Artificial *vs* Substantial Gauge Symmetries: a Criterion and an Application to the Electroweak Model

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Abstract

To answer the generalized Kretschman objection, we propose a criterion to decide if the gauge symmetry of a theory is artificial or substantial. It is based on the dressing field method of gauge symmetry reduction, a new simple tool from mathematical physics. Given this criterion we argue that the notion of spontaneous symmetry breaking is superfluous to the empirical success of the electroweak unification theory. Important questions regarding the context of justification of the theory then arise.

Keywords : Gauge symmetries, generalized Kretschman objection, electroweak theory, symmetry breaking, Higgs mechanism.

1 Introduction

Philosophical analysis of gauge symmetries, long overdue, gained particular interest in the past fifteen years. Several notions deserve scrutiny. One is the gauge principle, according to which imposing a local/gauge symmetry on a free theory turns it into an interaction theory. This was suggestively encapsulated by Yang's aphorism "Symmetry dictates interaction" [1], clearly one of the conceptual revolution of the last century. But soon philosophers of physics took hold of the celebrated principle and scrutinized it. And sure enough it was found, e.g in [2] (see also [3]), that actually gauge symmetry might not be the sole criterion constraining the space of admissible theories, and that others like renormalizability may even be more fundamental, with gauge symmetry as an epiphenomenon.

Nevertheless, it seems undeniable that gauge symmetries are a powerful heuristic guide to zero in on fruitful and ultimately empirically adequate theories of the fundamental interactions. So that they appear to tell something deep and important about Nature. But it took little time to raise a problem with this conclusion, which can be summarized by saying that there is a generalized Kretschmann objection applicable to gauge symmetries. As a reminder, shortly after Einstein delivered his General Theory of Relativity (GR), Erich Kretschman suggested in 1917 that the principle of general covariance was empty, unable to constrain the space of admissible theories, since any theory could be written as to be generally covariant. There has been a long and lively debate over the validity of Kretschman's objection and the relevance of general covariance in relativity theory, and much effort to determine if there is a demarcation criterion to distinguish artificial general covariance from substantial general covariance [4]. It happens that much of this discussion applies, mutatis mutandis, to gauge symmetries and that a generalized Kretschman objection [5; 6] can be raised against the gauge principle: Physicists have devised many ways to implement a gauge symmetry in a theory lacking it [7; 8], so if any theory can be turned into a gauge theory, how come that gauge symmetries are regarded as a fundamental insight into the hidden structure of Nature? Can we distinguish artificial and substantial gauge symmetries? Can one propose one or several demarcation criteria?

One of the goal of the present paper is to bring attention to a new simple mathematical tool to deal with gauge symmetries, the dressing field method, that might provide just such a criterion. Although it has no pretention to universality, it seems to me that it allows to capture a key insight as to what counts as a substantial gauge symmetry in connection to non-local properties of genuine gauge theories, and thus have some relevance to this debate. In section 2, I give a brief description of the method, which is easy to grasp in its gist, and refer to the published literature for technical details and elaborations.

As a relevant application, I then undertake an analysis of the electroweak unification. It is still common wisdom among practicing physicists to consider the notion of spontaneous symmetry breaking (SSB) as pivotal to the success of the theory. The idea is seen as a most important insight into the structure of physical reality. An opinion so widely shared and unquestioned that no modern textbook on gauge field theory or quantum field theory omits to devote the proper space to expose it, and that even prominent physicists and science popularizers convey it to the layman [9; 10], sometimes going as far as to suggest it is the most revolutionary discovery of XXth century theoretical physics [11; 12] (over relativity and quantum theory?).

But is it so? Since at least fifteen years philosophers of physics have voiced skepticism. Here I will join my voice to theirs: relying on the dressing field method, which gives a clear conceptual language to elucidate the real content of the electroweak unification, I will argue that the answer is a definitive "No": The empirical success of the Glashow-Weinberg-Salam model is entirely independent of the interpretation in terms of SSB.

It is to be hoped that this conclusion will come to be more widely acknowledged in the physics community. Unfortunately, it is common among scientists to be somewhat dismissive of the inquiries of philosophers.[1] An attitude for which little price is usually paid, in the short term. But in this case, not acknowledging the insight of philosophers of physics would certainly lead to an astoundingly long-lived misconception at the heart of particle physics to remain uncorrected for still some times, and important ensuing questions regarding the context of justification of the electroweak model to remain unasked, let alone answered.

2 The dressing field method of gauge symmetry reduction

The dressing field method has been devised as a mean to handle, i.e reduce, gauge symmetry in a way that differs markedly from either gauge fixing or SSB mechanisms, and bears some resemblance to the bundle reduction theorem from the differential geometry of fiber bundles [16]. It has been applied mainly to conformal geometry where it allows to recover tractor and twistor calculi (analogues for conformal manifolds of the Ricci and spinorial calculi on Riemanian manifolds) from a gauge reduction of the Cartan conformal geometry [17; 18]. It also uncovered a new class of gauge fields, called *non-standard* or *twisted* gauge fields, which generalize vector bundles sections and connections used in Yang-Mills theories to model matter fields and gauge potentials. See [19; 20] for a review with technical details and references. The method is fully developed within the geometry of fiber bundle, but its basics and application to physical models are easy to state.

