

Hawking Radiation and Analogue Experiments: A Bayesian Analysis

Radin Dardashti^{*1}, Stephan Hartmann^{†2}, Karim
Thébault^{‡3}, and Eric Winsberg^{§4}

¹*Interdisziplinäres Zentrum für Wissenschafts- und Technikforschung/
Philosophisches Seminar, Bergische Universität Wuppertal*

²*Munich Center for Mathematical Philosophy, Ludwig Maximilians Universität
Munich*

³*Department of Philosophy, University of Bristol*

⁴*Department of Philosophy, University of South Florida*

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Abstract

We present a Bayesian analysis of the epistemology of analogue experiments with particular reference to Hawking radiation. Provided such experiments can be ‘externally validated’ via universality arguments, we prove that they are confirmatory in Bayesian terms. We then provide a formal model for the scaling behaviour of the confirmation measure for multiple distinct realisations of the analogue system and isolate a generic saturation feature. Finally, we demonstrate that different potential analogue realisations could provide different levels of confirmation. Our results thus provide a basis both to formalise the epistemic value of analogue experiments that have been conducted and to advise scientists as to the respective epistemic value of future analogue experiments.

*email: Dardashti@uni-wuppertal.de

†email: S.Hartmann@lmu.de

‡email: karim.thebault@bristol.ac.uk

§email: winsberg@usf.edu

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1 Introduction

In a landmark paper, (Steinhauer 2016) reported the experimental observation of Hawking radiation in an acoustic analogue black hole (or ‘dumb hole’). This remarkable result has met with a strangely divergent response in the physics community, varying from the assertion that it provides ‘experimental confirmation of Hawking’s prediction’ (Jeff Steinhauer quoted in Haaterz), to dismissal simply as ‘an amusing feat of engineering’ that ‘won’t teach us anything about black holes’ (Daniel Harlow quoted in Quanta). What can we learn from such analogue black hole experiments? Are there circumstances in which they be taken to provide inductive support for conclusions about astrophysical black holes?

In this paper we will extend previous philosophical work characterising analogue black hole experiments as a form of ‘analogue simulation’ (Dardashti et al. 2017; Thébault 2016) via application of Bayesian

confirmation theory. In that previous analysis, emphasis was placed upon the qualitative claim that *universality arguments* can be used to link evidence about the ‘source’ dumb hole system to the ‘target’ black hole system. Here, we present a theorem that demonstrates how this claim can be qualitatively characterised in Bayesian terms. A more nuanced quantitative question in this context is *how many* different analogue systems one needs to construct to be confident in the astrophysical effect. With this in mind, a further focus in this paper will be upon the scaling behaviour of Bayesian confirmation measures in the context of multiple distinct experimental realisations of the analogue system. The second key result of this paper is a formal model for ‘multiple source’ analogue simulation displaying the generic feature of ‘saturation’ in confirmatory power with an increase in the number of sources.

Finally, and perhaps most intriguingly, there is the question of whether different potential analogue realisations could provide different levels of confirmation. This is established in third key result of the paper, a theorem proving that analogue experiments in which we are more confident about the fundamental physics teach us less about the target system than those about which we are less confident. Our results thus provide a basis to both formalise the epistemic value of analogue experiments that have been conducted, like that of Steinhauer, and to advise scientists as to the respective epistemic value of future analogue experiments.

2 Confirmation, Analogy and Experiment

The literature on analogical reasoning in science is fairly extensive, with particularly noteworthy contributions by Keynes (1921), Hesse (1963, 1964, 1974), Bailer-Jones (2009) and Bartha (2010, 2013).¹ Typically analogical arguments have the form of abstract speculative inferences regarding possible features of one system (‘the target’) based on known features of another system (‘the source’). Classic examples are Reid’s

¹See also Norton (2011) for an importantly different take on analogical arguments. Norton’s analysis focuses on analogical arguments that proceed via subsumption of the target system into a larger class of entities, including the source system. There are some parallels between the structure of such inferences and our claim that (defended below) that analogue experiments can be confirmatory given they are externally validated by universality arguments.

argument for the existence of life on other planets based upon life on earth (Reid and Hamilton 1850) or Hume's argument for animal consciousness based upon human consciousness (Hume 1978).

Analogical arguments evidently play an important *heuristic role* in scientific practice in that they provide 'cognitive strategies for creative discovery' (Bailer-Jones 2009, p. 56). The *epistemic role* of analogical arguments in science is, however, more controversial. In particular, the literature contains diverging answers regarding whether analogical arguments can provide Bayesian confirmation of a hypothesis regarding the target system. From a Bayesian perspective on confirmation, evidence for a hypothesis can count as confirmatory only if the probability of the hypothesis given the evidence together with certain background assumptions is larger than the probability of the hypothesis given only the background assumptions. In a detailed and nuanced treatment of the issue, Hesse (1974, pp. 208-19) suggests that analogical arguments *can in some cases* be confirmatory in a Bayesian sense, so long as the analogical relationship that holds is in terms of what she calls a 'material analogy'. That is, where there is a similarity relations of sharing at least one predicate between the target and source systems.²

Contrastingly, Bartha (2010, 2013) offers a persuasive argument that analogical arguments *cannot in principle* be confirmatory in a Bayesian sense. This is because the information encapsulated in an analogical argument is taken to already be part of the 'background knowledge', and thus the probability of a hypothesis regarding the target system to be identical before and after including the information encapsulated in an analogical argument. Rather one assumes that arguments by analogy to establish only the *plausibility* of a conclusion, and with it grounds for further investigation (Salmon 1990; Bartha 2010). On this analysis, it is not *in principle* possible for analogical arguments to confer inductive support for a hypothesis. That is, although analogical arguments can certainly be stronger or weaker, even the strongest possible analogical argument cannot confer confirmation in a Bayesian sense: they are abstract inferences that can only ever support plausibility claims rather than providing inductive evidence.

²Hesse (1974, p. 216) explicitly rules out the possibility of confirmation obtaining in cases where there is purely a 'formal analogy'. That is, where target and source are both interpretations of the same formal calculus but do not share material similarities.

Although worthy and insightful, the treatments of Hesse (1974) and Bartha (2010) are both largely irrelevant for the analysis of analogue experiments. This is because, in the first case, Hesse's notion of material analogy is too strict to accommodate the purely syntactic relation that the model of the target has to the model of source in the case of analogue experiments, such as the Steinhauer experiments. We are dealing with an *analogue simulation* that does not involve a material analogy in the sense of Hesse. Rather than a material relation between systems, we have a syntactic relation between models.³ Bartha's negative analysis of the prospect for confirmation via analogical argument is similarly inapplicable to analogue experiments. In this case because of the 'experimental' rather than the 'simulation' side of things. Analogue experiments unlike analogical arguments are essentially empirical: they involve learning about the world by manipulating it. In experiments such as those carried out by Steinhauer, we manipulate the source such that certain explicit modelling assumptions matching those for the target obtain. Analogue simulation thus resembles a form of experimentation, involving the 'programming' of a physical system such that it can be used to 'simulate' another physical system. Thus, we see that conclusions from the philosophical analysis of analogical argument should not be taken to be readily extendible to cases of analogue simulation in contemporary science. In particular, it is self-evidently the case that the background evidence problem for the Bayesian analysis of traditional arguments by analogy à la Hesse, is not longer relevant for analogue experiments. Analogue experiments unlike analogical reasoning explicitly involve the collection of new evidence.

