
Relations between units and relations between quantities

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Abstract

The proposed revision to the International System of Units contains two features that are bound to be of special interest to those concerned with foundational questions in philosophy of science. These are that the proposed system of international units ("New SI") can be defined (i) without drawing a distinction between base units and derived units, and (ii) without restricting (or, even, specifying) the means by which the value of the quantities associated with the units are to be established. In this paper, I address the question of the role of base units in light of the New SI: Do the "base units" of the SI play any essential role anymore, if they are neither at the bottom of a hierarchy of definitions themselves, nor the only units that figure in the statements fixing the numerical values of the "defining constants" ? The answer I develop and present (a qualified yes and no) also shows why it is important to retain the distinction between dimensions and quantities. I argue for an appreciation of the role of dimensions in understanding issues related to systems of units.

Keywords: Coherent system of units, Dimensional Homogeneity, Dimensions, Quantities, International System of Units, Units

[Note: There are four tables to be inserted and published with this paper. These are [. . .] appended at the end of this paper. They are numbered in order of appearance in the text, as requested in the JGPS guidelines for authors.]

1. Introduction

1.1 Background

It is expected that sometime soon, perhaps as early as 2018, we will finally have an international system of units in place in which *all* the units of the SI (Système International d'Unités) are defined in terms of "well-recognized fundamental constants of nature." [BIPM (2014a), BIPM (2011b), Quinn (2016) this issue, Borde (2016) this issue, Mills et al. (2011)] The specific change most discussed so far has been that the definition of the unit for the quantity mass, i.e., the kilogram, is to change, and that it is to change in such a way that it will no longer be defined in terms of a physical artefact. [e.g., Riordan 2015, Quinn 2016 this issue]

Changing the definition of the kilogram in this way also changes the network of interrelations amongst the definitions of the base SI units. After the proposed change, the definition of the kilogram, which is currently not dependent on the definition of any of the other base SI units, becomes dependent on, or interrelated with, the definitions for some of the other SI units. The kilogram is not the only unit whose definition will be changed by the proposed change to the SI, though: four of the base units of the SI will be redefined in terms of "constants of nature." These changes in *how* the base units are defined have been discussed, too; many such discussions focus on *how units in the SI are related to the "constants of nature"* in terms of which they are defined. I will be more interested here in how the proposed changes to the definitions of SI units impact *the relations between units of the SI*.

1.2 Definitions of Units in the SI

There are other differences between the proposed SI and the current SI that are even more fundamental, in that they have to do with the *means by which* the units are defined, i.e., the way in which the definitions are formulated (as is also mentioned in the papers by Quinn and Borde introducing this special issue). For completeness, I provide in this paper three tables showing the definitions of units in the current SI (Table 1) and in the proposed "New SI" (Tables 2 and 3), for reference in the discussion below.

The change in how the units are defined in the New SI touches on the very notion of the role of the base units in the SI. For, after the proposed change, there will be *two alternative formulations* of the definition of SI units, as follows:

1.2.1 Definition of the New SI without Base Units

In the first formulation of the definition of the proposed system of SI units (Table 3), in an unprecedented move, *no distinction is drawn between base units and derived units among all the SI units mentioned in the formulation*: the "system of SI units" is defined in terms of a collection of SI units that includes not only the seven base units of the current SI (length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity), but also the hertz, joule, coulomb, lumen and watt. The system of units is defined as a whole, in terms of seven constants of nature to be referred to as "the defining constants." The BIPM document describing the proposed change to the SI system is quite clear about there being no distinction between base and derived units in this formulation; as pointed out in the Draft 9th SI Brochure [BIPM 2013]:

"The use of seven defining constants is the simplest and most fundamental way to define the SI . . . In this way no distinction is made between base units and derived units; all units [mentioned in the definition] are simply described as SI units. This also effectively decouples the definition and practical realization of the units."

To explain what "practical realization of the units" means, I quote from the relevant BIPM document prepared to orient readers to the fundamental change associated with the realization of units in the "New SI":

"In general, the term "to realize a unit" is interpreted to mean the establishment of the value and associated uncertainty of a quantity of the same kind as the unit that is consistent with the definition of the unit. It is important to recognize that any method consistent with the laws of physics and the SI base unit definitions can be used to realize any SI unit, base or derived. [BIPM (n.d.) CCE-09-05]

Although the BIPM indicates it will provide specific methods for realizing the units of the New SI, the role of the methods for realizing units provided by the BIPM in the New SI is different than in previous SI systems: ". . . the list of methods given is not meant to be an exhaustive list of all possibilities, but rather a list of those methods that are easiest to implement and/or that provide the smallest uncertainties. " [BIPM (n.d.) CCE-09-05] The result of this "decoupling" of the definition of the units from the practical realization of the units is thus that "While the definitions may remain unchanged over a long period of time, the practical realizations can be established

by many different experiments, including totally new experiments not yet devised." [BIPM 2013, p. 9-10]

Thus the proposed system of international units ("New SI") can be defined (i) without drawing a distinction between base units and derived units, and (ii) without restricting (or, even, specifying) the means by which the value of the quantity associated with the unit is to be established. *Both these are striking changes*; many in philosophy of science will find it surprising that it is possible to specify a system of units without distinguishing base units from derived ones, and without specifying how the unit is to be established. (Or, at least, they will find it thought-provoking to consider how this might be done.) In order to provide a definition of the New SI in a more familiar formulation, an alternative definition has also been provided. The alternative (second) formulation does specify base units.

1.2.2 Definition of the New SI with Base Units

In the second formulation of the definition of the "New SI":

"[The SI is] defined by statements that explicitly define seven individual base units: the second, metre, kilogram, ampere, kelvin, mole, and candela. These correspond to the seven base quantities time, length, mass, electric current, thermodynamic temperature, amount of substance, and luminous intensity. All other units are then obtained as products of powers of the seven base units, which involve no numerical factors; these are called coherent derived units." [<http://www.bipm.org/en/measurement-units/new-si/>]

The definitions of the seven base units in this formulation of the definition of the proposed revision to the SI, or "New SI", are shown in Table 2. On this second, alternative, definition of the New SI, each of these seven definitions of an individual base unit is called an "explicit-constant formulation." Unlike in the first formulation of the definition of the "new SI" (shown in Table 3), each of the "explicit constant" definitions of a base SI unit identifies a quantity (e.g., time, length, mass, etc.) with which the unit is uniquely associated (Table 2). [BIPM (2011b)] For example, the metre is the unit of *length*; the second is the unit of *time*; the ampere is the unit of *electric current*, and so on for the seven base units and the associated seven quantities. Each of the units is then defined in terms of setting its magnitude "by fixing the numerical value of" a certain constant of nature.

While it is not unprecedented to define a unit by fixing the value of a constant of nature for a particular unit (e.g., metre) it is unprecedented to do so for the entire system of units. Those unfamiliar with the background leading up to the decision to take this approach who wish further explanation may find it in the papers in this special issue devoted to presenting the New SI. (Quinn 2016, Borde 2016) To further aid the reader here and for reference at later points in the paper, I provide, in Table 4, a selection from the BIPM's answers to FAQ's (Frequently Asked Questions) that I consider especially relevant to the logic of defining units via setting the values of constants of nature.