2.1 The basic mathematical setup of the method

First we need to lay the setup for a gauge theory. Consider a gauge theory on a m-dimensional spacetime manifold $(\mathcal{M}, g)$, whose gauge group of symmetry is $\mathcal{H} := \{\gamma : \mathcal{M} \to H\}$ with H a Lie group with Lie algebra $\mathfrak{h}$, and which by definition acts on itself via $\gamma_1^{\gamma_2} = \gamma_2^{-1}\gamma_1\gamma_2$.[2] Its basic space of fields is $\Phi = \{A, F, \varphi\}$, where F is the field strength (curvature 2-form $\in \Omega^2(\mathcal{M}, \mathfrak{h})$) of the gauge potential (connection 1-form $\in \Omega^1(\mathcal{M}, \mathfrak{h})$) A and φ is a matter field pertaining to a representation (ρ, V) of H. The gauge group acts on the space of fields, $\Phi \xrightarrow{\mathcal{H}} \Phi^\gamma$, as

$$\begin{aligned} A^\gamma &= \gamma^{-1}A\gamma + \gamma^{-1}d\gamma, & F^\gamma &= \gamma^{-1}F\gamma, \\ \varphi^\gamma &= \rho(\gamma^{-1})\varphi \qquad \text{and} & D\varphi^\gamma &= \rho(\gamma^{-1})D\varphi, \end{aligned} \tag{1}$$

where $D := d + \rho_*(A)$ is the covariant derivative implementing the minimal coupling between the matter field and the gauge potential.

Now, a physical theory is specified by its Lagrangian m-form $L(A, \varphi)$. In the case of a gauge theory, the Lagrangian is required to be $\mathcal{H}$-gauge invariant: $L(A^\gamma, \varphi^\gamma) = L(A, \varphi)$. A prototypical and almost minimal Yang-Mills

[1] One recalls the notorious example of Hawking and Mlodinow [13] and the controversial interview by Lawrence Krauss in The Atlantic [14] where he opinioned that "[...] the worst part of philosophy is the philosophy of science; the only people, as far as I can tell, that read work by philosophers of science are other philosophers of science. It has no impact on physics what so ever, and I doubt that other philosophers read it because it's fairly technical. And so it's really hard to understand what justifies it." He latter gave a retraction - sort of - in a column of Scientific American [15].

[2] Here $\mathcal{H}$ is to be seen as the pulled-back version of the group of vertical automorphisms of the underlying principal bundle $\mathcal{P}(\mathcal{M}, H)$ over $\mathcal{M}$, and not the group of transition functions between different trivializations of $\mathcal{P}$. So we deal with active rather than passive gauge transformation, which makes no formal difference.

Lagrangian is

$$\begin{aligned} L(A,\varphi,\varphi') &= L_{\text{YM}} + L_{\text{Scalar}} + L_{\text{Dirac}}, \\ &= \tfrac{1}{2}\,\text{Tr}(F \wedge *F) + \langle D\varphi, *D\varphi\rangle - m^2\langle\varphi,\varphi\rangle + \langle\varphi',\gamma\wedge *D\varphi'\rangle - m'\langle\varphi',\varphi'\rangle, \end{aligned} \tag{2}$$

where φ is a scalar field with mass m and φ' is a spinor field with mass m'. Here $\wedge$ is the wedge product of differential forms, $* : \Omega^p(\mathcal{M}) \to \Omega^{m-p}(\mathcal{M})$ is the Hodge star operator, while Tr and $\langle\ ,\ \rangle$ are bilinear forms on $\mathfrak{h}$ and V respectively. As for $\gamma = \gamma_\mu dx^\mu$, it is a one form whose components are Dirac's gamma matrices. A mass term for the gauge potential A, $\mu^2 Tr(A \wedge *A)$, failing to be gauge-invariant by virtue of (1), is forbidden so that a gauge interaction is a priori long range.

The core idea of the dressing field method consists in the following simple observation. Suppose the structure group H has some normal subgroup J, so that the gauge group $\mathcal{H}$ correspondingly has a normal subgroup $\mathcal{J}$. Suppose further that in the above setup of a gauge theory, one can extract a *local* field $u : \mathcal{M} \to J$, defined by its transformation property under $\gamma \in \mathcal{J}$: $u^\gamma = \gamma^{-1}u$. Such a field we call a *dressing field*. With it we can perform a change of field variable, $\Phi \to \Phi^u$, by forming the following *composite fields* :

$$\begin{aligned} A^u &: = u^{-1}Au + u^{-1}du, & F^u &= u^{-1}Fu, \\ \varphi^u &: = \rho(u^{-1})\varphi \qquad \text{and} & D^u\varphi^u &= \rho(u^{-1})D\varphi, \end{aligned} \tag{3}$$

where $D^u := d + \rho_*(A^u)$. These fields are $\mathcal{J}$-invariants variables. Notice that despite a formal similarity with (1), (3) are *not* gauge transformations. Indeed, by virtue of its defining transformation property, the dressing field is not an element of the gauge group: $u \notin \mathcal{H}$.

Taking advantage of the $\mathcal{H}$-gauge invariance of the Lagrangian, which holds as a strictly formal property, and of the formal similarity between (1) and (3), we can rewrite the gauge theory in terms of the $\mathcal{J}$-invariant variables:

$$\begin{aligned} L(A,\varphi,\varphi') &= L(A^u,\varphi^u,\varphi'^u), \\ &= \tfrac{1}{2}\,\text{Tr}(F^u \wedge *F^u) + \langle D^u\varphi^u, *D^u\varphi^u\rangle - m^2\langle\varphi^u, *\varphi^u\rangle + \langle\varphi'^u,\gamma\wedge *D^u\varphi'^u\rangle - m'\langle\varphi'^u, *\varphi'^u\rangle. \end{aligned} \tag{4}$$

This theory is therefore not a $\mathcal{H}$ gauge theory, as the prima facie form (2) of the theory would have us think, but a $\mathcal{H}/\mathcal{J}$-gauge theory.[3] Clearly the transformation of the composite fields (3) under the residual $\mathcal{H}/\mathcal{J}$-gauge symmetry depends on the behavior of the dressing field under this same symmetry. Some particularly important cases are described in [18].

