The Quantum Mechanics of Two Interacting Realities

Chris Allen Broka
(chris.broka@gmail.com)

Abstract.

We consider how two physical realities can be represented over a common set of spacetime coordinates. As an example we will utilize quantum electrodynamics since this is a familiar and well-understood theory. We will designate one world the 'red' one and the other the 'green' one. We will try to show how they can interact in a physically plausible way. We will also examine whether such an interacting theory is renormalizable. It will be shown that we can extend these ideas to the Standard Model. There are implications for this theory if we consider General Relativity and these will be discussed briefly.

Introduction.

There are many quantum field theories that explain physical processes that we see and measure around us. They, mostly, attempt to describe the physics of a single, discreet, reality. There is, of course, a very good reason for this – we have no compelling evidence for the existence of any other realities. All the same, it might be interesting to theorize about them. But this can only be done subject to the constraint that nothing implausible is proposed – anything that would plainly violate our everyday observations should not result from such speculation.

I will try to show that a theory of multiple realities can, in fact, be constructed using quantum electrodynamics (QED) as a simple and familiar example. We can imagine one reality – call it the 'red' one populated with 'red' electrons and 'red' photons. We will suppose that there is another, 'green,' reality populated with 'green' electrons and photons. There will exist a *common coordinate system* between them. For the moment we will take this idea in a very literal way and just imagine that the various fields are all functions of a common coordinate system defined over a shared Minkowski spacetime. If we are observers living in the 'red' reality we will imagine that the 'green' reality exists all around us and is defined over whatever spacetime coordinate system we decide to use. Ordinarily, we just cannot see this 'green' reality because its particles do not interact with our 'red' ones. We will introduce a new function, $c(\boldsymbol{x}, t)$, which reflects the degree to which the 'red' and 'green' realities interact with one another. The reader will want a more mathematically rigorous description of this and such will be forthcoming. We will always assume that the laws of physics are the same in both realities and that the two kinds of electrons have the same mass and charge in their respective realities.

QED in Two Realities.

We start out by writing the Lagrangian as it would look if these realities were always completely independent:

1) $$L_{\text{em}} = \overline{\psi_R}\,[\gamma^\mu[\mathrm{i}\,\partial_\mu - e\,A_{R\,\mu}] - \mathrm{m}]\psi_R - \tfrac{1}{4}F_R{}^{\mu\nu}F_{R\,\mu\nu} + \overline{\psi_G}\,[\gamma^\mu[\mathrm{i}\,\partial_\mu - e\,A_{G\,\mu}] - \mathrm{m}]\psi_G - \tfrac{1}{4}F_G{}^{\mu\nu}F_{G\,\mu\nu}.$$

The objects ψ_R, A_R, are understood to pertain to the 'red' reality. The 'G' subscript means they belong to the 'green' reality. A common spacetime coordinate system is shared by both the 'red' and 'green' particles. $F_R{}^{\mu\nu}$ is the electromagnetic field strength tensor appropriate to the 'red' world. $F_G{}^{\mu\nu}$ pertains to the 'green' reality. Now an interaction between these realities could occur if there were to take place a *mixing* of A_R and A_G in their interaction with the electron fields according to:

2) $A_{R\mu} \longrightarrow \left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2} [A_{R\mu} + c(\boldsymbol{x}, t)\ A_{G\mu}]$ and

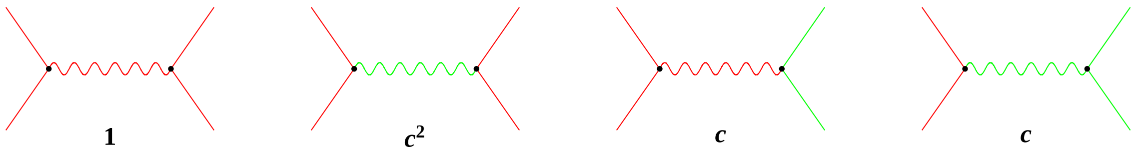
$A_{G\mu} \longrightarrow \left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2} [A_{G\mu} + c(\boldsymbol{x}, t)\ A_{R\mu}]$ where $c(\boldsymbol{x}, t)$ is taken to be a real scalar field.

Note that this mixing of quantum fields is confined to the photon fields. It is not applied to the electron fields. Nor is it applied it within the electromagnetic field strength tensors. When $c(\boldsymbol{x}, t) = 0$ there is no interaction. As $c(\boldsymbol{x}, t)$ becomes larger 'red' observers begin to experience some of the 'green' reality and vice-versa. $\left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2}$functions as a kind of normalization factor. Under the influence of this transformation the Lagrangian becomes:

3) $$L_{\text{em}} = \overline{\psi_R}\,[\gamma^\mu[\text{i}\,\partial_\mu - e\left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2}\left[A_{R\mu} + c(\boldsymbol{x}, t)\ A_{G\mu}\right]] - \text{m}]\psi_R - \tfrac{1}{4}F_R{}^{\mu\nu}F_{R\,\mu\nu}$$
$$+ \overline{\psi_G}\,[\gamma^\mu[\text{i}\,\partial_\mu - e\left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2}\left[A_{G\mu} + c(\boldsymbol{x}, t)\ A_{R\mu}\right]] - \text{m}]\psi_G - \tfrac{1}{4}F_G{}^{\mu\nu}F_{G\,\mu\nu}.$$

We will assume, for the moment, that $c(\boldsymbol{x}, t)$ is roughly constant over the spacetime volume of interest. Also note that $c(\boldsymbol{x}, t)$ is, right now, not a dynamical variable of this theory. It is like a physical "constant" that changes with time and space. While this new Lagrangian maintains local gauge invariance only under circumstances where $c(\boldsymbol{x}, t)$ is constant (1), it has the advantage of resulting, under these circumstances, in simple Feynman rules and a physics which, in many respects, corresponds with that we would like to see for a theory that doesn't grossly violate observed reality. In situations where $c(\boldsymbol{x}, t)$ varies things become more complicated. And we must address this problem since our theory would be either not interesting or not believable, physically, if $c(\boldsymbol{x}, t)$ could never change.

