Hume on the Social Construction of Mathematical Knowledge

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*Abstract*

Mathematics for Hume is the exemplary field of demonstrative knowledge. Ideally, this knowledge is *a priori* as it arises only from the comparison of ideas without any further empirical input; it is certain because demonstration consist of steps that are intuitively evident and infallible; and it is also necessary because the possibility of its falsity is inconceivable as it would imply a contradiction. But this is only the ideal, because demonstrative sciences are human enterprises and as such they are just as fallible as their human practitioners. According to the reading suggested here, Hume develops a radical sceptical challenge for mathematics, and thereby he undermines the knowledge claims associated with demonstrative reasoning. But Hume does not stop there: he also offers resources for a sceptical solution to this challenge, one that appeals crucially to social practices, and sketches the social genealogy of a community-wide mathematical certainty. While explaining this process, he relies on the conceptual resources of his faculty psychology that helps him to distinguish between the metaphysics and practices of mathematical knowledge. His account explains why we have reasons to be dubious about our reasoning capacities, and also how human nature and sociability offers some remedy from these epistemic adversities.

*Keywords*: certainty, demonstrative reasoning, mathematical practice, metaphysics of knowledge, faculties, scepticism, sceptical solution, sympathy

*I. Introduction*

One classification of knowledge Hume introduces is based on the “degrees of evidence” (T 1.3.11.2) that can be assigned to propositions and arguments. In the *Treatise* these degrees are called “knowledge”, “proof”and “probability”, a classification Hume retains in the first *Enquiry* (Section 6) except for replacing the label “knowledge” by “demonstration”. This first class of evidence is labelled in the *Treatise* in accordance with the *scientia*-ideal which requires demonstrative certainty in order to classify a conclusion as a piece of knowledge. This ideal which persisted pretty much until Newton’s revision of scientific standards – and one could argue indeed, until Hume’s explication of its epistemic ideal in the first *Enquiry*. [[1]](#footnote-1) Changing the label itself indicates a significant ideological shift from the *scientia*-ideal to that of science as a fallible and probabilistic enterprise, but here I am not going to explore this.[[2]](#footnote-2) Instead, I take the meaning of the two labels in Hume as identical and I will stick to the *Enquiry*’s usage because it is closer to our contemporary understanding.

Still speaking the language of the *Enquiry* (4.1), demonstrative knowledge can arise from an inquiry into “relations of ideas”, as opposed to proofs and probabilities arising from inquiries into “matters of fact”. But if we want to take a closer look at these two kinds of inquiry, it is worth switching to the outlook and language of the *Treatise* where Hume offers a more detailed introduction to their *principles*. He distinguishes seven philosophical relations, i.e. relations in which “we may think [it] proper to compare” ideas (T 1.1.5.1). Inquiry into matters of fact relies on three of them, “*identity, the situations in time and place*, and *causation*” (T 1.3.2.1). Whether these three relations hold between objects, or ideas thereof, depends not solely on the ideas themselves, but require further experimental inquiry to decide. Reasoning concerning the relations of ideas relies exclusively on the ideas themselves without the need for further experimental inquiry. Three out of the remaining four philosophical relations, “*resemblance, contrariety, degrees in quality*” (T 1.3.1.2) are liable to produce intuitive, i.e. immediately evident knowledge. The fourth relation of “*quantity and number*” can result in demonstrative knowledge that consists of successive intuitive steps making up complex mathematical reasoning.

So, mathematics for Hume is the exemplary, if not the only field,[[3]](#footnote-3) where demonstrative knowledge is attainable. Ideally, this knowledge is *a priori* as it arises only from the comparison of ideas without any further empirical input; it is *certain* because demonstration consist of steps that are intuitively evident and infallible; and it is also *necessary* because the possibility of its falsity is inconceivable as it would imply a contradiction.[[4]](#footnote-4) But this is only the ideal case, because

In all demonstrative sciences the rules are certain and infallible; but when we apply them, our fallible and uncertain faculties are very apt to depart from them, and fall into error. We must, therefore, in every reasoning form a new judgment, as a check or controul on our first judgment or belief; and must enlarge our view to comprehend a kind of history of all the instances, wherein our understanding has deceiv’d us, compar’d with those, wherein its testimony was just and true. Our reason must be consider’d as a kind of cause, of which truth is the natural effect; but such-a-one as by the irruption of other causes, and by the inconstancy of our mental powers, may frequently be prevented. By this means all knowledge degenerates into probability; and this probability is greater or less, according to our experience of the veracity or deceitfulness of our understanding, and according to the simplicity or intricacy of the question. (T 1.4.1.1)

So, even if demonstrative sciences are the only proper home of certainty, they cannot live up to their very own ideal, because they are human enterprises and as such they are just as fallible as their human practitioners. Consequently, demonstrative knowledge for Hume is an ideal and we can never be sure if we achieve it.

Explaining how and why demonstrative knowledge, due to “our fallible and uncertain faculties”, “degenerates into probability” is certainly an exercise in Hume’s “foundational project”, as Miren Boehm (2016) likes to call it. It shows the significance of the study of human nature for mathematics, and all the disciplines, in accordance with Hume’s announcement in the *Treatise*’s introduction. In this paper, however, I do not aim to reconstruct such an explanation; instead I will try to resolve the tension between Hume’s ideal of demonstrative knowledge and his denial of its attainability in practice. After all, we are offered an account of demonstrative knowledge only to find out that there is no such thing as demonstrative knowledge from a human standpoint. What then is the function of this distinction between the *nature* and the *practice* of demonstrative knowledge?