Insofar as the genuine physical degrees of freedom of a gauge theory is given by gauge invariant quantities, the dressing field method helps to easily exhibit the physical content of a gauge theory.

2.2 Artificial *vs* substantial gauge symmetry

Due to the many good properties of gauge theories (in relation e.g with renormalization), over time physicists have devised various ways to implement a gauge symmetry in a theory lacking it [5]. The Stueckelberg trick is the forefather of these and, and its generalization seems to be of some relevance still to contemporary studies [7]. It is easily illustrated on the historic example of the Proca model (1936) for massive electromagnetism. The Proca Lagrangian is

$$L(A) = \tfrac{1}{2}F \wedge *F + \mu^2 A \wedge *A \tag{5}$$

and describes a massive vector field, so that the theory has no $\mathcal{U}(1)$ gauge symmetry. Now Stueckelberg (1938) proposed to implement such a gauge symmetry by adding a compensating field, the Stueckelberg field B, satisfying $B^\gamma = B - \mu\theta$ while the vector field becomes a gauge field transforming as $A^\gamma = A - d\theta$, with $\gamma = e^{i\theta} \in \mathcal{U}(1)$. A minimal Stueckelberg Lagrangian is then,

$$L(A,B) = \tfrac{1}{2}F \wedge *F + \mu^2(A - \tfrac{1}{\mu}dB) \wedge *(A - \tfrac{1}{\mu}dB) \tag{6}$$

[3] The requirement of normality for J, while not strictly necessary, insures that H/J is still a group.

and is a $\mathcal{U}(1)$-gauge theory.

In spite of what would be infered from a superficial reading, the Lagrangian (5) and (6) actually describe the same theory. Indeed the $\mathcal{U}(1)$ gauge symmetry is *artificial*, its presence by design compensated by the degree of freedom of the field B. In the case at hand, the Stueckelberg field is actually an abelian dressing field: $u := e^{i/_m B}$ so that $u^\gamma = e^{i/_\mu(B-\mu\theta)} = \gamma^{-1}u$. The associated $\mathcal{U}(1)$-invariant composite field is then $A^u := A + iu^{-1}du = A - \frac{1}{\mu}dB$, whose field strength is $F^u = F$. So the Stueckelberg Lagrangian (6) is rewritten in terms of gauge invariant variables as:

$$L(A, B) = L(A^u) = \tfrac{1}{2}F^u \wedge *F^u + \mu^2 A^u \wedge *A^u, \tag{7}$$

which is nothing but the Proca Lagrangian, devoid of any gauge symmetry. One may think of the dressing field method as a reciprocal to the Stueckelberg trick: the latter aims at implementing an artificial gauge freedom, the former seeks to erase it to reveal the gauge-invariant content.

The above simple discussion is illustrative of an important point: if one can find in a gauge theory a *local* dressing field, meaning that its value at a spacetime point depends only on this point and no others, then the invariant composite fields in terms of which the theory can be rewritten are local variables. So one pays no price in erasing the gauge symmetry, which is then fully dispensable. I therefore propose the following criterion:

A local dressing field in a gauge theory signals that its gauge symmetry is artificial. (C1)

Gauge theories present a number of conceptual as well as technical challenges. Among those, the fact that the gauge variables have a nondeterministic evolution, and the hindrance gauge symmetry poses a priori to the quantification of the theory. Dirac has pondered long and hard about these difficulties in the context of electromagnetism, an abelian $\mathcal{U}(1)$-gauge theory. One solution he first proposed in a 1955 paper [21] and then developed in the 1958 fourth edition of his *Principles of Quantum Mechanics* [22] (section 80), was to reformulate the theory with gauge-invariant variables, which would qualify as physical variables.

In the following we use essentially the notations of Dirac's 1955 paper [21] while setting all fundamental constants to 1. Let ψ be the electron field and $A = (A_0, A_r)$ the electromagnetic gauge potential, subject to the $\mathcal{U}(1)$-gauge transformations $\psi' = e^{iS}\psi$ and $A' = A + dS$. Dirac introduces the new variables $\psi^* = e^{iC}\psi$ (Eq [16]) and the associated "covariant" derivative $d\psi^* - iA^*\psi^* = e^{iC}(d\psi - iA\psi)$, with $A^* = A + dC$ (Eq [21] and below). The phase factor is defined by $C(x) = \int c_r(x, x')A^r(x')d^3x'$, and in order for the new variables to be gauge invariant $c_r(x, x')$ must satisfy $\frac{\partial}{\partial x'_r}c_r(x, x') = \delta(x-x')$ (Eq [18]). Dirac then notices that the latter equation admits the Coulomb potential as a solution,[4] so that by proceeding with the quantification of the electromagnetic theory written in terms of his invariant variables he interprets ψ^* in the following way:

> "We can now see that the operator $\psi^*(x)$ is the operator of creation of an electron *together with its Coulomb field*, or possibly the operator of absorption of a positron *together with its Coulomb field*. It is to be contrasted with the operator $\psi(x)$, which gives the creation or absorption of a bare particle. *A theory that works entirely with gauge-invariant operators has its electrons and positrons always accompanied by Coulomb fields around them*, which is very reasonable from the physical point of view."

