Causality and Complementarity in Kant, Hermann, and Bohr

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Abstract

Kant's doctrine of transcendental idealism, as put forth in the first Critique, is best understood as a conceptual or epistemic doctrine. However critics of the conceptual understanding of transcendental idealism argue that it amounts to an arbitrary stipulation and that it does not do justice to the real ontological distinctions that mattered for Kant. Some stipulations are better than others, however. In this paper I argue that Kant's doctrine, though it should be understood 'merely epistemically', is nevertheless full of significance and is motivated through his long-running pre-critical struggle to discover first principles for metaphysical cognition. I further argue that an epistemic understanding of the doctrine of transcendental idealism provides a Kantian with a natural way of understanding the novel epistemic situation presented to us by modern physics and in particular by quantum mechanics. And I argue that considering Kant's philosophy in the light of the challenges posed by quantum mechanics illuminates, in return, several elements of his philosophical framework, notably the principle of causality, the doctrine of synthetic a priori principles in general, and most generally: the conceptual understanding of transcendental idealism itself. I illustrate this via an analysis of the views of the physicist Niels Bohr as well the views of the (neo-)Kantian philosopher Grete Hermann.

1 Introduction

Even before Hegel's death in 1831, the idea, traditional since the ancient Greeks, that philosophical speculation could furnish a general model for scientific thought had been waning in the Germanic world.[1] What were seen as the excesses of the romantic and idealist movements, combined with the rapid growth and manifest success of empirical science, had brought philosophy into disrepute. Some—the so-called 'vulgar materialists'—went so far as to call for philosophy to essentially identify itself with empirical science (Beiser 2014, pp. 182-188; Schnädelbach 1984, ch. 3).

Mid-nineteenth century neo-Kantianism arose partly as a reaction to these intellectual currents.[2] Championed, at first, primarily by natural scientists such as Helmholtz (Schnädel-

[1]I say 'Germanic world' rather than Germany so as to include those in surrounding nations, in particular Denmark, who were influenced by German thinkers of the period; see, for example, Stewart (2007).

[2]Neo-Kantianism as a philosophical movement began in the mid-nineteenth century. There were, however, neo-Kantian thinkers prior to this period as well. We will discuss one of these thinkers, Jakob Friedrich Fries, later in this essay.

bach, 1984, p. 103), the movement did not, as did speculative idealism, view it as philosophy's task to produce scientific knowledge by pure thought alone. Nor did these neo-Kantians, with the vulgar materialists, consider it philosophy's task merely to uncritically catalogue and systematise the results of empirical science. Rather, with Kant, they saw it as philosophy's essential, unavoidable, and enduring mission to enquire into the sources of our knowledge and the degree of its justification.[3] In other words they, and the philosophers such as Liebmann, Lange, and Cohen who took up their battle call (Zeman, 1997), saw the proper task of philosophy as consisting in the provision of an *epistemological foundation* for science. By the close of the nineteenth century, neo-Kantianism exerted a powerful influence on Germanic thought.

It was against this intellectual backdrop that many of the modern era's spectacular achievements in logic, mathematics, and physical science were made. Some of these were to eventually deal a heavy blow to the popularity of Kant's philosophy. Pure logic, Kant had argued, could never provide us with a genuine expansion of our knowledge; yet Frege's *Begriffsschrift* and the later systems that drew from it seemed to provide us with tools to do just this. Euclidean geometry was held by Kant to be both a synthetic and an a priori science. Yet Hilbert was able to show that it followed analytically from a set of basic axioms, and the development and physical application (in relativity theory) of non-Euclidean geometry showed that no one particular geometry could be regarded as a priori true. Arguably worst of all, the development of quantum theory seemed to tell against according a fundamental status to the principle of cause and effect. By the middle of the twentieth century it was widely held that Kant's philosophy had been definitively refuted.[4]

Yet the truth is more subtle. Many of the thinkers whose work had contributed to the demise in popularity of Kantian philosophy were, despite their divergences from him, substantially influenced by Kant. Frege, for example, is at pains to call attention to his agreement with Kant, which he claims far exceeds the extent of his disagreement (Frege, 1980 [1884], §89).[5] Hilbert was also influenced by Kant and in the epigraph to his seminal work on geometry invokes Kant in support of the spirit of his investigations (Hilbert, 1902, p. 1).[6] Reichenbach's conception of the relativised a priori, the conventionalisms of Poincaré, Schlick, and Carnap, the pragmatisms of C. I. Lewis and others, are best characterised, not as radical rejections of Kant's philosophical framework, but rather as attempts to re-explicate the basic Kantian idea that our theoretical frameworks include an element—what Kant had (mistakenly, according to these thinkers) called the synthetic a priori—that is conceptually and epistemologically privileged in some sense. Viewed as a research program (cf. Bitbol, 2017), one may say that Kant's transcendental approach to philosophy continued, and continued to evolve, albeit along multiple independent pathways, well into the last century. It is only with the rise of the Quinean holistic conception of science that these ideas are rejected in their totality.[7]