Even though each of the seven unit definitions in the second, "explicit-constant" formulation of the New SI contains just one of the seven "defining constants", *it is not the case that each base unit of the SI is considered uniquely associated with one of the "defining constants."* This is clear to see, by inspecting the definitions in Table 2: it can easily be seen that most of the definitions of a base unit involve other units, so that often several SI units are jointly involved in using a defining constant to define a unit. Thus, on either definition of the proposed SI system of units, or New SI, the relation between the individual units being defined and the seven "defining constants" is a matter of a *collection* of SI units being *jointly defined* by a *collection of seven constants* considered invariants of nature.

In the sections that follow, I discuss aspects of the two fundamental changes to the proposed "New SI" pointed out as (i) and (ii) above that relate to philosophy of science, especially to the topic of units, quantities, and relations between them. I then address the question of whether the distinction between base units and derived units is actually still needed after the proposed change to the SI is adopted as an international standard. My answer to this question also explains why we need to retain another distinction in that has been questioned in metrology and ignored in much of philosophy of science: the distinction between quantities and dimensions (in the sense of kinds of quantities). I will argue that there is actually a central role for dimensions in the formulation of the SI system, even though they do not appear prominently in the final formulation.

2. What happened to the "base units"?

Consider the form of the first definition of the New SI system of units, shown in Table 3 of this paper (and provided by BIPM to the public in "Draft 9th SI Brochure" [BIPM 2013]). There are no base units designated. However, if we chose, we could, hypothetically, identify seven quantities as especially selected to be the units defined by the seven "defining constants" of the SI. For

example, Hz and m/s are SI units¹ defined by the "defining constant" associated with the hyperfine splitting of Cs, and m/s is an SI unit defined by the speed of light in vacuum. (BIPM 2013, p. 12/29). However, neither of these is a "base" SI unit, in either the current or proposed SI systems.

There are still units designated as the "base units" in the "new SI", though: in terms of proper names, they are the same seven base units of the current SI: metre, kilogram, second, ampere, kelvin, mole and candela. In the first definition of the new SI, they are distinguished in a somewhat indirect, or implicit, way, in that the SI units hertz, joule, coulomb, lumen and watt are related to them (Table 3). *Whether using the "explicit-constant" formulation for the seven base units (Table 2), or the more implicit "defining constant" formulation (Table 3), the base units will be defined in terms of the exact numerical values that fundamental constants take on when they are expressed in certain of the (newly defined) SI units.*

Thus, what is at the base of *either* of the definitions of the New SI system of units are seven selected fundamental constants chosen in part for their stability and, thus, their capability to serve as invariant constants of nature. Their numerical values will have zero uncertainty. The reason that the numerical values of the fundamental constants will be exact in the new SI, rather than being the result of measurements containing some uncertainty, is that, in the "new SI", the uncertainty of the constants of nature chosen to define the SI units is zero as a consequence of *fixing* their numerical values with respect to the SI units. That is, *when expressed in the specified new SI units*, their *numerical* value is a certain fixed number. The setting of the numerical value of these constants when expressed in certain SI units effectively defines the SI units. The concisely worded FAQs on the subject developed by the BIPM in [BIPM 2011a] provide more explanation; some selected excerpts are provided in Table 4 of this paper.

This approach involves an unprecedented change in the *general organization of units*: in terms of logical priority, the setting of the numerical value of the fundamental constants is *logically* prior to the definition of the *new, redefined* SI units, since the newly defined SI units are defined in terms of them. The new SI units and the previous SI units will coincide in value *at the moment the change is made*, but there is no expectation that their values according to the current and proposed definitions will continue to coincide after the change, nor that they would have coincided prior to the change. Another subtle detail, which is bound to require some adjustment in pedagogy and in the formulation of investigations about measurement in philosophy of science is

¹ The quantity velocity is measured in m/s, which is identified as one of the SI derived units without a special name. (Other derived SI units without a special name are m^3 , m^2 , and m/s^2)

that, in the definitions of the new SI, the SI units in which the fundamental constants are expressed when their numerical values are fixed are not limited to the seven base units of the current SI with which so many are so familiar (although it is still the case that one can express any SI unit in terms of the seven base SI units alone).

What role do the "base units" of the SI play, then, if they are neither at the base of a hierarchy of definitions, nor the only units that figure in the statements fixing the numerical values of the "defining constants" that are at the base of the definitions?

3. Base units in the New SI International System of Units

As explained above, the New SI is striking in that it is the first to propose a definition of the SI system of units that does not distinguish between base SI units and derived SI units. It does, however, as noted above, include an (alternative) definition of the New SI system of units in terms of base units, too, and makes the claim that the two formulations are equivalent. This raises the question of whether the notion of base units is required. [Stock & Witt 2006, p. 586] One thing is not in question: *which* seven units are selected as the base units is not uniquely determined; there is an element of arbitrariness involved. [Sterrett 2009, p. 813; Mohr 2008, p. 132] However, the point about arbitrariness is distinct from the question of whether the notion of base units can be dispensed with. Mohr has raised the question of the need for the distinction between base units and derived units at all, in support of a proposal that one of the definitions of the proposed SI system not make the distinction. [Mohr 2008, p. 133]

We need to ask, then: if the distinction between base and derived SI units is not essential to providing a definition of the SI system, what are the base units used for, other than organizing the definition of the SI? One answer is that, since the inception of the SI, it seems, the concept of a "coherent system of units" has been characterized in terms of base units. When Burdun (1960) explicitly listed the provision of a "coherent system of units" as a requirement of a system of units, he clarified what he meant, and did so in terms of "basic units" and "derived units": "In selecting the basic units of the system it is necessary: 1) to provide a coherent system of units, i.e., to select basic units which would produce derived units by multiplication or division without introducing numerical coefficients; . . ." [Burdun, p. 913-914] Thus, if base units were not specified, this account of the coherence of the SI system of units could not be applied to produce a coherent system of units, or to verify that a certain system of units was coherent.

Yet one of the points celebrated about the new SI is that the definition of the SI can now be made without mentioning base units or derived units at all. [Borde 2005, Mohr 2008] The way I see to make sense of this is as follows: the "more fundamental" definition of the SI, the one made in terms of the seven "defining constants" without mentioning base units (Table 3), is one way to define the SI, and that definition can be used to identify *specific quantities* for the SI units (some base, some derived) that are mentioned in that definition. This is just what the definitions referring to artefacts such as the standard meter and the standard kilogram did for the base SI units (as Mohr also points out in [Mohr 2008, p. 133]) but this definition does so without mentioning any artefacts. This more fundamental definition also ties all the SI units that it does mention to the units that *happen to be the units previously identified as SI base units*, thus allowing the question of which units are coherent SI units to be answered in the same way as before. Thus the question of coherence of units of the "new SI" is answered, without having to be addressed anew.

It seems to me the new SI raises, in turn, however, a foundational question about *coherence of a system of units*. We have seen that there is a *practical value* in having the distinction between base units and derived units in place when it comes to specifying which units of the SI are coherent SI units: doing so makes it fairly simple to define a coherent SI unit, and, accordingly, simplifies confirming that a unit is a coherent SI unit. Does coherence of a system of units *require* the distinction, though? Raising this question leads in turn to the question as to what coherence of a system of units consists in, and why it is considered essential to a system of units.

4. Coherence of a System of Units

In a previous article I used a different characterization of the *coherence of a system of units* than Burdun gives, and one I think not uncommon, i.e., that "[a] system of units is coherent if the relations between the units used for the quantities is the same as the relation between the quantities in the fundamental equations of the science" [Sterrett 2009, p. 806]; the title of this paper comes from that characterization. In laying out the topic there, I found it important to discuss not only units and quantities, but also 'dimensions' (sometimes called "kinds of quantities.") Although in that article I referred only to the SI system and associated documents in place at that time (the current SI), many of the major points are still relevant, since the New SI likewise uses dimensions in its formulation of the distinction between base units and derived units [Draft 9th SI Brochure, section 1.3].