An appealing conclusion indeed. It is not hard to see that Dirac's scheme is an instance of the dressing field method. Indeed under gauge transformation of the gauge potential, the phase factor transforms as

$$\begin{aligned} C'(x) &= \int c_r(x, x')A'^r(x')d^3x' = C(x) + \int c_r(x, x')\tfrac{\partial S}{\partial x'_r}(x')d^3x', \\ &= C(x) - \int \tfrac{\partial}{dx'_r}c_r(x, x')S(x')d^3x' = C(x) - S(x). \end{aligned}$$

So $u = e^{iC}$ transforms under $\gamma = e^{iS} \in \mathcal{U}(1)$ as $u' = \gamma^{-1}u$, and is therefore an abelian dressing field, which means that ψ^* and A^* in Dirac's equation [16] and [21] are abelian instances of the composite fields φ^u and A^u in (3) above.

[4] Other solutions differing only by terms dependent on the gauge-invariant Maxwell-Faraday field strength F.

Should we then conclude, on the basis of the criterion (C1), that Dirac has revealed the $\mathcal{U}(1)$ gauge symmetry of electromagnetism to be artificial? We must resist that conclusion because here, contrary to what happened in the Stueckelberg model, gauge-invariance wasn't free; it could be achieved only at the price of locality. Indeed the dressing field $u = e^{iC}$ is clearly non-local, so that the gauge-invariant composite fields $\psi^* = \psi^u$ and $A^* = A^u$ in terms of which electromagnetism theory is rewritten are also non-local variables. It appears then that in classical or quantum electrodynamics, there is a trade-off between gauge symmetry and locality: either one works with local gauge variables, or with non-local gauge-invariant ones.

Such a trade-off is already familiar from the analysis of the Aharonov-Bohm (AB) effect [23; 24]. We recall that one setup of the effect is a modified double slit experiment involving electrons where a solenoid stands behind the first screen between the two slits. When a current traverses the solenoid, the interference pattern formed by the electrons on the second screen is shifted due to a phase factor depending only on the flux of magnetic field inside the solenoid: $e^{i\int_c A} = e^{i\int_s F}$ (c being a closed path from the source of electron beam through the two slits to a point on the final screen and enclosing the surface s including the solenoid). Yet, outside the solenoid - the only region accessible to the electrons - the electromagnetic field strength vanishes, $F = 0$, only the electromagnetic potential A is non-zero. So the latter is the only local variable that is available to maintain a semblance of explanation of the alteration of the behavior of the electrons via a local interaction between fields, A and ψ.

Of course the gauge non-invariance of A makes it a doubtful candidate as a genuine physical field, as many among physicists and philosophers alike have pointed out. Curiously it is not often stressed that if is also true for the spinor field ψ, either seen as a wave function for electrons or as the electron quantum field. Both field variables A and ψ should then be equally faulted for the difficulty in interpreting the AB effect in terms of local interactions of physical field. Therefore several authors didn't shy away from concluding that the AB effect forces us, not to accept the physicality of the gauge potential A (which was usually seen as a computational device in classical electromagnetism) as Aharonov and Bohm argued, but rather that there is such things as non-local electromagnetic properties represented by gauge invariant non-local variables.[5]

This conclusion extends to non-abelian Yang-Mills theories. A beautiful articulation is provided by Healey [25] who argues that the physical content of gauge theories is best represented by the path-ordered trace holonomies of the connection, also known as Wilson loops, which are gauge-invariant non-local variables. The trade-off gauge symmetry *vs* locality is indeed a characteristic features of genuine gauge theories, so that one may argue that what is probed, indirectly, by a substantial gauge symmetry is the existence of non-local physical phenomena. In complement to (C1), I therefore propose the following criterion:

A non-local dressing field in a gauge theory signals that its gauge symmetry is substantial. (C2)

In dirac's scheme, the dressing field exhibited is non-local and so are the gauge-invariant variables (the composite fields). In this case the initial $\mathcal{U}(1)$-gauge symmetry is substantial as it implies that there is non-local phenomena manifested in electromagnetic theory.

Adopting the viewpoint synthesized by the criteria (C1) and (C2), I now turn to an analysis of the Glashow-Weinberg-Salam electroweak theory.

3 The Electroweak Theory without Spontaneous Symmetry Breaking

The opinion that the notion of SSB is pivotal to the success of the electroweak unification is pervasive among physicists and rarely, if ever, questioned. But in the past fifteen years philosopher started to see the notion as suspicious. I here argue that their intuition was correct: the SSB interpretation of the electroweak theory is superfluous to its empirical success. Hints at this conclusion were scattered in the gauge field theory literature for years, from the mid-sixties onward, as we will show in the commentary section 3.2. But first, in the following section we prove the main point in sketching the treatment of the theory via the dressing field method. Further details and comments can be found in [19; 20].

[5] As Aharonov and Bohm also argued! To wit, in [24] p.1513 second paragraph:

"The observable physical effects in question must therefore be attributed to the potential integrals themselves. Such integrals, being not only gauge invariant, but also Hermitian operators, are perfectly legitimate examples of quantum-mechanical observables. They represent extended (non-local) properties of the fields [...]."