The development of quantum theory in the early part of the last century posed a particularly strong challenge to the Kantian philosophical viewpoint. Owing to the indeterminacy intrinsic to the theory, a common opinion expressed at the time was that this represents a

[3]*... sondern sie beabsichtigte nur, die Quellen unseres Wissens und den Grad seiner Berechtigung zu untersuchen, ein Geschäft, welches immer der Philosophie verbleiben wird, und dem sich kein Zeitalter ungestraft wird entziehen können* (Helmholtz, 1855, p. 5).

[4]See, for instance, Carnap's summary of the prevailing attitude toward Kant in Reichenbach (1958, p. vi).

[5]For more on the parallels between Kant and Frege, see Cuffaro (2012); Merrick (2006).

[6]For more on the Kantian aspects of Hilbert's thought, see Kitcher (1976).

[7]For more on these topics, see Coffa (1991); DiSalle (2002); Friedman (2009); Howard (1994); Murphey (2005).

definitive refutation of Kant's philosophy insofar as it shows that Kant's ascription of a priori status to the principle of causality cannot be correct. As with other developments in the mathematical and natural sciences during the period, however, the relationship between quantum theory and Kant's theoretical philosophy is far more rich and interesting than this. Cuffaro (2010), for instance, has shown that the physicist Niels Bohr's highly influential views on quantum mechanics are, in fact, broadly Kantian in the sense that they are motivated from considerations that arise naturally from within Kant's philosophical framework; further that Bohr's views remain broadly compatible with—and indeed are very much in the spirit of—a Kantian worldview adapted to address the situation presented to us by quantum theory. Indeed there has been a flowering of scholarship in recent years exploring the relationship between Kantian philosophy and quantum mechanics, and especially between Kant and Bohr (Bächtold, 2017; Bitbol, 2017; Chevalley, 1994; Kaiser, 1992; Kauark-Leite, 2017).

Most of this literature is aimed at clarifying the philosophical viewpoint of Bohr, or of related interpretations of quantum mechanics such as that of Werner Heisenberg (Camilleri, 2005). The primary contribution of this paper, on the other hand, will be to turn an eye back toward Kant. In the sequel it will be my contention that considering Kant's philosophy in the light of the challenges posed by quantum mechanics illuminates the significance of several elements of his philosophical framework. Foremost among these are the principle of causality, the doctrine of synthetic a priori principles in general, and most importantly and most generally: his central doctrine of transcendental idealism itself.

Transcendental idealism is made up of two claims: first, that space and time are the necessary subjective forms of all appearances for us, and second, that they do not attach to things as they are in themselves. With respect to the second claim in particular, there is dispute within Kant scholarship over whether to interpret it ontologically (see especially Guyer 1987), as a claim about how things as they exist independently of us in fact are, or merely epistemically (see especially Allison 2004), as a claim about what we are licensed to attach to our conception of things as they are in themselves—or in other words what we are in a position to determinately assert of them—given our particular epistemic limitations.

Allison's interpretation of Kant, at least in this general sense, is the correct one. However it has been criticised by commentators such as Ameriks (1992), among others, who argue that it amounts to little more than stipulation and that it does not sufficiently capture the real ontological significance that Kant seems to accord to the transcendental distinction between appearances and things as they are in themselves. Some stipulations are better than others, however, and in the sequel I will argue that Kant's doctrine of transcendental idealism, though it should be understood 'merely epistemically'—or rather: conceptually—is nevertheless full of significance. I will argue that his views in this regard are rooted in his long-running pre-critical struggle to discover the first principles for metaphysical cognition, i.e. cognition of things as they actually are in themselves. And I will further argue that an epistemic understanding of the doctrine of transcendental idealism provides a Kantian with a natural way of understanding the situation presented to us by quantum mechanics.