The usefulness and necessity of the concept of dimension has been questioned [e.g., Emerson 2005]. However, I will argue, the points that have been raised as reasons for questioning the need for the concept of dimension are mistaken, largely because they do not take into consideration the significance of the distinction between quantities and dimensions. In addressing the topic of whether the distinction between base units and derived units is needed, we shall see how important and useful the concept of dimension is, once we get clear about the distinction between 'quantity' and 'dimension' (a distinction which is made in both the current SI and the New SI).

More recently, another relevant article about the coherence of a system of units has appeared [de Courtenay 2015]. The two articles (my "Similarity and Dimensional Analysis" [Sterrett 2009] and de Courtenay's "The Double Interpretation of the Equations of Physics and the Quest for Common Meanings" [de Courtenay 2015]) are concerned with different aspects of the topic, but are complementary. Put briefly, whereas I was concerned with drawing out the consequences of what is built into a system of units that meets the constraint of being a coherent system of units, de Courtenay's article explains how that constraint is actually achieved. She also discusses why the constraint has been so important to modern science, what it conceals, and the situation we are left in even after it has been achieved. In the discussion below I thus refer to some points made by De Courtenay in "The Double Interpretation of the Equations of Physics and the Quest for Common Meanings", [de Courtenay 2015] as well as those I have made in my "Similarity and Dimensional Analysis." [Sterrett 2009]

4.1 Units, Quantities and Dimensions

I will use the characterization that a system of units is *coherent* if the relations between the units used for the quantities are the same as the relations between the quantities in the fundamental equations of the science. [Sterrett 2009, p. 806] It follows from this that if one is concerned with making sure that a certain system of units being formulated is to be coherent, then, they would need to know the relations between quantities prior to formulating a system of units. This would require being able to write the fundamental equations of science in a way that is *independent of a* system of units. One might well ask if this is indeed possible?

The answer is that, yes, it is possible to write the fundamental equations of science in a way that is independent of a system of units, if the fundamental equations are regarded as *quantity equations*. In [Sterrett 2009], I discussed quantity equations, contrasting Lodge's recognition of the fundamental equations of mechanics and physics as quantity equations with James Clerk

Maxwell's view of equations. On Maxwell's view of equations, a physical quantity appearing in a physical equation must be measurable, which means that the value of a quantity consists of two parts: a numerical part, and a unit with which all quantities of that kind can be compared. As I noted there:

"Maxwell explicitly discussed this conception of a quantity, while recognizing ambiguities in the notation of physical quantities as used in equations in scientific practice. He noted that symbols used as variables in equations of physics lent themselves to two different interpretations: (i) as denoting the lines, masses, times, and so on themselves, and (ii) 'as denoting only the numerical value of the corresponding quantity, the concrete unit to which it is referred being tacitly understood.' [Maxwell 1890, p. 241]

Each of these interpretations presents a problem, though . . . The first interpretation doesn't really apply during the process of performing the numerical calculations. . . . The second interpretation doesn't satisfy the requirement that 'every term [of an equation of physics] has to be interpreted in a physical sense.' Maxwell's way of resolving the ambiguity he identified was to take a sort of hybrid approach. . . . [Sterrett 2009, p. 804] "

Alfred Lodge's approach aimed to avoid the ambiguity that Maxwell identified (what de Courtenay [2015] calls "the double interpretation of the equations of physics") by regarding the equations of mechanics and physics as equations that "express relations among quantities" [Lodge 1888, pp. 281 - 283] , i.e., as *quantity equations*. This is the approach I took in [Sterrett 2009], in which I identified the feature of *coherence of a system of units* as the key to understanding why it is possible to use dimensionless parameters to establish similarity of physically similar systems. For my purposes in that paper, I, too, could avoid the ambiguity in the equations of physics that Maxwell identified, as Lodge did, by regarding the fundamental equations of mechanics and physics as quantity equations. Lodge pointed out that, understood as quantity equations, the fundamental equations of a science 'are independent of the mode of measurement of such quantities; much as one may say that two lengths are equal without inquiring whether they are going to be measured in feet or metres; and, indeed, even though one may be measured in feet and the other in metres.' [A. Lodge 1888, p. 281 - 283]

Guggenheim, a student of Lodge's, wrote a piece intended as a reference for others on the topic called "Units and Dimensions." [Guggenheim 1942] Guggenheim there states his view that "we are entitled to multiply together any two entities, provided our definition of multiplication is self-consistent and obeys the associative and distributive laws." He argues that it is "perfectly legitimate to multiply together any two physical entities, such as a length and a force. If the

reader naively asks: 'What, then, is the product of a foot and a pound?' I reply a 'foot-pound.'" [Guggenheim 1942] That view is now fairly well accepted. [de Boer 1994/5; VIM 3rd edition 2012] The VIM (Vocabulary of International Metrology) includes the term "quantity equation", giving the meaning as "mathematical relation between quantities in a given system of quantities, independent of measurement units." (VIM 1.22 at <http://jcgmbipm.org/vim/en/1.22.html>) de Boer explains the significance of a relation between quantities that is independent of measurement units, following up on Maxwell's arguments:

"Expressing the results of physics in terms of physical quantities has the advantage of giving a representation which does not depend on the choice of unit. When a particular length L is expressed in two different units $[L]$ and $[L']$ by the two expressions $L = \{L\}'x[L]$ and $L = \{L\}x[L']$. . . the *physical quantity* L itself is an invariant [as Maxwell argued.] This is an important argument and an essential reason in favour of using quantities and not numerical values in the theoretical description of physical phenomena; *Using physical quantities gives a representation which is invariant with respect to the choice of units.*" [de Boer 1995, p. 406]

This approach was used in discussions about the transition from CGS units to the Giorgi/SI units. Cornelius remarked in 1964 that using quantity equations led to a dispute as to whether the difference between the two systems lay in different units or different quantities. The answer, of course, was that the 'radical' change was a change in the form of the equations -- which led to a change in the coherence of units. [Cornelius et al., 1964, p. 1446] The terms "quantity calculus" and "quantity equation" are still used. Very recently, in discussing the possible roles the international prototype of the kilogram might still play after the proposed revisions to the SI are adopted, Davis likewise found it useful to do so [Davis 2011], citing de Boer's "On the History of the Quantity Calculus and the International System" [de Boer 1995]

De Courtenay, in contrast, faces the ambiguity head-on, rather than avoiding it. She shows Maxwell's view on equations to be something that is "now taken for granted by every student of physics when solving a problem and sliding from the algebraical solution to its numerical application." [de Courtenay 2015, p. 145-146] Maxwell's view of the equations of physics serves as the starting point of her inquiry into the double interpretation of equations; she proceeds to discuss how terminology has since evolved, and explains why in fact it is now appropriate to regard the "double interpretation" view as cogent:

"The smooth passage between the two interpretations was achieved by the construction of a *coherent* system of units which ensured that the equations had *exactly the same expression*

under both interpretations. The coherent system of units, which was to be disseminated worldwide by the metrological international organizations, reconciled the two formerly conflicting points of view within a framework that warranted, at last, the double interpretation of the equations of physics propounded by Maxwell . . . [de Courtenay 2015, p. 146]

