles that possess energies many orders of magnitude in excess of mc^2, will ascribe a value to these logarithmic factors not vastly different from unity. The charge and mass renormalization factors will then differ only slightly from unity, as befits a perturbation theory, in consequence of the small coupling constant for the matter and electromagnetic fields, $e^2/4\pi\hbar c = 1/137$. (Schwinger, 1949b, pp. 1439-1440)

It is important to notice that Schwinger considers that the divergences result from 'virtual processes involving particles with ultra-relativistic energies'. These virtual processes came about in perturbative calculations. They are basically the transitory states Dirac found in his second order perturbation theory (see chapter 5; see also chapter 7 for a treatment of virtual quanta). As Schwinger writes, in the 'language of perturbation theory' we have, in what regards the vacuum polarization, the creation and annihilation of transitory (virtual) electron-positron pairs. The most interesting aspect of Schwinger's view is that he considers that a possible future modification of the theory might exclude virtual states with energies 'many orders of magnitude in excess of mc^2'. This is Dirac's subtraction physics with a cut off. The difference is that Schwinger is hoping for a future more elaborated theory with a 'natural' cut off, while Dirac presents the cut off as a necessary 'patch' to maintain the calculation within a mathematical domain where the physical concepts make sense (in this case the abstraction of the electron as a point-like electron; see also chapter 5). To put it simply, Dirac (with Bohr) is seeing a conceptual inconsistency when considering integrals without an energy cut off, where Schwinger (as other renormalization physicists) sees basically a mathematical problem to be solved in a future better theory.

Back in Cornell (from the conference in Shelter Island) in early July 1947, Bethe gave a lecture on his non-relativistic calculation of the Lamb shift, which R. P. Feynman attended. According to Feynman:

He explained that it gets very confusing to figure out exactly which infinite term corresponds to what in trying to make the correction for the infinite change in mass. If there were … any modification whatever at high frequencies, which would make this correction finite, then there would be no problem at all to figuring out how to keep track of everything … if in addition this method were relativistically invariant, then we would be absolutely sure how to do it without destroying [relativistic invariance]. (Feynman, 1965, p. 170)

Feynman considered first the case of determining a relativistic cut-off for classical electrodynamics. Using his path-integral method, Feynman, following Bethe's idea, replaced a delta function appearing in the interaction term of the action by an invariant function dependent on a cut-off parameter that made all results finite; this procedure corresponding to a 'regularization' of the theory. Feynman would then renormalize the mass, putting the bare mass and the now finite electromagnetic mass under the umbrella of the experimental mass.

Feynman did not manage to derive the Dirac equation using his path-integral method. So, by 'guessing' he was able to use an invariant regularization method based on a cut-off with the Dirac theory of the electron (Mehra, 1994, pp. 229-234). Feynman developed his "little theory of electrodynamics in which the interaction is not exact on a delta function" (quoted in Schweber, 1994, p. 427) as if it was different from the conventional electrodynamical theory. In his 1948 paper on the relativistic cut-off, Feynman presented his method as "a model, for which all quantities automatically do come out finite" (Feynman, 1948, p. 1430). This does not mean that Feynman rejected the renormalization method. In his calculations he performed the mass and charge renormalization, but he saw his method as an "attempt to find a consistent modification of quantum electrodynamics" (Feynman, 1949b, p. 778). Because it was inconsistent, the correct physics had to be obtained by making the renormalization of mass and charge, and obtaining expressions independent of the cut-off parameter by making the cut-off parameter go to infinity after renormalization (Feynman, 1962, p. 145).

In between the Shelter Island and the Pocono conferences, Feynman, after developing a relativistic cut off procedure for classical electrodynamics, extended this approach to the case of a spinless relativistic particle, being able to obtain a relativistic generalization of the expression Bethe had used in his calculations. By applying this approach to the Dirac electron (following an intuitive procedure suggested by Bethe), Feynman was able to obtain Weisskopf's expression for the self-energy (now depending logarithmically on Feynmam's cut off parameter). In a letter (from late autumn) written to Bert and Mulaika Corber, Feynman commented on his ongoing work:

> There was so much talk around here about self-energy, that I thought it would be the easiest thing to calculate directly in my form. The result is exactly the same as one gets for ordinary perturbation theory … It therefore also gives infinity. I then altered the delta function in the interaction to be a sum of less sharp function. This corresponds to a kind of finite electron. Then the self-energy of a non-relativistic particle is finite. (Quoted in Schweber, 1994, p. 423)

At the Tenth Washington Conference on Theoretical Physics (held on 13-15 November 1947), Feynman attended a talk by Schwinger. Feynman was interested in a remark made by Schwinger, referring to the fact that, according to Feynman's recollections on the conference, "the discrepancy in the hyperfine structure of the hydrogen noted by Rabi, can be explained on the same basis as that of electromagnetic self-energy, as can the line shift of Lamb" (quoted in Mehra, 1994, p. 236). After the conference Feynman did the calculation of the anomalous magnetic moment using his approach.

In a letter to the Corbers from middle January 1948 we already see an outline of Feynman's regularization approach to the problem of the infinites in quantum electrodynamics:

> I have been working with a theory of electricity in which the delta function interaction is replaced by a less sharp function. Then (in quantum mechanics) the self-energy of an electron including the Dirac hole theory comes out finite … actually, the self-energy comes out finite and invariant and is therefore

representable as a pure mass … thus all mass cannot be represented as electrodynamic unless the cutoff is ridiculously short. The experimental mass is of course the sum of inertial and this electromagnetic correction … I think all the problems of electrodynamics can be unambiguously solved by this process: First compute the answer which is finite (but contains the cutoff logarithm). Then express the result in terms of the experimental mass. The answer still contains the cutoff but this time not logarithmically. Take the limit which now exists, as the cutoff goes to infinity. (Quoted in Schweber, 1994, p. 426)

In the meeting of the American Physical Society that took place at the end of January 1948, Schwinger reported on his results regarding the anomalous magnetic moment of a free electron and his results for the Lamb shift (published on his note from December 1947). In this talk Schwinger mentioned that his results for the anomalous magnetic moment for an electron in a magnetic field did not agree with the value obtained for an electron in a Coulomb field. Feynman mentioned then that he had got things right, i.e. the same result for the magnetic moment of an electron, in both cases of a free or bound electron (Mehra, 1994, pp. 238-239). In his approach Feynman was not working with Dirac's hole theory, but thinking in terms of paths (in space-time) and representing the positrons as electrons going backwards in time (Schweber, 1994, p. 428).

By the time of the Pocono conference Feynman had a working approach that enabled him to calculate the anomalous magnetic moment, Lamb shift and cross sections for diverse processes. What Feynman did not have was a way to deal with the vacuum polarization, which in Feynman's approach was connected with so-called closed loops (i.e. paths that give rise to an infinite polarizability of vacuum). In his presentation Feynman was still hoping that it was possible to get a consistent theory without using loops (Schweber, 1994, p. 443). That was not the case.