Indeed, the question of the epistemic constraints and preconditions for objective cognition is, as I will show, precisely the question which occupied physicists, such as Bohr, in their efforts to interpret quantum theory. And as I will also show, this is precisely the question which occupied the philosopher Grete Hermann, a neo-Kantian of the Friesian school, who produced a detailed commentary on the relationship between quantum mechanics and Kantian philosophy following a series of extended discussions with Werner Heisenberg and his then assistant Carl Friedrich von Weizsäcker. For reasons that are not altogether clear, Hermann's work remained largely obscure for many years.[8] Fortunately this appears to be changing, and there is now increased awareness of her work, largely due to the effors of Bacciagaluppi & Crull (2017). However to my knowledge there has to date been no detailed commentary on the significance of Hermann's analyses for Kantian philsophy. Thus a secondary aim of this paper will be to contribute to the scholarship on both Hermann and Kant by doing just this.

Before beginning let me note that my goal here is not to somehow definitively refute the ontological interpretation of transcendental idealism. Rather, my goal here is to show that, in addition to being more charitable to Kant and more consistent with his published writings (Allison, 2004), transcendental idealism understood epistemically is neither an artificial nor an arbitrary doctrine. For it can be motivated both by the questions that concerned Kant in his own intellectual development, and by the fact that these very questions come to the fore in the confrontation of quantum theory with experience. Indeed they were precisely the questions which concerned both physicists such as Bohr whose views contained Kantian influences, as well as professed neo-Kantians such as Hermann, in the crucial initial stages of the debate over the interpretation of the theory.

The paper will proceed as follows. In §2 I will introduce Kant's doctrine of transcendental idealism and in §3 I will show how it is motivated, for Kant, by his struggle to provide first principles for metaphysical cognition in the period leading up to the publication of the first Critique. In §4 I will discuss how Kant's search for the first principles of metaphysical cognition was transformed, during the critical period, into the search for the first principles of synthetic a priori cognition, focusing in particular on Kant's conception of the principle of causality. In §5 I will discuss the challenges faced by Kant's view which arise from the emergence of quantum theory. Then in §6 I will argue that causality strictly speaking remains valid within quantum mechanics, but that there is nevertheless a worry that it ceases to be relevant as a principle for the investigation of nature. In §7 I then consider the views of Grete Hermann, and in particular her response to the worry just mentioned and her understanding of the general situation regarding Kant's philosophy *vis á vis* quantum theory.

2 Transcendental Idealism and the 'Neglected Alternative'

In the *Critique of Pure Reason*, Kant identifies two distinct aspects of objective experience: *intuition*—the 'this', the 'that' of experience—and the *concepts* whereby one synthesises the manifold of the former. Concepts belong to the faculty of *understanding*, which we will discuss in §4. Intuition is mediated by the faculty of *sensibility*: our mind's capacity to be affected by objects (CPR, A19/B33).[9] The effect on sensibility of some object is called the *sensation* of it, and with sensation we associate the *empirical* aspect of our intuition.

We call "[t]he undetermined object of an empirical intuition" (CPR, A20/B34) an *appearance*. A shape against the far wall in a dark room, for instance, which only after some scrutiny is determined to be a chair, is before this determination merely the appearance

[8]Besides its mention by Heisenberg and von Weizäcker, Hermann's work is also briefly mentioned in Feyerabend (2015, p. 33, fn. 2), as well as in Jammer (1974, pp. 207-211). Thanks to Jamie Shaw for drawing my attention to Feyerabend's paper.

[9]References to the *Critique of Pure Reason* (CPR), or 'first Critique', will be to the Guyer-Wood translation (1998). Page numbers for Kant's works cited in this essay (with the exception of the first Critique) will be as they are in the standard German edition of Kant's works. In the case of the first Critique, page numbers are as in the first (1781) and second (1787) edition, where "A" denotes the first and "B" the second, as usual.

of something indeterminate.[10] There are two aspects to an appearance. First, there is its *matter*; i.e. *what* we sense in it. Second, there is an appearance's *form*, i.e. that which allows the manifold corresponding to the appearance to be ordered in certain relations. Appearance has two characteristic forms: space, associated with outer appearances, and time, associated with both inner and outer appearances. As forms of appearances, they are the *formal conditions* for appearances, in virtue of which they are known a priori as necessary relations according to which sensations must be ordered for subjects like us (CPR, A20/B34, A26/B42, A33/B49–50). They are also called 'pure' in virtue of not in themselves containing anything belonging to the matter of sensation (A20/B34).