The process involved in establishing a coherent system of units that proceeds starting from relations that are invariant under a change of units is laid out in de Courtenay 2015 [pp 146 -147]. These are "accepted physical relationships that can be stated in relations of proportions", and choices at this initial stage "determine the *structure* of the system." [p. 146] de Courtenay's account of what goes into producing a coherent system of units provides valuable background for a better understanding of the point in my previous paper. I had emphasized that coherence of a system of units is relative to the quantity equations one chooses to use, but I did so based upon looking at an historical episode, rather than providing a logical analysis of the process:

"At one time, the CGS system provided a coherent set of units for Newtonian mechanics, yet it was not clear what to say at that time about a coherent set of units for electromagnetism. There was a CGS system for Newtonian mechanics, a CGS-M system of units for magnetism, and a CGS-E system of units for electrical phenomena. It was shown in 1901 by Giorgi that the CGS system could be amended in alternate ways to provide a coherent set of units for the quantities in the equations describing electromagnetic phenomena, so that a choice had to be made. The decision made by the committees governing the SI system was to include a base unit for the physical quantity of electrical current; the SI system now provides a coherent set of units with respect to Maxwell's equations as well as for Newton's. When used for electromagnetism, the SI system (which follows Giorgi) and the older CGS systems are really different systems of units; for electromagnetic phenomena, unlike for mechanics, switching from one of these systems to the other involves more than a simple change of units, for there will be some equations whose form differs depending on which system one is using. Thus, some of the units for the older CGS systems are referred to as non-SI units." [Sterrett 2009, p. 809]

The question being investigated there was: what underlies inferences based upon dimensionless parameters, or more precisely, based upon the identity of the values of dimensionless parameters between two different systems? The answer was that, once a coherent system of units is in place, the fact that one is using a coherent system of units is relied upon (whether or not it is explicitly recognized), when using the numerical values that dimensionless parameters take on to establish similarity of physical systems. [Sterrett 2009; Sterrett 2016a, 2016b] That a ratio of

quantities is dimensionless is significant, because it says something about the relations between the quantities in it. The underlying logic involves dimensions as well as quantities and units, and further examination will reveal a role for dimensions in drawing a distinction between base units and derived units.

Dimensions are to be distinguished from units and from quantities, although they are frequently confused with both of them, in print as well as in discussion, sometimes even in textbooks. Dimensions have to do with a constraint on equations, a constraint which we know the equations of physics, if we've gotten them right, must be subject to. Dimensions can be used to formulate a necessary and sufficient criterion for an equation to be homogeneous -- even an equation containing undetermined quantities. The requirement that an equation be homogeneous is like the requirement that a sentence be grammatically correct; just as a sentence is not a proper sentence unless it is grammatically correct, so an equation is not a physical equation unless it is dimensionally homogeneous. Now, the question often arises as to whether one might just as well appeal to units for this purpose, and hence, whether it is possible to regard dimensions as superfluous (i.e., whether it is possible to hold that dimensions can be dispensed with, or, alternatively, to hold that dimensions just are units). Although this is often done, we shall see that things are not so easy, and that a lack of clarity about both units and dimensions is very lamentable and may have contributed to an ill-advised disregard for an appreciation of the role of dimensions.

Dimensions appear in the new SI (SI Brochure 9th edition, draft) just as they do in the current SI (SI Brochure 8th edition, 2014 update); they are referred to as "dimensions of quantities." Each SI base unit has a unique dimension associated with it; the dimensions of other quantities are constructed from products of powers of these dimensions. The SI brochure indicates as a rationale for providing dimensions only that "By convention physical quantities are organised in a system of dimensions." [BIPM 2013 section 1.2, p.3] This does not help much in understanding what dimensions are, nor in determining whether the fact that each base unit is associated with a unique dimension means that defining some units as base units is essential in the SI system. In order to understand what dimensions are, we go back to historical roots of the concept. I find Fourier's comments on dimensions clearer and more reliable than Maxwell's. And, since Maxwell credits Fourier as the first to state the theory of the dimensions of physical quantities, I take it that Maxwell means to be endorsing the points about dimension found in Fourier's *Analytical Theory of Heat*. How Fourier speaks about dimensions, and what he says about them, is helpful in clarifying the distinctions between dimensions, quantities, and units, and will lead us to the topic of base units.

Fourier speaks in terms of *exponents of dimension*, and he speaks of the dimension of (what we would call) a variable in the equation *with respect to* a unit, i.e., "the dimension of x *with respect to* the unit of *length*." For instance, a number representing a surface has dimension 2 [with respect to the unit of *length*], and one representing a solid has dimension 3 [with respect to the unit of *length*]. His discussion focuses on equations; an equation expresses a relation between magnitudes. He notes two things: (i) the terms of an equation can not be compared unless they have the same exponent of dimension; and (ii) "every undetermined magnitude or constant has one dimension proper to itself." (Fourier 1878: Section 160; p. 129) The idea in (ii) is that one can *deduce the dimensions* of an undetermined magnitude occurring in an equation from these two principles. That is, the *equation* expresses how the magnitudes and constants occurring in it are related to each other; and the *dimensions* of each magnitude or constant indicate how each of them is related to each of the units (for each "kind of unit"). Then, the requirement of homogeneity puts a collective constraint on the dimensions of the magnitudes and constants that occur in the equation.

Fourier identifies three units as being relevant in his analysis of equations in the theory of heat: length, duration, and temperature; these are independent of each other. (That they are independent on Fourier's account is clear from a table he presents in which he shows that the dimension of the magnitude he calls x with respect to length is 1, with respect to duration is 0, and with respect to temperature is 0. If we express this by saying that the dimension of l (length) is 1,0,0, we can express the situation with the others by saying that the dimension of t (duration) is 0,1,0; and that of v (temperature) is 0,0,1) Thus this collective constraint on the dimensions of the magnitudes and constants that occur in an equation in the theory of heat is actually the *conjunction of three separate constraints on the equation*: the dimension of every term of the equation with respect to the unit of length must be equal, *and* the dimension of every term of the equation with respect to unit of duration must be equal, *and* the dimension of every term of the equation with respect to the unit of temperature must be equal.

Now, in saying that (on Fourier's account) the dimensions of a certain magnitude or constant indicate how that magnitude or constant is related to each of the units he identifies, I do not mean that in order to carry out the dimensional analysis, one must refer to, or assume, any *particular* unit or system of units. Fourier's starting point is an equation relating magnitudes and constants, and, as he repeatedly emphasizes, such an equation is the same no matter what units one is using. The kind of equation he is talking about is in fact invariant with respect to changes of units, but the kind of changes he means are only changes that are a matter of size of the unit. Fourier

works out an example to illustrate that, although individual magnitudes in the equation will change to accommodate a change in (the size of) units, they will change in unison in such a way that the equation itself holds without modification. (Fourier 1878; p. 128 - 130) Thus the constraint of dimensional homogeneity on the equation involves reference to the existence of some units, but not any particular units, and the constraint itself is put in terms of exponents that refer to units very generally, i.e., as "unit of length", "unit of duration", and "unit of temperature."

To get clear on the respective roles of units and dimensions: on Fourier's account, the notion of dimension draws on the rather minimal assumption of the existence of a system of units in which the magnitudes and constants in the equation can be measured (or, perhaps just the possibility of the existence of one), but not on the existence of any particular choice of units. What features of a system of units is he relying on, though? As mentioned above, the three units Fourier uses in analyzing his equation (one unit each for length, duration, and temperature) are independent of each other. Does his analysis assume that *any* systems of units one might use has the feature that there is a set of independent units, too? What role do dimensions have in his reasoning?