*II. The metaphysics of mathematical knowledge*

In order to explicate this distinction, I will rely on the idea of a *faculty psychology* in Hume. There are at least two fundamentally different ways of representing the Humean mind, both inspired by the idea that it makes sense to find its place on the Newtonian globe. Terence Penelhum neatly summarizes the more widely accepted image:

If the never-ending changes in the physical world are all to be explained in terms of the attraction of material particles to one another, there is no room for the suggestion that the world itself, which merely *contains* them, exerts a force of its own. It is just the place where the events being described occur. Similarly, if the course of my mental history is determined by the associative attraction of my perceptions, so that they cause one another to arise, there seems no place, perhaps even no clear sense, to the suggestion that *I*, the mind or soul that *has* them, can exert any influence over their course. All it does is *include* them. … The denial of an independent real self is not an awkward consequence of Hume’s theory of knowledge, which requires us to say that it is not there because we cannot find it when we look for it (although this is true); it is a cornerstone of his system, required by the supposed fact of a science of man conceived in quasi-Newtonian terms. (Penelhum 2000, 131-132)

This widespread image connects the Humean mind to the world of Newton’s *Principia*. Hume’s perceptions are particulate building blocks of the mental universe held together by association in a way analogous with Newton’s gravity.

There is thus no place here for faculties proper, and Hume’s frequent talk about them is to be translated into, e.g., talk about “processes”, as Rachel Cohon does:

To say that the human mind possesses the faculty (that is, power) of reason is just to say that the process of linking perceptions in this characteristic way does or probably will occur in the mind. It is not to postulate any reasoning organ that carries out these tasks. Even in speaking of the faculty of reason, then, Hume speaks only of the reasoning process. (Cohon 2008, 67)

Thus Hume is turned into a natural historian of the mind telling “just-so” stories: his project is to describe processes instead of exploring causes and invoking them to explain *why* perceptions and actions follow one another in the order they do. But, among other things, this image of Hume’s project and the Humean mind does not seem to fit those passages referring to the causal and frequently transformative contribution of the faculties (e.g. T 2.2.6.1, 2.3.6.8, 3.3.1.7).

According to the alternative image,[[5]](#footnote-5) the Humean mind is composed of autonomous faculties identified by their characteristic active principles, whose interplay explains how and why sensations follow one another, transform into one another and result in actions. The operations of the mind are distributed throughout this mental architecture, instead of being carried out centrally. The image of faculties performing their tasks according to their principles in an unnoticed, unconscious manner provides a framework within which the idea of a unitary self cries out for an explanation.

These faculties can be characterized *exclusively functionally*, only by the characteristic activity they exert on specific kinds of perception and on each other. The focus on functions, as Hume sees it, is the only appropriate one for “a just and *philosophical* way of thinking”, because “the distinction which we sometimes make betwixt a *power* and the *exercise* of it, is entirely frivolous” (T 2.1.10.4, see also 1.3.14.34). Accordingly, the faculties of the mind can be studied and described only in terms of their actual functioning, i.e. through the exploration of the processes to which they contribute. So inquiry should not begin with the definition of faculties, and explanations should not proceed from those definitions, but from observation and experimental reasoning. Instead of arguing *from* faculties, one should argue *to* them; they are not the beginning but the aim of proper, experimental inquiry, because this inquiry begins with *observations* of behaviour and reveals, through *comparison* and *analogy*, their *systematic connections*, which reveal the *principles* that identify the characteristic *activities* of *faculties*.

The principles so identified are not scattered regularities, but are indeed *structured*, and in this important sense the anatomy of the human mind is analogous with the structure of the body (T 2.1.11.5). As some of these principles, just like certain organs of the human body, interact more closely, they can be conveniently subsumed under various *faculties*, so Hume is justified in talking freely, for example, about the universal principles of imagination, sympathy (T 1.1.4.1, 2.2.5.14), and other faculties, as well as their limits and imperfections. This is why talk about faculties is abundant throughout the text; sometimes they are referred to straightforwardly as the “organs of the human mind”, as in the case of the faculty which is responsible for producing passions, i.e. reflection (T 2.1.5.6).

I think this background of Humean faculty psychology is instrumental in explicating the Humean metaphysics of mathematical knowledge that gives an account of its *nature* independently of its *practice*. Knowledge arises from processing ideas, and faculties are responsible for processing them. Faculties are identified through their principles, i.e. the contributions they make to the chain of perceptions – just as reason does according to the opening block quote, where Hume claims “reason must be consider’d as a kind of cause, of which truth is the natural effect”. The undisturbed *natural functioning* of the principles of reason produces demonstrative knowledge. Even if the principles are never undisturbed, their normal functioning can be analysed from their actual functioning, and this is enough to reveal the nature of demonstrative knowledge through the principles of reason. So the natural functioning of a faculty can be reconstructed by exploring the nature of the ideas it processes *and* the way it works when undisturbed: reason is a cause whose natural effect is truth. But it “may frequently be prevented” from producing this effect by the interference of other faculties, i.e. “by the irruption of other causes”, and by the “inconstancy”, i.e. the fallibility, of its powers (T 1.4.1.1).