Given the foregoing, our analysis of the epistemology of analogue experimentation will draw upon the literature on the philosophy of experiment rather than analogical reasoning. In particular we will focus upon the idea of 'external validation' of an experiment as a precondition for confirmatory power of an experiment to stretch beyond the systems directly manipulated. Following Winsberg (2010) we can

³Hesse (1963), does in fact, rather presciently, consider the relevance of simulators in her account of models and analogies in science. Tantalisingly, she says that analogue machines (i.e. simulators): 'are useful and necessary as predictive models precisely in those cases where the material substance of parts of the analogue is *not essential* to the model, but where the mutual relations of the parts are essential' (p. 102) This connection is unfortunately not taken up in the 1974 Bayesian analysis.

make an important distinction between two different types of validation in the context of experimental science: i) An experimental result is *internally valid* when the experimenter is genuinely learning about the actual system they are manipulating – when, that is, the system is not being unduly disturbed by outside interferences; ii) An experimental result is *externally valid* when the information learned about the system being manipulated is relevantly probative about the class of systems that are of interest to the experimenters.

A nice illustration of this distinction is provided by experiments designed to learn about the thermal conductivity of the iron in Earth's core (Konôpková et al. 2016; Dobson 2016). The experiments were carried out *in the lab* using samples of iron that are placed in a laser-heated diamond-anvil cell. The pressure and temperature that the iron samples were subjected to were specifically matched to those relevant to the cores of Mercury-sized to Earth-sized planets. For these experiments to be internally valid the experimenters must be genuinely learning about the samples of iron that they are manipulating. That is, the measurements of thermal conductivity must correspond to the relevant property of the actual iron samples in the lab. For the experiments to be externally valid, these measurements must be 'relevantly probative' of the thermal conductivity of iron in the core's of Mercury-sized to Earth-sized planets. That is, there must be a basis to generalise from the (internally valid) observations regarding the lab based system (the 'source'), to relevant systems outside the lab (the 'target').

Clearly, an experiment that is not internally valid is of little or no epistemic value. If the experiment does not, in fact, allow us to make reliable inferences about the source system being manipulated, for example due to experimental error, then it is almost entirely uninformative. Furthermore, in most cases, if an experiment is internally valid, but not externally valid, then the results will also be of comparably little interest. Usually it is the larger class of target systems that we are interested in learning about. An internally valid experiment that allows us to learn about the thermal conductivity of iron in conditions that are not suitably aligned to those of the Earth's core would be of little use to geophysics. The epistemic power of experiments is usually predicated upon the combination of both internal and external validation.

The particular inferential importance of external validation can be

illustrated via reference to the Bayesian theory of confirmation: Call E that the proposition that the source system (i.e., the iron in the lab) has some value of thermal conductivity corresponding to the measured value. Given that E is true the experiment is internally valid and successful. Call H the proposition that this value of thermal conductivity is generalisable to a given class of target systems including the iron in the Earth's core (but excluding the source system). If and only if the experiment is externally valid do we have confirmation of H by E, that is $P(H|E) > P(H)$. Only externally validated experiments have any real inferential power: it is external validation that allows experiments to confer inductive support to hypotheses regarding target systems that we are not directly manipulating.

The key question in the *epistemology of analogue experimentation* is thus whether there are arguments that can provide external validation of the analogue experiments *qua* analogue experiments. By this we mean in addition to the necessary conventional external validation, can we provide arguments that the relevant source systems 'stand-in' for the target systems to which the analogical relationship refers.

3 Hawking Radiation and Universality

Hawking radiation (Hawking 1975) is a thermal phenomena that is predicted to be generically associated with black holes. Despite the absence of either a clear causal process behind the radiation or experimental evidence, it is widely believed in by theoretical physicists. In fact, the ability to recover Hawking radiation could even be taken as a theoretical test of prospective theories of quantum gravity, much like recovery of the Bekenstein-Hawking formula for black hole entropy (Wüthrich 2017). There are two connected reasons why physicists are so confident in the prediction of Hawking radiation. First, given the Unruh effect (Unruh 1976), which associates a temperature with acceleration, Hawking radiation seems to be directly implied by the equivalence principle.⁴ Second, starting from Hawking's original calculation a remarkable number of different derivations of the effect have been given.⁵ The con-

⁴Such a conclusion is, in fact, a little too quick since the equivalence principle holds only locally and Hawking radiation is a global effect. See (Helfer 2010).

⁵ See (Leonhardt and Philbin 2008; Thompson and Ford 2008; Wallace 2017).

sensus is that the effect is ‘remarkably robust’ to the addition of complicating factors to the original derivation. The overall implication is that very general theoretical constraints coming from quantum field theory and general relativity (two well tested theories) necessitate that something like Hawking radiation must exist. The purpose of this paper is not to address the evidential importance of such theoretical considerations. Rather, our focus is on the potential for analogue experiments to provide confirmatory evidence of a form akin to conventional experiments. This notwithstanding, questions of robustness will return to the fore in the context of a particular form of universality argument that will be found to be central for questions of external validation. Before then, it will be instructive to consider the basic elements of the original Hawking derivation of a radiative flux for astrophysical black holes in comparison with their sonic analogues.

Hawking’s analysis is performed in the context of a semi-classical approach to gravity. That is, we consider matter as described by quantum field theory and spacetime as described by a continuous classical geometry. Crucially, although the spacetime in question can have non-trivial curvature, it is not coupled to the quantum field. That is, there can be no ‘back-reaction’ between the quantum matter and classical geometry. For this modelling framework to be valid it is assumed that we are considering quanta of wavelengths much larger than the Planck length. Quanta of the order of the Planck length could be expected to ‘see’ the (presumed) non-classical and non-continuous structure of spacetime and would necessitate a quantum theory of gravity in their description. Quite general formal considerations can be used to show that in the semi-classical framework the vacuum state of a quantum scalar field defined at past null infinity need not appear as a vacuum state to observers at positive null infinity. In particular, it may contain a ‘particle flux’. What Hawking shows in his original paper is that for spacetime in which an astrophysical black hole forms there will be a particle flux which observers at positive null infinity will associate with the blackhole horizon. The asymptotic form of the expression for the particle flux is shown to depend only upon the *surface gravity* of the black hole denoted by κ_G . Surface gravity is essentially the force per unit mass that must be applied at infinity in order to hold a stationary zero angular momentum particle just outside the horizon (Jacobson 1996).

Hawking's calculation implies that a black hole has intrinsic properties that are connected to a non-zero *thermal* particle flux at late times. The precise relation takes the form:

$$\langle \hat{N}_\omega^{\text{Black Hole}} \rangle = \frac{1}{e^{\frac{2\pi\omega}{\hbar\kappa_G}} - 1} \quad T_{BH} = \hbar\kappa_G/2\pi. \quad (1)$$

One key feature of the derivation of the temperature is worth noting here since it will be very important in what follows. In the derivation of Hawking radiation an exponential gravitational red-shift means that the black hole radiation detected at late times must be taken to correspond to extremely high frequency radiation at the horizon. These 'trans-Planckian' modes are of wavelengths that are of precisely the kind that we presumed to exclude in using the semi-classical framework. There is thus a tension between the initial modelling assumptions and the details of the calculation. We will return to this issue shortly.