These new Feynman rules are similar to the familiar ones but with two important differences: Firstly, the vertices connecting an incoming and outgoing 'red' electron (or positron) line with a 'red' photon contribute with a coupling constant $e\left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2}$. It is likewise for the 'green' particles. Secondly, new vertices appear which connect incoming and outgoing 'red' electron (or positron) lines with a 'green' photon and incoming and outgoing 'green' electron (or positron) lines with a 'red' photon (*fig.1*). (In the first two cases we omit drawing the graphs with the outgoing electrons exchanged. But we know they are there.) These contribute with a coupling constant which is $e\,c(\boldsymbol{x}, t)\left(1 + c(\boldsymbol{x}, t)^2\right)^{-1/2}$. Consider the scattering of one 'red' electron off another in the presence of an interaction. To find the probability amplitude for this process (to second order in the coupling constant) we will sum the amplitudes corresponding to the usual Feynman diagrams and new diagrams in which it is a 'green' virtual photon that is being exchanged. Straightforward arithmetic shows that the overall coupling constant is still e. Thus the resulting amplitude is unchanged by the presence of the interaction. The contribution from the 'green' virtual photon compensates exactly for the reduction in the coupling strength of the normal interaction. This is encouraging – as long as we are dealing with interactions between 'red' particles and other 'red' particles, electromagnetism should continue to work normally in the 'red' world even if $c(\boldsymbol{x}, t)$ became different from zero. The same situation would obtain in the 'green' world. Suppose, instead, that we try to scatter 'red' electrons off of 'green' electrons. Now things are a little different. In each of the two relevant Feynman diagrams would be a vertex connecting either 'green' fermions with a virtual 'red' photon or 'red' fermions with a 'green' virtual photon. Arithmetic again yields a simple result. If we are 'red' observers looking at the behavior of 'red' electrons, we would have to conclude that the 'green' electrons had a charge that was only $2\,c(\mathbf{x},\,t)\left(1+\,c(\mathbf{x},\,t)^2\right)^{-1} e$. We would always assume that our 'red' electrons have charge e. If the 'green' electrons scatter abnormally, it must because they have a reduced charge. Also, since there are no vertices connecting an incoming 'red' electron with an outgoing 'green' electron, the scattering would be the same as that produced by two non-identical particles; this makes sense as we would not want to say that 'green' and 'red' particles are indistinguishable.



$\frac{1}{c^2+1}$ $\qquad$ $\frac{c^2}{c^2+1}$ $\qquad$ $\frac{c}{c^2+1}$ $\qquad$ $\frac{c}{c^2+1}$

fig.1

The Classical Limit.

We want to know what the physics resulting from this would look like to an ordinary, macroscopic, observer. And it is not clear how much more we can do in a quantum mechanical way. There, if we do not regard $c(\mathbf{x},\,t)$ as a constant, we have no easy way of doing the math. Let us look at Equation 3) from a semi-classical point of view. We must recall that, according to Dirac theory, the 4-current density in the 'red' world is given by $e\,\overline{\psi_R}\,\gamma^\mu\psi_R$, and by $e\,\overline{\psi_G}\,\gamma^\mu\psi_G$, in the 'green' one. Varying Equation 3) by $A_{R\mu}$ we find:

4) $$F_R{}^{\mu\nu}{}_{,\nu} = J^\mu\Big/\left(1+c(\mathbf{x},\,t)^2\right)^{1/2} + \tilde{J}^\mu\,c(\mathbf{x},\,t)\Big/\left(1+c(\mathbf{x},\,t)^2\right)^{1/2}$$

where J^μ denotes the 4-current density in the 'red' world, and $\tilde{J}^\mu$ that in the 'green' world. Varying by $A_{G\mu}$, we find a corresponding equation for things the 'green' world. Let us now vary Equation 3) by $\overline{\psi_R}$ so as to get the Dirac equation for the behavior of 'red' electrons. We find:

5) $$[\gamma^\mu[\mathrm{i}\,\partial_\mu - e\left(1+\,c(\mathbf{x},\,t)^2\right)^{-1/2}\left[A_{R\mu} +\, c(\mathbf{x},\,t)\,\,A_{G\mu}\right]] - \mathrm{m}]\psi_R = 0.$$

This tells us what effective "4-potential" the 'red' electron is responding to. We can perform the same exercise for the 'green' Dirac equation. We obtain, as a practical matter, a Lorentz force law for 'red' electron which reads:

6) $$m\,\ddot{x}_R{}^{\mu} = e\Big(F_R{}^{\mu}{}_{\nu}\Big/\left(1+c(\mathbf{x},\,t)^2\right)^{1/2} + \,F_G{}^{\mu}{}_{\nu}\,c(\mathbf{x},\,t)\Big/\left(1+c(\mathbf{x},\,t)^2\right)^{1/2} - \left(1+c(\mathbf{x},\,t)^2\right)^{-3/2}\left[c(\mathbf{x},\,t)\,\left(c\,(\mathbf{x},\,t)^{,\mu}\,A_{R\nu} - c(\mathbf{x},\,t)_{,\nu}\,A_R{}^{\mu}\right) - \left(c\,(\mathbf{x},\,t)^{,\mu}\,A_{G\nu} - c(\mathbf{x},\,t)_{,\nu}\,A_G{}^{\mu}\right)\right]\Big)\,\dot{x}_R{}^{\nu}.$$

And we will obtain a reversed version for the 'green' electron, having the 'R's and 'G's interchanged.

Equation 6) is actually rather remarkable as it shows that we can deduce useful things by *not* trying to use the quantized theory. Equation 6) follows from 5) in the most simple way. We *know* that Dirac's Equation – the one with A_μ as we are used to seeing it – gives us the familiar Lorentz force law when translated into the classical world. (It is actually rather hard to deduce this mathematically. But it is certainly true.) Thus by treating the strange term that appears in Equation 5) *exactly as if* it were A_μ (i.e. constructing an $F_{\mu\nu}$ from it) we arrive at Equation 6). And it must be true.