With respect to the intuitions of space and time, Kant's doctrine of *transcendental idealism* puts forward two theses. The thesis just asserted, that space and time are necessary subjective conditions for appearances, is traditionally called the 'subjectivity thesis'. Kant also makes the following claim: "Space represents no property ... that attaches to objects themselves and that would remain even if one were to abstract from all subjective conditions of intuition." (A26/B42). This, and the parallel claim for time (A33/B49), are together traditionally referred to as the 'nonspatiotemporality thesis'. Of the subjectivity and nonspatiotemporality theses, it is nonspatiotemporality which is more controversial. For on one interpretation of Kant's argument for nonspatiotemporality, Kant concludes that space and time cannot be determinations of things in themselves *because* they are necessary subjective conditions of intuition. Assuming this reading of Kant is correct, such a conclusion would only follow if subjectivity and spatiotemporality were mutually exclusive options. But some have argued that there is no reason to think so. In particular some have pointed out that the very fact that space and time are necessary forms of appearances may be taken as good evidence that things in themselves are spatiotemporal.

This objection, that Kant has 'neglected an alternative', is most often associated with Trendelenburg (Gardner, 1999, p. 107), but the objection was also made much earlier by others, for instance by Feder (Sassen, 2000, p. 140). Among contemporary commentators, Guyer puts it particularly forcefully:

> Transcendental idealism is not a skeptical reminder that we *cannot be sure* that things as they are in themselves *are* also as we represent them to be; it is a harshly dogmatic insistence that we *can be quite sure* that things as they are in themselves *cannot be* as we represent them to be. (Guyer, 1987, p. 333).

Guyer is actually slightly more charitable to Kant than it would appear from the above statement, for he argues that Kant does in fact provide an (ultimately unsuccessful) argument for the exclusion of Trendelenburg's alternative. We will pass over the details of Guyer's analysis of the deficiency in Kant's argument here, except to say that Guyer's criticism—and the neglected alternative objection more generally—presupposes an ontological reading of transcendental idealism that was not Kant's. Transcendental idealism is best interpreted as an 'epistemic' or 'conceptual' doctrine (Allison, 2004). That is, the nonspatiotemporality thesis does not amount to the claim that space and time are 'in reality' nonspatiotemporal—it is not an ontological thesis in this sense. Rather, it is best understood in the following way. On Kant's view, implicit in the concept of objective cognition are the subjective—i.e. epistemic—conditions under which an object is representable to us. The concept of a thing regarded as it is *in itself* by definition abstracts from such subjective conditions, and by definition, therefore, abstracts from the determinations of space and time.

[10]For a more thorough discussion of this point, see Harper (1984, p. 110-111).

This is then why we can have no objective cognition of a thing in itself (even though we can conceive of it as something indeterminate), or of its attributes—whatever these may be—for an 'object' that does not refer at least implicitly to the conditions under which it can be represented to us cannot be an object—at any rate not a cognisable one—for us.

Transcendental idealism, as expounded in the first Critique, is an epistemic or conceptual doctrine, but Kant's aim in expounding it is to clarify metaphysics. For although the concept of a thing, regarded as it is in itself, is stipulated to be as I have described it above, this conception is nevertheless *motivated* by metaphysical considerations.[11] These motivations are evident, I will now argue, when one considers Kant's project in the years leading up to the first Critique, i.e. during his long-running pre-critical struggle to discover the first principles of metaphysical cognition.[12]

3 First Principles for Metaphysical Cognition

Kant did indeed acknowledge a version of the neglected alternative objection almost a decade before the publication of the first Critique. In a famous letter to Marcus Herz of February 21, 1772,[13] addressing Lambert's objection to his conception of time,[14] Kant wrote:

> [Lambert objects that changes] are possible only on the assumption of time; therefore time is something real ... Then I asked myself: Why does one not accept the following parallel argument? Bodies are real (according to the testimony of outer sense). Now, bodies are possible only under the condition of space; therefore space is something objective and real that inheres in the things themselves. The reason lies in the fact that it is obvious, in regard to outer things, that one cannot infer the reality of the object from the reality of the representation ... (Zweig, 1967, p. 75).

