Time's Arrow in a Quantum Universe I:

On the Simplicity and Uniqueness of the Initial Quantum State

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Abstract

In a quantum universe with a strong arrow of time, we postulate a low-entropy boundary condition (the Past Hypothesis) to account for the temporal asymmetry. In this paper, I show that the Past Hypothesis also contains enough information to significantly simplify the quantum ontology and clearly define a unique initial condition in such a world.

First, I introduce *Density Matrix Realism*, the thesis that the quantum universe is described by a fundamental density matrix (a mixed state) that corresponds to some physical degrees of freedom in the world. This stands in sharp contrast to *Wave Function Realism*, the thesis that the quantum universe is described by a wave function (a pure state) that represents something physical.

Second, I suggest that the Past Hypothesis is sufficient to determine a unique and simple density matrix. This is achieved by what I call the *Initial Projection Hypothesis*: the initial density matrix of the universe is the projection onto the special low-dimensional Hilbert space.

Third, because the initial quantum state is unique and simple, we have a strong case for the *Nomological Thesis*: the initial quantum state of the universe is completely specified by a law of nature.

This new package of ideas has several interesting implications, including on the dynamic unity of the universe and the subsystems, the theoretical unity of statistical mechanics and quantum mechanics, and the alleged conflict between Humean supervenience and quantum entanglement.

Keywords: time's arrow, Past Hypothesis, Statistical Postulate, reduction, typicality, foundations of probability, quantum statistical mechanics, wave function realism, quantum ontology, density matrix, Weyl Curvature Hypothesis, Humean Supervenience

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1 Introduction

In recent debates about the metaphysics of the wave function, it is standard to assume that the universal wave function represents something physical. It may be interpreted as a field on the configuration space, a multi-field on physical space, something like a physical law, or an entity of a completely novel kind. Let us call this view *Wave Function Realism*.[1]

[1]See Albert (1996), Loewer (1996), Wallace and Timpson (2010), Ney (2012), North (2013), Maudlin (2013), Goldstein and Zanghì (2013), Miller (2014), Esfeld (2014), Bhogal and Perry (2015), Esfeld and Deckert (2017), Chen (2017a,b, ms). Notice that this is not how Albert, Loewer, and Ney use the term. For them, to be a wave function realist is to be a realist about the wave function and a fundamental

However, we may reject the assumption that there is a universal wave function that represents something physical. Indeed, it has been rejected by many people, notably by quantum Bayesians, and various anti-realists and instrumentalists. As a scientific realist, I do not find their arguments convincing. In previous papers, I have assumed and defended Wave Function Realism. However, in this paper I want to argue for a different perspective, for reasons related to the origin of time-asymmetry in a quantum universe.

To be sure, realism about the universal wave function is quite natural given standard quantum mechanics and various realist quantum theories such as Bohmian mechanics, GRW spontaneous collapse theories, and Everettian quantum mechanics. In those theories, the universal wave function is indispensable to the kinematics and the dynamics of the quantum system. However, as I would like to emphasize in this paper, our world is not just quantum-mechanical. We also live in a world with a strong arrow of time (entropy gradient). There are thermodynamic phenomena that we hope to explain with quantum mechanics and quantum statistical mechanics. A central theme of this paper is to suggest that quantum statistical mechanics is highly relevant for assessing the fundamentality and reality of the universal wave function.

We will take a close look at the connections between the foundations of quantum statistical mechanics and various solutions to the quantum measurement problem. When we do, we realize that we do not *need* to postulate a universal wave function. We only need certain "coarse-grained" information about the quantum macrostate, which can be represented by either a class of universal wave functions or a density matrix. A natural question is: can we describe the universe with a fundamental density matrix instead of a wave function?

The first step of of this paper is to argue that we can. I call this view *Density Matrix Realism*, the thesis that the universal quantum state is given by a fundamental density matrix that represents something physical. This idea may be unfamiliar to some people, as we are used to take mixed states to represent *epistemic uncertainty of the actual pure state* (a wave function). The proposal here is that the quantum state should be represented by a density matrix and not a wave function. This idea is not new in foundations of physics. We will then reformulate Bohmian mechanics, GRW theories, and Everettian quantum mechanics in terms of fundamental density matrices.

The second step is to point out that Density Matrix Realism allows us to unify quantum ontology with time-asymmetry in a new way. In classical and quantum statistical mechanics, thermodynamic time-asymmetry arises from a special boundary condition that is now called the Past Hypothesis.[2] I suggest that the information in the Past Hypothesis (with an implicit uniformity condition) is sufficient to determine a unique and simple fundamental density matrix. This can be done by

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high-dimensional space—the "configuration space." For the purpose of this paper, let us use *Wave Function Realism* to designate just the commitment that the wave function represents something physical.

[2]See Albert (2000).

postulating what I call the *Initial Projection Hypothesis*: the quantum state of the universe at t_0 is given by the unique projection on the special low-dimensional Hilbert space.

The third step is to show that, because of the simplicity and the uniqueness of the initial quantum state (now given by a fundamental density matrix), we have a strong case for the *Nomological Thesis*: the initial quantum state of the world is exactly specified by a law of nature.

As we shall see, this package of views has interesting implications for the dynamic unity of the universe and the subsystems, reduction of statistical mechanical probabilities to quantum mechanics, and Humean supervenience in a quantum world.

Here is the roadmap of the paper. First, in §2, I review the foundations of quantum mechanics and quantum statistical mechanics. In §3, I introduce the framework of Density Matrix Realism and illustrate it with a concrete example. In §4, I formulate the Initial Projection Hypothesis in the framework of Density Matrix Realism. In §5, I discuss their implications for dynamical and theoretical unification. In §6, I suggest that they provide a strong case for the Nomological Thesis and a new solution to the conflict between quantum entanglement and Humean supervenience.

2 Foundations of Quantum Mechanics and Statistical Mechanics

In this section, we first review the foundations of quantum mechanics and statistical mechanics. As we shall see in the next section, they suggest an alternative to Wave Function Realism.

2.1 Quantum Mechanics

Standard quantum mechanics is often presented with a set of axioms and rules about measurement. Firstly, there is a quantum state of the system, represented by a wave function ψ. For a N-particle quantum system in $\mathbb{R}^3$, the wave function is a (square-integrable) function from the configuration space $\mathbb{R}^{3N}$ to the spin space $\mathbb{C}^k$. Secondly, the wave function evolves in time according to the the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \tag{1}$$

Thirdly, the Schrödinger evolution of the wave function is supplemented with collapse rules. The wave function typically evolves into superpositions of macrostates, such as the cat being alive and the cat being dead. This can be represented by wave functions on the configuration space with disjoint macroscopic supports X and Y. During measurements, which are not precisely defined processes in the standard theory, the wave function undergoes collapses. Moreover, the probability that it collapses into any particular macrostate X is given by the Born rule:

$$P(X) = \int_X |\psi(x)|^2 dx \tag{2}$$

As such, quantum mechanics is not a candidate for a fundamental physical theory. It has two dynamical laws: the deterministic Schrödinger equation and the stochastic collapse rule. What are the conditions for applying the former, and what are the conditions for applying the latter? Measurements and observations are extremely vague concepts. Take a concrete experimental apparatus for example. When should we treat it as part of the quantum system that evolves linearly and when should we treat it as an "observer," i.e. something that stands outside the quantum system and collapses the wave function? That is, in short, the quantum measurement problem.[3]

Various solutions have been proposed regarding the measurement problem. Bohmian mechanics (BM) solves it by adding particles to the ontology and an additional guidance equation for the particles' motion. Ghirardi-Rimini-Weber (GRW) theories postulate a spontaneous collapse mechanism. Everettian quantum mechanics (EQM) simply removes the collapse rules from standard quantum mechanics and suggest that there are many emergent worlds, corresponding to emergent branches of the wave function, which are all real. My aim here is not to adjudicate among these theories. Suffice it to say that they are all quantum theories that remove the centrality of observations and observers.