It will be observed that this equation of motion does not respect local gauge invariance, nor should it. As has been mentioned, gauge invariance requires the constancy of $c(\boldsymbol{x}, t)$. And simply specifying a gauge will not help us here. We could require, for example, $\partial_\mu A_{R,G}{}^\mu = 0$. But this, alone, is insufficient. We could imagine adding a 4-vector, $\Lambda^{,\mu}$, to either A^μ and this would not disturb the gauge condition so long as $\Lambda^{,\mu}{}_\mu = 0$. It would, however, change Equation 6). The $A_{R,G}{}^\mu$ in this theory must be definite, unambiguous, and not subject to the addition of any factors. We would be better off endowing both of our photons with a vanishingly small mass. In effect we add terms $\epsilon^2 A_{R,G}{}^\mu A_{R,G_\mu}$ to the Lagrangians for our two photons (understanding that ϵ is so small that it can be taken to zero at the end of any practical calculation). The dynamical equations for the two A fields become Proca equations. This is invaluable both because it automatically ensures $\partial_\mu A_{R,G}{}^\mu = 0$ *and* also rules out the addition of any intrusive gradients to our A fields.

No assumptions regarding the constancy of $c(\boldsymbol{x}, t)$ have been made in deriving Equations 4) and 6) (and their two 'green' counterparts). These will be true under any circumstances. It seems likely that, under many circumstances, $c(\boldsymbol{x}, t)$ can be treated as, more-or-less, a constant. This allows us to make some simplifications to the mathematics. Since all we are interested in is the effective field that 'red' or 'green' electrons respond to, let us simplify matters by writing:

7) $F^{\mu\nu} = F_R{}^{\mu\nu} \Big/ \left(1 + c(\boldsymbol{x}, t)^2\right)^{1/2} + F_G{}^{\mu\nu}\, c(\boldsymbol{x}, t) \Big/ \left(1 + c(\boldsymbol{x}, t)^2\right)^{1/2}$ and

8) $\tilde{F}^{\mu\nu} = F_G{}^{\mu\nu} \Big/ \left(1 + c(\boldsymbol{x}, t)^2\right)^{1/2} + F_R{}^{\mu\nu}\, c(\boldsymbol{x}, t) \Big/ \left(1 + c(\boldsymbol{x}, t)^2\right)^{1/2}$.

It now becomes possible to write Maxwell's equations and the Lorentz force law, in the presence of an interaction, in a more compact form:

9) $F^{\mu\nu}{}_{,\nu} = J^\mu + 2\, \tilde{J}^\mu\, c(\boldsymbol{x}, t) \Big/ \left(1 + c(\boldsymbol{x}, t)^2\right)$

10) $F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0$

11) $\tilde{F}^{\mu\nu}{}_{,\nu} = \tilde{J}^\mu + 2\, J^\mu\, c(\boldsymbol{x}, t) \Big/ \left(1 + c(\boldsymbol{x}, t)^2\right)$

12) $\tilde{F}_{\alpha\beta,\gamma} + \tilde{F}_{\beta\gamma,\alpha} + \tilde{F}_{\gamma\alpha,\beta} = 0$

13) $m\, \ddot{x}_R{}^\mu = e\, F^\mu{}_\nu\, \dot{x}_R{}^\nu$

14) $m\,\ddot{x}_G^{\;\mu} = e\,\tilde{F}^{\mu}{}_{\nu}\,\dot{x}_G^{\;\nu}$

where $F^{\mu\nu}$denotes the classical electromagnetic field strength tensor, measured by the 'red' physicist, and $\tilde{F}^{\mu\nu}$ that measured similarly by the 'green' one.

Now the 4-divergences of the left-hand sides of Equation 4), and the 'green' version thereof, both vanish identically owing to the antisymmetry of $F_R{}^{\mu\nu}$ and $F_G{}^{\mu\nu}$. The current densities should also have vanishing 4-divergences. This implies that:

15) $c(\boldsymbol{x}, t)_{,\,\mu}\, J^{\mu} = 0$ and

$c(\boldsymbol{x}, t)_{,\,\mu}\, \tilde{J}^{\mu} = 0.$

Equations 15) put some definite constraints on what $c(\boldsymbol{x}, t)$ can do.

By J^{μ} we do mean $e\, \overline{\psi_R}\, \gamma^{\mu}\psi_R$ and likewise for the 'green' current. This may cause some confusion because it would seem that $J^0 = e\, \psi_R{}^{\dagger}\psi_R$ which would have to be negative everywhere. Of course, J^0 is, properly, to be understood as a field theoretic operator. (We have presented some arguments in terms of one-particle Dirac theory just to establish a few simple facts.) Considered as an operator J^0 would, in most places, have a positive expectation value in a world full of 'red' positrons and a negative one if the world were dominated by 'red' electrons. Thus J^{μ} assumes a role similar to that of a classical electromagnetic current. Really, by J^{μ} we mean $\langle\Psi|J^{\mu}|\Psi\rangle$ where $|\Psi\rangle$ designates the state of this twofold World in a kind of extended Fock space populated with both 'red' and 'green' particles. We assume this Fock space to have a vacuum state and that its basis states are constructed from it by the sequential action of the multiple creation operators that correspond to the 'green' and 'red' particles in our theory. We assume, also, that 'green' and 'red' operators always commute – they simply do not see one another and act on their respective Fock "subspaces" independently. We work in the Dirac Interaction Picture.

In most reasonable and electrically neutral worlds we can assume the electromagnetic currents to be zero in most places. At worst they will, in the classical limit, be non-zero only at the specific locations of 'red' and 'green' point electrons or positrons. Elsewhere $c(\boldsymbol{x}, t)$ is free to change subject to whatever other physics guides it.

The Role of $c(\boldsymbol{x}, t)$.