After the Pocono conference Feynman decided to write down his work in a set of papers. The first addressed the relativistic cut off for classical electrodynamics. The next an extension of this approach to the case of quantum electrodynamics. In this paper Feynman got the results for the self-energy obtained previously by Weisskopf and Bethe; and it included a discussion of the scattering of an electron by a (classical) potential. In this paper Feynman considered only processes in which the photons appeared only in the intermediate states of the perturbative calculations. Feynman found that he could simply take on equal footing the four polarizations of the photon, transverse, longitudinal, and scalar, in a relativistic and gauge invariant way. All this before the Gupta-Bleuler method. This state of affair was possible because Feynman was considering the case where all the light quanta are virtual. As W. Heitler stresses in his classical book on quantum electrodynamics, "we can compute the transition probability by choosing an initial state without longitudinal and scalar photons, ignoring the Lorentz condition in the meantime, and by calculating only the probabilities of final states which have no longitudinal and scalar photons" (Heitler, 1954, p. 130). In the case of Feynman's calculation we do not even have transverse photons in the initial and final states. In this paper Feynman had still not found a way to deal with the infinites related to the vacuum polarization. We can know about Feynman's progress on the vacuum polarization problem from a letter from late 1948:

In regards to "Q.E.D." as you put it, I don't have a cold dope. I can calculate anything, and everything is finite, but the polarization of the vacuum is not gauge-invariant when calculated. This is because my prescription for making the polarization integral converge is not gauge-invariant. If I threw away the obvious large gauge-dependent term (a procedure which I can not justify legally, but which is practically unambiguous) the result is a charge renormalization plus the usual Uehling term. The amount of charge renormalization depends logarithmically on the cut-off. The Uehling terms are practically independent on the cut-off and give the usual –1/5 in the Lamb shift. (Quoted in Mehra, 1994, p. 265)

By the end of January 1949 Feynman learned about the Pauli-Villars regularization procedure that enables a gauge-invariant regularization of the vacuum polarization. By the spring of 1949 he had all the elements together and published his two most important papers on quantum electrodynamics, where his approach was presented and put to use.

In his 'The theory of positrons', submitted on 8 April 1949, Feynman begins by presenting his idea of positrons as electrons moving backward in time. In the abstract Feynman writes:

> the problem of the behavior of positrons and electrons in given external potentials, neglecting their mutual interaction, is analysed by replacing the theory of holes by a reinterpretation of the solutions of the Dirac equation. It is possible to write down a complete solution of the problem in terms of the boundary conditions on the wave function, and this solution contains automatically all the possibilities of virtual (and real) pair formation and annihilation together with the ordinary scattering processes, including the correct relative signs of the various terms.
>
> In this solution, the "negative energy states" appear in a form which may be pictured (as [done] by Stückelberg) in spacetime as waves travelling away from the external potential backwards in time. (Feynman, 1949a, p. 749)

In here we see Feynman talking about solutions with appropriate boundary conditions, not the equations themselves. In the introduction we start seeing how Feynman thought about physical processes as described in his scheme:

> In the approximation of classical relativity theory the creation of an electron pair (electron A, positron B) might be represented by the start of two world lines from the point of creation, 1. The world lines of the positron will then continue until it annihilates another electron, C, at a world point 2. Between the times t_1 and t_2 there are then three world lines, before and after only one. However, the world lines of C, B, and A together form one continuous line albeit the "positron part" B of this continuous line is directed backwards in time. Following the charge rather than the particles corresponds to considering this continuous world line as a whole rather than breaking it up into its pieces. It is as though a bombardier flying low over a road suddenly see three roads and it is only when two of them come together and disappear again that he realizes that he has simply passed over a long switchback in a single road.
>
> This over-all space-time point of view leads to considerable simplification in many problems. (p. 749)

Feynman then relates his 'over-all space-time view' to his path integral approach to quantum mechanics. Feynman begins by "a brief discussion of the relation of the non-relativistic wave equation to its solution" (p. 750). It goes as follows. Starting with the Schrödinger equation $i\partial\psi/\partial t = H\psi$, if $\psi(x_1, t_1)$ is the solution at x_1 at time t_1, then the wave function for $t_2 > t_1$ is given by

$$\psi(x_2, t_2) = \int K(x_2, t_2; x_1, t_1)\psi(x_1, t_1)d^3x_1 \ ,$$

where the green function K is given by

$$K(2,1) = \sum_n \phi_n(x_2)\phi_n^*(x_1)\exp(-iE_n(t_2 - t_1))$$

(E_n and ϕ_n are the eigenvalues and eigenfunctions of the operator H in the case of a free particle). Feynman calls "K(2, 1) the total amplitude for arrival at x_2, t_2 starting from x_1, t_1. (It results from adding an amplitude, exp(iS), for each space time path between these points, where S is the action along the path)" (p. 750).

In the case of a particle in a weak external potential U (x, t), differing from zero for t between t_1 and t_2, we can expand K in increasing powers of U:

$$K(2, 1) = K_0(2, 1) + K^{(1)}(2, 1) + K^{(2)}(2, 1) + \cdots.$$

To zero order in U, K is that for a free particle: $K_0(2, 1)$. In first order of perturbation theory $K^{(1)}(2, 1)$ results from the action of the potential U at some time t_3 (between t_1 and t_2). From t_1 to t_3, and from t_3 to t_2 the particle is free. In this way it can be shown that $K^{(1)}(2, 1)$ is given by

$$K^{(1)}(2,1) = -i\int K_0(2,3)U(3)K_0(3,1)d\tau_3 .$$

In Feynman's explanation of the meaning of these formulas we see how his over-all space-time approach goes:

We can imagine that a particle travels as a free particle from point to point, but is scattered by the potential U. Thus the total amplitude for arrival at 2 from 1 can be considered as the sum of the amplitudes for various alternative routes. It may go directly from 1 to 2 (amplitude $k_0(2, 1)$) … or it may go from 1 to 3 (amplitude $k_0(3, 1)$), get scattered there by the potential (scattering amplitude –iU(3) per unit volume and time) and then go from 3 to 2 (amplitude $k_0(2, 3)$). This may occur for any point 3 … Again it may be scattered twice by the potential … It goes from 1 to 3 ($k_0(1, 3)$), gets scattered there (–iU(3)) then proceeds to some other point, 4, in space time (amplitude $k_0(4, 3)$) is scattered again (–iU(4)) and then proceeds to 2 ($k_0(2, 4)$). Summing over all possible places and times for 3, 4 find that the second order contribution to the total amplitude $k^{(2)}(2, 1)$ is

$$(-i)^2\int\int K_0(2,4)U(4)K_0(4,3)U(3)K_0(3,1)d\tau_3 d\tau_4 .$$ (p. 751)

After presenting his approach in the case of non-relativistic quantum mechanics Feynman addresses the relativistic case. Starting with Dirac's equation $(i\nabla - m)\psi = A\psi$, for a particle of mass m in an external potential A, the equation determining the propagation of a free particle is $(i\nabla_2 - m)\, K_+(2, 1) = i\delta(2, 1)$. In analogy to the non-relativistic case, the first order and second order corrections to $K_+(2, 1)$ are given by

$$K_+^{(1)}(2,1) = -i\int K_+(2,3)A(3)K_+(3,1)d\tau_3$$

$$K_+^{(2)}(2,1) = -\int\int K_+(2,4)A(4)K_+(4,3)A(3)K_+(3,1)d\tau_3 d\tau_4 .$$

The core part of this paper is the selection of the admissible solutions according to Dirac's hole theory (related to his 'positrons as electrons moving backward in time' view):