To simplify the discussions, I will use BM as a key example.[4] In BM, in addition to the wave function that evolves linearly according to the Schrödinger equation, there are particles with precise locations, $Q_1, Q_2, ..., Q_N$, which follow the guidance equation:

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\psi^* \nabla_i \psi}{\psi^* \psi} \tag{3}$$

Moreover, the initial particle distribution is given by the Quantum Equilibrium Hypothesis:

$$\rho_{t_0}(x) = |\psi(x, t_0)|^2 \tag{4}$$

By the equivariance theorem, if this condition holds at the initial time, then it holds at all time. Consequently, BM agrees with standard quantum mechanics with respect to the Born rule predictions (which are all there is to the observable predictions of quantum mechanics).

In BM, the wave function ψ is central to the quantum system. It not only has its own dynamics described by (1) but also guides particle motion via (3). Its connection to the empirical predictions of quantum mechanics is manifested in (2) and (4). For a universe with N elementary particles, let us call the wave function of the universe the *universal wave function* and denote it as $\Psi(\boldsymbol{q_1}, \boldsymbol{q_2}, ...\boldsymbol{q_N})$. Therefore, at least *prima facie*, the universal wave function Ψ seems central to the description of the kinematics and the dynamics of the universe as a whole.

[3]See Bell (1990), Myrvold (2017) for introductions to the quantum measurement problem.

[4]See Dürr et al. (1992) for a rigorous presentation of BM and its statistical analysis.

2.2 Statistical Mechanics

Let us now consider macroscopic systems such as gas in a box. This can be described by a system of N particles, with $N > 10^{20}$. If the system is governed by classical mechanics, although it is difficult to solve the system of equations exactly, we can still use classical statistical mechanics (CSM) to describe its statistical behaviors, such as approach to thermal equilibrium suggested by the Second Law of Thermodynamics. Similarly, if the system is governed by quantum mechanics, we can use quantum statistical mechanics (QSM) to describe its statistical behaviors. Generally speaking, there are two different views on CSM: the individualistic view and the ensemblist view. We will first illustrate the two views with CSM, which is more familiar and will be helpful for understanding the two views in QSM.

2.2.1 Elements of Classical Statistical Mechanics

Let us review the basic elements of CSM on the individualistic view.[5] For concreteness, let us consider a classical-mechanical system with N particles in a box $\Lambda = [0, L]^3 \subset \mathbb{R}^3$ and a Hamiltonian H.

1. Microstate: at any time t, the microstate of the system is given by a point on a $6N$-dimensional phase space,

$$X = (\boldsymbol{q_1}, ..., \boldsymbol{q_N}; \boldsymbol{p_1}, ..., \boldsymbol{p_n}) \in \Gamma_{total} \subseteq \mathbb{R}^{6N}, \tag{5}$$

where Γ_{total} is the total phase space of the system.

2. Dynamics: the time dependence of $X_t = (\boldsymbol{q_1}, ..., \boldsymbol{q_N}; \boldsymbol{p_1}, ..., \boldsymbol{p_n}; t)$ is given by the Hamiltonian equations of motion:

$$\frac{\partial \boldsymbol{q_i}}{\partial t} = \frac{\partial H}{\partial \boldsymbol{p_i}}, \frac{\partial \boldsymbol{p_i}}{\partial t} = -\frac{\partial H}{\partial \boldsymbol{q_i}}. \tag{6}$$

3. Energy shell: the physically relevant part of the total phase space is the energy shell $\Gamma \subseteq \Gamma_{total}$ defined as:

$$\Gamma = \{X \in \Gamma_{total} : E \leq H(x) \leq E + \delta E\}. \tag{7}$$

We only consider microstates in Γ.

4. Measure: the measure μ is the standard Lebesgue measure of volume $|\cdot|$ on $\mathbb{R}^{6N}$.

5. Macrostate: with a choice of macro-variables, the energy shell Γ can be partitioned into macrostates Γ_ν:

$$\Gamma = \bigcup_\nu \Gamma_\nu. \tag{8}$$

[5] Here I follow the discussion in Goldstein and Tumulka (2011).

6. Unique correspondence: every phase point X belongs to one and only one Γ_ν.

7. Thermal equilibrium: typically, there is a dominant macrostate Γ_{eq} that has the most volume with respect to μ:

$$\frac{\mu(\Gamma_{eq})}{\mu(\Gamma)} \approx 1. \tag{9}$$

   A system is in thermal equilibrium if its phase point $X \in \Gamma_{eq}$.

8. Boltzmann Entropy: the Boltzmann entropy of a classical-mechanical system in microstate X is given by:

$$S_B(X) = k_B \log(\mu(\Gamma(X))), \tag{10}$$

   where $\Gamma(X)$ denotes the macrostate containing X. The thermal equilibrium state thus has the maximum entropy.

9. Low-Entropy Initial Condition: when we consider the universe as a classical-mechanical system, we postulate a special low-entropy boundary condition, which David Albert calls *the Past Hypothesis*:

$$X_{t_0} \in \Gamma_{PH}\ ,\ \mu(\Gamma_{PH}) \ll \mu(\Gamma_{eq}) \approx \mu(\Gamma), \tag{11}$$

   where Γ_{PH} is the Past Hypothesis macrostate with volume much smaller than that of the equilibrium macrostate. Hence, $S_B(X_{t_0})$, the Boltzmann entropy of the microstate at the boundary, is very small compared to that of thermal equilibrium.

10. A central task of CSM is to establish mathematical results that demonstrate (or suggest) that for μ–most microstates satisfying the Past Hypothesis, they will approach thermal equilibrium (in reasonable time).

Above is the individualistic view of CSM in a nutshell. In contrast, the ensemblist view differs in several ways. First, on the ensemblist view, instead of focusing on the microstate of an individual system, the focus is on the ensemble of systems that have the same statistical state ρ.[6] ρ is a distribution on the energy shell, and it also evolves according to the Hamiltonian dynamics. The crucial difference lies in the definition of thermal equilibrium. On the ensemblist view, a system is in thermal equilibrium if:

$$\rho = \rho_{mc} \text{ or } \rho = \rho_{can}, \tag{12}$$

where ρ_{mc} is the microcanonical ensemble and ρ_{can} is the canonical ensemble.[7]

[6]Some ensemblists would further insist that it makes no sense to talk about the thermodynamic state X of an individual system.

[7]Instead of using the Boltzmann entropy, some ensemblists use the Gibbs entropy:

$$S_G(\rho) = -k_B \int_\Gamma \rho \log(\rho) dx.$$

2.2.2 Elements of Quantum Statistical Mechanics

The foundations of QSM have important similarities to and differences with the foundations of CSM. For concreteness, let us consider a quantum-mechanical system with N fermions in a box $\Lambda = [0, L]^3 \subset \mathbb{R}^3$ and a Hamiltonian $\hat{H}$.[8]

1. Microstate: at any time t, the microstate of the system is given by a normalized (and anti-symmetrized) wave function:

$$\psi(\boldsymbol{q_1}, ..., \boldsymbol{q_N}) \in \mathscr{H}_{total} = L^2(\mathbb{R}^{3N}, \mathbb{C}^k) \ , \ \parallel \psi \parallel_{L^2} = 1, \tag{13}$$

   where $\mathscr{H}_{total} = L^2(\mathbb{R}^{3N}, \mathbb{C}^k)$ is the total Hilbert space of the system.