Not long after the derivation of Hawking's radiation, it was proposed by Unruh that a similar thermal effect might exist in the context of sound in fluid systems (Unruh 1981). In particular, Unruh showed that the key elements of Hawking's calculation could be re-applied in the context of a semi-classical model of sound in fluids. The role of the spacetime geometry is now played by a 'bulk' fluid flow as described by continuum hydrodynamics. The role of the quantum field is then played by a quantized linear fluctuation within the fluid, a phonon. The modelling framework of continuum hydrodynamics is only valid provided fluid density fluctuations of the order of molecular lengths can be ignored. So for this semi-classical description to be adequate the wavelengths of the phonons must be much larger than the intermolecular distances. Unruh's brilliant insight was to recognise that there is a special class of analogue fluid systems for which the equations of semi-classical continuum hydrodynamics take a form isomorphic to those of semi-classical gravity. The role of the black hole event horizon is now played by an effective acoustic horizon where the inward flowing magnitude of the radial velocity of the fluid exceeds the speed of sound. The black hole is replaced by a *dumb hole*. Just as in the gravitational Hawking effect a black hole event horizon is associated with a late time *thermal photonic flux*, in the hydrodynamic Hawking effect Unruh showed that a dumb hole sonic horizon can be associated with a late time *thermal*

phononic flux.

In practice, detection of Hawking radiation in a fluid is extremely difficult and it has not been possible to construct an acoustic analogue black hole of the type originally proposed by Unruh. Rather, an alternative medium for experimentally realisable acoustic blackholes was proposed in terms of Bose-Einstein condensates (Garay et al. 2000). This is an exotic form of matter predicted by in the 1920s (Einstein 1924; Bose 1924) but not created experimentally until 1995 (Anderson et al. 1995). Crucially, in the limit of *weak coupling* and *no backreaction* density fluctuations in a BEC are described by the Gross-Pitaevskii equation. When variations in the density of the BEC occur on length scales much greater than the *healing length* the Gross-Pitaevskii equation can be used to derived an equation of the same form as that of for an irrotational fluid derived by continuum hydrodynamics. Following the same line of reasoning as Unruh’s original ideal fluid argument, (Garay et al. 2000) derived a BEC Hawking Effect using appeal to this hydrodynamic approximation to a BEC.

There are now a huge number of potential analogue realisations of the Hawking effect: phonons in superfluid liquid helium, ‘slow light’ in moving media, traveling refractive index interfaces in nonlinear optical media, laser pulses in nonlinear dielectric medium.⁶ To realize the Hawking effect it seems it is sufficient to have: i) a classical (effective) background with quantum fields living on it; and ii) an (effective) geometry with an (effective) causal horizon.

A crucial feature in the derivation of all these effects is the integration over extremely high energy ‘trans-Planckian’ modes. As noted above, in the black hole case these modes get included in the calculation due to an exponential redshift between the horizon (where they originate) and late times (where they are detected). Such a feature is generic to all models of Hawking radiation in which the modes originate near the horizon.⁷ Since the ‘trans-Planckian’ regime beyond the domain of applicability of the semi-classical modelling framework we are using, this problem of exponential redshift seems rather worrying.

⁶See (Philbin et al. 2008; Belgiorno et al. 2010; Unruh and Schützhold 2012; Liberati et al. 2012; Nguyen et al. 2015).

⁷It is worth noting here that whilst, the non-standard derivation of (Giddings 2016) does appear to allow one to avoid this feature, that of (Polchinski 1995), *prima facie*, does not (Harlow 2016, pp. 37-8).

In fact, according to some, the trans-Planckian problem is so serious as to cast doubt upon the Hawking calculation entirely. Unruh, for instance, even asks that ‘if the derivation relies on such absurd physical assumptions, can the result be trusted?’ (Unruh 2014, p. 534). The problem with ‘trans-Planckian’ modes has a direct analogue in both the continuum hydrodynamic and BEC derivations. In particular, the acoustic analogue of the gravitational redshift, means that in both cases we are including in our calculation phonons of wavelengths small enough to probe the regimes well beyond the inter-molecular length and healing length respectively.

Fortunately, there are good reasons to expect that the Hawking effect in both gravitational and analogue cases will be robust to disturbance from trans-Planckian modes. In particular, (Unruh and Schützhold 2005) have provided strong theoretical reasons to expect that, under certain conditions, any possible modifications to the Hawking flux by trans-Planckian modes will be negligible.⁸ Unruh and Schützhold show that a wide family of trans-Planckian effects can be factored into the calculation of Hawking radiation via a non-trivial dispersion relation. To lowest order and given certain modelling assumptions, Hawking radiation, both astrophysical and acoustic, is independent of the details of the underlying physics.

Following the influential account of (Batterman 2000), we can isolate two characteristic features of a universality argument: i) Details of microstructure of a given token system are largely irrelevant for describing behaviour generically exhibited by members of the system type; and ii) Many different system types, with physically distinct microstructure (e.g. fluids and magnets), exhibit the same behaviour. Given this definition, we can plausibly take the work of Unruh and Schützhold to serve as a model for any theoretical argument towards the universality of the Hawking effect in the sense that it gives conditions under which: i) Details of ‘trans-Planckian’ structure of a given (analogue) black hole system are largely irrelevant for describing the associated thermal behaviour; and ii) Many different realisations of the (analogue) black hole system, with distinct ‘trans-Planckian’ structure (e.g. black holes and

⁸For further work on these issues, using a range of different methodologies, see for example (Corley 1998; Himemoto and Tanaka 2000; Barceló, Garay, and Jannes 2009; Coutant, Parentani, and Finazzi 2012). For philosophical discussion see (Dardashti, Thébault, and Winsberg 2017) and (Gryb, Palacios, and Thébault 2018).

BECs), exhibit the Hawking effect. We thus have a precise sense of what it would mean for the Hawking effect to be universal, and one reasonable theoretical basis to expect it to be so. Whilst detailed analysis of the universality argument of Unruh and Schützhold, is an important task, we will not further pursue it here. Rather, in the remainder of this paper, we will assume that universality arguments for Hawking radiation are at least plausible, and pursue the consequences for the epistemology of analogue experimentation.

4 Validating The Steinhauer Experiments

As noted at the end of Section §2, the key question in the *epistemology of analogue experimentation* is whether there are arguments that can provide external validation of the analogue experiments *qua* analogue experiments. What we mean by this is whether we can provide arguments that the relevant source systems ‘stand-in’ for the target systems to which the analogical relationship refers. External validation in the context of a conventional experiment justifies us making inferences about target systems, like the iron in the core of the earth, based upon source systems, like an iron sample in a laser-heated diamond-anvil cell. External validation of an analogue experiment would justify a similar inferential leap from a source system, like a BEC analogue black hole, to a target system, like an astrophysical blackhole. Clearly, in order for such an external validation of the analogue experiments *qua* analogue experiment to be valuable, the experiment would need to also be internally validated and externally validated in the conventional sense. In this section we will briefly consider the sense in which the landmark experiments of Steinhauer (Steinhauer 2016) in all three senses. Our account mostly follows that of (Thébaud 2016).