We would now expect to choose for the special solution of [the equation for $K_+(2, 1)$], $K_+ = K_0$ where $K_0(2, 1)$ vanishes for $t_2 < t_1$ and for $t_2 > t_1$ is given by [the equation for K(2,1)] where ϕ_n and E_n are the eigenfunctions and energy values of a particle satisfying Dirac's equation … The formulas arising from this choice, however, suffer from the drawback that they apply to the one electron theory of Dirac rather than to the hole theory of the positron … the choice $K+ = K_0$ is unsatisfactory. But there are other solutions of [the equation for $K_+(2, 1)$]. We shall choose the solution defining $K_+(2, 1)$ so that $K_+(2, 1)$ for $t_2 > t_1$ *is the sum of [the equation for K(2, 1)] over positive energy states only*. … With this choice of $[K_+(2, 1)]$ our equations such as [the previous one for $K_+^{(1)}(2, 1)$] and [the previous one for $K_+^{(2)}(2, 1)$] will now give results equivalent to those of the positron theory. (p. 752)

In this paper Feynman only considered the case of several non-interacting particles, leaving the problem of interacting particles to his next paper.

In "Space-Time Approach to Quantum Electrodynamics", sent to publication on 9 May 1949, Feynman put forward the regularization procedure he had been developing within his over-all space-time view. Feynman addressed first the case of particles interacting instantaneously, which could then be easily generalized to the case of delayed interactions. As in the case of the previous paper Feynman considers

> the solutions of equations rather than the time differential equations from which they come. We shall discover that the solutions, because of the over-all space-time view that they permit, are as easy to understand when interactions are delayed as when they are instantaneous. (Feynman, 1949b, p. 771)

Using the methods of the previous paper, Feynman addresses the case of two interacting particles, considering first the non-relativistic case described by the Schrödinger equation. In the case of two free particles the amplitude is given by

$$K(x_a, x_b, t; x_a', x_b', t') = K_{0a}(x_a, t; x_a', t')K_{0b}(x_b, t; x_b', t')$$

where x_a' and x_b' are the positions of the particles at time t', and x_a and x_b the positions of the particles at a later time t. We can also define the amplitude

$$K_0(3, 4; 1, 2) = K_{0a}(3, 1)K_{0b}(4, 2)$$

that the particle a goes from x_1 at t_1 to x_3 at t_3 and that particle b goes from x_2 at t_2 to x_4 at t_4. According to Feynman

> When the particles do interact, one can only define the quantity K(3, 4; 1, 2) precisely if the interaction vanishes between t_1 and t_2 and also between t_3 and t_4. In a real physical system such is not the case. There is such an enormous advantage, however, to the concept that we shall continue to use it, imagining that we can neglect the effect of interactions between t_1 and t_2 and between t_3 and t_4. For practical problems this means choosing such long time intervals $t_3 - t_1$ and $t_4 - t_2$ that the extra interactions near the end points have small relative effects. As an example, in a scattering problem it may well be that the particles are so well separated initially and finally that the interaction at these times is negligible. Again energy values can be defined by the average rate of change of phase over such long time intervals that errors initially and finally can be neglected. Inasmuch as any physical problem can be defined in terms of scattering processes we do not lose much in a general theoretical sense by this approximation. (p. 771)

Feynman first gives an example of this approach in the case of an instantaneous Coulomb interaction, and then shows how it can be extended to the case of a delayed interaction. Considering a Coulomb potential e^2/r (where r is the distance between the particles), which is 'active' for only a short time interval Δt_0 at time t_0, the first order correction to the amplitude is given by

$$K^{(1)}(3, 4; 1, 2) = -ie^2 \iint K_{0a}(3, 5)K_{0b}(4, 6)r_{56}^{-1} \times K_{0a}(5, 1)K_{0b}(6, 2)d^3x_5 d^3x_6 \Delta t_0 ,$$

where $t_5 = t_6 = t_0$. Considering now the potential to be active all the time, according to Feynman, "the first-order effect is obtained by integrating on t_0, which we can write as an integral over both t_5 and t_6 if we include a delta-function $\delta(t_5 - t_6)$ to insure contribution only when $t_5 = t_6$" (p. 772). In this case the first order correction to the amplitude is given by

$$K^{(1)}(3,4;1,2) = -ie^2 \iint K_{0a}(3,5)K_{0b}(4,6)r_{56}^{-1} \times \delta(t_{56})K_{0a}(5,1)K_{0b}(6,2)d\tau_5 d\tau_6$$

where $d\tau = d^3xdt$. In the case of a delayed interaction the story goes as follows. Since "the Coulomb potential does not act instantaneously, but is delayed by a time r_{56}, taking the speed of light as unity. This suggests simply replacing $r_{56}^{-1}\delta(t_{56})$ in [the expression for $K^{(1)}(3, 4;1, 2)$] by something like $r_{56}^{-1}\delta(t_{56} - r_{56})$ to represent the delay in the effect of b on a."(p. 772). According to Feynman things are not that easy because "when this interaction is represented by photons they must be of only positive energy, while the Fourier transform of $\delta(t_{56} - r_{56})$ contains frequencies of both signs" (p. 773). Because of this, Feynman uses instead the expression

$$\delta_+(x) = \int_0^\infty e^{-i\omega x} d\omega/\pi = \delta(x) + (\pi i x)^{-1}.$$

This is not the whole story. After taking into account the contribution due to the case when $t_5 < t_6$ (which corresponds to a emitting the quantum that b receives), generalizing to an interaction described also by the vector potential, and adapting the formalism to the case of electrons described by the Dirac equation, Feynman arrives at the expression

$$(1 - \alpha_a \cdot \alpha_b)\delta_+(s_{56}^2) = \beta_a\beta_b\gamma_{a\mu}\gamma_{b\mu}\delta_+(s_{56}^2).$$

In this way, in the case of electrons interacting through an electromagnetic field, the amplitude is given by

$$K^{(1)}(3,4;1,2) = -ie^2 \iint K_{+a}(3,5)K_{+b}(4,6)\gamma_{a\mu}\gamma_{b\mu} \times \delta(s_{56}^2)K_{+a}(5,1)K_{+b}(6,2)d\tau_5 d\tau_6 .$$

Feynman calls it his fundamental equation for quantum electrodynamics. According to Feynman it "describes the effect of exchange of one quantum (therefore first order in e^2) between two electrons. It will serve as a prototype enabling us to write down the corresponding quantities involving the exchange of two or more quanta between two electrons or the interaction of an electron with itself" (p. 772). Feynman then gives a description of the meaning of the equation presenting a graphical support to his interpretation: a Feynman diagram. It goes as follows:

> It can be understood (see Fig. 1) as saying that the amplitude for "a" to go from 1 to 3 and "b" to go from 2 to 4 is altered to first order because they can exchange a quantum. Thus, "a" can go to 5 (amplitude ($K_+(5, 1)$) emit a quantum (longitudinal, transverse, or scalar $\gamma_{a\mu}$) and then proceed to 3 ($K_+(3, 5)$). Meantime "b" goes to 6 ($K_+(6, 2)$), absorbs the quantum ($\gamma_{b\mu}$) and proceeds to 4 ($K_+(4, 6)$). The quantum meanwhile proceeds from 5 to 6, which it does with amplitude $\delta_+(s_{56}^2)$. We must sum over all the possible quantum polarizations it and positions and times of emission 5, and of absorption 6. Actually if $t_5 > t_6$ it would be better to say that "a" absorbs and "b" emits but no attention need be paid to these matters, as all such alternatives are automatically contained in [the fundamental equation]. (pp. 772-773)

Feynman first applied his technique to the case of the electron's self-energy. Since the calculations turn out to be easier in the momentum-energy space, Feynman presented rules to calculate the amplitude working with momentum and energy variables. Feynman then shows how his regularization scheme works. In this paper Feynman also addresses the problem of vacuum polarization making use of the gauge invariant regularization procedure developed by Pauli and Villars.

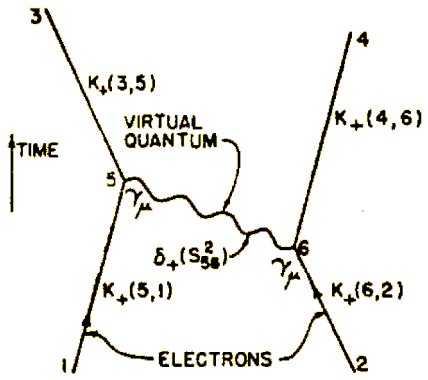

Figure 1: The fundamental interaction (described by Feynman's fundamental equation). Exchange of one quantum between two electrons.

To see regularization at work I will look into the problems of the electron's self-energy and photon's self-energy. In the case of the electron's self-energy, Feynman's 'fundamental formula' reduces to

$$K^{(1)}(2,1) = -ie^2 \int\int K_+(2,4)\gamma_\mu K_+(4,3)\gamma_\mu K_+(3,1)d\tau_3 d\tau_4 \delta_+(s_{43}{}^2)\,.$$

According to Feynman, this first order correction to the amplitude $K_+(2, 1)$, "arises because the electron instead of going from 1 directly to 2, may go (Fig. 2) first to 3, ($K_+(3, 1)$), emit a quantum (γ_μ), proceed to 4, ($K_+(4, 3)$), absorb it (γ_μ), and finally arrive at 2 (K+(2, 4)). The quantum must go from 3 to 4 $\delta_+(s_{43}{}^2)$)" (p. 773). Feynman shows how this expression is related with the self-energy of an electron, which turn out to be

$$\Delta E = e^2 \int (\bar{u}\gamma_\mu K_+(4,3)\gamma_\mu u)\exp(ip\cdot x_{43})\delta_+(s_{43}{}^2)d\tau_4\,.$$

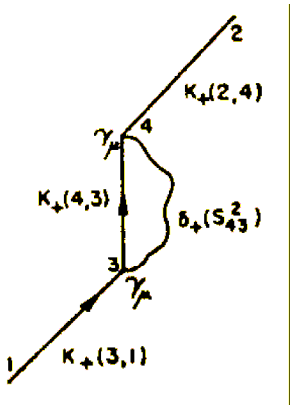

Figure 2: Interaction of an electron with itself

As mentioned, for easiness in the calculations Feynman works in the momentum-energy space. In this case the self-energy is the matrix element between $\bar{u}$ and u (taken from the plane wave solution for a free Dirac electron: u exp(−px)) of the matrix

$$(e^2/\pi i)\int \gamma_\mu (p-k-m)^{-1}\gamma_\mu k^{-2} d^4k\,.$$

According to Feynman:

The equation can be understood by imagining (Fig. 3) that the electron of momentum p emits (γ_μ) a quantum of momentum k, and makes its way now with momentum p − k to the next event (factor $(p - k - m)^{-1}$) which is to absorb the quantum (another γ_μ). The amplitude of propagation of quanta is k^{-2}. (There is a factor $e^2/\pi i$ for each virtual quantum). One integrates over all quanta. The reason an electron of momentum p propagates as $1/(p - m)$ is that this operator is the reciprocal of the Dirac equation operator, and we are simply solving this equation. Likewise light goes as $1/k^2$, for this is the reciprocal D'Alembertian operator of the wave equation of light. The first γ_μ represents the current which generates the vector potential, while the second is the velocity operator by which this potential is multiplied in the Dirac equation when an external field acts on an electron. (p. 775)

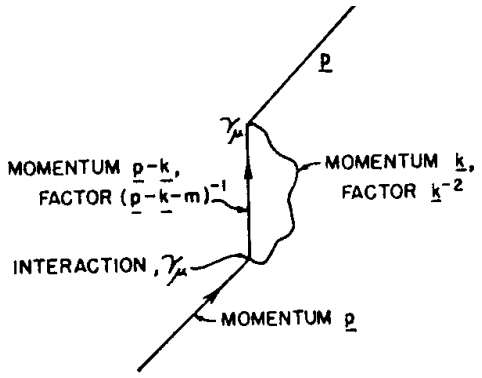

Figure 3: Interaction of an electron with itself. Momentum-energy space.

Up to this point, things are moving smoothly. However if we calculate the integral to obtain the self-energy it turns out, as already mentioned, to be infinite. Using contemporary notation, the problems in the calculation of the electron self-energy are all concentrated in this apparently harmless integral (Mandl & Shaw, 1984, p. 187):

$$ie_0^2\,\Sigma(p) = \frac{e_0^2}{(2\pi)^4}\int d^4k \frac{1}{k^2 + i\varepsilon}\frac{2p - 2k - 4m_0}{(p-k)^2 - m_0^2 + i\varepsilon}\,.$$

As it stands this integral is divergent. From Lorentz invariance Σ(p) can be put in the form

$$\Sigma(p) = A + (p - m)B + (p - m)\Sigma_c(p)\,,$$

where m is the electron's (experimental) mass, and, in particular, A = Σ(p) when $\gamma^\mu p_\mu = m$ (Mandl & Shaw, 1984, p. 189). This term provides a correction $\delta m = -e_0^2 A$ of electromagnetic origin to the bare mass m_0 of the electron, which can be seen as resulting from the interaction of the electron with its own field. In this way, at the level of quantum electrodynamical applications, the electron's mass that is experimentally measured corresponds to a renormalized mass where the electron's self-energy is taken into account.