This is the same objection which Guyer puts as follows: "Why doesn't the indispensable role of space and time in our experience prove the transcendental realism rather than idealism of space and time themselves?" (Guyer, 1987, p. 349). Kant's answer, that it is "obvious" that such an inference is invalid, seems unsatisfactory. But before dismissing this as mere dogmatism we should attempt to understand exactly why Kant thinks it is obvious. This is possible if we place Kant's 1772 letter in its proper context. Kant's preoccupation, as early as 1755, was with the question of how one might make metaphysics legitimate.[15] For Kant, although the aim of metaphysics is purportedly rational certainty with regards to its propositions, in practice its methods are often arbitrary and hypothetical in character. His (preparatory) project at this stage is thus to provide a principled *epistemic grounding* for metaphysics, now to be thought of as a *science*.

[11] Perhaps a better way to think of Kant's concerns is that they are 'meta-metaphysical'.

[12] Kant's 'pre-critical' period refers to the period of Kant's philosophy up until the publication of the *Critique of Pure Reason* in 1781.

[13] The letter is famous because it marks the beginning of Kant's so-called 'silent decade' and poses many of the questions that he will later take up in the 1781 Critique.

[14] Compare also the "Elucidation" to the Aesthetic, A36-37.

[15] In characterising Kant's views in this way, I depart from commentators, such as Caranti, who view Kant's philosophy (both early and late) as primarily concerned with providing an answer to Cartesian skepticism (Caranti, 2007, Ch. 1).

Thus, in the *New Elucidation* of 1755, Kant declares that his intention is to "establish two new principles of metaphysical cognition" (*New Elucidation*, 1:387). "By their means", he tells us, "you may acquire no inconsiderable power in the realm of truths." (*New Elucidation*, 1:416). In his *Only Possible Argument* of 1763, Kant declares that he will supply—not an actual demonstration—but the only sure way in which a demonstration of God's existence may proceed: "What I am furnishing here is the materials for constructing a building ..." (*Only Possible Argument*, 2:66). In his 'Prize Essay' of the same year, Kant outlines the rules by which metaphysics must proceed in its investigations and "by which alone the highest possible degree of metaphysical certainty can be attained" (*Inquiry*, 2:285). Within metaphysics, Kant tells us, "One's chief concern will be to arrive only at judgements about the object which are true and completely certain" (ibid.). Rationally certain cognition with regard to the propositions of metaphysics (traditionally construed) is the goal which by the first Critique Kant will ultimately reject as unachievable. Yet at this stage, Kant is still hopeful that metaphysically certain cognition is possible if only one can purify metaphysics' methods.

Kant's solution to the problem was to take its most mature form in his *Innaugural Dissertation* of 1770. Here we are told that we will achieve our goal if we first identify and abstract from the form and principles of *sensible* cognition. In this way we will elucidate the form and principles of *intellectual* cognition (the proper concern of metaphysics). Intellectual (or 'rational') cognition is cognition which transcends the limitations imposed by our sensibility. It is subject only to the laws of the understanding: "whatever cognition is exempt from such subjective conditions relates only to the object. It is thus clear that things which are thought sensitively are representations of things *as they appear*, while things which are intellectual are representations of things *as they are*." (*Dissertation*, 2:392).

In order to obtain intellectual cognition, one must abstract from all conditions related to sensibility.[16] This includes even the form and principles of sensible cognition. Thus, of space and time, Kant tells us "that these notions are not *rational* at all, and that they are not *objective* ideas of any connection, but that they are appearances, and that, while they do, indeed, bear witness to some common principle constituting a universal connection, they do not expose it to view." (*Dissertation*, 2:391). Note that Kant is here using the term 'objective' differently from the way he will later use it in the first Critique. In the Dissertation, 'objective' is meant in the sense of pertaining to the noumenal, or mind-independent, world. But in the Critique, as we will see shortly, Kant is not hesitant to grant 'objective' status to representations that are dependent on the faculty of sensibility *as such*, so long as they are not dependent on any particular sense impressions.

In any case, and again anticipating the neglected alternative objection, Kant tells us that the forms of sensibility must be excluded even though they seem to provide good evidence for some underlying analogous connection between things as they are in themselves: "the *form* of the same representation is undoubtedly evidence of a certain reference or relation in what is sensed, though properly speaking it is not an outline or any kind of schema of the object ..." (*Dissertation*, 2:393). "[E]mpirical concepts do not," Kant tells us, "in virtue of being raised to greater universality, become intellectual in the *real sense*, nor do they pass

[16]Kant writes, at 2:394, "a concept of the understanding *abstracts* from everything sensitive, but it is *not abstracted* from what is sensitive." This is explained in the sentence immediately preceding: "The former expression indicates that in a certain concept we should not attend to the other things which are connected with it in some way or other, while the latter expression indicates that it would be given only concretely, and only in such a way that it is separated from the things which are joined to it." He goes on: "For this reason, it is more advisable to call concepts of the understanding 'pure ideas', and concepts which are only given empirically 'abstract concepts'."

beyond the species of sensitive cognition; no matter how high they ascend by abstracting, they always remain sensitive." (*Dissertation*, 2:394).