There are many things this theory cannot tell us about $c(\boldsymbol{x}, t)$. The most important of these is whether it should be treated as a dynamical variable of the theory – one with its own place in the Lagrangian of our twofold reality – or as a completely external variable. First, let's suppose it's the latter way. Then $c(\boldsymbol{x}, t)$ is rather like a physical "constant" that happens to vary with space and time.

Suppose that $c(\boldsymbol{x}, t)$ is, initially, zero everywhere but that it becomes a bit bigger than zero in a small area where both a 'red' and a 'green' physicist have an electron of their own type under observation. As the 'red' electron starts to move under the influence of its 'green' counterpart, and vice-versa, both physicists will be amazed that 4-momentum is not being conserved. But there is no reason why it should be. Conservation of 4-momentum follows from Noether's Theorem and relies on the independence of the Lagrangian from space and time. By allowing $c(\boldsymbol{x}, t)$ to change with space and time we have destroyed this invariance. Conservation of 4-momentum will only hold if $c(\boldsymbol{x}, t)$ is constant everywhere. In a small area where $c(\boldsymbol{x}, t)$ is constant there will be a conserved 4-momentum but it will be the sum of the 4-momenta present in both worlds plus any interaction energy between the variously colored particles involved. Charge conservation also becomes an ambiguous concept when $c(\boldsymbol{x}, t)$ changes.

Maybe $c(\boldsymbol{x}, t)$ ought to be regarded as a dynamical variable of this theory rather than as something that has to be introduced in an arbitrary way. It would then be possible to define a rigorously conserved 4-momentum. One could incorporate $c(\boldsymbol{x}, t)$ into the Lagrangian 3) by any number of means. Suppose we try the simplest one:

16) $L_{\text{real}} = L_{\text{em}} + \kappa\, c(\boldsymbol{x}, t)_{,\mu}\, c(\boldsymbol{x}, t)^{,\mu}$ (where κ is a real constant).

If we assume $c(\boldsymbol{x}, t)$ is always quite small we end up with a wave equation for $c(\boldsymbol{x}, t)$ having a source term proportional to:

17) $J^{\mu} A_{\text{G}\mu} + \tilde{J}^{\mu} A_{\text{R}\mu}$.

It is not obvious that such an equation leads us to any productive physics. We would need simultaneous knowledge of both the 'red' and 'green' realities to evaluate it in any particular case. And we can, of course, propose other 'kinetic' terms for $c(\boldsymbol{x}, t)$, if we prefer those, and end up with a different theory. In any case, such a theory would lead to a conserved 4-momentum derivable from L_{real}. But this would no longer resemble that which we conventionally recognize as 4-momentum. It would contain terms involving $c(\boldsymbol{x}, t)$.

We do not want a theory that blatantly contradicts observed reality. Adopting something like Equation 16) might lead to consequences that would have been noticed long ago unless we arrange things (e.g. κ) in such a way that those consequences would always be so small as to be imperceptible. And that would not lead to an interesting theory. Also, Equations 15) already put severe constraints on what $c(\boldsymbol{x}, t)$ can do. Imposing any further dynamical constraints on it might confine us to a very uninteresting theory. We are, perhaps, better off regarding $c(\boldsymbol{x}, t)$ as something like a physical "constant" that varies, according to its own unknown physics. If $c(\boldsymbol{x}, t)$ does not become very large in very many places, we might well not have noticed it. Also it seems to seldom fluctuate much over atomic time and distance scales. If it did, this would result in easily noticed disturbances to our atoms' behavior.

The Importance of Congruence Between the Realities.

Referring back to Equations 15) we notice some interesting things. The 4-gradient of $c(\boldsymbol{x}, t)$ is constrained only where J^{μ} or $\tilde{J}^{\mu}$ differs from zero. Where they do not, $c(\boldsymbol{x}, t)$ is free to change as it wishes. Suppose that both J^{μ} and $\tilde{J}^{\mu}$ differ from zero in some area. This places two constraints on the 4-gradient of $c(\boldsymbol{x}, t)$ and would restrict more stringently the forms an interaction could take. Of course, if $J^{\mu} = \tilde{J}^{\mu}$ the number of constraint equations drops back to one. It should then be easier for an interaction to take place. The less different the two realities are the more freedom $c(\boldsymbol{x}, t)$ has to change. And, if we want to consider a kinetic term (as in Equation 16)) we see that the source term, Equation 17), would usually average out to zero if the 'red' and 'green' worlds were completely different and unrelated. If the two realties are rather similar the source term may have a better chance of becoming large in certain locations.

Is Such a Theory Renormalizable?

I consider this in the simple case where $c(\boldsymbol{x}, t)$ may be treated as a constant over the volume of spacetime where the interactions of interest are taking place. The 'red' and 'green' photon loops that figure in calculating the vertex correction and electron self-mass terms sum to results that differ in no essential way from those encountered in normal QED. Of course, there are twice as many particles to keep in mind. But, otherwise, nothing important is changed and we can renormalize these in the usual way.

The fermion loops that renormalize the photon propagators – the vacuum polarization terms – require a more careful treatment. These loops can and do link incoming 'red' photon lines to outgoing 'green' photon lines and vice-versa. There is, accordingly, some amplitude for a 'red' photon to be created at one vertex only to be absorbed as a 'green' photon somewhere else. We were very happy when the second-order diagrams in *fig. 1* showed that 'red' electrons would see each other's charges as e no matter what $c(\boldsymbol{x}, t)$ did. We are less happy when we inspect *fig. 2* and find the intrusion of an additional factor $4\left(c(\boldsymbol{x}, t)^2/(1 + c(\boldsymbol{x}, t)^2)\right)$ coming from the one-loop diagrams. This problem shows up at the e^4 order. The closed loops diverge and must be regularized.