3.1 The electroweak model treated via dressing

The gauge group postulated a priori for the model is $\mathcal{H} = \mathcal{U}(1) \times \mathcal{SU}(2) = \{\alpha \times \beta : \mathcal{M} \to U(1) \times SU(2)\}$. The the space of field is $\Phi = \{A, F, \varphi\}$, where $A = a+b$ is the gauge potential 1-form with curvature $F = f_a + g_b$, and φ is a $\mathbb{C}^2$-scalar field. The latter couples minimally with the gauge potential via the covariant derivative $D\varphi = d\varphi + (g'a + gb)\varphi$, with g', g the coupling constants of $U(1)$ and $SU(2)$ respectively. The gauge group acts as:

$$\begin{aligned}
&a^\alpha = a + \tfrac{1}{g'}\alpha^{-1}d\alpha, \qquad && b^\alpha = b, \qquad && \text{and} \quad \varphi^\alpha = \alpha^{-1}\varphi,\\
&a^\beta = a, && b^\beta = \beta^{-1}b\beta + \tfrac{1}{g}\beta d\beta, && \text{and} \quad \varphi^\beta = \beta^{-1}\varphi.
\end{aligned}$$

The $\mathcal{H}$-invariant Lagrangian form of the theory is,

$$\begin{aligned}
L(a, b, \varphi) &= \tfrac{1}{2}\,\mathrm{Tr}(F \wedge *F) + \langle D\varphi,\ *D\varphi\rangle - U(\varphi)\,\mathrm{vol},\\
&= \tfrac{1}{2}\,\mathrm{Tr}(f_a \wedge *f_a) + \tfrac{1}{2}\,\mathrm{Tr}(g_b \wedge *g_b) + \langle D\varphi,\ *D\varphi\rangle - \left(\mu^2\langle\varphi,\varphi\rangle + \lambda\langle\varphi,\varphi\rangle^2\right)\mathrm{vol},
\end{aligned} \tag{8}$$

where $\mu, \lambda \in \mathbb{R}$ and vol is the volume form on spacetime $\mathcal{M}$. As it stands nor a nor b can be massive, and indeed L contains no mass term for them. While at least one massless field is expected in order to carry the electromagnetic interaction, the weak interaction is short range so its associated field must be massive. Hence the necessity to reduce the $\mathcal{SU}(2)$ gauge symmetry in order to allow a mass term for the weak field.

As the usual narrative goes, this is achieved via SSB: if $\mu^2 < 0$, the electroweak vacuum given by $U(\varphi) = 0$ seems degenerate as it appears to be an $\mathcal{SU}(2)$-orbit of non-vanishing vacuum expectation values for φ. When the latter settles randomly - spontaneously - on one of them, this breaks $\mathcal{SU}(2)$ and generates mass terms for the weak fields with which it couples. Oddly, in order to exhibit the physical modes of the theory it is claimed that a convenient choice of gauge is necessary, the so-called *unitary gauge* (see e.g [26]). But how come we are allowed to use a gauge freedom if it is supposedly broken?

We suggest that a better approach and a more satisfactory interpretation is provided by the dressing field method. Indeed it is not hard to find a dressing field in the electroweak model. Considering the polar decomposition in $\mathbb{C}^2$ of the scalar field

$$\varphi = u\eta, \quad \text{with} \quad u \in SU(2) \quad \text{and} \quad \eta := \begin{pmatrix} 0 \\ \|\varphi\| \end{pmatrix} \in \mathbb{R}^+ \subset \mathbb{C}^2, \qquad \text{one has} \quad \varphi^\beta \quad \Rightarrow \quad u^\beta = \beta^{-1}u \tag{9}$$

Thus, u is a $\mathcal{SU}(2)$-dressing field that can be used to construct the $\mathcal{SU}(2)$-invariant composite fields:

$$\begin{aligned}
&A^u = u^{-1}Au + \tfrac{1}{g}u^{-1}du = a + (u^{-1}bu + \tfrac{1}{g}u^{-1}du) = a + B,\\
&F^u = u^{-1}Fu = f_a + u^{-1}g_b u = f_a + G, \qquad \text{with } G = dB + gB^2,\\
&\varphi^u = u^{-1}\varphi = \eta, \qquad \text{and} \qquad D^u\eta = u^{-1}D\varphi = d\eta + (g'a + gB)\eta.
\end{aligned} \tag{10}$$

Since u is *local* so are the composite fields above. Therefore, by virtue of criterion (C1) we conclude that the $\mathcal{SU}(2)$-gauge symmetry of the model is *artificial*, so that the theory defined by the electroweak Lagrangian (8) is actually a $\mathcal{U}(1)$-gauge theory, described in terms of local $\mathcal{SU}(2)$-invariant variables:

$$L(a, B, \eta) = \tfrac{1}{2}\,\mathrm{Tr}(F^u \wedge *F^u) + \langle D^u\eta,\ *D^u\eta\rangle - \left(\mu^2\eta^2 + \lambda\eta^4\right)\mathrm{vol}\,. \tag{11}$$

We already reached our main conclusion: Since the $\mathcal{SU}(2)$-gauge symmetry is artificial the interpretation of the model in terms of SSB is superfluous, and indeed impossible when expressed in the form (11). We could then stop here. But at this point it is not clear that as it stands our analysis reproduces all the phenomenology usually obtained via the standard interpretation. In what follows we show that it is indeed so: It is done simply by proceeding to the natural step of analyzing the residual and substantial $\mathcal{U}(1)$-gauge symmetry of the model, which is very easily done from the viewpoint of the dressing field method.