2. Dynamics: the time dependence of $\psi(\boldsymbol{q_1}, ..., \boldsymbol{q_N}; t)$ is given by the Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi. \tag{14}$$

3. Energy shell: the physically relevant part of the total Hilbert space is the subspace ("the energy shell"):

$$\mathscr{H} \subseteq \mathscr{H}_{total} \ , \ \mathscr{H} = \text{span}\{\phi_\alpha : E_\alpha \in [E, E + \delta E]\}, \tag{15}$$

   This is the subspace (of the total Hilbert space) spanned by energy eigenstates ϕ_α whose eigenvalues E_α's belong to the $[E, E + \delta E]$ range. Let $D = \text{dim}\mathscr{H}$, the number of energy levels between E and $E + \delta E$.

   We only consider wave functions ψ's in $\mathscr{H}$.

4. Measure: the measure μ is given by the standard Lebesgue measure on the unit sphere in the energy subspace $\mathscr{S}(\mathscr{H})$.[9]

5. Macrostate: with a choice of macro-variables (suitably "rounded" *à la* Von Neumann (1955)), the energy shell $\mathscr{H}$ can be orthogonally decomposed into macro-spaces:

$$\mathscr{H} = \oplus_\nu \mathscr{H}_\nu \ , \ \sum_\nu \text{dim}\mathscr{H}_\nu = D \tag{16}$$

   Each $\mathscr{H}_\nu$ corresponds more or less to small ranges of values of macro-variables that we have chosen in advance.

6. Non-unique correspondence: typically, a wave function is in a superposition of macrostates and is not entirely in any one of the macrospaces. However, we

Since $S_G(\rho_t)$ is stationary under the Hamiltonian dynamics, it is not the right kind of object for understanding the approach to thermal equilibrium in the sense of the Second Law, as we would like to have an object that can change, and, in particular, increase with time.

[8]Here I follow the discussions in Goldstein et al. (2010a) and Goldstein and Tumulka (2011).

[9]In cases where the Hilbert space is infinite-dimensional, we should use a Gaussian measure, which is not translation-invariant.

can make sense of situations where ψ is (in the Hilbert space norm) very close to a macrostate $\mathscr{H}_\nu$:

$$\langle\psi| P_\nu |\psi\rangle \approx 1, \tag{17}$$

where P_ν is the projection operator into $\mathscr{H}_\nu$. This means that almost all of $|\psi\rangle$ lies in $\mathscr{H}_\nu$.

7. Thermal equilibrium: typically, there is a dominant macro-space $\mathscr{H}_{eq}$ that has a dimension that almost equal to D:

$$\frac{\dim\mathscr{H}_{eq}}{\dim\mathscr{H}} \approx 1. \tag{18}$$

A system with wave function ψ is in equilibrium if the wave function ψ is very close to $\mathscr{H}_{eq}$ in the sense of (17): $\langle\psi| P_{eq} |\psi\rangle \approx 1$.

Simple Example. Consider a gas consisting of $n = 10^{23}$ atoms in a box $\Lambda \subseteq \mathbb{R}^3$. The system is governed by quantum mechanics. We orthogonally decompose the Hilbert space $\mathscr{H}$ into 51 macro-spaces: $\mathscr{H}_0 \oplus \mathscr{H}_2 \oplus \mathscr{H}_4 \oplus ... \oplus \mathscr{H}_{100}$, where $\mathscr{H}_\nu$ is the subspace corresponding to the macrostate that the number of atoms in the left half of the box is between $(\nu - 1)\%$ and $(\nu + 1)\%$ of n. In this example, $\mathscr{H}_{50}$ has the overwhelming majority of dimensions and is thus the equilibrium macro-space. A system whose wave function is very close to $\mathscr{H}_{50}$ is in equilibrium.

8. Boltzmann Entropy: the Boltzmann entropy of a quantum-mechanical system with wave function ψ that is very close to a macrostate ν is given by:

$$S_B(\psi) = k_B \log(\dim\mathscr{H}_\nu), \tag{19}$$

where $\mathscr{H}_\nu$ denotes the subspace containing almost all of ψ in the sense of (17). The thermal equilibrium state thus has the maximum entropy:

$$S_B(eq) = k_B \log(\dim\mathscr{H}_{eq}) \approx k_B \log(D) = S_B(mc), \tag{20}$$

where eq denotes the equilibrium macrostate and mc the micro-canonical ensemble.

9. Low-Entropy Initial Condition: when we consider the universe as a quantum-mechanical system, we postulate a special low-entropy boundary condition on the universal wave function—the quantum-mechanical version of *the Past Hypothesis*:

$$\Psi(t_0) \in \mathscr{H}_{PH}\ , \dim\mathscr{H}_{PH} \ll \dim\mathscr{H}_{eq} \approx \dim\mathscr{H} \tag{21}$$

where $\mathscr{H}_{PH}$ is the Past Hypothesis macro-space with dimension much smaller than that of the equilibrium macro-space.[10] Hence, the initial state has very low entropy in the sense of (19).

[10] The Past Hypothesis macro-space is thus finite-dimensional. So we can use the Lebesgue measure on the unit sphere as the typicality measure for # 10.

10. A central task of QSM is to establish mathematical results that demonstrate (or suggest) that for μ−most (maybe even all) wave functions satisfying the Past Hypothesis, they will approach thermal equilibrium (in reasonable time).

Above is the individualistic view of QSM in a nutshell. In contrast, the ensemblist view of QSM differs in several ways. First, on the ensemblist view, instead of focusing on the wave function of an individual system, the focus is on an ensemble of systems that have the same statistical state $\hat{W}$, a density matrix.[11] As a statistical density matrix, $\hat{W}$ can be defined from a uniform distribution on the unit sphere in the Hilbert space:

$$\hat{W} = \int_{\mathscr{S}(\mathscr{H})} \mu(d\psi) \left|\psi\right\rangle \left\langle\psi\right|. \tag{22}$$

It evolves according to the von Neumann equation:

$$i\hbar \frac{d\hat{W}(t)}{dt} = [\hat{H}, \hat{W}]. \tag{23}$$

The crucial difference between the individualistic and the ensemblist views of QSM lies, again, in the definition of thermal equilibrium. On the ensemblist view, a system is in thermal equilibrium if:

$$W = \rho_{mc} \text{ or } W = \rho_{can}, \tag{24}$$

where ρ_{mc} is the microcanonical ensemble and ρ_{can} is the canonical ensemble.

For the QSM individualist, if the microstate ψ of a system is close to some macro-space $\mathscr{H}_\nu$ in the sense of (17), we can say that the macrostate of the system is $\mathscr{H}_\nu$. However, we can also represent the macrostate by a density matrix $\hat{W}_\nu$ generated from the unit sphere in $\mathscr{H}_\nu$ with a uniform distribution $\mu(d\psi)$.

$$\hat{W}_\nu = \int_{\mathscr{S}(\mathscr{H}_\nu)} \mu(d\psi) \left|\psi\right\rangle \left\langle\psi\right|. \tag{25}$$

In (25), there is a clear sense that $\hat{W}_\nu$ is defined with a choice of measure and from the wave functions on the unit sphere of the Hilbert space $\mathscr{H}_\nu$. But different measures can give rise to the same density matrix. What is essential and intrinsic to a density matrix is its geometrical meaning in the Hilbert space—a projection operator. When $\mathscr{H}_\nu$ is finite-dimensional, we can simply think of $\hat{W}_\nu$ as the normalized identity operator on $\mathscr{H}_\nu$ and define it without using a measure:

$$\hat{W}_\nu = \frac{I_\nu}{dim\mathscr{H}_\nu}, \tag{26}$$

where I_ν is the identity operator on $\mathscr{H}_\nu$.

[11]Similarly to the situation in CSM, some ensemblists would further insist that it makes no sense to talk about the thermodynamic state of an individual system.

2.2.3 CSM, QSM, and Microstate Dispensability

As mentioned above, there are several differences between CSM and QSM. First, the classical phase point X lies entirely in a macrostate Γ, while a wave function ψ typically does not belong entirely to any macro-space $\mathscr{H}_\nu$ (#6). Second, Boltzmann entropy in CSM counts the volume of regions in phase space, while Boltzmann entropy in QSM counts dimensionality of subspaces in the Hilbert space (#8). Third, an important class of typicality results in CSM holds for μ-most initial phase points, while some typicality results in QSM holds for every initial wave function (but only for most Hamiltonians) (#10).