Due to imperfection and interference faculties are prone to error, and this is one reason why reflection and internal observation are not reliable sources of knowledge about them (T I.10). What Hume needs to rely on is the “experimental method of reasoning” in order to reveal the principles of human nature. His method is a version of analysis and synthesis:[[6]](#footnote-6) accordingly, principles are specified as a result of analysis by the comparison and analogy of crucial phenomena, and once they are found, they can be deployed in the process of synthesis, in explaining “the nature of the ideas we employ, and of the operations we perform in our reasonings” (T I.4). Natural functioning can be analysed from the comparison of several instances of actual and potentially disturbed functioning: disturbances can be explored comparatively from what is changing from one instance to another due to imperfections, inconstancies, or to external influences. Comparing several potentially disturbed operations of a faculty can reveal, at least fallibly, what its undisturbed functioning would look like,[[7]](#footnote-7) and on this comparative and analogical basis natural functioning can be inferred even if never observed.[[8]](#footnote-8) The description of the natural functioning of reason is thus construed on the basis of the actual operations of the faculty that the analysis of relevant phenomena takes as input.

Now we can turn to the metaphysics of mathematics as reflected in Hume’s account of its branches (geometry, arithmetic and algebra), and explore their differences through the respective faculties contributing to the production of mathematical truths. Let’s consider geometry first. As Hume famously claims, its propositions *cannot* attain demonstrative certainty, mainly because geometry for Hume, as Henry Allison (2008, 84) aptly put it, is for the eye rather than the mind – at least as far as the origins of its *ideas* are concerned. For Hume the ideas of geometrical reasoning are ideas of figures and diagrams actually drawn “upon paper or any continu’d surface” (T 1.2.4.25), whose definition is “fruitless without the perception of such objects” (T 1.2.4.22 (App.)). Being copied from impressions of the senses, these ideas are prone to the imperfections of perception and those arising from the process of geometric construction. Consequently, geometry takes “the dimensions and proportions of figures … roughly, and with some liberty” (T 1.2.4.17).

These imperfections can be corrected, but only to the extent that our faculties allow, so they cannot be perfect. Our corrections are always susceptible of further corrections, and they remain human corrections, i.e. limited by our senses, imagination, our instruments and the care we can take while making them (T 1.2.4.23). These are, as Hume sees it, *essential* to the nature of geometry, so

in vain should we have recourse to … a deity whose omnipotence may enable him to form a perfect geometrical figure … As the ultimate standard of these figures is deriv’d from nothing but the senses and imagination, ’tis absurd to talk of any perfection beyond what these faculties can judge of (T 1.2.4.29).

As the ideas deployed in geometrical reasoning are inexact, [[9]](#footnote-9) the further steps of our reasoning inherit this property too. Even if geometrical reasoning proceeds through the comparison of ideas without further experiential input, and so it should be deemed demonstrative (T A 650), it relies on the comparison of inexact ideas, therefore its conclusions are conceivably false. If they are conceivably false, then by Hume’s standards they are also possibly false – and if they are possibly false, they are not demonstrable, because “whatever we conceive is possible, at least in a metaphysical sense: but wherever a demonstration takes place, the contrary is impossible, and implies a contradiction.” (T A 650) It is thus conceivable that seemingly right lines are curved, and also that a triangle be composed of seemingly right but curved lines in which case its three angles are not equal to two right angles. So, as Hume summarizes, the demonstrativity of geometrical reasoning is compromised by its input ideas because “An exact idea can never be built on such as are loose and undetermined” (T 1.2.4.27). Even if geometrical reasoning itself “excels both in universality and exactness”, still it “never attains a perfect precision and exactness”, because of “the loose judgments of the senses and imagination” (T 1.3.1.4). Therefore, Hume rightly refers to geometry as “the *art*” rather than the science of fixing the proportions of figures (T 1.3.1.4).

Due to the inexact nature of the ideas involved, the geometrical ideas of fundamental mathematical *relations* also suffer from similar imperfections, because they are also “deriv’d merely from appearances” (T 1.3.1.6). Determining the proportions of quantity is a core element in any mathematical reasoning including geometry, so Hume’s first question to ask geometers is “what they mean when they say one line or surface is *equal* to, or *greater* or *less* than another” (T 1.2.4.18), because it is by these relations that “the mind distinguishes in the general appearance of its objects” (T 1.2.4.23), i.e. by these relations the mind judges their equality and inequality. The geometrical idea of *equality* on which these judgments depend also arises from the appearance of objects, so “the ideas which are most essential to geometry, viz. those of equality and inequality, of a right line and a plane surface, are far from being exact and determinate, according to our common method of conceiving them” (T 1.2.4.29)

Euclid’s postulates invoke straight lines, circles, right angles, parallels and congruence, none of which can be constructed and perceived with perfect precision. Beyond a certain point of resolution we can never know if two parallel right lines are indeed right or slightly curved, or if they are indeed parallel or slightly convergent. Not surprising then, the *axioms* of geometry are problematic too, and Hume concludes that geometry is “built on ideas, which are not exact, and maxims, which are not precisely true” (T 1.2.4.17). Consequently, geometrical propositions can apply for membership only in the category of “proofs” (T 1.2.4.17), but not of demonstrations. They may be “entirely free from doubt and uncertainty” (T 1.3.11.2), but they are not necessarily, only possibly so.