To clarify: the fact that spatiotemporal relations necessarily attach to sensible concepts seems to provide some warrant for also attaching these (or analogous) determinations to the concept of the thing in itself considered apart from the conditions of our sensibility. Nevertheless we certainly *cannot* say that it follows necessarily from this that things as they are in themselves are related to each other in any particular way, let alone spatiotemporally. We are not warranted, therefore, to attach these attributes to our conception of the thing in itself. It may be helpful to use a mathematical analogy here. In mathematics we refrain from attaching a certain property to a concept even though we know that this property holds for some particular instances of the concept. Instead, we include in our concept only that which can determinately be affirmed of the concept in general. For instance, we say that matrix multiplication is non-commutative since it is not the case in general that for two matrices, A and B, that $AB = BA$, even though, for some particular matrices, AB does in fact equal BA. It is similar with Kant's conception of the thing in itself, not in the sense that we can attach spatiotemporal attributes to some but not all particular things in themselves (whatever that would mean), but in the sense that our concept of the thing in itself, for Kant, can only be composed of properties we are in a position to determinately affirm of it as such. But if our concept of the thing in itself as such is composed only of what we can determinately affirm of it, then spatiotemporal attributes certainly may not be attached to this concept.

In fact the difference between sensible and intelligible concepts is a difference in *kind*, not in degree (cf. Allison, 2004, p. 17), for space and time attach necessarily to the objects of sensible cognition, but necessarily do not attach to the objects of intelligible cognition (*Dissertation*, 2:394).[17] And since the object of metaphysics is to attain intelligible cognition, anything pertaining to sensible cognition—no matter to how high a degree of abstraction it is raised—must be excluded. Thus in the Dissertation Kant offers to metaphysicians the "principle of reduction" (*Dissertation*, 2:413), which asserts that any concept of the understanding to which one predicates anything belonging to space and time must not be asserted objectively (in the sense of the Dissertation), i.e., asserted as having objective validity independently of all actual—or possible—experience of it. One might object that the bar that Kant sets for objective cognition here is high indeed, but one should keep in mind Kant's project and goals: the attainment of metaphysically certain cognition. Given these goals, Kant's standards are in my view appropriate.[18]

[17]Compare this also with A43/B60 of the Critique, where Kant writes: "Even if we could bring this intuition of ours to the highest degree of distinctness we would not thereby come any closer to the constitution of objects in themselves. For in any case we would still completely cognize only our own way of intuiting, i.e., our sensibility, and this always only under the conditions originally depending on the subject, space and time; ..."

[18]This high bar for metaphysical cognition and the preceding discussion clarify, in my view, what the actual relation between the subjectivity and nonspatiotemporality theses of transcendental idealism is, and how the argument for both theses progresses (see §2 above). In the transcendental exposition of the concept of space, Kant begins with the claim that geometry is a science "that determines the properties of space synthetically and yet *a priori*." (B40). He then asks: "What then must the representation of space be for such a cognition of it to be possible?" (B40). It cannot be a concept, "for from a mere concept no propositions can be drawn that go beyond the concept" (B41); i.e., if space were a concept we could not make sense of the synthetic nature of geometrical propositions. The only other alternative is that space is an intuition. And since geometry is a science that proceeds independently of any actual experience, space must be an a priori intuition. Kant then goes on to make the apparently unjustified claim that this can be true of space "Obviously not otherwise than insofar as it has its seat merely in the subject" (B41) (the subjectivity thesis). This may be obvious to

In any case, once we have purged our concepts of spatiotemporal elements, the question then becomes one of the formal principle of the intelligible world. In particular, the question is "to explain how it is possible *that a plurality of substances should be in mutual interaction with each other*, and in this way belong to the same whole, which is called a world" (*Dissertation*, 2:407). The principle that is left to elucidate, that is, is the principle of causality (in a generalised sense) or what Kant elsewhere calls the principle of the determining ground.[19] We need not discuss Kant's more detailed comments on this principle here, but it is worth noting that, just as in his earlier works, Kant does not actually provide the needed elucidation. Rather, his goal in the Dissertation is merely to offer this principle and the principle of reduction as the tool with which and the ground upon which, respectively, one can begin to tackle the problems of metaphysics.[20]