Residual $\mathcal{U}(1)$-symmetry By its very definition $\eta^\beta = \eta^\alpha = \eta$, so it is already a fully $\mathcal{H}$-gauge invariant scalar field which then qualifies as an observable. As a rule, the $\mathcal{U}(1)$-residual gauge transformations of the $\mathcal{SU}(2)$-invariant composite fields depends on the $\mathcal{U}(1)$-gauge transformation of the dressing field u. One finds that

$$\varphi^\alpha \quad \Rightarrow \quad u^\alpha = u\widetilde{\alpha}, \qquad \text{where} \qquad \widetilde{\alpha} = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{pmatrix}.$$

Therefore $B^\alpha = (b^\alpha)^{u^\alpha} = b^{u\widetilde{\alpha}} = \widetilde{\alpha}^{-1}u^{-1}bu\widetilde{\alpha} + \frac{1}{g}\widetilde{\alpha}^{-1}(u^{-1}du)\widetilde{\alpha} + \frac{1}{g}\widetilde{\alpha}^{-1}d\widetilde{\alpha} = \widetilde{\alpha}^{-1}B\widetilde{\alpha} + \frac{1}{g}\widetilde{\alpha}^{-1}d\widetilde{\alpha}$. Given the decomposition $B = B_a\sigma^a$, where σ^a are the hermitian Pauli matrices and $B_a \in i\mathbb{R}$, we have explicitely

$$B = B_a\sigma^a = \begin{pmatrix} B_3 & B_1 - iB_2 \\ B_1 + iB_2 & -B_3 \end{pmatrix} =: \begin{pmatrix} B_3 & W^- \\ W^+ & -B_3 \end{pmatrix}, \qquad \text{and} \qquad B^\alpha = \begin{pmatrix} B_3 + \frac{1}{g}\alpha^{-1}d\alpha & \alpha^{-2}W^- \\ \alpha^2 W^+ & -B_3 - \frac{1}{g}\alpha^{-1}d\alpha \end{pmatrix}.$$

The fields $W^\pm$ transform tensorially, it is then possible for these two fields to be massive. B_3 transforms as a $\mathcal{U}(1)$-gauge potential, but with a different coupling constant, making it another massless field together with the initial $\mathcal{U}(1)$-gauge potential a. But consider the $\mathcal{U}(1)$ transformation of the $\mathcal{SU}(2)$-*invariant* covariant derivative:

$$D^u\eta = d\eta + (g'a + gB)\eta = \begin{pmatrix} gW^-\eta \\ d\eta - gB_3\eta + g'a\eta \end{pmatrix}, \qquad \text{and} \qquad (D^u\eta)^\alpha = \begin{pmatrix} g\alpha^{-2}W^-\eta \\ d\eta - gB_3\eta + g'a\eta \end{pmatrix}.$$

We see that a $\mathcal{U}(1)$-invariant combination of a and B_3 appears. So, considering (a, B_3) as a doublet in $\mathbb{C}^2$, one is invited to perform the natural change of variables

$$\begin{pmatrix} A \\ Z^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} a \\ B_3 \end{pmatrix} = \begin{pmatrix} \cos\theta_W a + \sin\theta_W B_3 \\ \cos\theta_W B_3 - \sin\theta_W a \end{pmatrix},$$

where $\cos\theta_W = {}^{g}/\sqrt{g^2+g'^2}$ and $\sin\theta_W = {}^{g'}/\sqrt{g^2+g'^2}$ (θ_W being known as the Weinberg, or weak mixing, angle). By construction the 1-form Z^0 is fully $\mathcal{H}$-gauge invariant, thus observable and potentially massive. Now, still by construction, $A^\beta = A$ and $A^\alpha = A + \frac{1}{e}\alpha^{-1}d\alpha$ with coupling constant $e = {}^{gg'}/\sqrt{g^2+g'^2}$. So A transforms as a $\mathcal{U}(1)$-gauge potential, it can thus be interpreted as the massless mediator of the electromagnetic interaction whose coupling constant e is the elementary electric charge.

The electroweak theory (11) is then expressed in terms of the gauge invariant fields η, Z^0 and of the $\mathcal{U}(1)$-gauge fields $W^\pm, A$:

$$L(A, W^\pm, Z^0, \eta) = \tfrac{1}{2}\operatorname{Tr}(F^u \wedge *F^u) + d\eta \wedge *d\eta - g^2\eta^2\, W^+ \wedge *W^- - (g^2 + g'^2)\eta^2\, Z^0 \wedge *Z^0 - \left(\mu^2\eta^2 + \lambda\eta^4\right)\mathrm{vol}\,. \tag{12}$$

The next natural step is to expand the $\mathbb{R}^+$-valued scalar field η around its *unique* groundstate η_0,[6] given by $U(\eta_0) = 0$, as $\eta = \eta_0 + H$ where H is the gauge-invariant Higgs field. Mass terms for Z^0, $W^\pm$ and H depending on η_0 appear from the couplings of the electroweak fields with η and from the latter's self interaction.[7] The theory has two qualitatively distinct phases. In the phase where $\mu^2 > 0$, η_0 vanishes and so do all masses. But in the phase where $\mu^2 < 0$, the groundstate is non-vanishing: $\eta_0 = \sqrt{-\mu^2/2\lambda}$. The masses of the fields $Z^0, W^\pm$ and H are then $m_{Z^0} = \eta_0\sqrt{(g^2 + g'^2)}$, $m_{W^\pm} = \eta_0 g$ and $m_H = \eta_0\sqrt{2\lambda}$. All physical predictions of the electroweak theory are indeed preserved in our treatment: masses are gained through a phase transition of the unique electroweak vacuum.

As a satisfactory concluding aside, notice the fact that this approach to the electroweak model allowing to dispense with the idea of spontaneous breaking of a gauge symmetry offers a neat reconciliation with the so-called Elitzur theorem [28], stating that in lattice gauge theory a gauge symmetry cannot be spontaneously broken.

[6] According to Westenholz [27] the very meaning of the terminology "*spontaneous* symmetry breaking" lies in the fact that the manifold of vacua is not reduced to a point.

[7] Since A does not couple to η directly it is masslessness. The two photons decay channel of the Higgs boson involves intermediary leptons, not treated here but whose inclusion in our scheme is straightforward.