For $k \to \infty$ the previous integral is logarithmically divergent (this is the famous ultra-violet divergence). A way out is to make a "change in the fundamental laws"

(Feynman, 1961, p. 137): the photon propagator $1/k^2$ is multiplied by a relativistically invariant convergence factor, assumed by Feynman to be $c(k^2) = -\lambda^2/(k^2 - \lambda^2)$. This change has to be seen as a formal calculational device, a mathematical trick to get rid of the logarithmic divergence in the integral. If we try to see it as a new theory distinct from the one derived from classical electrodynamics we obtain a non-hermitian interaction Lagrangian that implies that probability is not conserved. Also from a physical point of view the use of this convergence factor is equivalent to considering "an additional interaction of the electron-positron field with a vector field whose quanta have mass λ and whose propagators are $-(k^2 - \lambda^2)^{-1}$," (Schweber, 1961, p. 519). With this prescription it is possible to calculate the integral. With this regularization procedure we have

$$A = -\frac{3m}{8\pi^2} \ln \frac{\Lambda}{m},$$

where Λ is a cut-off parameter (Mandl & Shaw, 1984, 191). It turns out that the only contribution from the self-energy which is not renormalized is the finite integral $\Sigma_c(p)$. From this term the radiative correction (due to the electron self-energy) to the lowest-order calculations is obtained. If we stopped here, we would have an experimentally measurable radiative correction dependent on an arbitrary cut-off parameter Λ.[19] To get things right, after the renormalization we have to make the cut-off parameter go to infinity, so that the radiative correction term "remains well-defined and finite in this limit and independent of the details of the regularization procedure" (Mandl & Shaw, 1984, p. 191). This method to overturn the problem of infinites in quantum electrodynamics was summarized by Feynman in one of his quantum electrodynamical rules: "(1) Put in an arbitrary cutoff factor $c(k^2) = -[\lambda^2/(k^2 - \lambda^2)]$ for each propagator $1/k^2$. (2) Express everything in $m_{exp} = m - \delta m$. (3) Take the limit as $\lambda \to \infty$ and keep m_{exp} fixed." (Feynman, 1962, p. 143).

4 Different views on renormalization

It is usually held that the divergence problem in quantum electrodynamics is due to a failure of the theory at ultra-relativistic energies, that is, to the fact that there is no upper bound to the energy of the virtual quanta that are exchanged during interactions. This might lead to the idea that the cut-off parameter serves like a "boundary line separating the knowable region from the unknowable" (Cao & Schweber, 1993, p. 52). But since there is no indication on where to put this cut-off, it seems that "we cover our ignorance by calculating only quantities which are independent of the exact value of the cut-off" (Teller, 1988, p. 87). This procedure results in a change from the approximative regularized version of the theory to a recovered quantum electrodynamics with renormalized mass and charge. This means changing "the status of the cutoff from a tentative, and tantalizing, threshold energy to a purely formalistic device" (Cao & Schweber, 1993, p. 53). Even if Feynman was trying to achieve a consistent regularized theory, and published his method as a provisional one while searching for a "correct

[19] This problem does not arise with δm because we consider it to be 'absorbed' in the experimental measurable mass $m_{exp} = m_0 + \delta m$, which is seen as an amalgamation of the bare mass and the electromagnetic mass, and whose magnitude – as a phenomenological parameter – is determined not by the theory but from experiments.

form of f_+ [the function that substitutes the delta function appearing in the interaction term] which will guarantee energy conservation" (Feynman, 1949b, p. 778), it seems that it ended up being what Bethe had envisaged from the beginning: a mathematical calculational device designed to overcome the divergence problems in some integrals. Also, as previously mentioned, from a physical point of view the regularized theory is completely different from quantum electrodynamics. It does not have a divergence problem because of the presence of an auxiliary vector field. This field can be seen as a formal mathematical device if, and only if, after the renormalization we recover a cut-off independent theory. If we tried to maintain the regularized theory, so that we did not have to deal with the problem of infinites in the calculations, we would be working not with quantum electrodynamics but with another (inconsistent) field theory.

We see in the case of Feynman (like previously with Schwinger) a lack of understanding of the possible implications at a conceptual level of the regularization scheme and renormalization. As we have seen Feynman as looking for a consistent modification of quantum electrodynamics. Contrary to Dirac (with Bohr), Feynman does not relate the regularization to structural aspects of the theory (i.e. to the adoption in Dirac's equation of a point-like electron) that might imply (even if in an inconsistent way) an upper bond to the possible energy exchanges.

Contrary to a common view I think that the divergence of the integrals and the use of the cut-off trick do not reveal where the theory stops being good and a 'true' theory should come into play. It reveals structural problems in the construction of the theory, that are impossible to ignore when we have some integrals, that should (from a physical point of view) provide small radiative corrections to lower-order calculations, and end up blowing apart in a proliferation of infinites. In quantum electrodynamics the description of the interaction between 'particles' (like photons and electrons) as quanta of the Maxwell and the Dirac fields is given by the perturbative expansion of the scattering matrix that describes the interaction (and it is in the terms of the S-matrix that the divergent integrals appear). Considering the second-order term of the scattering matrix $S_2(x_1, x_2)$ in configuration space, the mathematical expression of the terms related with the divergent part of the electron's self-energy are dependent on $\delta(x_1 - x_2)$. This means that "all the divergences in $S_2(x_1, x_2)$ come from terms proportional to $\delta(x_1 - x_2)$ and to its derivatives which differ from zero only in the infinitesimal neighbourhood of the point $x_1 = x_2$" (Bogoliubov & Shirkov, 1959, p. 299). The divergence problem does not arise solely from the fact that there is in the theory no upper bound to the energy of the virtual quanta. It results from the 'coincidence' in the theory of the inexistence of a natural limit to the energy of virtual quanta and from the local character of the interaction between the fields in quantum electrodynamics.

One other aspect of the mass renormalization procedure is that when we go beyond the lowest-order approximations, and the mass renormalization is needed, there is a mismatch between the conceptual basis of the theory and its description of matter: conceptually quantum electrodynamics is developed from the idea of two independently quantized fields – one of them describing matter – that interact. The electrons (and positrons) are described in quantum electrodynamics as quanta of the quantized Dirac field, having a (bare) mass associated with them. At the same time the higher-order calculations require considering the mass of the electron as having a non detachable contribution from the Maxwell field. The observed or experimental mass of the electron results in the applications of the theory from a simultaneous contribution from the bare mass of quanta of the Dirac field and the self-energy of these quanta due to the interaction of the Dirac and Maxwell fields.

That the divergence in the calculation of the electron's self-energy reveals structural problems related to distinct aspects of the theory does not mean that the theory does not have other less noticed limitations, also related to the divergence of integrals. This can be seen on a closer look at the second-order calculation of the photon self-energy. Again we have a divergent integral and again a regularization scheme is used and a renormalization is made.

The (second-order) photon self-energy leads to a modification of the photon propagator: $D_{\mu\nu}'(k)= g_{\mu\nu}D(k)+ D(k)\Pi_{\mu\nu}(k)D(k)$, where $\Pi_{\mu\nu}(k)$ is a quadratically divergent integral and D(k) is the bare photon propagator. Considering the requirement of Lorentz and gauge invariance, the second-order tensor $\Pi_{\mu\nu}(k)$ must have the form $(g_{\mu\nu}k^2 - k_\mu k_\nu)\Pi(k^2)$ (Jauch & Rohrlich, 1976, p. 189). As it stands, for $k^2 = 0$ we have $\Pi_\mu^{\ \mu}(0) \neq 0$. This would mean that the propagator we obtain taking into account this second-order correction is not the propagator for a zero mass photon but the propagator for a massive neutral vector boson (Sakurai, 1967, p. 275). To recover our photon we must recall that $\Pi_{\mu\nu}(k)$ must be gauge invariant. Imposing this condition, we must have $\Pi_{\mu\nu}(k)k^\nu = 0$. From this we obtain the ambiguous result that the quadratically divergent integral

$$\Pi_\mu^{\ \mu}(0) = -\frac{2\alpha}{\pi}\frac{1}{i\pi^2}\int \frac{p^2 + 2m^2}{(p^2 + m^2)^2} d^4p,$$

must be identically zero (Jauch & Rohrlich, 1976, p. 190). The only way to circumvent this situation is to consider that "the integral is, strictly speaking, meaningless, since it is divergent" (Schweber, 1961, p. 552).