Steinhauer used a BEC of ^{87}Rb atoms confined radially by a narrow laser beam. The horizon was created by a very sharp potential step which is swept along the BEC at a constant speed. Significantly the length scales are such that the hydrodynamic description of a BEC is appropriate so long as we take into account small corrections due to the quantum potential. That is, the width of the horizon is of the order of a few times bigger than the healing length. The main experimental re-

sult consists of an aggregate correlation function computed based upon an ensemble of 4,600 repeated experiments which were conducted over six days. Given some reasonable assumptions (for example modes at different frequency are assumed to be independent of each other) the experiments can be interpreted as establishing an ‘entanglement witness’ to Hawking radiation in a BEC.

Internal Validation: Was Steinhauer genuinely learning about the physics of the particular sonic horizon within the particular ^{87}Rb BEC that he was manipulating? Various sources of internal validation are apparent from the description of the experimental set up given, not least the repetition of the experimental procedure nearly five thousand times.

External Validation (Conventional): Can the particular sonic horizon that was constructed, within the particular ^{87}Rb BEC, stand in for a wider class of systems? For example, all BEC sonic horizons. Given this set of systems obeys the ‘reasonable assumptions’ of the Steinhauer experiments, such as modes at different frequency are assumed to be independent of each other, then we can also externally validate the experiments in the conventional sense.

External Validation (Analogue): If accepted, the theoretical universality arguments of Unruh and Schützhold would function as external validation for the Steinhauer experiments. That is, they give us a *theoretical basis* to take the source system of the Steinhauer experiments to stand in for a wider class of target systems, including astrophysical black holes.

Our central hypothesis is that given such tripple validation, the Steinhauer experiments can confer inductive support to hypotheses Hawking radiation in astrophysical black holes. The remaining task for this paper is to show how important qualitative and quantitate features of this inductive support can be modelled within the framework of Bayesian confirmation theory.

5 Bayesian Analysis

5.1 Single Source Confirmation

The key claim that we wish to investigate is the sense in which analogue ‘dumb hole’ experiments can provide inductive support for a hypothesis regarding black holes given we believe the appropriate universality arguments. In what follows we give a Bayesian network representation of the proposed inferential structure of analogue simulation defended in (Dardashti et al. 2017) and show that the evidence in the source system can provide confirmation of hypotheses regarding the target system in certain circumstances.⁹

Let us start with the representation of the target system \mathcal{T} . We denote by M a propositional variable that takes the two values:

M : The modelling framework \mathcal{M} provides an empirically adequate description of the target system \mathcal{T} within a certain domain of conditions $D_{\mathcal{M}}$.

$\neg M$: The modelling framework \mathcal{M} does not provide an empirically adequate description of the target system \mathcal{T} within a certain domain of conditions $D_{\mathcal{M}}$.

The adequacy of the modelling framework \mathcal{T} depends on whether the background assumptions which justify the empirical adequacy of the modelling framework obtain. We denote with X_M the random variable with the values:

X_M : The background assumptions $x_M = \{x_M^1, x_M^2, \dots, x_M^n\}$ are satisfied for system \mathcal{T} .

$\neg X_M$: The background assumptions $x_M = \{x_M^1, x_M^2, \dots, x_M^n\}$ are not satisfied for system \mathcal{T} .

The role of the background assumptions is to define and justify the domain of conditions for the model. These assumptions involve knowledge, both theoretical and empirical, that goes beyond what is encoded

⁹For models of confirmation in terms of the Bayesian framework see (Hartmann and Sprenger 2010; Bovens and Hartmann 2004) or for the hypothetic-deductive framework see (Betz 2013). Throughout this paper, we follow the convention that propositional variables are printed in italic script, and that the instantiations of these variables are printed in roman script.

within the model. Such knowledge need not be in the form of a simple, unified framework. Rather the background knowledge of the people who build and use models can contain an incompletely integrated set of explicit and tacit ideas about when a particular modelling framework will be adequate for a particular purpose and to a particular desired degree of accuracy.

With this in mind, we can introduce the random variables A and X_A for the source system \mathcal{S} . Where A is a propositional variable that takes the two values:

A : The modelling framework \mathcal{A} provides an empirically adequate description of the source system \mathcal{S} within a certain domain of conditions $D_{\mathcal{S}}$.

$\neg A$: The modelling framework \mathcal{A} does not provide an empirically adequate description of the source system \mathcal{S} within a certain domain of conditions D_A .

and X_A is the random variable with the values:

X_A : The background assumptions $x_A = \{x_A^1, x_A^2, \dots, x_A^k\}$ are satisfied for system \mathcal{S} .

$\neg X_A$: The background assumptions $x_A = \{x_A^1, x_A^2, \dots, x_A^k\}$ are not satisfied for system \mathcal{S} .

The systems \mathcal{T} and \mathcal{S} are assumed to differ in terms of their material constituency and the fundamental laws governing their dynamics. This means that the background assumptions behind the models \mathcal{M} and \mathcal{A} can reasonably be assumed to be very different. Given this, it is justified, *prima facie*, to assume that X_M and X_A are probabilistically independent. Furthermore, we have assumed that the source system is empirically accessible meaning we can gain empirical evidence regarding (at least) some of its consequences. We can encode this by introducing a variable E corresponding to the two values, E , the empirical evidence obtains, and $\neg E$, the empirical evidence does not obtain.

We can represent all the variables introduced thus far as well as the probabilistic dependancies using a Bayesian network (Bovens and Hartmann 2004). The random variables are represented as ‘nodes’ in the network (i.e. circles) and the probabilistic dependancies as directed

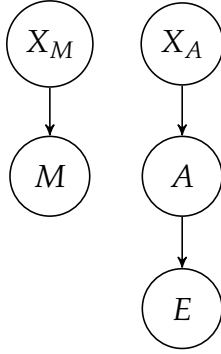


Figure 1: One source system without external validation.

edges (i.e. arrows). We draw an arrow between two nodes when the variable in the ‘parent node’ has a direct influence on the variable in the ‘child node’. Probabilistic independence is represented implicit by the absence of an arrow between two nodes. The entire set up thus far is represented by the Bayesian network in Figure 1.

Before we continue, let us first translate the discussion from before about external validation into the Bayesian framework. As mentioned before the equations of the target model are syntactically isomorphic to the equations that are suppose to govern the behaviour of the source model. This is so, even though one arrives at them from different starting points (semi-classical gravity vs. e.g. semi-classical continuum hydrodynamics) and by making different background assumptions. The question of externally validating the model (both conventional and analogue) translates into the question of the domain of applicability of the model. That is, whether the mathematical model is an empirically adequate description of one specific BEC horizon¹⁰ (the one experimented upon), all BEC horizons, or even including the black hole horizon. Whether this is the case or not is therefore not determined by the mathematical equations, as they are the same, but by the ability of the experiment to probe the respective background assumptions.