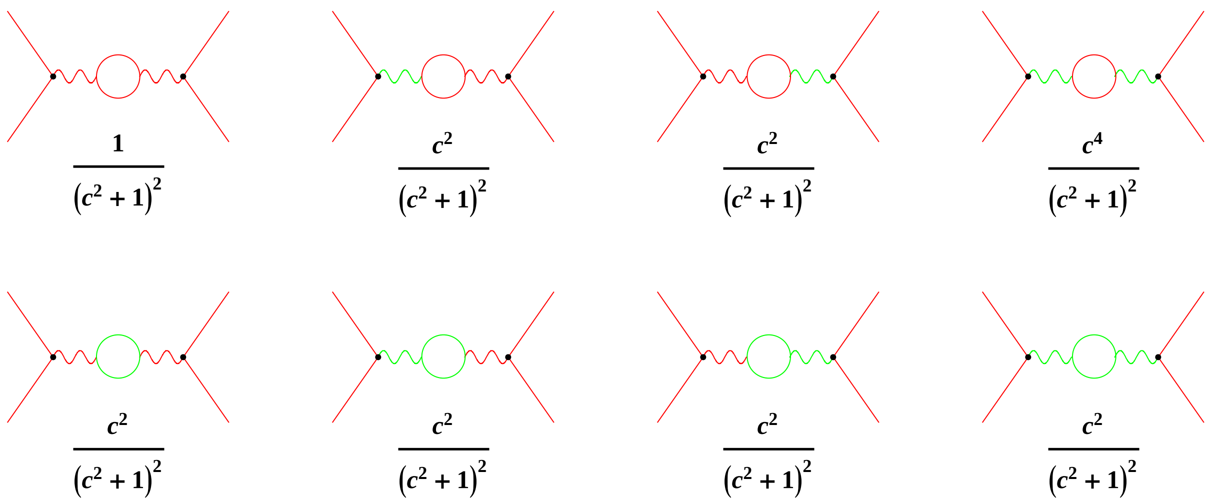


fig.2

A propagator represents the amplitude for a particle to be created at one spacetime point and absorbed at another. It is meant to be evaluated in its free-field theory and no interactions should be allowed for it once it has been renormalized. A 'red' photon must therefore always be absorbed as 'red.' But there are many ways in which this can happen. Let us look at *fig. 3*. At the one-loop level everything seems fine. But we see now just what the problem is at the two-loop level. Again we find the factor $4\left(c(\boldsymbol{x}, t)^2/(1 + c(\boldsymbol{x}, t)^2)\right)$. By renomalizing this situation (there are a variety of methods we can imagine using) we absorb it into the new physical charge, e_{phys}, which is what we actually measure in the laboratory. This accommodates the problem that seemed to stem from *fig. 2*. Electron scattering will then proceed through finite diagrams just like the (loop-less) two left-most diagrams in fig. 1. but with e replaced by e_{phys}. "Red' virtual photon lines will stay 'red' and 'green' ones 'green.' It should be pointed out that a real ($k^\mu\ k_\mu = 0$) photon will always retain its color. Moving at c these states are frozen, so to speak, in time.