As I'll show, in Fourier's analysis (as in the SI brochures for both the current SI and the New SI), dimensions are not the same thing as units. We can see that there is an intimate relation between them, though, in spite of the fact that they are not the same thing, by reflecting upon Fourier's analysis of (dimensional) homogeneity of equations. Recall that Fourier speaks of the dimension of a magnitude *with respect to* a unit of a certain quantity for a particular magnitude. Thus, *dimensions cannot be the same thing as units*, because dimensions stay the same throughout many changes in size of units. For example, in Fourier's analysis, as he exhibits in the form of a table, the dimension of the quantity "surface conductivity h " with respect to length is -2 [Fourier 1878, section 161; p. 130], and the exponent of the dimension will stay the same for any size of unit for length one chooses. The exponent of dimension of a given magnitude with respect to the unit of length thus implicitly assumes the *possibility of a unit for length* and indicates how each of the magnitudes involved in an equation is related to the unit for length.

Yet, in general, it is not a requirement of a system of units that it contain a unit for length, nor for any particular quantity. Fourier's analysis is thus not an analysis of the most general case, for it assumes a system of units with some special features, i.e., that the system contains units for the magnitudes of length, duration, and temperature. [He mentioned that a total of five were needed for physics.] Perhaps that assumption was not inappropriate for his purposes. However, we are interested here in a more general question. Fourier's formulation is very useful in that it lends itself to generalization: he spoke of the exponents of the dimensions of the unit for length, for

duration, and for temperature. This way of speaking does allow us to generalize away from a specific set of units, by talking about "unit for length", "unit for duration" and "unit for temperature." What are the things that we wish to generalize over (length, duration, temperature) called then, if not units? They might be called kinds of quantities or magnitudes, but they are distinguished among other quantities in that they are selected as belonging to a small group of kinds of quantities or magnitudes that are of some special relevance to the kind of equation being analyzed. We already saw that for Fourier, they are independent of each other.

As Fourier lays out his analysis of the equations of the theory of heat in terms of the exponents of the dimension of each of three selected magnitudes, the formalization he uses provides a canonical format for associating exponents of dimensions with each magnitude or constant that might occur in an equation in the theory of heat. Using the term dimension to generalize, we would then say that Fourier used the three *dimensions* for length, duration and temperature in his analysis, and assumed that units for each of the three quantities of length, duration and temperature could be identified, although he did not refer to any specific system of units. Such a formulation permits generalizing from those three specific dimensions. We might put it this way: the role that the dimensions for units of length, duration, and temperature have is to provide a logical tool for implementing the grammatical constraint of dimensional homogeneity on the equation, independently of reference to a system of units. A dimensional equation consists of sums of products of dimensions with exponents; it is independent of the choice of units one uses, and so is invariant with respect to changes in (sizes of) units. It is very easy to blur things here and conflate dimensions with units or quantities. Many have identified dimensions with either units or quantities, as a matter of convenience. It may be there are some contexts in which that does no harm. However, I will argue for the importance of distinguishing dimension from either units or quantities in foundational or methodological investigations.

Since the dimensions of a quantity do not always determine which quantity it is, dimensions are not quantities. Lodge made a point of this, too, noting that *work* and *moment of force* are not the same quantity, but they have the same dimensions of quantity. The point bears emphasis: there are quantities we consider distinct which, when subjected to the kind of analysis Fourier described, yield the same exponents of dimension. This is not a reductio of the notion of dimension, however, as Emerson's comments appear to imply. [Emerson 2005] Rather, these examples merely illustrate the basic fact that dimensions are not the same thing as quantities; they show nothing regarding the uselessness of dimensions. Dimensions have an essential role to play in formulating the constraint of dimensional homogeneity on equations. For this purpose, a few quantities determined to be independent are designated as base dimensions. The set of

independent base dimensions selected for the analysis then provides a canonical format for expressing the exponent of dimensions for any quantity, just as a set of selected vectors provides a canonical format for expressing every vector, or factorization by prime numbers provides a canonical format for expressing every positive integer. If by analogy with base (or basis) vectors, we call this set of independent dimensions the base dimensions, it is then straightforward to write an equation for a given dimension that expresses the constraint that all the terms in the equation have the same exponent of (that particular) dimension. We can write such an equation for each of the dimensions. A dimension is thus associated with a quantity: for each dimension, a certain quantity is selected to serve the role that length, duration, and temperature (which Fourier called a "kind of unit") served in Fourier's analysis when he spoke of the exponents of dimensions of magnitudes and constants *with respect to* length, duration, and temperature. It is true that, as Fourier puts it, each magnitude or constant in an equation [of the theory of heat] has "one dimension proper to itself", a fact that he regards as derived from "primary notions" about quantity. (Fourier 1878; p. 128) But, as we've seen, this does not mean that there is a single quantity associated with each dimension; that point is entirely consistent with the existence of two different quantities that have the same dimensions. [Sterrett 2009; Lodge 1888]. As I have written on an earlier occasion:

"One of the principles used for the [current] SI System is that the dimension of every quantity, whether base or derived, is unique; that is, there is only one such dimension of canonical form associated with each quantity Q. Since the number of derived quantities is unlimited, and the number of dimensions of canonical form is unlimited [as well], one may ask whether there is a unique quantity associated with each dimension of canonical form. The answer is no: more than one quantity may have a given dimension associated with it, just as more than one quantity may have the same units (*heat capacity* and *entropy* are considered physically distinct quantities, though they are both measured in joule/Kelvin; *electric current* and *magnetomotive force* are both measured in amperes.)

Thus, one cannot infer from a dimension, the quantity with which that dimension is associated, for the quantity is not uniquely determined." [Sterrett 2009; p. 813]

To summarize: the *concept of dimension* is important to preserve, because of the role it has in the criterion for the homogeneity of equations in physics. Whereas, the *concept of quantity* is important because of the role that quantities have in the quantity equations expressing the fundamental laws of science.

4.2 Quantity Equations and Dimensional Equations

On Lodge's approach (as with the approach I took in [Sterrett 2009]), quantity equations express the fundamental laws of physics as relations between quantities. Quantity equations must be dimensionally homogeneous, but the constraint of dimensional homogeneity on an equation is like a grammatical constraint on a statement; just as there is much more to a statement than its grammatical structure, so there is much more to an equation of physics than homogeneity of dimensions. Another way to put this is to point out that the dimensional equation one would write to show that the quantity equation is dimensionally homogeneous is not necessarily as informative as the quantity equation itself is. For example, consider the equation $s = a t^2$ where s is a distance, a is an acceleration, and t is a time, considered as a quantity equation. The dimensional equation is $[L] = [L] [T]^2/[T]^2$ or $[L] = [L]$.

Where do dimensional equations fit in the logical scheme of developing a system of units? They are not the beginning point, for dimensional equations are *relative* to the quantity equations expressing the fundamental laws of the science one is concerned with. (We saw that, even in Fourier's presentation of how to determine the exponents of dimensions of the units for various quantities, a necessary prior step was identifying the three quantities with respect to which the exponents of dimension were to be determined.) Dimensional equations are also relative to the selection of *which* of the quantities in those quantity equations is designated for use as a *base dimension* in the dimensional analysis.