As Hume’s arguments testify, the falsity of the axioms can be conceived, and if they are possibly false, geometry cannot be demonstrative. And they are indeed possibly false, not because of the possible dysfunctions of our reasoning faculty – that is external to the nature of geometrical knowledge. They are possibly false due to the *natural functioning of our senses and imagination*, that are ill-suited to deliver exact ideas that are needed for the demonstrativity of geometry. As long as we are with Hume saying that geometrical figures cannot be defined without reference to the actual perception of such objects, we cannot form an idea of a perfectly exact geometry: “The first principles are founded on the imagination and the senses: The conclusion, therefore, can never go beyond, much less contradict these faculties.” (T 1.2.4.31) So uncertainty, inexactness, lack of demonstrative knowledge arises from the very nature of the ideas geometry is concerned with, and so it belongs to the very nature, and thus to the metaphysics of geometry itself.

The metaphysics of arithmetic and algebra looks different, because their ideas are not derived from the impressions of the senses, and thus we have in the faculties of the mind appropriate foundations for demonstrative knowledge. Arithmetic and algebra are founded on the idea of a discrete ‘unit’ (see e.g. de Pierris 2015, 179),

’Tis evident, that existence in itself belongs only to unity, and is never applicable to number, but on account of the unites, of which the number is compos’d. Twenty men may be said to exist; but ’tis only because one, two, three, four, *&*c. are existent; and if you deny the existence of the latter, that of the former falls of course. ’Tis therefore utterly absurd to suppose any number to exist, and yet deny the existence of unites. (T 1.2.2.3)

And the idea of a unit makes *equality* exact in the field of numbers, the only field of knowledge where

We are possest of a precise standard, by which we can judge of the equality and proportion of numbers; and according as they correspond or not to that standard, we determine their relations, without any possibility of error. When two numbers are so combin’d, as that the one has always an unite answering to every unite of the other, we pronounce them equal; and ’tis for want of such a standard of equality in extension, that geometry can scarce be esteem’d a perfect and infallible science. (T 1.3.1.5)

So equality can be justly pronounced of two numbers, but not two geometrical figures, and an inference to the equality of quantities as represented by numbers is demonstrative because of the exact idea of a unit.

Now the question is: where does the idea of a unit come from? A plausible account, I think can be construed by invoking Hume’s account of a “distinction of reason” (T 1.1.7.17-18), i.e. viewing objects “in different aspects, according to the resemblances, of which they are susceptible”. The idea of a unit can be seen as arising from resemblance relations between objects: being a unit is a certain *aspect* in which objects can resemble one another; it is just like other aspects such as shape and colour. ‘Unit’ is thus an abstract idea, i.e. a particular idea whose “revival set” (see Garrett 1997, 104) consists of objects such that any of them may represent others as units within this set, making them available for the formation of a custom of comparing objects in this aspect. The relevance of these resemblances on which the idea of a unit is founded is apparent when we make *comparisons concerning proportions of quantity and number* – the only context, I guess, where ‘unit’ as an aspect of objects matters. So we get the idea of a discrete unit when comparing “objects, which admit of *quantity*, or *number*” (T 1.1.5.6).

Ideas arising from distinctions of reason are, as Don Garrett (1997, 63) points out, ideas without corresponding origins in sense impressions, so they do not suffer from the imprecision of sense impressions that has proven to be so toxic for the demonstrativity of geometrical reasoning. Furthermore, as Wayne Waxman (1996, 134) says, aspects of resemblance are “bound up with mental activity … and can have no reality independently thereof”. We never experience anything as a discrete unit, not more than as an independent shape or colour; comparison produces a complex idea consisting of two particulars resembling in their aspects as *units*. So here we have a case (not so unique)[[10]](#footnote-10) where an idea is produced from a comparison of two ideas, due to the active contribution of the faculties, without having a corresponding impression first.

Drawing a distinction of reason is an active contribution of the mind. The faculty of imagination is the home of unreflective, *natural relations* that serve as “connecting principles” (T 1.1.5.1) that are causally active in producing associative transitions between ideas. So we know that “wherever the mind constantly and uniformly makes a transition without any reason, it is influenc’d by these relations” (T 1.3.6.12). The faculty of reason, however, can operate on “any particular subject of comparison, without connecting principles” (T 1.1.5.1). Its field of competence is not that of facilitating transitions between perceptions, but making comparisons between ideas and thereby putting them into relations with each other. These relations can be taxonomized “under seven general heads” (T 1.1.5.2), and comparing ideas according to the proportions of quantity and number is included in this list of “philosophical relations”. If *reason* could not compare objects in this philosophical relation, there could be no resemblance between objects *as* units, and there could be no distinctions of reason necessary for this idea to be drawn.[[11]](#footnote-11)

Given that the idea of a unit arising from a distinction of reason does not bear the imprecision of the ideas derived directly from the senses, it is not surprising that this idea is a solid foundation of “algebra and arithmetic as the only sciences, in which we can carry on a chain of reasoning to any degree of intricacy, and yet preserve a perfect exactness and certainty” (T 1.3.1.5). Although the reasoning we deploy in geometry and in arithmetic and algebra is of the same kind, due to the difference between the origins of ideas that constitute the raw material of reasoning in these fields, only those of arithmetic and algebra are susceptible of demonstrativity. So the metaphysics of these fields of mathematical inquiry is determined by the nature of the ideas they rely on.

*III. The practices of mathematical knowledge*

Now this metaphysical division between the branches of mathematics can be blurred if we explore Hume’s way to account for the practices of mathematical knowledge production, i.e. the actual processes through which mathematical insights are elevated in a community to the status of knowledge. From this practical angle the difference between the natures of the branches are hardly of consequence. This is due to Hume diminishing the significance of geometry’s imperfections on the one hand, and mitigating the conceit some mathematicians might derive from the seemingly impeccable demonstrativity of arithmetic and algebra. In practice, geometry gains and arithmetic loses something in demonstrativity.