Let us first consider the external validation in the ‘conventional’ sense, i.e. in this context the question whether the experiment on ⁸⁷Rb BEC, can stand in for a wider class of systems, namely all BEC sonic horizons. This will allow us to clarify the difference between the two kinds of external validation. By deriving the relevant equations, the

¹⁰This is always taken to be within the realm of validity of the hydrodynamic approximation.

background assumptions were not dependent on the specifics of the BEC used.¹¹ So the majority of the background assumptions x_A of the specific ^{87}Rb BEC experiment will be shared with any experiment on non- ^{87}Rb condensates. Some background assumptions may, of course, still differ as they may be effects of some specific BEC that is not shared by others that may influence the adequacy of the derivational assumptions. However, this will still keep a majority of shared background assumptions, which in the Bayesian network will now function as a common cause for the adequacy of the model for ^{87}Rb BEC and the other non- ^{87}Rb BECs. External validation in the ‘conventional’ sense then amounts to probing, through investigation (e.g. robustness analysis), the background assumptions which are not shared by all BECs. However, even in the absence of this further investigation, we can already assume that (under certain assumptions) the observation on the ^{87}Rb BEC will confirm the claim with respect to the larger class of BECs. This is due to the known shared background assumptions. So in the case of ‘conventional’ external validation we can observe a probabilistic dependence even in the absence of external validation.

Let us contrast this now with the case of external validation in the analogue sense. The assumption that X_A and X_M in Fig. 1 are probabilistically independent is equivalent to us having reasons to believe that the target model and the source model share no background assumptions. This is a legitimate assumption as these models originate, unlike the case of ‘conventional’ validation, from different theories.

The universality argument now functions as an inferential link *between certain background assumptions* of both the model of the source system and the model of the target system. That is, while we expect the majority of the background assumptions to remain independent the universality argument provides a link between some background assumptions. In this sense evidence in \mathcal{A} may support a subset of the background assumptions X_M . The inductive support that such an externally validated experiment would provide for M will depend on how certain we are about the adequacy of the related background assumptions. If we have already independent grounds on which to assign high probabilities to x_A^1 or x_M^2 , then there is not much added in terms of inductive support we gain. In the context of simulating Hawking ra-

¹¹See Faccio (2013) for an elementary derivation of the result.

diation via ‘dumb holes’, the universality argument provides the link between the background assumptions, which is here the independence of the phenomenon in each system from the respective influences of the higher energy theories. The independence claim, however, is probably the least supported of the background assumption and is thus the route via which analogue simulation can provide strong inductive support for black hole Hawking radiation. So unlike the case of ‘conventional’ external validation, we will not be able to observe a probabilistic dependence between the two systems without externally validating it first.

Before we turn to the details of the Bayesian analysis we can already observe several features of our approach. First, the question of external validation (in the ‘conventional’ and analogue sense) translates into a question of probing the respective background assumptions. Second, whether we have external validation of the analogue kind is not necessarily related to the question whether we have external validation of the ‘conventional’ kind. The reason for this is that the background assumptions that are being probed by the latter kind of validation need not be the relevant background assumptions. And finally, it is a consequence of the Bayesian account that both kind of validations come in degrees.

Let us now turn to the details of the Bayesian analysis. In order to make explicit calculation tractable we will subsume both the universality arguments and the common background assumptions within a single variable X . The binary value X has the values:

X : universality arguments in support of common background assumptions hold.

$\neg X$: universality arguments in support of common background assumptions do not hold.

So X expresses a rather general claim, which can plausibly be assumed to be uncertain. If we were certain about X , the inference from \mathcal{A} to \mathcal{M} would be blocked. We will say more about this later. We will also subsume the remaining background assumptions, that is those that are not addressed by X , under the nodes M and A .

Under the conditions of our assumptions, the simplified Bayesian network given in Figure 2 will then adequately model the chain of inferences involved in analogue simulation supported by universality arguments. We would like to show that E confirms M within a Bayesian the-

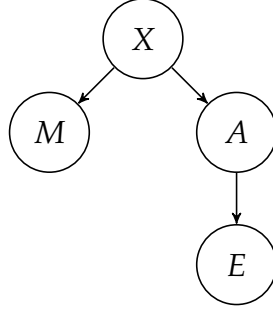


Figure 2: Simplified network for one source system with universality arguments.

ory of confirmation. This requires that one proves that $P(M|E) > P(M)$. For this purpose we need to specify all prior probabilities of the ‘parent node’ in the Bayesian network (i.e., X) and the conditional probabilities for the other ‘child nodes’, given the values of their parents.

Let us simplify our notation by using the following shorthand:

$$\begin{aligned}
 P(X) &=: x & P(M|X) &=: m_x \\
 P(A|X) &=: a_x & P(E|A) &=: e_a.
 \end{aligned}$$

The probabilities of the corresponding negated propositions are denoted with a bar, viz. $P(A|\bar{X}) =: a_{\bar{x}}$, $P(\bar{A}|X) =: \bar{a}_x$ and $P(\bar{A}|\bar{X}) =: \bar{a}_{\bar{x}}$.

The first central assumption is that the prior probability of X lies in the open interval $(0,1)$, however, as we discussed in Sect. 3, we have theoretical arguments, in particular the result by Unruh and Schützhold, in favor of X . So a rational agent would assign:

$$\frac{1}{2} < x < 1. \tag{2}$$

The conditional probabilities are then constrained by the following conditions:

$$m_x > m_{\bar{x}} \tag{3}$$

$$a_x > a_{\bar{x}} \tag{4}$$

$$e_a > e_{\bar{a}}. \tag{5}$$

The conditions (2) to (4) encode probabilistically the relevant elements of the universality arguments since they allow for the possibility of a

background assumption that supports both M and A . The statement (5) encodes probabilistically that the empirical evidence actually plays the role of evidence in favour of the model A .

With this, the following theorem holds (the proof is in Appendix A):

Theorem 1: $P(M|E) > P(M)$, if the conditions (2), (3), (4) and (5) are satisfied.

The satisfaction of Theorem 1 implies that E confirms M within a Bayesian analysis of confirmation. Within the framework of analogue simulation, provided we have universality arguments with prior probability that is neither unity or zero, confirmation of a hypothesis regarding the target system can obtain based upon evidence relating to the source system. It is important to note again, that having independent grounds on which to support one of the common background assumptions will ‘block’ the inductive support E can give for M as that background assumption already has a large marginal probability. This does not pose a problem for this account but offers a way to distinguish between those circumstances in which the novel empirical evidence E can provide substantial inductive support for M and those circumstances it cannot be used for that purpose.

An important implication of the Bayesian analysis relates to the role of the syntactic isomorphism. The structure of the Bayesian network is such that the syntactic isomorphism is not explicitly represented. Furthermore, based upon the network, even if no syntactic isomorphism obtains between the modelling frameworks \mathcal{M} and \mathcal{A} , one could sensibly talk about confirmation of M by E , provided there exists some non-empty set of shared background assumptions. The key point is that in such circumstances although confirmation of M would indeed obtain, there would be no ‘analogue simulation’. As discussed above, the role of the isomorphism is to guarantee that there will be a term within the modelling language of \mathcal{M} that is counterpart to the term within \mathcal{A} that refers to \mathcal{E} . Without such a term within \mathcal{M} there would be no sense in which \mathcal{S} is acting as a simulator for the behaviour of \mathcal{T} . For analogue simulation with confirmation to obtain we require both universality arguments and a syntactic isomorphism.