Thus, in a rational reconstruction of dimensional equations, the logical order of things seems to be: 1. identifying the quantity equations expressing relations between quantities; 2. showing that there can be a system of units for the quantities associated with those quantity equations; 3. selecting the quantities upon which the dimensional analysis yielding the dimensional equation is to be based, and 4. writing the dimensional equations that result from performing a dimensional analysis of a quantity equation one is interested in. The specific dimensional equation that results will depend upon the choices in 1 and 3.

4.3 Constructing Systems of Units

Now, the rational reconstruction of dimensional equations just discussed does not include actually developing a coherent system of units. There is another step, the one de Courtenay calls the final and most crucial step:

" Only in the end do the units of the base magnitudes, called base units, get selected. The units of the derived magnitudes are then fixed as derived units expressed in terms of the base units through the structure, or the other equations, without introducing any numerical coefficient (i.e., by setting all the coefficients equal to one). It is this last step that is all important: it established the bridge between the two interpretations and at the same time conceals the gap between them because it ensures that the equations interpreted in terms of magnitudes and in terms of measures have exactly the same expression." [de Courtenay 2015, p. 146-147]

However, in later sections of the paper, she points out that the gap still exists, in that "the mathematical equations of physics, in their theoretical use, are primarily to be understood as equations between magnitudes" which is very much like understanding them as quantity equations. Her explanation makes it clear that achieving the goal of a coherent system of units involves activities prior to this last step: identifying the quantity equations (from observation, experiment and analysis), and choosing which magnitudes to designate as base magnitudes. This is meant to describe the current and past SI systems. These reflections reinforce the value of "thinking in terms of requirements placed on a *system* of units, rather than in terms of requirements placed on the units chosen for each kind of quantity individually", which was also helpful in understanding the historical case of the change from CGS units to the Giorgi/SI units I discussed in [Sterrett 2009]. In that case, which was a revision of a system rather than the construction of a new one from scratch, thinking in terms of coherence of a *system* of units rather than about the change to individual units led to understanding that a change in the form of the quantity equations could bring about a change in the coherence of units. [Sterrett 2009, p. 809]

For any coherent system of units, then, the establishment of the system of units, including the definitions of the units, is a holistic affair that starts with the quantity equations. Since it is part and parcel of the international system of units that it be a coherent system of units, really, the units in most earlier versions of the SI are in some sense collectively defined. Yet there is a difference between the New SI and its predecessors due to the definitions no longer including any artefacts in the New SI: now that no base unit is defined in terms of an artefact, that reason for defining base units is gone. The other important change effected by the New SI, the decoupling of the definition and the "practical realization" of the units, removed another reason one might need to designate some of the SI units as base units. There was another objective that did call for designating some of the units as base units, and that reason did not go away -- however, it is parasitic on the fact that the previous systems employed a distinction between base units and

derived units: the objective of keeping the values of the base quantities continuous at the moment of transition from the use of the current SI to the use of the New SI.

However, referring to de Courtenay's description of what is involved in establishing a coherent set of units, we see that even though the designation of base units does not occur until the last step, the designation of base *magnitudes* occurs in a previous step. The 'last step' of the process taken to conceal the gap between what she calls magnitude equations and measure equations does not involve a free choice as to which units to designate as base units. Rather, on her account, the designation of base units is determined by the prior choice of which magnitudes were selected to be base magnitudes, since base units are "the units of the base magnitudes." [de Courtenay 2015, p 147]. Thus, the "magnitude equations" and choice of base magnitudes are really what organize the base units. Then, the relation between the derived and base units is structured as needed to accord with the magnitude equations.

As I employ Lodge's idea of quantity equations rather than Maxwell's "double interpretation", my discussion is slightly different, but its bottom line is analogous: prior to the selection of base units, one possible designation of which units to call the base units is already available, due to the choice of *base dimensions*, which is logically prior. I do not see, however, that the choice of base units is actually constrained by this. (According to the draft SI brochure "The choice of the base units was never unique." [BIPM 2013, sec 1.3]) By selecting the units of the base dimensions to be the base units, though, the rest of the process is made much easier than by making any other selection for the base units. *The dimensions help organize the units by providing a formalism in which relations between quantities, with respect to a set of quantity equations expressing the fundamental relations of a science, can be more perspicuously studied. Dimensions, however, do not appear prominently in the formulation of the system of units, once it has been constructed.*

5. Conclusion

We are now ready to address the question of whether there is a need for the distinction between base quantities and derived quantities in a system of coherent units, as opposed to abandoning the distinction. The answer is both a qualified yes and a qualified no.

The answer to the question of whether the distinction is needed is (a qualified) yes. Obviously it is possible to construct a definition of the SI that does not designate any units as base units, as evidenced by the "defining constants", or "single scaling statement" formulation of the new SI. Hence it may seem unequivocal that it is possible to define a coherent system of units without

designating some of them as base units. However, it seems to me that some distinction between base and derived units necessarily occurs in the logical underpinnings of a coherent system of units, in that a choice of base units, even if only a provisional one, has to be made in the steps taken prior to the definition of units. This is because the formalism developed to understand how quantities are related to each other -- the concept of exponents of dimension, or dimensional analysis -- does involve the notion of base units. (Recall that Fourier used "the unit for length", "the unit for duration" and "the unit for temperature" as base units in performing a dimensional analysis.) This choice of base units, which is needed to carry out the dimensional analysis of quantity equations in order to determine the dimensions of quantities, is logically prior to defining the (value of the) units. There is some freedom of choice in selecting which dimensions to designate as base dimensions and, from a logical standpoint, one need not designate any particular dimension as a base dimension. In fact, in the new SI, on the "definitional constants", or "single scaling statement" definition there does not even seem to be a *logical* constraint anymore that would prevent someone from choosing to designate one set of units as base units when using the new SI for one application, and then to designate another set of base units for another problem or situation. There are, of course, often practical constraints and disadvantages associated with doing so.

It is worthwhile reflecting on why this "yes" answer is so. Because the starting point for formulating a coherent system of units is a collection of quantity equations that expresses relations between quantities, the relations between any quantity included in these equations and the selected base quantities will be available, even if not explicitly stated. The quantity equations are actually discussed in the SI Brochure, which says that "The system of quantities used with the SI, including the equations relating the quantities, is just the set of quantities and equations that are familiar to all scientists, technologists, and engineers. They are listed in many textbooks and in many references, but any such list can only be a selection of the possible quantities and equations, which is without limit." There are various international standards cited, but there is now a single standard comprising them: "the ISO/IEC 80000 Standards, Quantities and Units, in which the corresponding quantities and equations are described as the International System of Quantities." A telling detail is found in a footnote: "In these equations the electric constant ϵ_0 (the permittivity of vacuum) and the magnetic constant μ_0 (the permeability of vacuum) have dimensions and values such that $\epsilon_0\mu_0 = 1/c^2$, where c is the speed of light in a vacuum. Note that the electromagnetic equations in the CGS-EMU, CGS-ESU and Gaussian systems are based on a different set of quantities and equations in which the magnetic constant and electrical constant have different dimensions, and may be dimensionless." [BIPM 2013, p. 2] This note illustrates an important point about the history of the SI: the quantity equations and system of quantities

developed for these standards already incorporate a concern for being suitable for use in developing a coherent a system of units. This just reflects the fact that the development of coherent systems of units was a holistic affair that included the quantity equations; sometimes the logical order of things even ran backwards, from an already established unit in use to reformulation of quantity equations to accommodate using it. Using the quantity equations that have been developed in these standards, no matter what units are designated as base units, the relations between any SI unit and the selected base units will be deducible; the relations between the base units and derived SI units do not have to be *created* by another statement of definition that stipulates how they are related. The theory of dimensions can be used to articulate what those relations are. A remark in the SI brochure puts things slightly differently, though I think it reflects the same point: "The dimension of a derived quantity provides the same information about the relation of that quantity to the base quantities as is provided by the SI unit of the derived quantity as a product of powers of the SI base units." [BIPM 2013, p. 2]

The answer to the question (as to whether there is a need for the distinction between base quantities and derived quantities in a system of coherent units) is a qualified no in the following sense: a definition of the units of the SI that distinguishes some of the units as base units is no longer needed, in the sense of requiring an *additional* step of selecting base units, defining them, and stating the relations between SI units and the base units.