By 1781, Kant had been awoken from his "dogmatic slumbers" (*Prolegomena*, 4:260), professedly through his reading of Hume, with regard to the principle of causality. However the legitimacy of the principle of causality had been undermined only within the domain of metaphysics. Its use had not, in Kant's mind, been undermined within the domain of natural science. He writes, in 1783:

> The question was not whether the concept of cause was right, useful, and even indispensable for our knowledge of nature, for this Hume had never doubted; but whether that concept could be thought by reason *a priori*, and consequently whether it possessed an inner truth, independent of all experience (*Prolegomena*, 4:258–259).

He continues: "This was Hume's problem. It was a question concerning the *origin* of the concept, not concerning its indispensability in use. Were the former decided, the conditions

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Kant, but it nevertheless requires some elucidation. Kant provides the needed elucidation at A26. The first step is the argument for nonspatiality. He writes: "Space represents no property ... that attaches to objects themselves and that would remain even if one were to abstract from all subjective conditions of intuition." Kant's justification for this is that "neither absolute nor relative determinations can be intuited prior to the existence of the things to which they pertain, thus be intuited *a priori*." To clarify: recall that Kant has already established that space is an a priori intuition. Put aside, for now, the question of whether or not space is a necessary subjective condition, and suppose space were a necessary condition of *objects* (as they exist independently of us). How could we determine this a priori (as, by hypothesis, we do in geometry)? Since space is not a concept, we would have to *intuit* it a priori; and since it is the determination of an object existing independently of us, we would have to intuit it without being affected by the object. But we do not have the capacity to intuit without being affected (B72). Thus there would be no way for us to know that space was a necessary determination of objects, independently of our actual experience of them. Thus we *could not know*, a priori, that the intuition of space is a necessary condition of things as they are in themselves. We have no *warrant*, therefore, for attaching space to our conception of things as they are in themselves. On the other hand, we can make sense of the synthetic a priori status of the propositions of geometry if space is a necessary *subjective* condition of our intuition (the subjectivity thesis). For in that case we *can* intuit space in advance of *actual* (though not without reference to possible) experience. In broad outlines the structure this argument is similar to that identified by Guyer (1987) (as opposed, for instance, to the structure identified by Falkenstein 1995, ch. 9, who locates the argument for nonspatiotemporality mainly in the 'metaphysical exposition' section of the Transcendental Aesthetic). But *pace* Guyer it should be clear that such an argument only really makes sense (and obviously its conclusion only follows) given an epistemic or conceptual reading of transcendental idealism.

[19]Cf. *New Elucidation*, §2, Proposition IV, 1:391-393.

[20]Kant does offer a sketch of what one must do in order to provide a demonstration of the community of substances: "Granted that the inference from a given world to the unique cause of all its parts is valid, then, if, conversely, the argument proceeded in the same way from a given cause, which was common to all the parts, to the connection between them and, thus, to the form of the world (although I confess that this conclusion does not seem as clear to me), then the fundamental connection of substances would not be contingent but necessary, for all the substances are *sustained by a common principle*." (*Dissertation*, 2:409).

of its use and the sphere of its valid application would have been determined as a matter of course" (ibid.).

Synthetic a priori cognition (a term Kant introduces in the first Critique) is that cognition in which two (or more) concepts are cognized, in advance of experience, as necessarily connected in some way to one another. 'Ampliative' cognition of this kind is what metaphysics seeks. But by the time of the Critique, Kant is convinced that such cognition is impossible without a reference to the forms of our sensibility, space and time.[21] Even the principle of causality has validity only with regard to appearances, which are always given in space and time. Thus, once we carry out what is required by the Dissertation's principle of reduction, metaphysical cognition—cognition of the intelligible world, i.e., of things as they are independently of *all* conditions for the (possible or actual) experience of them—must be given up forever. This does not undermine, however, the idea that the concept of cause is indispensable in some sense, nor does it imply that synthetic a priori cognition is impossible. Thus Kant's project now becomes to explicate that sense and to show *how* synthetic a priori cognition is possible and to investigate its limits. These limits are *possible* experience, and since the *form* of possible experience is given a priori, synthetic a priori knowledge is possible with respect to it. A useful *corollary* to the results of this investigation is a grounding for the theoretical sciences; i.e., an answer to the issue that Hume *did not* intend to raise: the validity and scope of principles, such as that of causality, *within* the theoretical sciences.[22] Metaphysics, regarded as a system of synthetic a priori constitutive principles which *transcend* experience, is declared impossible. Metaphysics is instead transformed into a system of *methodological* principles for the investigation of nature.[23]