Let’s consider again geometry first. It is important to note that the problem of exactness arises only beyond a certain point of resolution.[[12]](#footnote-12) With simple, medium-sized geometrical figures the problem does not arise:

’Tis evident, that the eye, or rather the mind is often able at one view to determine the proportions of bodies, and pronounce them equal to, or greater or less than each other, without examining or comparing the number of their minute parts. Such judgments are not only common, but in many cases certain and infallible. When the measure of a yard and that of a foot are presented, the mind can no more question, that the first is longer than the second, than it can doubt of those principles, which are the most clear and self-evident. (T 1.2.4.22)

So in these cases there is an intuitive judgment from which demonstrative reasoning can proceed, and the certainty of geometrical propositions so reached is not compromised. These are the cases in which geometry does not “err” because it does not “aspire” to “absolute perfection”. *Prima facie*, the case is similar in arithmetic too, where

We might proceed, after the same manner, in fixing the *proportions* of *quantity* or *number*, and might at one view observe a superiority or inferiority betwixt any numbers, or figures; especially where the difference is very great and remarkable. As to equality or any exact proportion, we can only guess at it from a single consideration; except in very short numbers, or very limited portions of extension; which are comprehended in an instant, and where we perceive an impossibility of falling into any considerable error. In all other cases we must settle the proportions with some liberty, or proceed in a more *artificial* manner. (T 1.3.1.3)

And these are the cases where geometry diverges from arithmetic and algebra: where the proportions are to be settled by processing ideas in an “artificial manner”, i.e. by reasoning and reflective comparison.

 An artificial manner is required where proportions of numbers and extensions are *beyond the purview of intuition*, i.e. where they are too complex, too big or too small. When in geometry the proportion of two figures are too complex, or if they are too big, then they are to be analyzed into “the easiest and least deceitful appearances” so we can determine that the angles of a chiliagon is “equal to 1996 right angles … And this is the nature and use of geometry to run us up to such appearances, as, by reason of their simplicity, cannot lead us into any considerable error.” (T 1.3.1.6) So in geometrical analysis we reduce complex diagrams to actual simple diagrams on which intuitive judgment can be formed, and then steps of intuitive reasoning can be built. We are safe in this geometrical reasoning until we do not go beyond the intuitive certainty of the senses, “the ultimate standard of these figures” (T 1.2.4.29). And we cannot go beyond that point, because we cannot form adequate geometrical ideas that are sensitive to infinitesimal differences. In geometrical practice this inability does not make much difference: there we are concerned with intuitive certainties or appearances reducible to them. The lack of exactness, to repeat, has consequences only for the metaphysics of geometrical knowledge by showing the difference between the nature of geometry and of arithmetic and algebra – and thereby between the functioning of the faculties responsible for them.

Due to the lack of exact ideas, the fallibility of geometrical reasoning is built into its raw material. Arithmetic and algebra are doing better in this respect because the exact idea of a ‘unit’ provides a safe haven for them. Concerning small numbers, even if I cannot form a distinct idea (in the sense of an image) “of the thousandth and ten thousandth part of a grain of sand”, I still “have a distinct idea of these numbers and of their different proportions” (in the sense of a distinction of reason) (T 1.2.1.3). Concerning large numbers, although we cannot conceive an adequate idea of them, we do have an “adequate idea of the decimals, under which the number is comprehended” (T 1.1.7.12). This case is an “instance parallel to” the case of general ideas: the particular idea of the decimals is made general in its representation of its revival set of resembling decimals. So we can analyze large numbers by relying on the idea of ‘decimal units’: we can conceive them in decimals thereby going back to simple empirical contents whose particular idea brings to mind the revival set of its resembling ideas. This is how abstract mathematical talk can go beyond what can be empirically tested. So just as in the case of geometry, we can trace back these calculations from abstract mathematical terms, through abstract ideas to clear cases of intuition on the “exact proportion” of “very short numbers”. The only reason why reasoning on numbers is impeccably demonstrative is that, unlike reasoning on figures, it relies on an exact idea (‘unit’) – otherwise the reasoning process itself is the same in both fields.

It is important to note that even when exact proportions are intuitively clear, there is only “an impossibility of falling into any *considerable* error” (T 1.3.1.3, emphasis added, see also 1.3.1.6). Not only in geometry, but also in arithmetic and algebra Hume can justly say that albeit our reasoning concerning proportions are “sometimes infallible, they are not always so; nor are our judgments of this kind more exempt from doubt and error, than those on any other subject.” (T 1.2.4.23) The *common* fallibility of all branches of mathematics arises not from the nature of the ideas of their respective subject matters, but from the nature of their practitioners.

Even if we “reduce” a complex mathematical problem or procedure “to the most simple question” (T 1.4.1.3), this reduction, or analysis, itself is fallible, because “we ought always to correct the first judgment, deriv’d *from* the nature of the object, by another judgment, deriv’d *from* the nature of the understanding”, and this latter judgment is to be further corrected by a judgment *on* the nature of the understanding itself (T 1.4.1.5, emphasis added). The outcome of our demonstrative reasoning must be judged by a probabilistic judgment on the reliability of our faculties, which in turn must be judged by a further probabilistic judgment on the probabilistic judgment on the reliability of our faculties.