This is how the issue appears to me, for the reasons stated in this paper. More importantly, the two striking features of the new SI that I identified near the outset of this paper as features I expect many philosophers of science to find surprising present an opportunity. These features were that "the proposed system of international units ("New SI") can be defined (i) without drawing a distinction between base units and derived units, and (ii) without restricting (or, even, specifying) the means by which the value of the quantity associated with the unit is to be established." These challenge naive notions about measurement, such as that values of units are tied to artefacts, or that definitions of quantities are tied to specifically defined processes, which one often finds in philosophical discussions of measurement. On the one hand, as many have noted, the new SI is the realization of a vision that physicists of the nineteenth century had outlined, and so, in some sense, it is not new. On the other hand, however, this is the first time we have a system of units realizing that vision. It is an occasion for philosophers of science to re-examine presumptions about measurement and the logic of the equations of physics. I also hope that my study in this paper will encourage an appreciation of what I feel is an essential part of the language of science: the theory of dimensions.

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References

- Burdun, G. D. (1960) International System of Units. *Measurement Techniques* 3(11), 913-919.
- BIPM (Bureau International des Poids et Mesures) (2013) "Draft 9th SI Brochure"
- BIPM (Bureau International des Poids et Mesures) (2011a) "FAQs, Frequently Asked Questions about the New SI (Revised October 2011) " <http://www.bipm.org/en/measurement-units/new-si/faqs.html> downloaded 28 June 2015.
- BIPM (Bureau International des Poids et Mesures) (2011b) "Future revision of the SI: "explicit-constant formulation [of units]" downloaded 28 June 2015 <http://www.bipm.org/en/measurement-units/new-si/explicit-constant.html>
- BIPM (Bureau International des Poids et Mesures) (2014a) "Resolution 1" in "Resolutions adopted by the CGPM at its 25th meeting (18-20 November 2014)" <http://www.bipm.org/en/CGPM/db/25/1/>
- BIPM (Bureau International des Poids et Mesures) (n.d.) Mise en Pratique CCE-09-05 <http://www.bipm.org/cc/CCEM/Allowed/26/CCEM-09-05.pdf>
- Cornelius, P., W. de Groot and R. Vermeulen. (1964) Quantity Equations and System Variation in Electricity, *Physica*, 30, 1446-1452
- Davis, R. S. (2011) The international prototype of the kilogram after redefinition of the International System of Units. *Phil. Trans. R. Soc. A* , 369, 3975 -3992
- de Boer, J. (1995) On the history of quantity calculus and the International System, *Metrologia*, 31, 405– 429.
- Emerson, W H (2005). On the concept of dimension. *Metrologia*, 42, L21-L22.
- Fourier, Joseph and Alexander Freedman (transl.) (1878) *The Analytical Theory of Heat*. Cambridge, UK: Cambridge University Press
- Guggenheim, E. A. (1942) Units and Dimensions. *The Philosophical Magazine* 33 (222), 479-296.
- Lodge, A. (1888) The Multiplication and Division of Concrete Quantities, *Nature* 38, 281-283
- Mills, Ian M., Peter J. Mohr, Terry J. Quinn, Barry N. Taylor, and Edwin R. Williams (2011). Adapting the International System of Units to the twenty-first century. *Phil. Trans. R. Soc. A* , 369, 3907-3924
- Mohr, Peter J (2008) Defining units in the quantum based SI. *Metrologia* 45, 129 - 143.

Quinn, Terry (2016) The origins of the Metre Convention, the SI, and the development of modern metrology. (this volume)

Riordan, Sally (2015) The objectivity of scientific measures. *Studies in History and Philosophy of Science*, 50, 38 - 47.

de Courtenay, Nadine (2015) The double interpretation of the equations of physics, in Schlaudt, Oliver, and Lara Huber, Eds. *Standardization in Measurement: Philosophical, Historical and Sociological Issues*. London, UK: Pickering & Chatto Publishers. , pp.

Sterrett, Susan G. (2016a) Physically similar systems: a history of the concept. In Lorenzo Magnani, Tommaso Bertolotti (eds.) *Springer Handbook of Model-Based Science* (Springer, Cham), in press

Sterrett, Susan G. (2016b) Experimentation on analogue models. In Lorenzo Magnani, Tommaso Bertolotti (eds.) *Springer Handbook of Model-Based Science* (Springer, Cham), in press

Sterrett, Susan G. (2009) Similarity and Dimensional Analysis. In A. W. M. Meijers (Ed.) *Handbook of the Philosophy of Science, Volume 9: Philosophy of technology and engineering sciences*. 799 - 823 Amsterdam: North Holland.

Stock, Michael and Thomas J Witt (2006) CPEM 2006 round table discussion 'Proposed changes to the SI', *Metrologia*, 43, 583 - 587.

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[Note to editor: notice that Table 3 and Table 4 each span two pages]

Tab. 1. Base quantities and base units used in the (current) SI

Tab. 2 Definition of "new SI" in terms of seven "defining constants"

**Tab. 3 Base quantities and base units - proposed revision to the SI
Definition of "new SI" in "explicit-constant" formulation**

Tab. 4 Selected Excerpts from BIPM's "FAQs about the New SI"

Tab. 1. Base quantities and base units used in the (current) SI

Ref: "A concise summary of the International System of Units, the SI"
published by the BIPM, dated March 2006 www.bipm.org

Base quantity	Base unit	Symbol for unit	Definition of unit
length	metre	m	The metre is the length of the path travelled by light in vacuum during a time interval of $1/299\,792\,458$ of a second.
mass	kilogram	kg	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
time, duration	second	s	The second is the duration of $9\,192\,631\,770$ periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.
electric current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
thermodynamic temperature	kelvin	K	The kelvin, unit of thermodynamic temperature, is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water.
amount of substance	mole	mol	<ol style="list-style-type: none"> 1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilograms of carbon 12. 2. When the mole is used, the elementary entities must be specified, and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
luminous intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of $1/683$ watt per steradian.