I would like to highlight another difference that becomes obvious in the context of additional-ontology quantum theories. In these theories, the physical state of the world is given by two things: (A) a quantum state, represented by a universal wave function Ψ, and (B) the state of the additional ontology: particles (in BM), flashes (GRWf), or mass densities (GRWm and Sm).[12] If we were to take away the microstate Ψ from the physical state, we would still be left with the additional ontology, and we can still specify a quantum macrostate $\mathscr{H}$ or W as in (22). For example, in BM, the state of the world at a time t is given by the pair $(Q(t), \Psi(t))$, where $Q(t)$ is the complete list of N particle locations in $\mathbb{R}^3$. If we were to (figuratively speaking) take away $\Psi(t)$, we would still be left with $Q(t)$. That is, we would still be left with (in addition to the quantum macrostate) microscopic ontology that constitutes experimental devices and observers.

Thus, QSM in these quantum theories have the following curious property: we can dispense with its statistical-mechanical microstate (ψ in QSM) and still be left with a microscopic ontology. Let us call this property Microstate Dispensability.

The situation is quite different on CSM. If we were to take away the microstate $X = (\boldsymbol{q}_1, ..., \boldsymbol{q}_N; \boldsymbol{p}_1, ..., \boldsymbol{p}_n)$ from the state description, nothing would be left over to describe the microscopic ontology, which consists in point particles. Thus, CSM for Newtonian particle systems does not possess Microstate Dispensability. That is a crucial difference between CSM and QSM. This becomes important in §6 in the context of Humean supervenience.

3 Density Matrix Realism

According to Wave Function Realism, the quantum state of the world at any time is described by a universal wave function Ψ and it corresponds to some physical degrees of freedom. On this view, Ψ is both the microstate of QSM and the dynamical object of QM. It evolves by the Schrödinger equation (1), and in the case of BM it also determines particle motions via the guidance equation (3).

On the other hand, we usually use W, a density matrix, to represent our ig-

[12]These are sometimes called *primitive ontology* or *primary ontology*. But these labels are usually used for ontologies in the three-dimensional physical space. Here I do not want to prejudge the issue. In the case of BM, I allow that the configuration point can be interpreted as either a single particle in a high-dimensional space or N particles in the three-dimensional physical space.

norances over Ψ, the actual wave function of the system. W is understood to correspond to a macrostate in QSM. In some cases, W is also easier for calculation than Ψ, such as in the case of GRW collapse theories where there are multiple sources of randomness.

However, can we describe the universe with W instead of Ψ? In this section, I show that this indeed can be done.[13] I call this new framework *Density Matrix Realism*. To show that it is possible, I will use W-Bohmian Mechanics as a concrete example and explain how a fundamental density matrix can be empirically adequate for describing a quantum world. We can similarly construct W-GRW theories and W-Everett theories. I will also provide a physical interpretation of W as a fundamental object.

3.1 Example: W-Bohmian Mechanics

We will illustrate the differences between Wave Function Realism and Density Matrix Realism by thinking about two different Bohmian theories.

In standard Bohmian mechanics (BM), an N-particle universe at a time t is described by $(Q(t), \Psi(t))$. The universal wave function $\Psi(t)$ is governed by the Schrödinger equation (1), and the particle configuration $Q(t)$ evolves according to the guidance equation (3). Moreover, BM postulates a Quantum Equilibrium Hypothesis (4) at a temporal boundary of the universe. Given the centrality of Ψ in BM, Wave Function Realism is a natural interpretation of the ontology of the quantum state.

Alternatively, we can formulate a Bohmian theory with only W and Q. This was rigorously introduced as W-Bohmian Mechanics (W-BM) in Dürr et al. (2005).[14] On W-BM, an N-particle universe at time t is described by by $(W(t), \Psi(t))$. The fundamental density matrix $W(t)$ is governed by the von Neumann equation (23). Next, the particle configuration $Q(t)$ evolves according to an analogue of the guidance equation (W-guidance equation):

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\nabla_{q_i}\mathrm{tr}_{\mathbb{C}^k}W(q,q',t)}{\mathrm{tr}_{\mathbb{C}^k}W(q,q',t)}(q = q' = Q), \tag{27}$$

where $\mathrm{tr}_{\mathbb{C}^k}$ denotes the partial trace over the spin components. Finally, we can impose a boundary condition similar to that of the Quantum Equilibrium Hypothesis:

$$P(Q(t_0) \in dq) = \mathrm{tr}_{\mathbb{C}^k}W(q,q,t_0)dq. \tag{28}$$

[13]The possibility that the universe is described by a density matrix is not new. It has been suggested by multiple authors and explored to various extents. For some recent examples, see Dürr et al. (2005) Wallace and Timpson (2010) and Wallace (2011, 2012).

[14]Even though we are only discussing the universal quantum state, W-BM also has implications for their study on the conditional density matrices of subsystems. This becomes relevant in §4 when we discuss the dynamical unity in W-BM. See Dürr and Lienert (2014) for an extension of conditional density matrices to the Bohm-Dirac model, a relativistic version of BM with spin. See Maroney (2005) for another discussion about fundamental density matrices in BM.

Since the system is also equivariant, if the probability distribution holds at t_0, it holds at all times.[15]

With the defining equations—the von Neumann equation (23) and the W-guidance equation (27)—and the Bohmian boundary condition (28), we have a theory that directly uses a density matrix $W(t)$ to characterize the trajectories $Q(t)$ of the universe's N particles.

W-BM is empirically equivalent to BM with respect to the macroscopic quantum phenomena, that is, pointer readings in quantum-mechanical experiments. This follows from (28), which is analogous to the Quantum Equilibrium Hypothesis. With the respective dynamical equations, both BM and W-BM generate equivariant Born-rule probability distribution over macroscopic measurement outcomes.

3.2 Other Examples: Everettian and GRW Theories

W-BM is a simple quantum theory that is compatible with Density Matrix Realism. In this theory, we can be a realist about the universal density matrix $W(t)$—it represents some physical degrees of freedom. What about other quantum theories, such as Everettian and GRW theories? Is it possible to "replace" their universal wave functions with universal density matrices? We will show that this is also possible.

For the Everettian theory with no additional ontology (S0), we can postulate that the fundamental state is given by a density matrix $W(t)$ that evolves by the unitary von Neumann equation (23).

For the Everettian theory with an additional mass-density ontology $m(x,t)$, which was introduced as Sm by Allori et al. (2010), we can still use the von Neumann equation for $W(t)$. Next, we can define the mass-density function directly in terms of $W(t)$:

$$m(x,t) = \mathrm{tr}(M(x)W(t)), \tag{29}$$

where $M(x) = \sum_i m_i \delta(Q_i - x)$ is the mass-density operator, which is defined via the position operator $Q_i\psi(q_1, q_2, ...q_n) = q_i\psi(q_1, q_2, ...q_n)$. This allows us to determine the mass-density ontology at time t via $W(t)$.[16]

For the GRW theory with just the quantum state $W(t)$, we can follow Goldstein et al. (2012) and write down its stochastic evolution as follows:

$$i\hbar\frac{\partial W(t)}{\partial t} = -\frac{i}{\hbar}[H, W(t)] + \lambda \sum_{k=1}^{N} \int d^3x \Lambda_k(x)^{1/2} W(t) \Lambda_k(x)^{1/2} - N\lambda W(t), \tag{30}$$

where the commutator bracket represents the unitary evolution and the further

[15] Equivariance holds because of the following continuity equation:

$$\frac{\partial \mathrm{tr}_{\mathbb{C}^k} W(q,q,t)}{\partial t} = -\mathrm{div}(\mathrm{tr}_{\mathbb{C}^k} W(q,q,t)v),$$

where v denotes the velocity field generated via (27.)