This explains why “[t]here is no algebraist nor mathematician so expert in his science, as to place entire confidence in any truth immediately upon his discovery of it, or regard it as any thing, but a mere probability” (T 1.4.1.2). The apparent demonstrative certainty of our mathematical reasoning degenerates into probability, and we are left with fallible and uncertain conclusions even in our demonstrative sciences. This is admittedly an unfortunate epistemic situation from which *mathematical practice* offers two partial remedies.

An *individual* remedy for a mathematician is to go back and check again his reasoning, because “[e]very time he runs over his proofs, his confidence increases”. This is a common practice we all apply in our calculations:

We frequently correct our first opinion by a review and reflection; and pronounce those objects to be equal, which at first we esteem’d unequal; and regard an object as less, tho’ before it appear’d greater than another. Nor is this the only correction, which these judgments of our senses undergo; but we often discover our error by a juxtaposition of the objects; or where that is impracticable, by the use of some common and invariable measure, which being successively apply’d to each, informs us of their different proportions. And even this correction is susceptible of a new correction, and of different degrees of exactness, according to the nature of the instrument by which we measure the bodies, and the care which we employ in the comparison. (T 1.2.4.23)

So, individual corrections are one way the mathematician can increase his certainty without eliminating the reasons of doubt about the reliability of his calculations. Social processes are more effective in reducing doubt, because his certainty is increased “still more by the approbation of his friends; and is rais’d to its utmost perfection by the universal assent and applauses of the learned world.” (T 1.4.1.2) Even if all demonstrative knowledge remains probabilistic in this approach too, the certainty of mathematical reasoning reaches its peak due to the acknowledgement of epistemic peers.

This is the *social* remedy from the scepticism that arises from degrading demonstrative knowledge into probability, even if “’tis evident, that this gradual encrease of assurance is nothing but the addition of new probabilities, and is deriv’d from the constant union of causes and effects, according to past experience and observation.” (T 1.4.1.2) The popularity of a mathematical proof (or demonstration) among experts in the field naturally speaks in favour of the proof. This feature is not peculiar to mathematical conclusions, because as Hume points out: “I feel all my opinions loosen and fall of themselves, when unsupported by the approbation of others” (T 1.4.7.4). What makes mathematics peculiar is that, in addition to individual reasoning, the approbation of the mathematician’s close circle of “friends”, and the approval of the scholarly community seem to be the *only* sources of justification in matters of mathematics.

Experience can have an at most inexact word in the field of demonstrative reasoning only in geometry, and our causal and probabilistic knowledge of human nature teaches us that in cognitive matters individuals are more fallible than communities are. So what is it that communities can provide as a cure for these sceptical challenges? The sentiment of *approbation* (T 3.3.1.9) plays a crucial epistemic role here. It is especially significant for Hume in moral contexts, and so it forges a fruitful connection between natural and epistemic virtues. What are the sources of approbation relevant in generating mathematical certainty? “All the sentiments of approbation, which attend any particular species of objects, have a great resemblance to each other, ’tho deriv’d from different sources” (T 3.3.5.5). Yet, the different sources also resemble to each other in that they are somehow useful or agreeable. The approbation attached to a mathematical proof may be derived from these common sources too, like for example, from the *beauty* of the proof. What is common to “*beauty* of all kinds” is that it “gives us a peculiar delight and satisfaction” (T 2.1.8.1), and mathematical proofs are certainly capable of “such an order and construction of parts, as either by the primary constitution of our nature, by custom, or by caprice, is fitted to give a pleasure and satisfaction to the soul.” (T 2.1.8.2)

Another source of approbation is a different kind of *pleasure* we – or even more so: relevant epistemic peers – can derive from the solution of mathematical problems. This pleasure arises from qualities not universally shared among mathematical solutions, only among those that incite curiosity and provide an occasion to exercise the mathematician’s “genius and capacity” (T 2.3.10.3), because “in an arithmetical operation, where both the truth and the assurance are of the same nature, as in the most profound algebraical problem, the pleasure is very inconsiderable, if rather it does not degenerate into pain” (T 2.3.10.2). Truth is an admirable prey only in those mathematical proof-hunts that are sufficiently difficult to “stretch our thought” (T 2.3.10.3).

A third source of approbation is the perfection of *reason*, or its particular fitness as testified by the solution itself: “There is a sentiment of esteem and approbation, which may be excited, in some degree, by any faculty of the mind, in its perfect state and condition” (T 3.3.4.4), and this is especially so with respect to qualities which capacitate “a man best for the world, and carries him farthest in any of his undertakings”. (T 3.3.4.5) Reason qualifies high in a list of those qualities, because

Men are superior to beasts principally by the superiority of their reason; and they are the degrees of the same faculty, which set such an infinite difference betwixt one man and another. All the advantages of art are owing to human reason; and where fortune is not very capricious, the most considerable part of these advantages must fall to the share of the prudent and sagacious. (T 3.3.4.4)

So a mathematical proof indicates the perfection of reason, the faculty so useful in taking people further in their endeavours, and thereby it commands approbation.

Now, the function of approbation in generating mathematical certainty can be reconstructed with resources borrowed from the Humean theory of the generation of *pride*. The end product of the social process of generating mathematical certainty is *belief* – “a peculiar manner of conception” which entails an “addition of force and vivacity” (T 1.4.1.8) to the conclusions of the proof. Belief, which is “*more properly an act of the sensitive, than of the cogitative part of our natures*” (T 1.4.1.8), is a feeling attached to the conclusion and its cause is the other persons’ approbation that increases certainty (and pride, but that aspect is not important here) (T 2.2.2.27).