3.2 There is no SSB in the electroweak model and we long suspected it

It turns out that several authors where close to formulating such a gauge-invariant account of the electroweak theory. Even before the theory was proposed, in 1965 - barely a year after his celebrated paper - Higgs hinted at a gauge invariant formulation of the mechanism that ended-up bearing his name by working on a abelian toy model, see section IV in [29]. In 1966, two years after his own celebrated contribution with Guralnik and Hagen, Kibble suggested a similar analysis but working on both abelian and non-abelian models [30]. Just before the conclusion of his paper he writes,

> "We note certain characteristic features of our model. It is perfectly possible to describe it without ever introducing the notion of symmetry breaking, merely by writing down the Lagrangian (66) [i.e, the one written with gauge-invariant variables]. Indeed if the physical world were really described by this model, it is (66) rather than (64) [i.e, the Lagrangian written in terms of gauge variables] to which we should be led by experiment."

With insight it is clear that both Higgs and Kibble were using instances of the dressing field method: see equation (22) and below (dressing and composite fields) in [29] as well as equations (9)-(59) (dressing fields) and (16)-(61) (composite fields) in [30].

But then the Glashow-Weinberg-Salam model was proposed the next year, using the BEHGHK[8] mechanism with its original interpretation in terms of SSB. So these important insights from Higgs and Kibble were eclipsed, and the view of the SSB a real physical phenomenon which had happened in the early universe gain currency. Even notorious names in physics keep perpetuating this narrative. In panel discussion during a large conference on the foundation of quantum field theory gathering physicists as well as historians and philosophers of physics in Boston in 1996, the following exchange took place [31].

> Nick Huggett: [...] And second, what is the mechanism, the dynamics for spontaneous symmetry breaking supposed to be? Are there answers to these questions?
>
> [...]
>
> Huggett: My worry is there's supposed to be a transition from an unbroken symmetry to the [current] state.
>
> Sidney Coleman: With what with temperature? A transition with what, with changing the fundamental parameters of the theory or with -
>
> Huggett: Right. I mean, isn't this a dynamic evolution, something that happens in the history of the universe?
>
> Coleman: Oh, it happens with temperature, yeah. Typically at high temperature you're very far from the ground sate but the density matrix or whatever has the symmetry. Have I got it right, Steve? You were one of the first to work this out.
>
> Steven Weinberg: Yeah, it doesn't always happen, but it usually happens.
>
> Coleman: Yes, typically at high temperature the density matrix has a symmetry which then disappears as the temperature gets lower. But its also true for ordinary material objects. [...] The difference between the vacuum and every other quantum mechanical system is that it's bigger. And that's from this viewpoint the only difference. If you understand what happens to a ferromagnet when you heat it up above the Curie temperature, you're a long way towards understanding one of the possible ways it can happen to the vacuum state.

Yet, the treatment of the electroweak model through the bundle reduction theorem - see e.g [27; 32; 33] - already cast some doubts on the interpretation of the SSB as a dynamical phenomenon. Indeed, thus formulated it appears that the model can naturally be rewritten on a $U(1)$-subbundle of the initial $U(1) \times SU(2)$-bundle.

As far as I know the first to give a fully $\mathcal{SU}(2)$-gauge-invariant formulation of the electroweak theory where Fröhlich, Morchio and Strocchi in 1981 [34]. Their account it actually fully equivalent to ours, but much less synthetic and systematic: they are working on individual scalar component of all the fields involved! See their equations

[8] Brout-Englert-Higgs-Guralnik-Hagen-Kibble, to honor all contributors.

(6.1) describing the composites fields (including dressed electron and neutrino). Subsequently, and especially in the last 10 years, several searchers independently rediscovered the gauge invariant description formulated essentially as in the above treatment, but without the conceptual clarity given by the dressing field method, as often the dressing field was mistaken for an element of the gauge group [35] (see in particular equations (6)-(7) and comment in between) or the interpretive shift was not fully embraced [36; 37]. Interestingly, the textbook by Rubakov gives essentially the dressing treatment of the abelian Higgs model, but stick to the usual treatment of the electroweak model using the unitary gauge, see [38] chapter 6. Some improvement in conceptual clarity is found in [39]. But it is the paper by Masson and Wallet [40] who first really appreciated the interpretive shift that comes with the invariant formulation, and as a matter of fact it was a precursor to the development of the dressing field method. Unfortunately it never get published.

In parallel, in the last few years, philosophers of science have questioned the orthodoxy of the SSB in the electroweak model, noticing the invariant formulation [41–45]. Earman first raised the issue in striking terms:

> "But what exactly is accomplished [in the BEHGHK mechanism] is hidden behind the veil of gauge redundancy. The popular presentations use the slogan that the vector field has acquired its mass by 'eating' the Higgs field. [...] The popular slogan can be counterbalanced by the cautionary slogan that neither mass nor any other genuine attribute can be gained by eating descriptive fluff. None of this need be any concern for practicing physicists who know when they have been presented with a fruitful idea and are concerned with putting the idea to work. But it is a dereliction of duty for philosophers to repeat the physicists' slogans rather than asking what is the content of the reality that lies behind the veil of gauge."
>
> Earman 2004, [46] pp189-190.

Shortly latter, he reiterated:

> "Readers of Scientific American can be satisfied with these just-so stories. But philosophers of science should not be. For a genuine property like mass cannot be gained by eating descriptive fluff, which is just what gauge is. Philosophers of science should be asking the Nozick question: What is the objective (i.e., gauge invariant) structure of the world corresponding to the gauge theory presented in the Higgs mechanism?"
>
> Earman 2004, [47] pp1239.