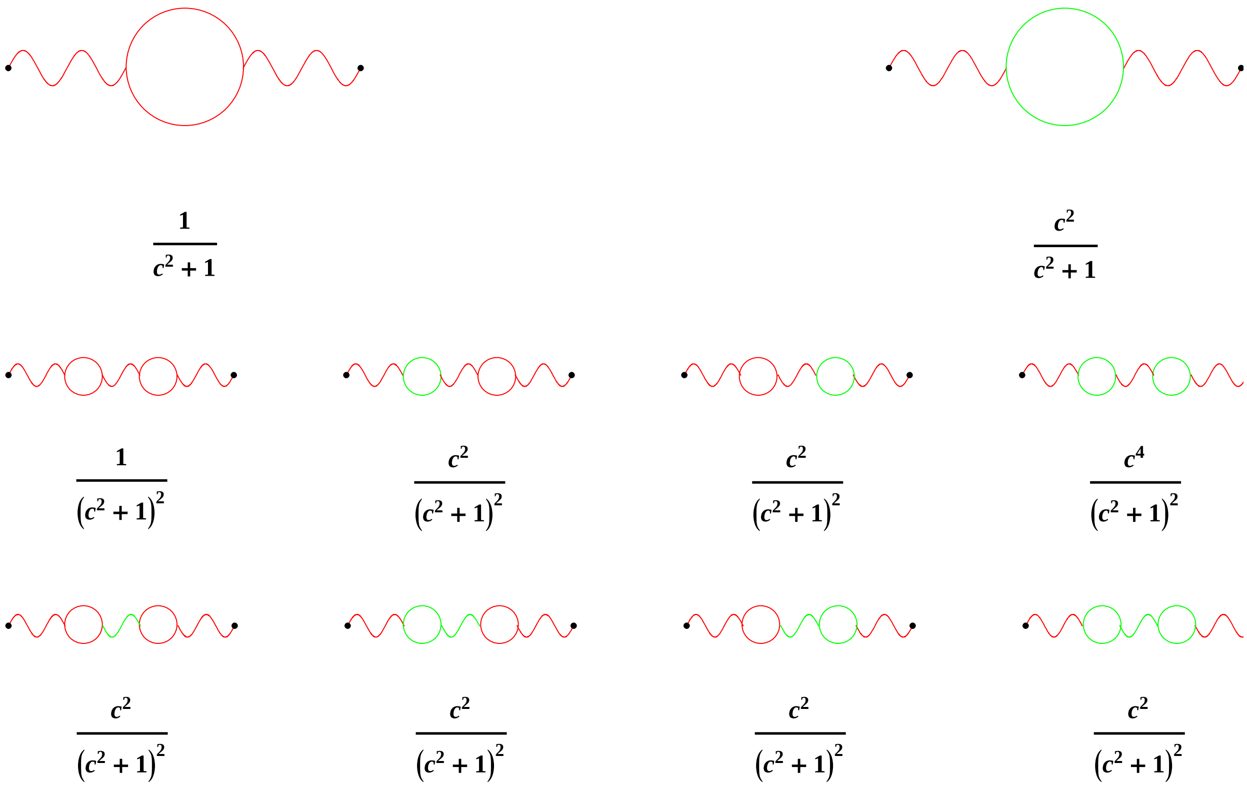


fig.3

Actually, there is an easier way of arriving at the same result. We can simply rotate A_R and A_G into $(A_1 + A_2)/\sqrt{2}$ and $(A_1 - A_2)/\sqrt{2}$, respectively. (We are not mixing anything or doing any strange physics here. We are just giving new names to old things.) Reexpressed in terms of these fields we get a new Equation 3). The kinetic terms for the $A_{R,G}$ fields stay, formally, unchanged. The newly written fermionic terms give rise to vertices such that A_1 photon lines, although they may be interspersed with 'red' and 'green' electron loops (whose net contributions always sum to one), never turn into A_2 lines, and likewise for the A_2. So both the A_1 and A_2 photon lines may be renormalized *exactly* as they are in normal QED. After this the result is simply rewritten in terms of A_R and A_G.

$c(\mathbf{x}, t)$ under Various Circumstances.

Suppose that both worlds are always identical – $\psi_R = \psi_G$, $A_R = A_G$, always. Equation 3) now describes a situation void of any distinction between 'red' and 'green' particles. It describes a single reality with an electromagnetic coupling constant that depends on $c(\mathbf{x}, t)$. If this is constant everywhere we can just reset e and recover perfectly normal physics. If $c(\mathbf{x}, t)$ varies we end up with a strange world in which the electromagnetic interaction changes from place to place. We do not seem to live in such a world and this simple possibility is ruled out unless $c(\mathbf{x}, t)$ never varies by more than an unnoticeable amount. Dropping the requirement that $\psi_R = \psi_G$ leads to the same situation but with 'red' and 'green' electrons that do not behave as identical particles but still interact through a common photon. We are better off assuming that our two worlds are not constrained to be identical.

If $c(\mathbf{x}, t)$ were to become just slightly different from zero over a defined area and time, 'red' observers within this area would be able to "see" the 'green' and 'red' photons emitted by vibrating 'green' electrons in the 'green' world. These would become more apparent as $c(\boldsymbol{x}, t)$ increased. Now it might be possible for $c(\boldsymbol{x}, t)$ to become less than zero. If this happened the 'green' elections would appear to be positively charged – a strange, but not unimaginable, circumstance.

Even if $c(\boldsymbol{x}, t)$ became different from zero in some spacetime volume, this would not affect the local electromagnetic interactions between 'red' particles and other 'red' particles, and 'green' ones with 'green' in that volume. If we lived in the 'red' reality, we would not see our 'red' atoms fall apart if $c(\boldsymbol{x}, t)$ changed. This is, of course, very encouraging if we want this idea to be considered plausible. But this is not to say that $c(\boldsymbol{x}, t)$ would be devoid of observable consequences, even in our 'red' world. Consider the decay of 'red' positronium. We can easily write down the necessary Feynman diagrams. We find that the overall rate of its decay, in an area where $c(\boldsymbol{x}, t)$ is non-zero, is reduced by a factor of $1 - (c(\boldsymbol{x}, t))^2/\left(1 + c(\boldsymbol{x}, t)^2\right)^2$. In particular, the rate at which it will decay into two 'red' photons is reduced by a factor of $1/\left(1 + c(\boldsymbol{x}, t)^2\right)^2$. If we are 'red' observers looking at all this from outside the high $c(\boldsymbol{x}, t)$ area we will not be able to see the green photons resulting from this process. We will only see our 'red' positronium decaying, somewhat slowly, into normal 'red' photons sometimes, into only one 'red' photon other times, and, occasionally, into nothing at all! A rather disconcerting, but potentially observable, situation. In this situation neither 4-momentum nor spin will always be conserved, according to the 'red' observer.

Suppose that a 'red' observer ventured into a spacetime volume where $c(\boldsymbol{x}, t) = 1$. Within that volume his atoms would function normally. If their electrons vibrated they would they would give off 'red' and 'green' photons in equal measure. He could also see 'green' objects, within that area, just as if they were his familiar 'red' objects. Suppose that, outside this area, far away, where $c(\boldsymbol{x}, t) = 0$, there are vibrating 'red' and 'green' electrons which, of course, are giving off only 'red' and 'green' photons, respectively. The observer inside the $c(\boldsymbol{x}, t) = 1$ region would be able to see the light from both of these. But its intensity would be reduced by a factor of 1/2 in both cases.

Let us examine a still more radical case. Imagine that $c(\boldsymbol{x}, t) \to \infty$ in a small spatiotemporal region with it being zero everywhere without. There a 'red' observer could respond only to 'green' photons. If he ventured into such a region he would see himself surround by the 'green' reality – he would respond to the 'green' photons hitting his retina. He could no longer respond to 'red' photons from "his" world. An observer in the 'green' reality could see him since, as the electrons in his body vibrated, they would give off 'green' photons. If he left this region, or if $c(\boldsymbol{x}, t)$ returned to 0, neither could see the other again. We might wonder if he could breathe in this region – maybe there is no oxygen in the other reality. Surely he could, as 'red' oxygen molecules would diffuse into his region where they would interact with him as 'green' molecules which his now-'green' lungs could process. Perhaps he sees a friendly 'green' observer in the other reality. Could he shake hands with him? No. If he tried to reach his hand out of the interaction region it would simply find itself back in 'red' reality and be able to interact only with 'red' things. But suppose this strange region of spacetime were surrounded by a small area of milder interaction where $c(\boldsymbol{x}, t)$ was only, say, about 1. There both 'red' and 'green' atoms could interact and a handshake might be possible. It would probably be a strange affair. The forces that repel my hand as I try to pass it through yours are a complicated combination of electrostatic, dispersion, and Pauli exchange forces. These latter would be absent since 'red' and 'green' fermions are not identical. I am unsure what form an interaction between 'red' and 'green' matter would take under macroscopic circumstances such as these. But it would be peculiar. (2)

The Standard Model.