[16] Thanks to Roderich Tumulka and Matthias Lienert for discussions here.

terms represent deviations from the unitary evolution. $\Lambda_k(x)$'s are the collapse rate operators.[17]

For the GRW theory (GRWm) that include both the quantum state $W(t)$ and a mass-density ontology $m(x,t)$, we can combine the above steps: $W(t)$ evolves by (30) and $m(x,t)$ is defined by (29). We can define GRW with a flash-ontology (GRWf) in a similar way, by using $W(t)$ to characterize the distribution of flashes in physical space-time.

3.3 Some Intepretations of W

For the sake of simplicity and without loss of generality, let us return to the example of W-BM. Since $W(t)$ plays an essential role in the dynamics of guiding particle motions, it ought to be possible to understand $W(t)$ as a concrete physical object. In debates about the metaphysics of the wave function, realists have offered several interpretations of Ψ. Wave function realists, such as Albert and Loewer, have offered a concrete physical interpretation: they suggest we understand the quantum state primarily not as an object in the Hilbert space but as $\Psi(q)$, a function on the configuration space. They then offer arguments for thinking that it is to be interpreted as a physical field on the configuration space, and that the configuration space is more fundamental than the low-dimensional physical space.[18]

Can we give a similar concrete physical interpretation of $W(t)$? Let us start with a mathematical representation of the density matrix $W(t)$. It is defined as a positive, bounded, self-adjoint operator $\hat{W} : \mathscr{H} \to \mathscr{H}$ with $\mathrm{tr}\hat{W} = 1$. For W-BM, the configuration space $\mathbb{R}^{3N}$, and a density operator $\hat{W}$, the relevant Hilbert space is $\mathscr{H}$, which is a subspace of the total Hilbert space, i.e. $\mathscr{H} \subseteq \mathscr{H}_{total} = L^2(\mathbb{R}^{3N}, \mathbb{C}^k)$. Now, the density matrix $\hat{W}$ can also be represented as a function

$$W : \mathbb{R}^{3N} \times \mathbb{R}^{3N} \to \mathrm{End}(\mathbb{C}^k) \tag{31}$$

The range denotes the space of linear maps from $\mathbb{C}^k$ to itself. (Notice that we have already used this representation in (27) and (28).) This representation enables us to provide a similar concrete interpretation of the density matrix: it can be interpreted as a field on $\mathbb{R}^{6N}$ or a"multi-field" on $\mathbb{R}^{3N}$ that assigns quantities (linear maps from spin space to itself) to every ordered pair of points (q,q') on $\mathbb{R}^{3N}$. For 3D-Fundamentalists, they can interpret it as assigning quantities to every ordered

[17] A collapse rate operator is defined as follows:

$$\Lambda_k(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{(Q_k-x)^2}{2\sigma^2}},$$

where Q_k is the position operator of "particle" k, and σ is a new constant of nature of order 10^{-7} m postulated in current GRW theories.

[18] In Chen (2017b), I argue against this view and suggest that there are many good reasons—internal and external to quantum mechanics—for taking the low-dimensional physical space-time to be fundamental.

pair of N-regions on $\mathbb{R}^3$, the physical space.[19]

In §6, we introduce a new interpretation of W as a law of nature, in the context of the new Initial Projection Hypothesis.

4 The Initial Projection Hypothesis

We have established that Density Matrix Realism is an alternative to Wave Function Realism when it comes to the interpretations of quantum mechanics. In the previous section, we have given some concrete examples of how to reformulate quantum theories in terms of fundamental density matrices.

In §2, we have mentioned the identification of quantum macrostates with density matrices. In the individualistic framework of QSM, density matrices can be used to represent our epistemic uncertainties over the actual microstate—the wave function. A density matrix is compatible with many possible wave functions that the system could have.

In CSM and QSM, a fundamental postulate is added to the time-symmetric dynamics: the Past Hypothesis, which is a low-entropy boundary condition on the initial microstate of the universe. In this section, we will first discuss the microstate versions of the Past Hypothesis. Then we will formulate it directly on the "macrostate"—the density matrix. Finally, we point out some parallels between the "macrostate" version of the Past Hypothesis and Penrose's Weyl Curvature Hypothesis.

4.1 Standard Versions of the Past Hypothesis

The history of the Past Hypothesis goes back to Ludwig Boltzmann. His research was on the statistical-mechanical origin of thermodynamic time-asymmetry. In his time, physicists were confronted with the question of reconciling macroscopic time-asymmetry (of thermodynamics) with microscopic time-symmetry (of Hamiltonian mechanics). The Hamiltonian dynamics is time-reversal invariant but the macroscopic equations are not. Two bricks of different temperatures upon contact (but otherwise isolated from the environment) tend to approach thermal equilibrium. Entropy never decreases. How does temporal asymmetry arise from time-symmetric fundamental laws? Boltzmann ingeniously pointed out that there is an asymmetry in the phase space: there is a dramatic disproportion of volume of macrostates. As we stated in §2.2.1 # 7, the thermal equilibrium state takes up the overwhelming majority of the phase space volume on the energy hypersurface,[20] while other macrostates take up much smaller volume. If a non-equilibrium system undertakes a random walk (on some level of coarse-graining) in the phase space, it will most likely get into microstates that sit in larger and larger macrostates and eventually

[19]For discussions about the multi-field interpretation, see Forrest (1988), Belot (2012), Chen (2017), Chen (ms.) section 3, and Hubert and Romano (2017).

[20]This was only rigorously proved in the last century.

reach thermal equilibrium. This seems to explain the entropy gradient of the Second Law of Thermodynamics.

However, Boltzmann was criticized (notably by Zermelo) that his explanation must have smuggled in assumptions about time-asymmetry, for it worked just in the other way. Just as in the future there will be more higher-entropy points to go to, in the past there would be more higher-entropy points to come from. So Boltzmann's argument from the asymmetry of macrostate volume did not explain the Second Law. Instead, his argument predicted a time-symmetric history, where the system would sit in a local minimum of entropy and was in higher entropy both in the past and in the future. This is called the *reversibility objection*.

In reply, Botlzmann once suggested that we add a boundary condition: the universe started in a special state of very low-entropy. Richard Feynman agrees, "For some reason, the universe at one time had a very low entropy for its energy content, and since then the entropy has increased."[21] Such a low-entropy initial condition will block the reversibility objection, as there would be nothing before the initial moment to retrodict to. David Albert has called this condition the *Past Hypothesis* (PH).

In CSM, PH takes the form of §2.2.1 #9. In QSM, PH takes the form of §2.2.2 #9.[22] That is, PH severely constrains the initial microstates to be within a ridiculously small region in phase space or a particular low-dimensional subspace in Hilbert space. However, it does not pin down a unique microstate, either in CSM or QSM. There is still a continuous infinity of possible microstates compatible with the initial low-entropy macrostate.

I should mention that for PH to work as a successful explanation for the Second Law, it has to be on a par with other fundamental laws of nature. Moreover, since there are anti-thermodynamic exceptions even for trajectories starting from the PH macrostate, it is crucial to impose another law about a uniform probability distribution over the initial macrostate. David Albert calls it the Statistical Postulate (SP). It corresponds to the measures that we specified in §2.2.1 #4 and §2.2.2 #4. We used those measures to state the typicality statements in #10. Barry Loewer calls the joint system—the package of laws that includes PH and SP in addition to the dynamical laws of physics—the Mentaculus Vision.[23]

4.2 Introducing the Initial Projection Hypothesis

Standard versions of the Past Hypothesis make use of the low-entropy macrostate to constrain the microstate description of the system (a phase point in CSM or a state vector in QSM). This is natural from the perspective of Wave Function Realism, for which the state vector (the wave function) represents the physical degrees of

[21] Feynman et al. (2015), 46-8.