In this process the “principles of authority and sympathy” play the central roles (T 2.1.11.9). If we submit a mathematical proof, its approvals increase our certainty and give us pleasure, because “they concur with our own opinion, and extol us for those qualities, in which we chiefly excel.” (T 2.1.11.13) But it *does* matter where approvals come from. It must come from an *authority*, i.e. from someone we acknowledge as an epistemic peer or superior, because “we receive a much greater satisfaction from the approbation of those, whom we ourselves esteem and approve of, than of those, whom we hate and despise” (T 2.1.11.11). In mathematical contexts we assign such authority to those we deem mathematically competent peers, i.e. those whose mathematical reasoning we approve of. So approbation emerges in the social exchange of giving and asking for approbations. Our certainty in our proof depends on approbation by those whom we (and others) approve as epistemic peers or superiors.

So the relevance of approbation arises from social generated epistemic authority. The second step on the way to mathematical certainty is *sympathy*, which is responsible for the transfer of approbation from epistemic peers, so the force and vivacity, i.e. our belief in of our initially shaky conclusion can increase. Our certainty grows with the approbation of others through sympathy. Here again the epistemic community is important. Were there is no social vicinity among epistemic peers (as mutually acknowledged experts in the field), sympathy could not function so as to effectively transfer this sentiment in the learned world (see T 3.3.1.14-15). Therefore, approbation is not only communicated through sympathy; it also forges the connections necessary for the communication of the sentiment itself. And these are the connections upon which a scholarly community, or “the learned world” as a community of sympathy can emerge.

By connecting mathematical certainty to the approbation of others Hume gives an essentially social character to mathematical knowledge production. No matter the different metaphysical and epistemic pedigree of its branches, it is their practices that determine the certainty of mathematical conclusions:

For these conclusions are equally just, when we discover the equality of two bodies by a pair of compasses, as when we learn it by a mathematical demonstration; and tho’ in the one case the proofs be demonstrative, and in the other only sensible, yet generally speaking, the mind acquiesces with equal assurance in the one as in the other. (T 2.3.10.2)

In practice geometrical reasoning is elevated to the same level of certainty as our reasoning in arithmetic and algebra.[[13]](#footnote-13) The limitations of our practices, arising from human nature, condemn all branches of mathematics to eventual fallibility and uncertainty. But that is not the main lesson we should draw here. What really matters is that despite this fallibility we *generate* certainty through our predominantly social practices. So geometry *in practice* can be as certain and demonstrative as arithmetic is *in principle* demonstrative, but in practice, fallible. Taken together, the metaphysics and practice of mathematics leads us to pretty much the same epistemic grounds in all branches of mathematics.

It is important to note that the resources Hume offers for a solution to his sceptical worries do not point to a *straight solution*: they do not add up to a solution that makes the sceptical problem disappear by providing a justification of demonstrative reasoning that meets the initial sceptical challenge.[[14]](#footnote-14) A straight solution could argue that even if the sceptical worries are legitimate, they can be answered by presenting a source of demonstrativity that was neglected when formulating the initial challenge. But Hume thinks that “’Tis impossible, upon any system, to defend either our understanding or senses; and we but expose them further when we endeavour to justify them in that manner” (T 1.4.2.57).

Instead, his remedies add up to a *sceptical solution*: [[15]](#footnote-15) Hume accepts the sceptical conclusion that undermines demonstrative reasoning as “a malady, which can never be radically cur’d” (T 1.4.2.57). Besides admitting this, he also provides resources for showing how we still can live a peaceful epistemic life – despite our inability to provide the sort of justification for our mathematical beliefs and practices that the sceptical challenge requires. Hume’s sceptical solution provides a practical way of bypassing the uneasy epistemic consequences of the sceptical conclusion by showing that mathematical certainty actually arises more from agreement than from reasoning. He shows that our epistemic stance to mathematics is rationally unjustified – but also that we have dominantly social practices responsible for generating certainty about mathematical beliefs, and thereby elevating them to the status of demonstrative knowledge. Consequently, the sceptical worries turn out to be less pressing than they initially seemed: they are bypassed by practices that are themselves not rational, but can be rationally accounted for. The Humean sceptical solution shows that we do not need to aspire for a rational justification of mathematical knowledge claims, because the appeal to a social route to mathematical certainty reveals how mathematics can be “a science founded on scepticism”.[[16]](#footnote-16)

*V. Conclusion*

In conclusion let me point out two lessons of the present discussion. According to the reading suggested here, Hume develops a radical sceptical challenge for mathematics, and thereby he undermines the knowledge claims associated with demonstrative reasoning.[[17]](#footnote-17) But Hume does not stop there: he also offers a sceptical solution to this challenge, one that appeals to social practices and sketches the social genealogy of mathematical certainty.[[18]](#footnote-18) His account explains why we have reasons to be dubious about our reasoning capacities and how human nature and sociability offers some remedy from our epistemic adversities.

The present exposition also suggests an answer to the question asked at the beginning: What is the function of the distinction drawn between the *nature* and the *practice* of mathematical knowledge? The answer is: exploring them contributes to Hume’s project of the “anatomy of human nature”. The point of exploring the metaphysics of mathematical knowledge in terms of human faculties involved in producing it finds its place in the context of Hume’s *faculty psychology*. Even if these faculties are always pathological because they are prone to mistakes and never undisturbed, and they may never fulfil their role beyond doubt, in the context of Hume’s project it is worth asking what their undisturbed natural functioning would look like. And similarly: exploring the social processes through which an individual’s mathematical beliefs are turned into common certainties contributes to this anatomy by exposing human sociability as a remedy against our epistemic imperfections.