The pragmatic view is that we need a 'functioning' theory that is gauge invariant and provides a zero mass for the photon in the lower terms of the perturbation expansion of the S-matrix that are used in practice. This, when evaluating $\Pi_{\mu\nu}(k)$, can be done by taking into account the divergent integral $\Pi_\mu^{\ \mu}(0)$ and subtracting it from $\Pi(k^2)$, which leads to a logarithmically divergent integral. Using a gauge invariant regularization scheme we have $\Pi(k^2)= C + k^2\Pi^f(k^2)$, where $\Pi^f(k^2)$ is a finite correction term that as $\Lambda \rightarrow \infty$ "tends to a well-defined finite limit which is independent of the detailed form of the regularization procedure" (Mandl & Shaw, 1984, p. 187), and C is logarithmically divergent as $\Lambda \rightarrow \infty$ (Sakurai, 1967, p. 277). With this procedure we obtain a regularized photon propagator that includes second-order photon self-energy effects.

The next step is to incorporate the regularized constant C in a parameter of the theory whose value is experimentally determined, so that we can take the cut-off limit to infinity and recover quantum electrodynamics from the regularized 'theory'. Considering, for example, the second-order correction to the electron-electron or Møller scattering[20] due to the self-energy of the photon, the change in the Møller scattering amplitude amounts to

$$\frac{e^2(\bar{u}_1'\gamma_\mu u_1)(\bar{u}_2'\gamma_\mu u_2)}{ik^2} \rightarrow$$

[20] I will look into some general aspects of the Møller scattering in the next chapter.

$$e^2(1-C)\frac{e^2(\bar{u}_1'\gamma_\mu u_1)(\bar{u}_2'\gamma_\mu u_2)}{ik^2}+ie^2\Pi^f(k^2)(\bar{u}_1'\gamma_\mu u_1)(\bar{u}_2'\gamma_\mu u_2)\text{ . (Sakurai, 1967, p. 278)}$$

The trick is to consider the (infinite) constant 1 – C not as a correction factor related to the photon propagator itself, but, as can be seen more clearly in the limit $k^2 \to 0$, as a correction to the charges of the electrons which interact via the photon. In this way we relate this (infinite) correction to the coupling constant. In the limit $k^2 \to 0$ the modified photon propagator is given by $D'_F(k) = (1 - C)\, D_F(k)$. We renormalize the theory considering that $(1 - C)^{½}$ is a correction to the unobservable electron bare charge e_{bare}, so that what is observed is $e_{obs} = (1 - C)^{½}\, e_{bare}$. The distinctive flavour of this renormalization procedure when compared with the mass renormalization is that we are, so to speak, transferring the problems of the photon to the electrons.

Looking closely at the second-order self-energy correction to the Møller scattering, the infinity arising in the photon propagator is absorbed by the charges of the electrons 'located' at both 'vertices' of the interaction. This is possible because the description of the scattering by an S-matrix perturbative approach is done in a way that what appears in between the initial and final asymptotic states is not described as a process occurring in time, but the situation is such that "the S-matrix describes the scattering in the operational spirit of Heisenberg's matrix mechanics. It gives transition probabilities which correspond to measurable relative frequencies. But it treats the scattering itself as a black box" (Falkenburg, 2007, p. 131). Implicit in the procedure is a notion of time lapse between the initial and final asymptotic states (formally taken to be infinitely apart). What we do not have is a classical-like description of the photon propagation as something taking time to happen: in the renormalization procedure there is no possibility for the photon propagation to be seen as related to a causal temporal order connecting the electrons evolved in the scattering. In the applications of the theory, the Minkowski space-time loses any possible operational meaning related to space and time measurements, and becomes a sort of configurational space that is part of the machinery that enables to calculate energy-momentum cross-sections. There is, as I said, an idea implicit of temporal order in the scattering (also present in the ordering of operators in the terms of the S-matrix) but no temporal description of the process as something related to the exchange of a 'signal' propagating at light speed. All these are pretty much well-known aspects of quantum electrodynamics. But I think that the full implication of this situation has not been considered previously. The charge renormalization is possible only by not requiring a temporal description of the processes in the applications of quantum electrodynamics. But if a temporal description was (somehow) intended, it is clear that it would be incompatible with the charge renormalization procedure, because we can only have charge renormalization in an overall temporal description of the interaction going on inside an unobservable 'black box' (in the next chapters I will explore this situation a bit more).

Up to this point, I have been considering lowest order radiative corrections and the necessary mass and charge renormalization. This is not the whole story. I have not mentioned how Feynman's approach relates to the S-matrix calculations, the renormalization to all orders of the S-matrix, and what to make of Dyson's 1952 result about the divergence of the series expansion of the S-matrix; in the process of doing so I will return to the views that several physicists have had regarding renormalization.

Dyson gave a more formal structure to Feynman's approach. Considering the perturbative solution of the Tomonaga-Schwinger equation in terms of a unitary operator, Dyson realized that when taking the limits for an initial state in the infinite past and a final state in the infinite future, Schwinger's unitary operator was identical to

the Heisenberg S-matrix. Following Feynman's symmetrical approach between past and future, Dyson used a chronological operator P( ) that enabled him to present the S-matrix in the form

$$S(\infty) = \sum_{n=0}^{\infty} (-i/\hbar c)^n [1/n!] \int_{-\infty}^{+\infty} dx_1 \ldots \int_{-\infty}^{+\infty} dx_n P(H^I(x_1), \ldots, H^I(x_n)),$$

where $H^I(x)$ is the term in the Hamiltonian corresponding to the interaction between the Maxwell and Dirac fields (Dyson, 1948, p. 492). In the case of electron-electron scattering, the second-order term of this expansion is given by Feynman's fundamental equation (taking into account Pauli's exclusion principle).[21]

This Dyson did after the summer of 1948, having already talked substantially with Feynman and assisting in particular Schwinger's lectures at Ann Arbor in that summer. On arriving at the Institute for Advanced Study in Princeton in early September 1948, Dyson wrote home that

> on the third day of the journey a remarkable thing happened; going into a sort of semi-stupor as one does after 48 hours of bus-riding, I began to think very hard about physics, and particularly about the rival radiation theories of Schwinger and Feynman. Gradually my thoughts grew more coherent, and before I knew where I was, I had solved the problem that had been in the back of my mind all this year, which was to prove the equivalence of the two theories. Moreover, since each of the two theories is superior in certain features, the proof of the equivalence furnished incidentally a new form of the Schwinger theory which combines the advantages of both. (Quoted in Schweber, 1994, p. 505)

Afterwards Dyson confronted the question of whether the perturbative approach could be made finite to every order. Dyson was able to show inductively that if all divergences had been removed in a particular order n then they could be removed in order n + 1. Since this was the case in lowest order by using renormalization procedures, this would also be the case to whatever order we actually achieved when making a calculation. In this way the perturbative expansion of the S-matrix is renormalizable to all orders (Dyson, 1949). However as we have already seen, the perturbative expansion of the S-matrix is divergent. What to make of this with respect to renormalization?