Emphasizing Dirac's constrained Hamiltonian formalism as a systematic way to extract the gauge-invariant quantities of a gauge system he asks:

> "What is the upshot of applying this reduction procedure to the Higgs model and then quantizing the resulting unconstrained Hamiltonian system? In particular, what is the fate of spontaneous symmetry breaking? To my knowledge the application has not been carried out. [...]
>
> While there are too many what-ifs in this exercise to allow any firm conclusions to be drawn, it does suffice to plant the suspicion that when the veil of gauge is lifted, what is revealed is that the Higgs mechanism has worked its magic of suppressing zero mass modes and giving particles their masses by quashing spontaneous symmetry breaking. However, confirming the suspicion or putting it to rest require detailed calculations, not philosophizing."
>
> Earman 2004, [46] pp190-191.

Here Earman's preferred formalism wasn't used, but we have seen that the dressing field method allows to easily lift the "veil of gauge" and that in so doing Earman's suspicion was fully vindicated. I hope that this resolution will give philosophers of physics satisfaction.

4 Closing statement, open questions

The dressing field method puts forward a reasonable criterion to decide if the gauge symmetry of a theory is artificial or substantial. If a gauge theory contains a local dressing field, it can be rewritten in terms of local gauge-invariant composite fields. Nothing is then lost in erasing the gauge symmetry, so one can argue that it was artificial, stemming from an uneconomic - a "non-Ockhamized" - choice of variables. If on the contrary a gauge theory contains only a non-local dressing field, then its gauge symmetry can be erased only at the price of rewriting it in terms of non-local gauge-invariant variables. The trade-off between gauge symmetry and locality is characteristic of a substantial gauge symmetry, which signals the existence of non-local physical phenomena (as now classical analysis of the AB effect exemplify).

Armed with this criterion one shows that the $\mathcal{SU}(2)$-gauge symmetry in the electroweak model is artificial, canceling the need for the notion of gauge SSB, and that only the residual $\mathcal{U}(1)$-gauge symmetry is substantial. Provocatively, one could say that the substantial gauge group of the Standard Model of particle physics is therefore not $\mathcal{U}(1) \times \mathcal{SU}(2) \times \mathcal{SU}(3)$, but merely $\mathcal{U}(1) \times \mathcal{SU}(3)$!

It is the job of both mathematical physicists and philosopher of physics to prune a theory from any superfluous notion that pertains to the context of discovery so has to reveal its core conceptual and technical structure, and to clear the horizon of its context of justification. Here we conclude that the notion of gauge SSB pertains to the context of discovery of the electroweak unification: it has historical interest and has been a valuable heuristic guide to the correct theory. But it cannot belong to the context of justification.

A puzzling facts remains: How are we to understand that the artificial $\mathcal{SU}(2)$ formulation of the model - such as suggested by the gauge principle - is much more structurally simple than the substantial $\mathcal{U}(1)$ - and phenomenologically clearer - formulation? Let us continue the quotation of Kibble from [30]:

> "Indeed if the physical world were really described by this model, it is (66) [the Lagrangian written with gauge-invariant variables] rather than (64) [the Lagrangian exhibiting gauge symmetry] to which we should be led by experiment. The only advantage of (64) is that it is easier to understand the appearance of an exact symmetry than of an approximate one. Experimentally, we would discover the existence of a set of four vector bosons with different masses but whose interactions exhibited a remarkable degree of symmetry. We would also discover a pair of scalar particles forming an apparently incomplete multiplet under the group describing this symmetry. In such circumstances it would surely be regarded as a considerable advance if we could recast the theory into a form described by the symmetric Lagrangian (64)."

But given that the gauge symmetry is in this circumstance artificial, it must me clarified in what respect it is an advance. And if $\mathcal{SU}(2)$ is artificial and as such should not tell us anything important, it is a remarkable feat that the model guessed from it eventually had such predictive power.[9] Are we to believe that the distinction artificial *vs* substantial gauge symmetry does not capture all important theoretical differences and must be reconsidered?

I find this unlikely. My guess is that it remains to determine what constitutes the proper context of justification for the electroweak theory. The gauge principle associated with the substantial $\mathcal{U}(1)$-symmetry is clearly insufficient. And if a phenomenological a posteriori reconstruction is possible, it does not illuminates the key ideas or principles that might explain the form of the theory. Actually the question stands: is there such principles that would make the theory something other than a raw fact? Renormalizability of the quantum theory may come to mind as a powerful constraining factor, but is it to be elevated to such a high position in the explicative hierarchy? Effective field theory physicists would disagree. It is admitted that the Standard Model should be a low energy limit of a more fundamental theory. The governing principle we search for may then be part of the new framework in terms of which the latter is expressed. Could it be a new geometric framework, such a non-commutative geometry or transitive Lie algebroids? Could it be a firmer mathematical foundation for quantum field theory, such as the algebraic formulation or category theory? Reversing the logic, it may be that pondering on what explains the form

[9]The model was proposed in 1967 and predicted the neutral weak current, a given relation between the masses of the $W^{\pm}$ and the Z^0 as well as the existence of a scalar boson. The neutral weak current was discovered in 1973 in the Gargamelle experiment at CERN. The $W^{\pm}$ bosons were discovered in January 1983 and the Z^0 boson in May 1983, also at CERN. Finally the scalar boson's discovery at the LHC was announced in July 2012.

of the electroweak unification could provide hints on this as yet unidentified framework and on what lies beyond the Standard Model.

These questions can be genuinely explored only if the orthodoxy of SSB, a context of justification turned into a common wisdom, is challenged. Philosopher of physics have spearheaded that challenge in the past fifteen years. It is to be hoped that the community of physicists catches up quickly.

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