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1. On the evolution of *scientia*-ideal see Demeter, Láng, Schmal (2015). For a useful discussion of this transition see Lakatos (1978, 201-203), and more recently Gaukroger (2014). [↑](#footnote-ref-1)
2. I discuss some aspects of this process in relation to Hume’s philosophy, and Hume’s explication of epistemic standards in Demeter (2016). [↑](#footnote-ref-2)
3. David Owen (1999, 107), rightly I think, points out that Hume in the *Treatise* limits demonstrative knowledge to arithmetic and algebra. Recently Matias Slavov (2017), rightly again I think, points out in the context of the *Enquiry*, syllogisms and definitional truths are also listed in this class. The present paper focuses exclusively on the *Treatise*. [↑](#footnote-ref-3)
4. This summary is consistent with the common wisdom that for Hume mathematical truths are analytic. There are others (most notably Coleman 1979) who maintain that mathematics for Hume is synthetic *a priori*. [↑](#footnote-ref-4)
5. Prominent Hume scholars taking seriously the idea of a faculty psychology in Hume include Garrett (2015), Millican (2009), and I also made an attempt to explicate one such stance in Demeter (2016). [↑](#footnote-ref-5)
6. Or so I argue in detail in Demeter (2016). Sturm (2015) offers an enlightening discussion of analysis and synthesis in the broader context of the history of early modern human sciences. [↑](#footnote-ref-6)
7. Compare the following passage: “we must distinguish exactly betwixt the phaenomenon itself, and the causes, which I shall assign for it; and must not imagine from any uncertainty in the latter, that the former is also uncertain. The phaenomenon may be real, tho’ my explication be chimerical. The falsehood of the one is no consequence of that of the other.” (T 1.2.5.19) [↑](#footnote-ref-7)
8. For the significance of analogy and comparison in Hume’s method see e.g. T I.10, 3.2.3.4n71 *Enquiry* 4.12, 8.13. [↑](#footnote-ref-8)
9. The term “inexact idea” might require some clarification because of Hume’s dictum that all perceptions are “determin’d in its degrees of both quantity and quality” (1.1.7.4). Several ways can be suggested as to how ideas could be inexact while sticking to this dictum. 1) Ideas can be inexact copies of impressions without violating the Copy Principle. Even if my impressions of Edinburgh are very detailed, my idea of Edinburgh may be inexact due to the liberty of the imagination to rearrange ideas. 2) Inexact ideas can be abstract ideas with vague borders for their “revival set” (for the term see Garrett 1997, 104). For example, the abstract ideas "straight" and "curved" require a general "appearance" in spatial disposition, and it might be indeterminate whether some arrangements of individually unextended minima are "straight" or "curved." 3) an idea might be inexact because, although we retain the same term and treat it as identical over time, it in fact varies somewhat in size and/or disposition of parts without our taking note of it. Such fluctuations would, of course, be a potential hindrance to precise reasoning. A more detailed discussion would exceed the scope of this paper. I am grateful to Don Garrett for helpfully pointing this out to me. [↑](#footnote-ref-9)
10. For example, the idea of a “necessary connection”, and our idea of “causation” along with it, does not derive purely from a corresponding impression. There is a contribution on the mind’s part, i.e. custom, that plays a crucial role in producing this idea. For an interpretation along these lines cf. Buckle 2002, esp. 213-214. [↑](#footnote-ref-10)
11. There is an increasing literature discussing the intricacies of “natural” and “philosophical” relations in Hume, notable recent contributions include Schliesser 2010, Bebee 2011, and most recently Millican 2017. The limitations of the present context do not allow for a detailed discussion. [↑](#footnote-ref-11)
12. For a useful background discussion see Badici 2011, 462-463. [↑](#footnote-ref-12)
13. This may be one way to explain why the *Enquiry* (4.1, 7.1-2, 12.20) lists all of them as demonstrative. [↑](#footnote-ref-13)
14. For this reason the placement of Hume’s sceptical arguments should not pose a problem to the interpretation suggested here. It is true: Hume’s appeal to the significance of the approbation of epistemic peers occurs before one of Hume's main arguments (T 1.4.1.3) to the conclusion that knowledge degenerates into probability and also the subsequent argument that probability reduces to nothing, and so they seem to already take into account the social factors that pointed out here. But given that Hume’s solution is a sceptical one, this does not undermine the present interpretation: even if the resources of a sceptical solution are at hand, the sceptical problem is still a legitimate problem. This is what Hume does elsewhere too. For example, his sceptical worries about causation in the *Enquiry* are preceded by the elements of its skeptical solution: the chapter on induction already contains what he needs for solving his sceptical problem about causation. [↑](#footnote-ref-14)
15. The distinction between straight and sceptical solutions to sceptical problems comes from Kripke 1982, 66. [↑](#footnote-ref-15)
16. This is what Garrett (1997, 237), rightly I think, also deems possible contrary to Passmore (1952, 151). [↑](#footnote-ref-16)
17. Up to this point my reading is congenial to the one offered in Meeker 2007. [↑](#footnote-ref-17)
18. Hume’s challenge and the social character of his sceptical solution can find parallels in Kripke’s construal of Wittgenstein’s rule-scepticism (Kripke 1982), especially if it is interpreted with sociological inclinations as in Bloor 1997, Kusch 2006. [↑](#footnote-ref-18)