It is of interest to see whether this idea can be generalized to a more realistic physical model. We will examine, briefly, the Standard Model. We will employ the notation familiar from (3).

Since we are considering two realities we just double the Lagrangian to include both the 'red' and 'green' fermion fields. More interesting is L_{scalar} - the one that contains the Higgs boson. There will now be two of these - a 'red' one and a 'green' one. They will share the same properties and symmetry-breaking V[φ] potential and couple in the usual way to the gauge fields of their own color. We will assume that, when $c(\boldsymbol{x}, t)$ differs from zero, all the gauge fields transform according to $f_R \to (f_R + c(\boldsymbol{x}, t)\ f_G) \Big/ \left(1 + c(\boldsymbol{x}, t)^2\right)^{1/2}$ and vice-versa for the f_Gs both in respect of L_{scalar} and the Lagrangians that describe the 'red' and 'green' fermionic fields. The kinetic terms for the gauge fields are, as before, left unchanged.

What results is a theory that differs from the conventional Standard Model in only two ways (besides the obvious fact that there are now two colors of each particle to keep track of). We end up with interaction terms from L_{fermion} that give rise to vertices where, for instance, a 'red' neutrino goes in emerging as a 'red' electron and a 'green' W^+. A similar analysis pertains as in *fig. 1*. From L_{scalar} (both 'red' and 'green') comes the term:

18) $(4\, c(\boldsymbol{x}, t)/(1+ \left(c\,(x,\ t)^2\right) [(g'^2 + g^2)\, Z_{R\mu}\, Z_G{}^{\mu} + g^2\, (W_R{}^-{}_\mu\, W_G{}^{+\mu} + W_G{}^-{}_\mu\, W_R{}^{+\mu})]$

which we are not sure how to interpret physically. It is encouraging to see that the 'red' and 'green' physical photons resulting from this variation of the Standard Model do not acquire any mass or couple in abnormal ways. And the masses of the other particles are not affected by $c(\boldsymbol{x}, t)$.

An observer scattering 'red' neutrinos off of 'red' electrons would see no change regardless of what $c(\boldsymbol{x}, t)$ did. But there would still be consequences if $c(\boldsymbol{x}, t)$ changed. Consider the β-decay of a 'red' neutron. It is mediated by the release of a W^- boson from a 'red' d quark which then becomes a 'red' u quark. This boson then becomes an electron and an antineutrino. If $c(\boldsymbol{x}, t)$ were different from zero the decay into a 'red' electron and antineutrino would proceed unchanged. But decay could also proceed through different channels into a 'green' electron and antineutrino pair. So the rate of β-decay would be increased by a factor of $1 + 4\, c(\boldsymbol{x}, t)^2/\left(1 + c(\boldsymbol{x}, t)^2\right)^2$. If the resulting 'green' electron and antineutrino moved out into a $c(\boldsymbol{x}, t) = 0$ area the 'red' physicist, looking at all of this, would conclude that a neutron just turned into a proton without producing anything.

The Common Coordinate System and General Relativity.

The treatment we have so far given this common coordinate system has been purely intuitive. We can do a little better by supposing that a team of 'red' physicists decides to work with a similar team of 'green' physicists. (How they do this I don't know.) Using their preferred coordinate system the 'red' physicists mark off as many spacetime points as they can with little 'red' placards. One might read '(2,4,1,1)' and it would designate an event in the 'red' world. And other 'red' physicists are checking about with yardsticks and stopwatches to determine $g_{\text{R}\mu\nu}$ for their world. Things are fortuitously arranged such that $c(\boldsymbol{x}, t)$ becomes large enough just around each of these placards so that, if only for a moment, the 'green' physicists can just dimly see the placards. They immediately write the same numbers on green placards in their same spacetime places and other 'green' physicists go about measuring $g_{\text{G}\mu\nu}$. If the two metrics agree they can be said to have established a common coordinate system. This would work even if both spacetimes were curved in identical ways. This defines a mapping of the coordinates of one space onto those of the other. We will always require it to be continuous.

Things become more complicated when the 'red' and 'green' spacetimes have different geometries. And this is a case we cannot ignore because we have already concluded that these two realities must be different in order for interesting physics to take place. If they are different their geometries cannot very well be the same. We will never be able to get the metrics to agree. We might best fall back upon our thought experiment and let the placards read as they read, thereby being done with it. But how would these metrics be affected were $c(\boldsymbol{x}, t)$ to become large over a significant region?

Perhaps the metrics mix, rather like the A_μ fields we have considered already. Difficulties arise regarding what type of volume elements, $\sqrt{-\left|g_{R,G}\right|}$, we need to include for each term in our Lagrangian. And to what terms do we apply this mixing? All of this seems to lead to endless complications and may not be a good way to go. Perhaps it is better to suppose that the metrics never mix.

Even such a conservative approach leads to a few interesting problems. Let us suppose that the differently colored metrics always remain independent and unconnected. The Lagrangian will contain a kinetic term for the 'red' geometry that would be $\frac{1}{16\pi} R_R \sqrt{-|g_R|}$, and $\frac{1}{16\pi} R_G \sqrt{-|g_G|}$ for the 'green' one. The 'red' and 'green' fermionic Lagrangians will also be constructed using only $g_{\mathrm{R}\mu\nu}$ and $g_{\mathrm{G}\mu\nu}$, respectively. The $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ terms will be treated similarly. This leads to Einstein equations that work normally and independently in both worlds. If we ignore the contributions from electromagnetism, gravity would be independent in the two realities; there could be a black hole in the 'green' world and the 'red' particles would not directly notice it. But it would be incorrect to think that these worlds do not interact gravitationally. If $c(\boldsymbol{x}, t)$ became large the behavior of the stress-energy tensor for ψ_R would change depending on behavior of $A_{\mathrm{G}\mu}$ and this, in turn, would change the evolution of $g_{\mathrm{R}\mu\nu}$ (and vice-versa for $g_{\mathrm{G}\mu\nu}$). Also, the requirement that $\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\, g_{\mu\nu}$ tells us that the Dirac matrices will be different in the two realities. This will change, somewhat, the form electromagnetic interactions will take between 'green' and 'red' particles.