To recapitulate, in this section we have demonstrated that confirmation via analogue simulation obtains within a Bayesian analysis pro-

vided there exists an inferential connection between the conditions of applicability of the target and system models. That is, if there exists a binary variable that is assumed to be positively correlated with the empirical adequacy of both the source and target models, then evidence in favour of the model of the source system can be used to make inferences about the target system. This, in-and-of-itself, is not a particularly surprising result, and certainly the demonstration of such in principle inferential relations is not an external validation of the framework for analogue simulation that is being proposed. Rather, we take the results of this section to: i) demonstrate the internal consistency of the informal arguments towards confirmation via analogue simulation given in (Dardashti et al. 2017); and ii) provide a powerful evaluative and heuristic tool for the analysis of analogue simulation as it exists within contemporary scientific practice. Two natural directions of further development are: i) the identification and evaluation of potential cases of confirmation via analogue simulation based upon different forms of analogue external validation in other scientific examples; and ii) the refinement of the Bayesian model to include cases within more than one analogue system. The second of these will be pursued in the following section.

5.2 Multiple Source Confirmation

One important application of analogue simulation is in the context of universality arguments. In such cases the source system is ‘multiply realisable’ in that there are various different physical systems that can be used to implement the analogue simulation. Such a notion of ‘multiply realisability’ is intended to be something more than the variation of the material constitution of the source system. Such variation would involve keeping fixed the ‘nomological behaviour’ of the source system but changing the material constitution. Rather, the situation we are considering is when one varies the modelling frameworks used to construct the analogy, and in doing so considers equations that are syntactically isomorphic but extensionally distinct.

For example, consider again the dumb hole case. Rather than making use of the syntactic isomorphism between BEC and gravitational models we can draw inferences based upon analogue black holes constructed out of traveling refractive index interfaces in nonlinear optical media or

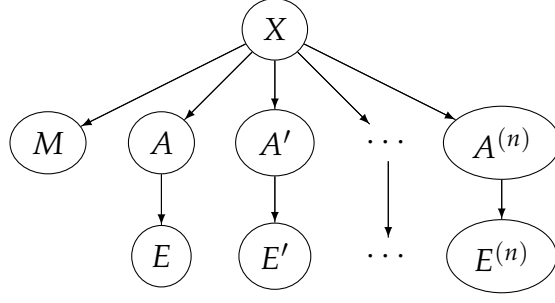


Figure 3: n -source system.

‘slow light’ in moving media (Carusotto et al. 2008). This is to vary both the material constitution and the nomological behaviour of the analogue system.

With such examples in mind, we can extend the analysis of the previous section to consider the case when we have multiple sources each providing independent evidence for the target system modelling framework. The expectation would be that adding more source systems should increase the degree of confirmation, but that this increase will eventually reach some ‘saturation point’. This matches the intuition that, given some non-zero (or one) prior probability for the truth of the universality arguments, a small set of different successful realisations of the source system would be enough to provide strong evidence in favour of a hypotheses of regarding analogue behaviour in the target system.

Consider a Bayesian network for an n -source system (Figure 3). The question we would like to answer is how does the confirmation measure change as one increases the number of different analogue systems providing us with evidence. Following the same line of reasoning as the last section we assume:

$$a'_x > a'_{\bar{x}} \quad , \quad e'_{a'} > e'_{\bar{a}'} \quad (6)$$

$$a''_x > a''_{\bar{x}} \quad , \quad e''_{a''} > e''_{\bar{a}''} \quad (7)$$

⋮

$$a_x^{(n)} > a_{\bar{x}}^{(n)} \quad , \quad e_{a^{(n)}}^{(n)} > e_{\bar{a}^{(n)}}^{(n)} \quad (8)$$

We can now calculate the difference measure of confirmation, which is defined as

$$\Delta^{(n)} := P(\mathbf{M} | \mathbf{E}, \mathbf{E}', \dots, \mathbf{E}^{(n)}) - P(\mathbf{M}) \quad (9)$$

and obtain $\Delta^{(n)} > 0$ (see Appendix B).

It can further be shown that:

Theorem 2: $\Delta^{(n)}$ is a strictly increasing function of the number of source systems.

This theorem implies that as the number of different analogue systems providing evidence increases so does the degree of confirmation.¹² Again, this is not a particularly surprising result. Given that confirmation via analogue simulation obtains for a single source system, one would expect that adding in more and more (independent) source systems would allow one to increase the degree of confirmation. The feature that is most interesting is not the fact that $\Delta^{(n)}$ is strictly increasing, but rather the functional form of this increase. In particular, the natural intuition is that as the number of source systems increases the increase in the degree of confirmation would eventually saturate. One of the chief virtues of the Bayesian model for analogue simulation with multiple source systems is that it allows us to give an analytical expression for such a saturation point.

First, let us consider how $\Delta^{(n)}$ changes in the large n limit. A little analytical work (again see Appendix B) allows us to show that:

$$\lim_{n \rightarrow +\infty} \Delta^{(n)} \rightarrow \bar{x}(m_x - m_{\bar{x}}) = N_{\text{sat}}. \quad (10)$$

This means that the maximum amount of confirmation one can obtain by adding in more and more sources is bounded by some finite number, N_{sat} , determined by the prior probabilities \bar{x} , m_x and $m_{\bar{x}}$. Beyond this point, there is vanishingly small added value (in terms of confirmation) achieved by adding in more source systems. Two features of N_{sat} are worth remarking on. First, the higher the prior probability of X the lower the saturation point will be. This makes sense because the more sure we are of X to start with, the lower the limit on the extra information we can learn from $E, E', \dots, E^{(n)}$. Second, the higher the relative likelihood of M given X to M given $\neg X$ (i.e. $m_x - m_{\bar{x}}$), the higher the saturation point. This makes sense because the stronger the relationship between X and M the more we can potentially learn from $E, E', \dots, E^{(n)}$.

¹²Theorem 2 does not depend on the choice of this particular confirmation measure and will also hold if we move to another confirmation measure (Fitelson 1999).