[22] See Wallace (2011, 2012) for detailed discussions about how to formulate PH as constraints for the classical and quantum microstates.

[23] For developments and defenses of the nomological account of the Past Hypothesis and the Statistical Postulate, see Albert (2000), Loewer (2007), Wallace (2011, 2012) and Loewer (2016)

freedom of the system. The initial state of the system is described by a normalized wave function $\Psi(t_0)$. It has to lie in the special low-dimensional Hilbert space $\mathscr{H}_{PH}$ with $dim\mathscr{H}_{PH} \ll dim\mathscr{H}_{eq}$. (It is important to point out that, in the position space representation, the wave function is a complicated function on the configuration space.) Moreover, there are many different choices of initial wave functions in $\mathscr{H}_{PH}$. Furthermore, for stating the typicality statements, we also need to specify a measure μ on the unit sphere of $\mathscr{H}_{PH}$. For the finite-dimensional case, it is just the uniform Lebesgue measure.

Let us now formulate a boundary condition in the framework of Density Matrix Realism. A fundamental density matrix $W(t)$ should encode the same amount of information as the statistical density matrix that we use to represents epistemic ignorance over the actual wave function. We know that standard versions of PH more or less pin down a unique macrostate—the special low-entropy macrostate. In QSM, this corresponds to $\mathscr{H}_{PH}$, the special subspace of the total Hilbert space. This naturally gives rise to a normalized projection operator onto that space. Just as in (26), we can specify the projection onto $\mathscr{H}_{PH}$ as:

$$\hat{W}_{PH}(t_0) = \frac{I_{PH}}{dim\mathscr{H}_{PH}}, \tag{32}$$

where t_0 represents a temporal boundary of the universe, I_{PH} is the identity operator on $\mathscr{H}_{PH}$, dim counts the dimension of the Hilbert space, and $dim\mathscr{H}_{PH} \ll dim\mathscr{H}_{eq}$. Given Boltzmannian arguments, we will also call t_0 the initial time.

I propose that we add the following postulate to any quantum theory in the framework of Density Matrix Realism:

Initial Projection Hypothesis: The initial quantum state of the universe is $\hat{W}_{PH}(t_0)$.

I would like to make three observations about the content of the Initial Projection Hypothesis (IPH).

First, IPH defines a unique initial quantum state. The quantum state $\hat{W}_{PH}(t_0)$ is informationally equivalent to the constraint that PH imposes on the initial microstates. Assuming that PH selects a unique low-entropy macrostate, $\hat{W}_{PH}(t_0)$ is singled out by the data in PH.[24] As such, IPH breaks time-symmetry; $\hat{W}_{PH}(t_0)$ will gives rise to time-asymmetric thermodynamic behaviors.

Second, we do not need to impose an additional probability / typicality measure on the Hilbert space, as $\hat{W}_{PH}(t_0)$ is mathematically equivalent to an integral over projection onto each normalized state vectors (wave functions) compatible with PH *with respect to a Lebesgue measure*. Of course, we are not defining $\hat{W}_{PH}(t_0)$ in terms of state vectors. Rather, we are thinking of $\hat{W}_{PH}(t_0)$ as a geometric object in the Hilbert space: the (normalized) identity operator on $\mathscr{H}_{PH}$. (It can be further interpreted as having the same status as a physical field (§3.3) or a physical law (§6).)

[24]Standard versions of PH may be vague about the exact initial low-entropy macrostate. In that case, we can let empirical observations and theoretical cosmology to pin down a unique initial macrostate. In that case, IPH would make the boundary condition more exact than standard versions of PH.

Third, $\hat{W}_{PH}(t_0)$ is simple. We can give three reasons. First, the identity operator on a low-dimensional Hilbert space is probably the simplest mathematical object we can associate with that Hilbert space. Second, since a density matrix is compatible with many different wave functions and different choices of measures on the Hilbert space, it has to contain very little information. Third, we can argue from the informational equivalence between the Past Hypothesis and the initial density matrix $\hat{W}_{PH}(t_0)$ (32). Some Humeans[25] have given good arguments that the Past Hypothesis is an additional (Humean) law of nature. As a law in the Humean best system, the Past Hypothesis has to be a very simple postulate. Hence, the initial density matrix $\hat{W}_{PH}(t_0)$, which is informationally equivalent to the Past Hypothesis, is also simple.[26] As we shall see in §6, the simplicity of $\hat{W}_{PH}(t_0)$ will be a crucial ingredient for a new version of Quantum Humeanism (§6.2).

To simplify the discussions in later sections, let us write down the modified theory of W-BM after adding IPH as a fundamental postulate:

(A) $\hat{W}_{PH}(t_0) = \frac{I_{PH}}{dim \mathscr{H}_{PH}}$

(B) $P(Q(t_0) \in dq) = \mathrm{tr}_{\mathbb{C}^k} W_{PH}(q, q, t_0) dq,$

(C) $i\hbar \frac{\partial \hat{W}}{\partial t} = [\hat{H}, \hat{W}],$

(D) $\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\nabla_{q_i} \mathrm{tr}_{\mathbb{C}^k} W_{PH}(q,q',t)}{\mathrm{tr}_{\mathbb{C}^k} W_{PH}(q,q',t)} (q = q' = Q).$

Let us call this theory W_{PH}-BM. It is worth pointing out that given the initial quantum state $\hat{W}_{PH}(t_0)$, there is a live possibility that for every particle at t_0, the velocity is zero. However, even in this possibility, as long as the initial quantum state "spreads out" later, as we assume it would, the particle configuration will typically start moving at a later time. This is true because of equivariance (see footnote #15).[27]

4.3 Connections to the Weyl Curvature Hypothesis

It is worth pointing out some connections between our Initial Projection Hypothesis (IPH) and the Weyl Curvature Hypothesis (WCH) proposed by Penrose (1979). Thinking about the origin of the Second Law of Thermodynamics in the early universe with high homogeneity and isotropy, and the relationship between space-time geometry and entropy, Penrose proposes a hypothesis:

> I propose, then, that there should be complete lack of chaos in the initial *geometry*. We need, in any case, some kind of low-entropy constraint on the initial state. But thermal equilibrium apparently held (at least very closely so) for the *matter* (including radiation) in the early stages.

[25] For example, Albert (2000), Loewer (2007) and Loewer (2016).

[26] This is of course compatible with the fact that $\hat{W}_{PH}(t)$ will become complicated at later times. But because von Neumann equation is deterministic, we have a simple way of deriving the quantum state at any later time from that at the initial time.

[27] Thanks to Shelly Goldstein and Tim Maudlin for discussions here.

> So the 'lowness' of the initial entropy was not a result of some special matter distribution, but, instead, of some very special initial spacetime geometry. The indications of [previous sections], in particular, are that this restriction on the early geometry should be something like: *the Weyl curvature C_{abcd} vanishes at any initial singularity*. (Penrose (1979), p.630, emphasis original)

The Wyel curvature tensor C_{abcd} is the traceless part of the Riemann curvature tensor R_{abcd}. It is not fixed completely by the stress-energy tensor and thus has independent degrees of freedom in Einstein's general theory of relativity. Since the entropy of matter distribution is quite high, the origin of thermodynamic asymmetry should be due to the low entropy in geometry, which corresponds very roughly to the vanishing of the Weyl curvature tensor.