First I will look again into Schwinger's view on the renormalization. Schwinger considered that the infinites meant that the theory breaks down at ultra-relativistic energies. Under this view, to Schwinger

> [renormalization] is the clear separation of what we don't know–but which affects our experiments in a clear limited way–from what we do know and where we can calculate in detail. In fact, I insist that all

[21] The S-matrix program was originally developed by W. Heisenberg as an alternative to quantum field theory. His idea was to sidestep the problem of divergences in quantum field theory – in his view due to the point-like interaction between fields – by considering only what he saw as measurable quantities (Miller, 1994, p. 97). Heisenberg's idea was to retain only the basic elements of quantum field theory, like the conservation laws, relativistic invariance, unitarity, and others, and to make the S-matrix the central element of a new theory (Pais, 1986, p. 498). This was not done because in practise it was not possible to define an S-matrix without a specific use of the theory it was intended to avoid (Cushing, 1986, p. 118). The S-matrix later reappeared in mainstream physics with Dyson's use of it as a calculational tool. In Dyson's view the "Feynman theory will provide a complete fulfilment of Heisenberg's S-matrix program. The Feynman theory is essentially nothing more than a method of calculating the S-matrix for any physical system from the usual equations of electrodynamics" (quoted in Cushing, 1986, p. 122).

theories are like this. –People may not want to face up to it, there is always an area beyond where the theory either breaks down or where other phenomena come into play that you don't know about. They do not upset everything in the area you can control, and you isolate that from it: That's what renormalization is really about. Not sweeping infinities away but isolating the unknown part and recognizing its limited influence.

I am not sure that I was at all interested in the mathematical question of convergence to all order. I don't think that is a physical question. I have a feeling even then that I did not take renormalization too seriously. If in fact the theory had been not renormalizable at the 27th stage or whatever have you, I would have said "O.K. That's good" because here is a place where what we don't know, namely what happens at very large energies, enters the theory and will learn something. It wasn't essential to me that the theory be renormalizable to all orders. That was nice to get the theory going to lowest order. What would be even more interesting is if it didn't work. I wasn't very caught up in all these all order questions. (Quoted in Schweber, 1994, p. 366)

We see that for Schwinger it was not problematic that the series expansion of the S-matrix is divergent (even if he does not mention it explicitly, I think this agrees with the view he presents). However, we see that there is not in Schwinger a connection between the mathematical problems facing the theory and a limitation on its applicability in a way to be consistent with the underlying concepts (like the idea of a point-like electron), as we can see in Dirac's subtraction physics.

My view is that Dyson's 1952 new divergence does not change much whatever view we decide to have on renormalization. We can still think that there is a breakdown at high energies or/and problems in the type of description of the interaction between the fields adopted in quantum electrodynamics. Depending on what we make of Dyson's divergence we can see it related, or not, to the renormalized infinites. I tend to see a relation, because, in my view, both are related with limitations in the description of the interactions in quantum electrodynamics, and both are manifestations of an improper use of the mathematics beyond the physical content of the theory.

In the previous chapter I defended that we can relate the divergence of the S-matrix series expansion with a tentative application of the theory beyond the possibilities provided by the input physical assumptions used to set up the theory as a perturbative approach. Now I put together the elements for the equivalent point in the case of renormalization. For this I will return to Dirac's subtraction physics and to Bohr's views on the problem of infinities in quantum electrodynamics. In a nutshell Dirac concluded that an external electromagnetic field had an effect of polarization of the distribution of negative-energy electrons. The calculation of the density matrix of the sea electrons, in the simple case of an external electrostatic field, gave a logarithmically divergent result. Dirac considered that we cannot assume that the theory applies when it is a question of energies greater than the order of $137mc^2$. So, he used a cut-off to render the results finite. With the finite result in his hands, Dirac concluded that there is no induced electric density except at the places where the electric density producing the field is situated, and at these places the induced electric density cancels a fraction of 1/137 of the electric density producing the field. This means that the electron's charge that is measured is smaller than the real charge.

For me the crucial aspect can be found in Dirac's argument for the need of a cut off. He says that

quantum mechanics does not seem applicable to phenomena which involve distances of the order of the classical radius e^2/mc^2, since the present theory can in no manner discuss the structure of the electron … such distances, considered as electron wavelengths, correspond to energies of the order $(\hbar c/e^2)(mc^2)$ [$\cong 137mc^2$]. (Dirac, 1934a, pp. 136-137)

As already mentioned, this can be seen as a Bohrian sentence, in the sense of taking into account the input physical assumptions in the theory of a point-like electron to justify disregarding energies greater than the order of $137mc^2$. To appreciate properly this point let us look again into Bohr's views on quantum electrodynamics:

> The foundations of the present treatment of atomic phenomena are the discoveries of the ultimate electrical particles and the elementary quantum of action, which rely upon quite separate lines of experimental evidence and at the present stage of atomic theory are introduced in essentially different and independent ways.… The possibility of treating the elementary particles and the quantum of action as independent foundations of the theory of the electronic constitution of atoms rests essentially upon the fact that the atomic dimensions, as deduced from quantum-mechanics and symbolized by the "radius" of the hydrogen atom $a = h^2/4\pi^2e^2m$ [(2)], are very large compared with the electron diameter given by [$d = e^2/mc^2$ (1)]. Obviously, *this is a necessary condition for considering the electron as a charged material point in the fundamental mechanical equations*…. Notwithstanding its fertility, the attack on atomic problems in which the particle idea and the quantum of action are considered as independent foundations is of *an essentially approximative character* … The possibility of treating radiation phenomena and other effects of the finite propagation of forces to a considerable extent rests entirely on the smallness of the two dimensionless constants of atomic theory, the fine structure constant $\alpha = 2\pi e^2/hc$ and the ratio between the masses of the electron and the proton $\beta = m / M$. Thus as will be seen from (1) and (2), it is the small value of α which is responsible for the smallness of the ratio between d and a, which is just equal to α^2. It is just this circumstance which affords a justification for the neglect of the radiative reaction in a description of the stationary states including the fine structure ... *the attempts to treat the radiation effects on rigorous lines by considering the atoms and the electromagnetic field as a closed quantum-mechanical system led to paradoxes arising from the appearance of an infinite energy of coupling between atoms and field*. The solution of these difficulties will certainly claim a formalism in which the elementary particles and the quantum of action appear as inseparable features…. It is important, however, to examine more closely to what extent the present theory offers a reliable guidance for the analysis of the phenomena ... In this procedure, in which the radiation field is not considered as part of the system under investigation ... By a proper application of the quantum mechanical formalism it has been possible … the treatment of such problems as the width of spectral lines and the retardation effects in the interaction of electrons bound in atoms. Still, *the condition for such applications is that the effects in question can be treated as small perturbations* of the phenomena to be expected if the finite propagation of forces would be neglected. *Due to the smallness of the constant α, mentioned above, this condition is widely fulfilled in problems of atomic constitution, since even for the electrons most firmly bound in atoms of high nuclear charge, "orbital" dimensions and spectral wave-lengths are very large compared with the classical electron diameter.* (Bohr, 1932b, p. 62-67 [my emphases])