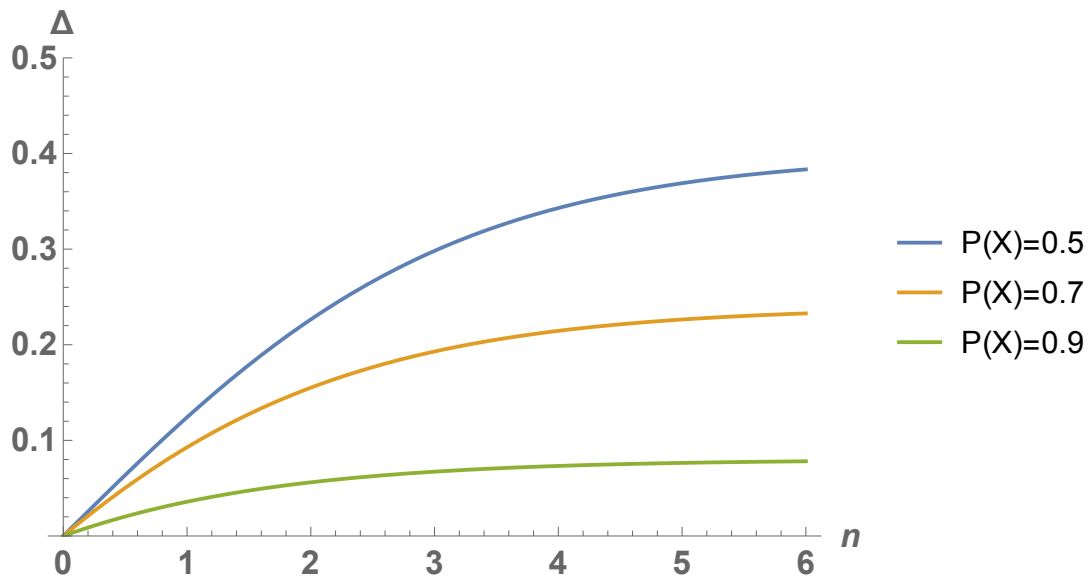


Figure 4: Confirmation measure dependence and saturation point.

A further interesting feature that we can examine is the speed with which the saturation point is approached. We can examine this ‘rate of saturation’ by plotting $\Delta^{(n)}$ for a set of prior probabilities of X .¹³ As can be seen from Figure 4, the higher the prior probability of X , the quicker the saturation point is reached. Strikingly, for the values of the parameters considered, we find that given a prior of greater than 0.5 for X , saturation can be reached after only three or four successful analogue experiments.

This result is in tune with scientific intuitions regarding analogue simulation in the context of universality arguments. Consider, in particular the dumb hole Hawking radiation case. There has been, thus far, only one implementation of a source system that is reported to display the quantum Hawking effect: the Bose-Einstein condensate experiments of (Steinhauer 2016).¹⁴ Given initial confidence in the universality arguments, if another different implementation of a source system dis-

¹³See Equation (23) of Appendix B. Here we have assumed for simplicity that $\gamma^{(k)} = c$ for all k with $c > 1$. c measures both the likelihood of $A^{(k)}$ given X and the likelihood of $E^{(k)}$ given $A^{(k)}$. The stronger the dependence of these the stronger the exponential increase of $\Delta^{(n)}$ with n .

¹⁴We should note that the earlier experiments of (Weinfurter et al. 2013) show classical aspects of Hawking radiation in an analogue system using surface waves. Modelling the inferential relationship between the classical and quantum experiments is an interesting open question.

playing the Hawking effect was achieved, that should surely radically increase the belief in the astrophysical Hawking effect. However, once a few such examples were constructed, one would quickly stop gaining new insight. Conversely, given initial skepticism regarding the universality arguments, a second implementation of the dumb hole source system would not radically increase the belief in the astrophysical Hawking effect. Furthermore, in such circumstances it would only be after a diverse and extensive range of implementations of source systems that one would stop believing that new examples gave new information.

5.3 Confirmation Dependence on Source System

Although most of the proposed analogue models of black hole Hawking radiation have not been tested yet, there can be significant differences in our prior belief regarding the adequacy of these models. One reason for this is that the modelling may rely on a strong theoretical basis in one system but a rather conjectural basis in the other. Another reason is that one may have good control of the experimental setup, such that one has more reason to rely on the adequate realization of the various idealizing assumptions involved in the derivation of the model. This will have as an effect that we would assign different marginal probabilities to the analogue models. A question thus arises, namely how do the variations in the marginal probabilities one assigns to the adequacy of the analogue model affect the confirmation behaviour of the analogue setup.

For simplicity, we will only consider the variation for the one-source model (Fig. 2). One can now show (see App. C for details):

Theorem 3: $\frac{\partial \Delta}{\partial a} < 0$, if conditions (2) to (5) hold,

where the derivative is with respect to $a := P(A)$.

This plausible result implies that an assignment of a higher probability to the adequacy of the analogue model will have the effect of a decrease in the confirmation of the adequacy of the target model by the observation of the analogue Hawking effect. Or to put it differently: the more certain we are about the adequacy of the model we are experimenting on, the less effective is the evidence obtained there. Significantly, this result has direct implications for the respective epistemic

value of future analogue experiments. In particular, all else being equal, it implies that scientists will learn more by conducting future analogue experiments using media about which we are *less certain* regarding the fundamental physics, than those using media about which we are more confident.

6 Conclusion and Prospectus

History is replete with examples of ‘transformative’ technology having a profound and lasting impact on the methodological foundations of science. Much recent literature in the philosophy of science has focused on the sense in which computer simulation should (or should not) be taken to have had such an impact.¹⁵ Analogue simulation is a new inferential tool found at the cutting edge of modern science that we see good reasons to take as potentially transformative. Building upon the initial analysis of (Dardashti et al. 2017), in this paper we have applied a Bayesian analysis to explicate the structure of inferences that analogue simulation can and cannot allow us to make.

Our three principal results are: i) that ‘single source’ confirmation via analogue simulation can obtain under certain conditions; ii) that ‘multiple source’ confirmation via analogue simulation displays the generic feature of saturation in confirmatory power; iii) analogue experiments in which we are more confident about the fundamental physics provide less confirmation than those about which we are more confident. Our results provide a basis to both formalise the epistemic value of analogue experiments that have been conducted and to advise scientists as to the respective epistemic value of future analogue experiments.

A Proof of Theorem 1

Let us start with the Bayesian network depicted in Figure 1. We have to show that

$$P(M|E) = \frac{P(M, E)}{P(E)} > P(M). \quad (11)$$

¹⁵See for example (Frigg and Reiss 2009; Parker 2009; Winsberg 2010; Beisbart and Norton 2012).

The joint probability can be obtained in the following way¹⁶:

$$\begin{aligned}
P(M, E) &= \sum_{\bar{X}, A} P(X, M, A, E) \\
&= \sum_{\bar{X}, A} P(X)P(M|X)P(A|X)P(E|A) \\
&= x m_x (a_x e_a + \bar{a}_x e_{\bar{a}}) + \bar{x} m_{\bar{x}} (a_{\bar{x}} e_a + \bar{a}_{\bar{x}} e_{\bar{a}}) \\
&= x m_x \alpha + \bar{x} m_{\bar{x}} \beta
\end{aligned} \tag{12}$$

where we have defined

$$\alpha := a_x e_a + \bar{a}_x e_{\bar{a}} \tag{13}$$

$$\beta := a_{\bar{x}} e_a + \bar{a}_{\bar{x}} e_{\bar{a}}. \tag{14}$$

Similarly we obtain

$$\begin{aligned}
P(E) &= \sum_{X, A, M} P(X, M, A, E) \\
&= x \alpha + \bar{x} \beta
\end{aligned} \tag{15}$$

and

$$P(M) = x m_x + \bar{x} m_{\bar{x}}. \tag{16}$$

Defining the difference measure $\Delta := P(M|E) - P(M)$, we need to show that Δ is larger than zero. After some algebraic manipulation one obtains

$$\begin{aligned}
\Delta &= \frac{x m_x \alpha + \bar{x} m_{\bar{x}} \beta - (x m_x + \bar{x} m_{\bar{x}})(x \alpha + \bar{x} \beta)}{x \alpha + \bar{x} \beta} \\
&= \frac{\bar{x} x (m_x - m_{\bar{x}})(\alpha - \beta)}{x \alpha + \bar{x} \beta}.
\end{aligned} \tag{17}$$

It is easy to show that

$$\alpha - \beta = (a_x - a_{\bar{x}})(e_a - e_{\bar{a}}), \tag{18}$$

so it follows that

$$\Delta = \frac{\bar{x} x (m_x - m_{\bar{x}})(a_x - a_{\bar{x}})(e_a - e_{\bar{a}})}{x \alpha + \bar{x} \beta}. \tag{19}$$

¹⁶See (Bovens and Hartmann 2004, Sect. 3.5) on the general methodology of reading joint probabilities from Bayesian networks.