Tab. 2. Definition of "new SI" in terms of seven "defining constants"

[Ref: Draft 9th SI Document, 16 December 2013]

The international system of units, the SI, is the system of units in which the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom (^{133}Cs)hfs is exactly 9 192 631 770 hertz,

the speed of light in vacuum c is exactly 299 792 458 metre per second,

the Planck constant h is exactly $6.626\,069\,57 \times 10^{-34}$ joule second,

the elementary charge e is exactly $1.602\,176\,565 \times 10^{-19}$ coulomb,

the Boltzmann constant k is exactly $1.380\,648\,8 \times 10^{-23}$ joule per kelvin,

the Avogadro constant N_A is exactly $6.022\,141\,29 \times 10^{23}$ reciprocal mole,

the luminous efficacy K_{cd} of monochromatic radiation of frequency 540×10^{12} hertz is exactly 683 lumen per watt,

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to the relations $\text{Hz} = \text{s}^{-1}$ (for periodic phenomena), $\text{J} = \text{kg m}^2 \text{s}^{-2}$, $\text{C} = \text{A s}$, $\text{lm} = \text{cd sr}$, and $\text{W} = \text{kg m}^2 \text{s}^{-3}$. The steradian, symbol sr, is the SI unit of solid angle and is a special name and symbol for the number 1, so that $\text{sr} = \text{m}^2 \text{m}^{-2} = 1$.

**Tab. 3. Base quantities and base units - proposed revision to the SI
Definition of "new SI" in "explicit-constant" formulation**
[Ref: Section 1.4, Draft 9th SI Brochure]

Base quantity	Base unit	Symbol for unit	Definition of unit
length	metre	m	The metre, symbol m, is the SI unit of length; its magnitude is set by fixing the numerical value of the speed of light in vacuum to be exactly 299 792 458 when it is expressed in the SI unit for speed ms^{-1} .
mass	kilogram	kg	The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be exactly 6.626 069 57 10^{-34} when it is expressed in the SI unit for action $\text{J s} = \text{kg m}^2 \text{s}^{-1}$.
time, duration	second	s	The second, symbol s, is the SI unit of time; its magnitude is set by fixing the numerical value of the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom to be exactly 9 192 631 770 when it is expressed in the SI unit s ⁻¹ , which for periodic phenomena is equal to Hz.
electric current	ampere	A	The ampere, symbol A, is the SI unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be exactly 1.602 176 565 10^{-19} when it is expressed in the SI unit for electric charge $\text{C} = \text{A s}$.
thermodynamic temperature	kelvin	K	The kelvin, symbol K, is the SI unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant to be exactly 1.380 648 8 10^{-23} when it is expressed in the SI unit for energy per thermodynamic temperature $\text{J K}^{-1} = \text{kg m}^2 \text{s}^{-2} \text{K}^{-1}$.

amount of substance	mole	mol	The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be exactly 6.022 141 29 10 ²³ when it is expressed in the SI unit mol ⁻¹ .
luminous intensity	candela	cd	The candela, symbol cd, is the unit of luminous intensity in a given direction; its magnitude is set by fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency 540 1012 Hz to be exactly 683 when it is expressed in the SI unit kg ⁻¹ m ⁻² s ³ cd sr = lm W ⁻¹ = cd sr W ⁻¹ .

Derived units are defined as products of powers of the base units. When this product includes no numerical factors other than one, the derived units are called *coherent derived units*. The base and coherent derived units of the SI form a coherent set, designated the *set of coherent SI units*.

Tab. 4. Selected Excerpts from BIPM's "FAQs about the New SI" [Ref: http://www.bipm.org/en/measurement-units/new-si/faqs.html]		
FAQ	Question	Answer
8	How can you fix the value of a fundamental constant like h to define the kilogram, and e to define the ampere, and so on? How do you know what value to fix them to? What if it emerges that you have chosen the wrong value?	<ul style="list-style-type: none"> • We do not fix – or change – the <u>value</u> of any constant that we use to define a unit. The values of the fundamental constants are constants of nature and we only fix the <u>numerical value</u> of each constant when expressed <u>in the New SI unit</u>. By fixing its numerical value we define the magnitude of the unit in which we measure that constant. • • Example: If c is the <u>value</u> of the speed of light, $\{c\}$ is its <u>numerical value</u>, and $[c]$ is the <u>unit</u>, so that $c = \{c\}[c] = 299\,792\,458$ m/s then the value c is the product of the number $\{c\}$ times the unit $[c]$, and the value never changes. However the factors $\{c\}$ and $[c]$ may be chosen in different ways such that the product c remains unchanged. • <p>In 1983 it was decided to fix the number $\{c\}$ to be exactly 299 792 458, which then defined the unit of speed $[c] =$ m/s. Since the second, s, was already defined, the effect was to define the metre, m. The number $\{c\}$ in the new definition was chosen so that the magnitude of the unit m/s was unchanged, thereby ensuring continuity between the new and old units.</p>
9	OK, you actually only fix the <u>numerical value</u> of the constant expressed in the <u>new unit</u> . For the kilogram, for example, you choose to fix the numerical value $\{h\}$ of the Planck constant expressed in the new unit $[h] = \text{kg m}^2 \text{s}^{-1}$. But the question remains: suppose a new experiment shortly after you change the definition suggests that you chose a wrong numerical value for $\{h\}$, what then?	<ul style="list-style-type: none"> • After making the change, the mass of the international prototype of the kilogram (the IPK), which defines the current kilogram, has to be determined by experiment. If we have chosen a "wrong value" it simply means that the new experiment tells us that the mass of the IPK is not exactly 1 kg in the New SI. • • Although this situation might seem to be problematic, it would only affect macroscopic mass measurements; the masses of atoms and the values of other constants related to quantum physics would not be affected. But if we were to retain the current definition of the kilogram, we would be continuing the unsatisfactory practice of using a reference constant (i.e. the mass of the IPK) that considerable evidence suggests to be changing with time compared to a true invariant such as the mass of an atom or the Planck constant. [. . .] • <p>The advantage of the new definition would be that we will know that the reference constant used to define the kilogram is a true invariant.</p>

10	Each of the fundamental constants used to define a unit has an uncertainty; its value is not known exactly. But it is proposed to fix its numerical value exactly. How can you do that? What has happened to the uncertainty?	<ul style="list-style-type: none"> • The present definition of the kilogram fixes the mass of the IPK to be one kilogram exactly with zero uncertainty, $u_r(m_{\text{IPK}}) = 0$. The Planck constant is at present experimentally determined, and has a relative standard uncertainty of 4.4 parts in 10^8, $u_r(h) = 4.4 \times 10^{-8}$. • • In the new definition the value of h would be known exactly in the new units, with zero uncertainty, $u_r(h) = 0$. But the mass of the IPK would have to be experimentally determined, and it would have a relative uncertainty of about $u_r(m_{\text{IPK}}) = 4.4 \times 10^{-8}$. Thus the uncertainty is not lost in the new definition, but it moves to become the uncertainty of the previous reference that is no longer used . . •
11	The unit of the Planck constant is equal to the unit of action, $\text{J s} = \text{kg m}^2 \text{s}^{-1}$. How does fixing the numerical value of the Planck constant define the kilogram?	Fixing the numerical value of h actually defines the unit of action, $\text{J s} = \text{kg m}^2 \text{s}^{-1}$. But if we have already defined the second, s , to fix the numerical value of the caesium hyperfine splitting frequency $\Delta\nu(^{133}\text{Cs})_{\text{hfs}}$, and the metre, m , to fix the numerical value of the speed of light in vacuum, c , then fixing the magnitude of the unit $\text{kg m}^2 \text{s}^{-1}$ has the effect of defining the unit kg .
15	Are not the proposed definitions of the base units in the New SI circular definitions, and therefore unsatisfactory?	No, they are not circular. A circular definition is one that makes use of the result of the definition in formulating the definition. The words for the individual definitions of the base units in the New SI specify the <i>numerical value</i> of each chosen reference constant to define the corresponding unit, but this does not make use of the result to formulate the definition.