Contrary to others that look into the renormalizable infinites in quantum electrodynamics from the perspective of a putative better theory, Bohr looks from within quantum electrodynamics. As we have seen, for example in the case of the Klein paradox, to Bohr it is not that the negative-energy solutions are non-physical or some other thing. It is simply that we cannot disregard the atomicity of matter, and when we do that, by considering mathematically possible potentials that are physically impossible when taking into account part of the conceptual basis of the theory (i.e. the atomicity of matter), we get into trouble. We obtain strange mathematical results without any clear physical meaning. The same holds in the case of the renormalizable infinities. In the structure of the theory we have a point-like description of the electron. This means we can not consider distances were we might have in some way to talk about the internal structure of the electron (whatever this might mean). When considering high-energy interactions we would be so to speak poking into the electron, i.e. going beyond the conceptual basis of the theory. The infinities show not where a better theory is needed but where we are stretching the mathematics beyond the physical basis of the theory. In quantum electrodynamics, according to Bohr, it only makes sense to consider distances larger than the so-called "electron diameter", which implies taking a limited range for

the energy in interactions. This in practice corresponds to using a Dirac type cut off in the necessary expressions (i.e. Feynman's regularization), even if it turns out to be a mathematically inconsistent procedure (as others in the theory).

5 Conclusions

The objective in this chapter was twofold. One of them was to address renormalization from the perspective of the spacio-temporal description of physical processes in quantum electrodynamics. As we have seen implicit in the charge renormalization procedure is the fact that we are considering an overall space-time description. In practice this means that we are not really describing the physical processes in time. The charge renormalization is possible exactly because of this. We implement a view of the physical processes as if from outside space-time and we move around infinities that should be related to the electromagnetic mediation between charged particles, i.e. related to delayed interactions, and 'by hand' put the infinities where it is more convenient. In this case the infinity due to the photon self-energy is attributed to the charge of the particles and 'renormalized'.

The other objective of this chapter was to dig into the history of renormalization to see if from an encounter with the original moments where the developments were being made it was possible to find a 'new' perspective on renormalization that might not be part of the contemporary philosophical views on renormalization; the objective was not to present a detailed study of the different contemporary views.[22]

The view regarding renormalization presented here is based on Bohr's ideas. As we have seen, according to Bohr

> the existence of the electron even in classical theory imposes an essential limitation on the applicability of the mechanical and electromagnetic concepts. Indeed, the finite propagation of electromagnetic forces brings with it the existence of a fundamental length, the so-called "electron diameter" defining a lower limit for the extension of the region where the idealization according to which the electron is considered as a charged material point is justifiable. (Bohr, 1932a, p. 377)

We must recall that in the structure of quantum electrodynamics is inscribed, through the Dirac equation, a point-like description of the electron (see chapter 3). This idealization of the electron as a point-like particle implies according to Bohr limitations to quantum electrodynamics:

> the difficulties inherent in any symbolism resting on the idealization of the electron as a charged material point appear also most instructively in the recent attempt of Heisenberg and Pauli to build up a theory of electromagnetic fields on the lines of quantum mechanics. (Bohr, 1932a, p. 378)

I regard Dirac's 'subtraction physics' as an example of a procedure that identifies and overcomes in an imperfect way one of these difficulties.[23] As we have seen, according to Dirac

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[22] In particular I do not address the renormalization group approach (on this subject see e.g. Huggett and Weingard, 1995).

[23] To Bohr the difficulties of quantum electrodynamics were, in particular, "Dirac's unobservable negative energies, the ambiguity of force indicated by the Klein paradox, the unmeasurable magnetic moment of the electron, the uninterpretable spin, and the unresolved infinities" (Moyer, 1981, p. 1061).

quantum mechanics does not seem applicable to phenomena which involve distances of the order of the classical radius e^2/mc^2, since the present theory can in no manner discuss the structure of the electron … such distances, considered as electron wavelengths, correspond to energies of the order $(\hbar c/e^2)(mc^2)$ [$\cong 137mc^2$]. (Dirac, 1934a, pp. 136-137)

In this way Dirac made use of a cut-off in the applications of the theory, corresponding to a maximum value of the energy of the order $137mc^2$:

we cannot assume that the theory applies when it is a question of energies greater than the order of $137mc^2$, and the most reasonable way to proceed seems to be to limit arbitrarily the domain of integration to a value of the momentum … corresponding to electron energies of the order indicated. (Dirac, 1934a, p. 141)

With this procedure we would be avoiding a conflict between one of the input physical assumptions of the theory (the point-like electron) and applications not taking into account this physical assumption.

Does this means that the theory sets from the inside its experimental domain of application? It seems that this was Bohr's view. According to a letter of Dirac from 1931,

Bohr is at present trying to convince everyone that the places where relativistic quantum theory fails are just those where one would expect it to fail from general philosophical consideration. (quoted in Moyer, 1981, p. 1060)

This is not the view being defended here. To clarify this point let us consider for example Newton's theory of gravitation. As it is well known, Newton's theory predicts a particular numerical value in relation to the drift of the perihelion of mercury, which, contrary to Einstein's gravitation theory, is not in good agreement with observation. However from the internal perspective of Newton's theory there is nothing strange about this result: it is physically meaningful (i.e. it goes along the line of different results provided by the theory). What is happening is that we are facing a limit of application of the theory in what regards 'saving the phenomena'. The situation with the renormalizable infinities is different; it is not related to the experimental/observational domain of applicability of quantum electrodynamics (i.e. it is not related with 'saving the phenomena'). In my view it represents a locus of 'divergence' between the physical assumptions of the theory and its mathematical applications; contrary to the view that Dirac attributes to Bohr (I think correctly), the limitations in the application of physical concepts do not affect or enable to define clearly the experimental/observational domain of applicability of the theory. When addressing appropriately the problem of infinities by the renormalization procedure it does not affect the domain of application of the theory.

How do we identify situations where a mismatch occurs between for example the physical idealization of a point-like electron and applications that go beyond this idealization? In my view when we obtain results that are not physically meaningful (without taking into account any comparison with observations). This is particularly clear when for example we expect on *physical grounds* to calculate a very small correction to the electron mass due to its self-energy and it turns out that the result is divergent. This does not imply that in all theories we must take a divergent result as a sign of a mismatch between the physical input assumptions and the mathematical applications of the theory. I have no general argument pointing to this, and I do not even explore this possibility. As mentioned, in this work the objective is only to look into a

few conceptual-mathematical problems of quantum electrodynamics not related to ‘saving the phenomena’.

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