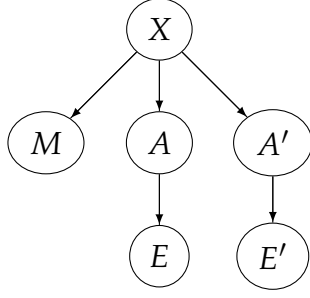


Figure 5: 2-source system.

So if (3), (4) and (5) are satisfied it follows that $\Delta > 0$, which needed to be shown.

B Proofs for n Source Systems

To see how the previous theorem can be generalized to the n source systems represented in Figure 3 let us consider first the 2-source system represented in Figure 5.

We need to show that $P(M|E, E') = P(M, E, E')/P(E, E') > P(M)$. Let us start with the following joint probability

$$\begin{aligned}
P(M, E, E') &= \sum_{X, A, A'} P(X, M, A, E, A', E') \\
&= \sum_{X, A, A'} P(X)P(M|X)P(A|X)P(E|A)P(A'|X)P(E'|A') \\
&= x m_x \alpha \alpha' + \bar{x} m_{\bar{x}} \beta \beta'
\end{aligned} \tag{20}$$

where α' and β' is defined identically to (13) and (14) with e and a replaced with e' and a' .

Similarly we obtain

$$\begin{aligned}
P(E, E') &= \sum_{X, A, A'} P(X)P(E|A)P(A|X)P(E'|A')P(A'|X) \\
&= x \alpha \alpha' + \bar{x} \beta \beta'.
\end{aligned} \tag{21}$$

Defining as before $\Delta' := P(M|E, E') - P(M)$ it follows that

$$\Delta' = \frac{x\bar{x}(m_x - m_{\bar{x}})(\alpha \alpha' - \beta \beta')}{x \alpha \alpha' + \bar{x} \beta \beta'}. \tag{22}$$

Now $\alpha \alpha' - \beta \beta'$ is larger than zero iff $(\alpha \beta)/(\alpha' \beta') > 1$. This in turn is the case when $\alpha - \beta > 0$ and $\alpha' - \beta' > 0$. Both of these conditions are satisfied due to (4) to (6).

So it follows that (22) is larger than zero.

It is now straightforward to generalize to the n -source system represented in Figure 3. For the n -source system we need to show that $\Delta^{(n)} = P(M|E, E', \dots, E^{(n)}) - P(M) > 0$. It follows from the above consideration that

$$\Delta^{(n)} = \frac{x \bar{x} (m_x - m_{\bar{x}}) (\prod_{k=0}^n \alpha^{(k)} - \prod_{k=0}^n \beta^{(k)})}{x \prod_{k=0}^n \alpha^{(k)} + \bar{x} \prod_{k=0}^n \beta^{(k)}} \quad (23)$$

with $\alpha^{(0)} = \alpha$ and $\beta^{(0)} = \beta$. We have again $\Delta^{(n)} > 0$ once (4) to (8) are satisfied.

Let us define $\gamma^{(n)} := \prod_{k=0}^n \alpha^{(k)} / \beta^{(k)}$. Since $\alpha^{(k)} > \beta^{(k)}$ for all k , $\gamma^{(n)}$ increases as n increases. Furthermore, we have

$$\frac{\partial \Delta^{(n)}}{\partial \gamma^{(n)}} = \frac{x \bar{x} (m_x - m_{\bar{x}})}{(x \gamma^{(n)} + \bar{x})^2} > 0. \quad (24)$$

So as n increases, i.e. as the number of analogue systems providing evidence increases, so does the amount of confirmation. Setting $\kappa := x \bar{x} (m_x - m_{\bar{x}})$ we obtain the large n behaviour:

$$\lim_{n \rightarrow +\infty} \Delta^{(n)} = \lim_{x \rightarrow +\infty} \kappa \frac{\gamma^{(n)} - 1}{x \gamma^{(n)} + \bar{x}} \rightarrow \frac{\kappa}{x}. \quad (25)$$

C Proof of Theorem 3

We need to show how the confirmation measure (19) varies as our belief regarding the adequacy of the analogue model varies. That is, we need to calculate

$$\begin{aligned} \frac{\partial \Delta}{\partial a} &= \frac{\partial a_x}{\partial a} \frac{\partial \Delta}{\partial a_x} + \frac{\partial a_{\bar{x}}}{\partial a} \frac{\partial \Delta}{\partial a_{\bar{x}}} + \frac{\partial x}{\partial a} \frac{\partial \Delta}{\partial x} \\ &= \frac{1}{x} \frac{\partial \Delta}{\partial a_x} + \frac{1}{\bar{x}} \frac{\partial \Delta}{\partial a_{\bar{x}}} + \frac{1}{(a_x - a_{\bar{x}})} \frac{\partial \Delta}{\partial x}, \end{aligned} \quad (26)$$

where we have used $a := x a_x + \bar{x} a_{\bar{x}}$ in the last step. Taking the derivatives yields

$$\begin{aligned}\frac{\partial \Delta}{\partial a_x} &= \frac{k x \bar{x}}{(x \alpha + \bar{x} \beta)^2} \left(x \alpha + \bar{x} \beta - (a_x - a_{\bar{x}})(e_a - e_{\bar{a}}) x \right), \\ \frac{\partial \Delta}{\partial a_{\bar{x}}} &= -\frac{k x \bar{x}}{(x \alpha + \bar{x} \beta)^2} \left(x \alpha + \bar{x} \beta + (a_x - a_{\bar{x}})(e_a - e_{\bar{a}}) \bar{x} \right), \\ \frac{\partial \Delta}{\partial x} &= \frac{k(a_x - a_{\bar{x}})}{(x \alpha + \bar{x} \beta)^2} \left(x \alpha + \bar{x} \beta - x(1+x)\alpha - x\bar{x}\beta \right),\end{aligned}$$

where we have used the shorthand $k := (m_x - m_{\bar{x}})(e_a - e_{\bar{a}})$. Plugging these back into (26) and after some manipulation one obtains

$$\frac{\partial \Delta}{\partial a} = \frac{k}{(x \alpha + \bar{x} \beta)^2} \left(\bar{x}(1 + \bar{x}) \beta - x(1 + x) \alpha \right). \quad (27)$$

Assuming that (3) and (5) hold makes k positive and assuming (2), (4) and (5) hold makes the large bracket in (27) negative, i.e. it leads to $\partial \Delta / \partial a < 0$.

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