WCH is an elegant way of encoding the initial low-entropy boundary condition in the classical spacetime geometry. It is a version of PH that also dispenses with SP. From this perspective, we can think of IPH as another way of encoding the initial low-entropy boundary condition in part of ontology—in our case, in the fundamental quantum state $\hat{W}_{PH}(t_0)$. It not only dispenses with SP, but it also uses the PH data to pin down a unique initial quantum state (although additional ontology such as Bohmian configuration can still be randomly distributed). In contrast, $C_{abcd} \to 0$ at the initial singularity only partially fixes the initial geometric data. In any case, the most interesting connection is that WCH and IPH are both ways of unifying (in some suitable sense) statistical mechanics (SM) with another branch of physics: WCH unifies SM with GR; IPH unifies SM with QM. This establishes an indirect connection between GR and QM (via SM), which could be fruitful to explore in quantum gravity research.

5 Unification

The Initial Projection Hypothesis is formulated in the framework of Density Matrix Realism: the initial density matrix is given by (32). Density Matrix Realism, plus the postulate of IPH, leads to two important kinds of unification, which we will explore in this section.

5.1 Dynamic Unity of the Universe and the Subsystems

First, Density Matrix Realism, independently of IPH, can harmonize the dynamics of the universe and the subsystems.

Let us start with a quantum-mechanical universe U. Suppose that it occupies a (finite or infinite) spatial region $R_U \subseteq \mathbb{R}^3$. U consists in subsystems S_i's that occupy spatial regions $R_{S_i} \subseteq \mathbb{R}^3$. As an idealization, they partition R_U:

$$R_U = \bigcup_i R_{S_i} \tag{33}$$

For a universe that can be partitioned into quasi-isolated subsystems (interactions with the environment effectively vanish), the following is a desirable property:

Dynamic Unity The dynamical laws of the universe are the same as the effective laws of most quasi-isolated subsystems.

This property is desirable, but not indispensable. It is desirable because law systems that apply both at the universal level and at the subsystem level are unifying and explanatory. This would be accepted, I believe, by Humeans and anti-Humeans alike.[28] However, it can be traded off with other properties that law systems can have.[29]

Thus, it is desirable, other things being equal, if quantum theories also display Dynamic Unity. However, BM formulated with a universal wave function violates this property. To be sure, Dynamic Unity is valid in the simplest kind of Bohmian theory—for N spin 0 particles and scalar-valued wave functions. Suppose that the universe is partitioned into two subsystems S_1 and S_2. We have a universal wave function $\Psi(q_1, q_2)$, where q_1, q_2 are the configuration variables for the subsystems. Because there are particles with precise positions, we can define conditional wave functions. For example, for S_1, we can define its conditional wave function:

$$\psi_{cond}(q_1) = C\Psi(q_1, Q_2), \tag{34}$$

where C is a normalization factor and Q_2 is the actual configuration of S_2. In general, $\psi_{cond}(q_1)$ does not evolve linearly by the Schrödinger equation. During quantum measurements, it undergoes collapses, just as von Neumann's measurement axioms tell us. However, when S_1 and S_2 are suitably decoupled, and between measurements, $\psi_{cond}(q_1, t)$ evolves, effectively, according to its own Schrödinger equation.

Just like $\Psi(q_1, q_2)$, $\psi_{cond}(q_1, t)$ can always give rise to the probability density according to the Born rule and the velocity field for the particles (in S_1) according to the guidance equation:

$$P(Q_1 \in dq_1 | Q_2) = |\psi_{cond}(q_1)|^2 dq_1, \tag{35}$$

$$\frac{dQ_{1i}}{dt} = \frac{\hbar}{m_{1i}} \mathrm{Im} \frac{\nabla_i \psi_{cond}}{\psi_{cond}}(Q_1) \tag{36}$$

(35) corresponds to the probability formula (4). (36) corresponds to the guidance equation for the (universal) wave function defined in (3).

However, since BM is described by $(\Psi(t), Q(t))$, it does not contain actual values of spin. It follows that when there are spin or other degrees of freedom, we

[28]See Hicks (2016) for an interesting discussion about quasi-isolated subsystems in the context of Humean supervenience. However, our accounts differ in some important aspects. Unlike Hicks, I do not want to define quasi-isolated subsystems as those that satisfy Dynamic Unity. They can fail to have it. Moreover, I want Dynamic Unity to be a property that comes in degrees, rather than an on/off thing.

[29]In personal communications, Matthias Lienert suggests that a failure of Dynamic Unity might be expected from the perspective of relational mechanics. So someone who is sympathetic to that project would be less inclined to regard Dynamic Unity as a desirable feature.

cannot define conditional wave functions in an analogous way. This means that for a Bohmian universe governed by a spinor-valued universal wave function, the subsystems are not governed by conditional wave functions. Thus, the subsystem particles do not always follow velocity fields defined from the subsystem guidance equation (36).[30]

Dürr et al. (2005) point out that, instead of defining conditional wave functions, we can still define conditional density matrices for these situations:

$$\hat{W}_{cond} = \frac{\mathrm{tr}_2(|\Psi\rangle\langle\Psi|\hat{1}\oplus\hat{P}_{q_2}(dq_2))}{\mathrm{tr}(|\Psi\rangle\langle\Psi|\hat{1}\oplus\hat{P}_{q_2}(dq_2))}(q_2 = Q_2), \tag{37}$$

where $\hat{P}_{q_2}$ is a PVM on $\mathbb{R}^{3N}$. Moreover, when the system is suitably decoupled from the environment, the conditional density matrix follows the von Neumann equation (23), which also dictates the time evolution of the universal density matrix of W-BM. Furthermore, the conditional density matrix *always* gives rise to the velocity field for particles in the subsystem in a way similar to (27):

$$\frac{dQ_{1i}}{dt} = \frac{\hbar}{m_{1i}}\mathrm{Im}\frac{\nabla_{q_{1i}}\mathrm{tr}_{\mathbb{C}^k}W_{cond}(q_1,q_1')}{\mathrm{tr}_{\mathbb{C}^k}W_{cond}(q_1,q_1')}(q_1 = q_1' = Q_1). \tag{38}$$

This is in contrast to the situation in BM. In BM, subsystems do not always have conditional wave functions or effective wave functions, and thus the conditional guidance equation (36) is not always valid. Therefore, W-BM has more Dynamic Unity than BM. Other things being equal, that is a (defeasible) reason for preferring W-BM to BM, and a reason for preferring Density Matrix Realism to Wave Function Realism.

5.2 Unification of Statistical Mechanics and Quantum Mechanics

When we add IPH to Density Matrix Realism, such as in the case of W_{PH}-BM, we achieve another kind of unification: between statistical mechanic and quantum mechanics. As we pointed out in §4, standard versions of CSM and QSM require the addition of both the Past Hypothesis (PH) and the Statistical Postulate (SP) to the dynamical laws. The situation is quite different in our framework.

For example, in W_{PH}-BM, we incorporate PH into the simple and unique initial quantum state $W_{PH}(t_0)$. The low-entropy boundary condition is now built into the dynamics of the theory. In §4.2 (A)—(D), we rewrote every dynamical equation and probability formula in terms of $W_{PH}(t_0)$.

Notice that the equations of W_{PH}-BM are no longer time-translationally invariant, which reflects the fact that time-asymmetry is built into the architecture of the theory. This might seem to be an undesirable feature to some people, as we would often like to separate the time-symmetric part from the time-asymmetric part of the theory.

[30]This is compatible with the fact that for subsystems that are *suitably decoupled* from the envioronment, they still have effective wave functions. See Dürr and Lienert (2014), Lemma.

But notice that the actual world we live in is *time-asymmetric*. So our fundamental theory has to account for that phenomena. If the simplest and the most unifying way to write down a theory can still be cleanly separated into parts that are time-symmetric from parts that are not, then it is a nice thing. But if it cannot be done, it is not a big strike against the theory. As I think of it, symmetries and invariances are only defeasible indicators for simplicity and parsimony. We ought to judge the theory on a case-by-case basis; we should compare theories to each other and not merely against abstract principles of symmetries.[31]

W_{PH}-BM also eliminates the need for SP and thus the need for a separate statistical-mechanical probability / typicality in addition to the quantum equilibrium measure (B). This is a great achievement of the new framework, as it has been a conceptual puzzle how to unify statistical-mechanical probabilities with quantum-mechanical probabilities. I used W_{PH}-BM as an illustration. But obviously similar unification carries over to GRW-type theories and Everet-type theories formulated with a fundamental density matrix satisfying the Initial Projection Hypothesis. A similar possibility has been explored in the context of GRW jumps by Albert (2000) and in the context of microstate versions of PH by Wallace (2011, 2012).