The derivation of Equations 15) becomes problematic as well. When we set the left side of Equation 4) to zero by taking its divergence with respect to x^μ, we have to take the divergence involving covariant differentiation given in terms of $g_{\mathrm{R}\mu\nu}$ (since that is the metric in terms of which $-\frac{1}{4}F_R{}^{\mu\nu}F_{\mathrm{R}\mu\nu}$ was defined in the first place). But the divergence of $\tilde{J}^\mu$ only vanishes necessarily when we differentiate it using $g_{\mathrm{G}\mu\nu}$. Designate the covariant divergence of J^μ, calculated using the 'green' metric tensor as $\nabla_G J$ and its counterpart as $\nabla_R \tilde{J}$. These terms will not necessarily vanish (although they will be very small if the metrics are quite similar). We must amend Equations 15) to read:

15') $c(\boldsymbol{x}, t)_{,\mu} J^\mu = f(\boldsymbol{x}, t)\,(\nabla_G J + c(\boldsymbol{x}, t)\, \nabla_R \tilde{J})$ and

$c(\boldsymbol{x}, t)_{,\mu} \tilde{J}^\mu = f(\boldsymbol{x}, t)\,(\nabla_R \tilde{J} + c(\boldsymbol{x}, t)\, \nabla_G J)$ where $f(\boldsymbol{x}, t) = c(\boldsymbol{x}, t)\,(1 + c(\boldsymbol{x}, t)^2)/\,(-1 + c(\boldsymbol{x}, t)^2)$.

Now, in any region where $c(\boldsymbol{x}, t) = 0$, Equations 15) apply automatically. And where $J^\mu = \tilde{J}^\mu = 0$ there are no constraints on $c(\boldsymbol{x}, t)$ at all. An interesting situation arises if we set $\tilde{J}$ to zero. Then we find $c(\boldsymbol{x}, t)_{,\mu} J^\mu = 0$ along with the additional requirement $\nabla_G J = 0$. This requirement places some restrictions on J and/or the 'green' 4-geometry. Obviously, Equations 15') break down if $c(\boldsymbol{x}, t) = \pm 1$. In these cases we find:

19) $c(\boldsymbol{x},\, t)_{,\,\mu}\, (J^\mu \mp \tilde{J}^\mu) \pm 2\, \nabla_G\, J = 0$, and the additional constraint $\nabla_G\, J = \mp\; \nabla_R\, \tilde{J}$.

Of course, we can easily extend this theory to encompass as many additional realities as we might like. If there were two extra realities we would need three $c(\boldsymbol{x},\, t)$s, and more if there were others.

References and Footnotes.

1) If $c(\boldsymbol{x},\, t)$ is everywhere constant this theory is locally gauge invariant (except if it becomes -1 in which case the math breaks down). If $c(\boldsymbol{x},\, t)$ varies gauge invariance is lost. Suppose that ψ_R and ψ_G can undergo independent local phase rotations, $\psi_R \to e^{-i\,\xi_R}\, \psi_R$ and $\psi_G \to e^{-i\,\xi_G}\, \psi_G$, where the ξs must be considered abitrary functions of the common coordinate system. We can, most easily, just assume one of these, say ξ_G, is zero. We must require the A fields to change according to $A_{\mathrm{R}\mu} \to A_{\mathrm{R}\mu} + \Lambda_{R,\mu}$ and $A_{\mathrm{G}\mu} \to A_{\mathrm{G}\mu} + \Lambda_{G,\mu}$; it is essential that they change by gradients so the $-\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$ terms will not be altered and that the Lorentz force laws will remain unchanged. This requires that $\partial_\mu\, \xi_R = f(\boldsymbol{x},\, t)\, \partial_\mu\, \Lambda_R$, where $f(\boldsymbol{x},\, t) = e\, (1\text{-}\, c\,(\boldsymbol{x},\, t)^2)\, \left(1 + c\,(x,\; t)^2\right)^{-1/2}$. This, in turn, leads to the requirement $\partial_\nu\, f\,(\boldsymbol{x},\, t)\, \partial_\mu\, \Lambda_R = \partial_\mu\, f\,(\boldsymbol{x},\, t)\, \partial_\nu\, \Lambda_R$ which will limit Λ_R and, with it, ξ_R to few, if any, choices. We could require that $\xi_R = \xi_G$ but this would only lead to a similar situation with $f(\boldsymbol{x},\, t)$ replaced by $e\, (1 + c(\boldsymbol{x},\, t))\, \left(1 + c\,(x,\; t)^2\right)^{-1/2}$.

2) I might be asked whether we should regard this extra reality, and our own, as Everett branches derived from a common past. This might not seem an unreasonable possibility (assuming that Everett is right); both realities would share the same laws of physics and, automatically, a common coordinate system. And both realities would be quite similar – something we have suggested may be conducive to $c(\boldsymbol{x},\, t)$ becoming large. But this is not plausible for a simple reason: Suppose we place the quantum mechanical measurement that bifurcates these branches at some point in the common Minkowski coordinate system and draw a future-pointing light cone from it. The distinction between 'red' and 'green' worlds only occurs within this light cone. Outside, it would be a single 'monochrome' world. Suppose that a 'monochrome' particle were to move into the aforementioned light cone. What would happen? Would it turn 'red' or 'green'? If so, into which and why? If it stayed 'monochrome' how would it interact with the colored particles? There is nothing in our Lagrangian that tells us this. We could, instead, suppose that there were always two 'red' and 'green' realities with things outside the light cone always the same. But then we end up with the unacceptable situation where the charge of our electrons changes with $c(\boldsymbol{x},\, t)$.

3) Quigg, C. Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, Benjamin/Cummings, 1983.