6 The Nomological Thesis

In previous sections, we have developed the framework of Density Matrix Realism and added the Initial Projection Hypothesis to that framework. We have argued that the initial quantum state in such theories would be simple and unique. This lends support of the following thesis about the status of the initial quantum state:

The Nomological Thesis: The initial quantum state of the world is nomological, i.e. it is completely specified by a law of nature.

Is it plausible to think of $W_{PH}(t_0)$ as nomological? That depends on whether IPH should be thought of as a law of nature. However, as we have argued, IPH is informationally equivalent to PH. Many people (Humeans and non-Humeans) have given good arguments that PH is on a par with the fundamental dynamical laws of physics. (If PH is not a law, then what is it, and how can it explain the Second Law of Thermodynamics?) There is a strong case for the nomological status of PH. Thus, we also have a strong case for the nomological status of IPH.

These considerations provide good reasons for accepting the Nomological Thesis. As a parallel to Albert and Loewer's Mentaculus Vision, let us call the following package of laws the *Quantaculus Theory*:

1. The fundamental dynamical laws of quantum mechanics, formulated in terms of fundamental density matrices.

[31]Thanks to David Albert for discussions about this point. Regarding the connection between symmetries and structures, I have learnt much from personal communications with Jill North and Ted Sider.

2. The Initial Projection Hypothesis.

The Quantaculus Theory has only two ingredients, as the Initial Projection Hypothesis eliminates the need for a statistical-mechanical probability measure (§5.2). This is a significantly improvement.

The Quantaculus Theory can be interpreted in non-Humean ways or Humean ways. On the non-Humean proposal, we can think of the initial density matrix as an additional law of nature that explains the distribution of particles, fields, or flashes. Since IPH is already very simple, the plausibility of the Nomological Thesis no longer depends on the time-independence of the quantum state.[32] On the Humean proposal, we can think of the initial density matrix as being completely specified by a law of nature that supervenes on a separable mosaic. This leads to a straightforward reconciliation between Humean supervenience and quantum entanglement. In this section, we will first review this alleged conflict. Then we will propose a new version of Quantum Humeanism that resolves the conflict.

6.1 The Humean Conflict with Quantum Entanglement

According to HS, the fundamental ontology of the world consists in:

> "a vast mosaic of local matters of particular fact, just one little thing and then another...We have geometry: a system of external relations of spatio-temporal distances between points...And at those points we have local qualities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated. For short, we have an arrangement of qualities. And that is all. There is no difference without difference in the arrangement of qualities. All else supervenes on that." (Lewis, 1986 p. ix)

According to HS, then, the "vast mosaic of local matters of particular fact" is a *supervenience base* for everything else in the world, the *metaphysical ground floor* on which everything else depends.[33] On this view, laws of physics are nothing over and above the "mosaic." They are just the simplest and most informative summaries of the local matters of particular fact. For example, in classical mechanics, we can think of the Newtonian laws of motion as mere summaries of the particle trajectories. The Newtonian laws are not additional facts in the world, for they supervene on the complete history of the N-particle universe.

Perhaps the most often-cited counterexample to HS comes from the entanglement phenomena in QM. A consequence of Humean Supervenience is that the

[32]See Goldstein and Zanghì (2013) for a discussion about the Nomological Thesis that is motivated by the Wheeler-DeWitt equation in quantum gravity.

[33]Humeans probably wants to formulate this thesis with some stronger relation, such as reduction or grounding, than the traditional supervenience relation, for the latter would more accurately capture the notion of asymmetric dependence relation of tfhe modal and nomological facts on the non-modal and non-nomological facts. But in any case, reduction and grounding arguably imply supervenience.

complete physical state of the universe is determined by the properties and spatiotemporal arrangement of the local matters (suitably extended to account for vector-valued magnititudes) of particular facts. It follows that there should not be any state of the universe that fails to be determined by the properties of individual space-time points.[34] However, the quantum state of our world–represented by the wave function–contains, as a matter of empirical fact, entanglement relations, which are not determined by the properties of space-time points.

The consideration above suggests a strong *prima facie* conflict between HS and quantum physics. On the basis of quantum non-separability, Tim Maudlin has proposed an influential argument against HS.[35]

6.2 A New Version of Quantum Humeanism

Among many things, the Quantaculus Theory offers a way out of the conflict between quantum entanglement and Humean supervenience. Suppose that a Humean accepts the Quantaculus Theory, then she can easily Humeanize it by treating the laws (including the IPH) as the axioms in the best system that summarize a separable mosaic. Let us use W_{PH}-BM as an example:

The W_{PH}-BM mosaic: particle trajectories $Q(t)$ on physical space-time.

The W_{PH}-BM best system: four equations—the simplest and strongest axioms summarizing the mosaic:

(A) $\hat{W}_{PH}(t_0) = \frac{I}{dim \mathscr{H}_{PH}}$

(B) $P(Q(t_0) \in dq) = \mathrm{tr}_{\mathbb{C}^k} W_{PH}(q, q, t_0) dq$,

(C) $i\hbar \frac{\partial \hat{W}}{\partial t} = [\hat{H}, \hat{W}]$,

(D) $\frac{dQ_i}{dt} = \frac{\hbar}{m_i} \mathrm{Im} \frac{\nabla_{q_i} \mathrm{tr}_{\mathbb{C}^k} W_{PH}(q, q', t)}{\mathrm{tr}_{\mathbb{C}^k} W_{PH}(q, q', t)} (q = q' = Q)$.

Notice that (A)—(D) are very simple and informative statements about $Q(t)$. They are expressed in terms of $\hat{W}_{PH}(t)$, which can be derived via a simple law (C) from $\hat{W}_{PH}(t_0)$. Moreover, as we have argued in §4.2, $\hat{W}_{PH}(t_0)$ is very simple.

Here we have exploited the fact that W_{PH}-BM satisfies Microstate Dispensability (§2.2.3): even after removing the quantum state (W) from the mosaic, there are still particles (Q's) in the mosaic that provide an adequate ontological basis. It is obvious that we can do similar "Humeanization" maneuvers on the fundamenteal density matrix in other quantum theories with additional ontologies—GRWm, GRWf, and Sm—since they also satisfy Microstate Dispensability.

This version of Quantum Humeanism stands in contrast to the current approaches in the literature: Albert (1996), Loewer (1996), Miller (2014), Esfeld (2014), Bhogal and Perry (2015), Esfeld and Deckert (2017) and Chen (ms). In a future paper, I will discuss their similarities and differences as well as the different demands they place on the Humean view.

[34] This is one reading of David Lewis. Tim Maudlin (2007) calls this thesis "Separability."

[35] See Maudlin (2007), Chapter 2.

7 Conclusion

I have argued for a new package of views: Density Matrix Realism, the Initial Projection Hypothesis, and the Nomological Thesis. Each is interesting in its own right, and they do not need to be taken together. However, together they seem to fit in rather nicely in the Quantaculus Theory. They provide alternatives to standard versions of realism about quantum mechanics and a new way of resolving the conflict between quantum entanglement and Humean Supervenience.

The most interesting feature of the new framework, I think, is that it unifies the foundations of quantum mechanics and quantum statistical mechanics. With the unification of the Past Hypothesis and quantum ontology, it suggests that the arrow of time is intimately related to the quantum-mechanical phenomena in nature, and vice versa. This could be fruitful for future research.

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