4 Kant's Synthetic A Priori Principles

The 'useful corollary' that I mentioned in the previous section—i.e. the grounding for the theoretical sciences that falls out of an investigation into the scope and limits of synthetic a priori cognition—is primarily found in the Transcendental Analytic section of the Critique, and especially in the Analytic of Principles.[24] Recall from §2 that for Kant, possible experience comprises two distinct aspects: intuition, for which space and time are its pure forms, and *concepts of the understanding*, which correspond to rules for synthesising the manifold of intuition. The concept 'chessboard', for instance, corresponds to a rule whereby this particular bit of white, that particular bit of black, etc. can be associated in one representation. When we synthesise, i.e. combine, some particular manifold of intuition according to the particular rule for a concept, we say that the manifold has been subsumed under the concept and cognised in accordance with it (A68-69/B93-94). A *pure* concept of the understanding is called a *category* (A80/B106); it is one of a set of 'meta-concepts' that all

[21] Cf. A5/B9.

[22] Kant writes, at B20: "In the solution of the above problem there is at the same time contained the possibility of the pure use of reason in the grounding and execution of all sciences that contain a theoretical *a priori* cognition of objects". Cf. also: *Prolegomena*, 4:280.

[23] I believe that the reviews that Kant received after the publication of the first edition of the Critique made it manifest to him that he had not been sufficiently clear with regard to this aspect of his project. See, for example, the Feder-Garve review (Sassen, 2000, pp. 53-58), and Kant's response (*Prolegomena*, 4:372-380). Thus I believe that at least part of the aim of the second edition emendations of the Critique was precisely to clarify this aspect of the Critique's goal, namely to delimit the sphere and conditions for the possibility of synthetic a priori cognition.

[24] I do not claim that this is *all* there is to be found in this section of the Critique, of course.

empirical concepts necessarily presuppose. Like the pure forms of intuition, the categories are a priori.

The application of a category to the manifold of intuition is governed by the *schema* of that category (A137-147/B176-B187); i.e. the rule by which the manifold is determined in accordance with the category. For example, the pure schema of magnitude is number (A142-143/B182). In apprehending the manifold corresponding to an object, determinate intuitions—'instants'—of time are produced with each successive act of synthesis. Through these is produced a *time series* from which one judges as to the extent of the object apprehended (A145/B184). Through an analysis of the use of the categories in accordance with their characteristic schema, we are entitled to make a number of synthetic a priori judgements regarding the objects of cognition in general. These are called synthetic a priori *principles* (A159-235/B198-294) by Kant. Of particular importance for our discussion in the next and subsequent sections, wherein we will be considering the relevance of quantum theory for Kant's philosophy, is the principle of causality. Kant calls this a 'dynamical principle' since it is a principle governing the connection of appearances in time. It tells us, according to Kant, that the changes undergone by an object of cognition are ordered uniquely and objectively according to a necessary rule (A191/B236). This is in contrast to the series of subjective perceptions of the object through which we apprehend it.

To illustrate: suppose I lean against a fence at the bank of a river, and watch a piece of wood as it is carried downstream by the current.[25] At time t_1, I watch as it comes into view from around the bend in the river some yards upstream. I then daydream for a while. Eventually, I notice (at t_2) that the log has travelled some distance from the place where I first spotted it. At t_3, I recall to myself the motion of the wood down the river that I half-consciously observed while daydreaming, after which I continue to watch as it disappears into the forest (t_4). Later that afternoon, I recall that what aroused me from my daydream was a sparrow alighting on the log (at t_5). If we list these representations in the order in which they are actually perceived, then this is a *subjective ordering*:

$$t_1, t_2, t_3, t_4, t_5.$$

I can also give them an *objective ordering*, however, according to which the motion of the log must have actually proceeded in time:

$$t_1, t_3, t_5, t_2, t_4.$$

To determine this objective ordering, I must discover a rule for the log's motion, for in general "appearance, in contradistinction to the representations of apprehension, can thereby only be represented as the object that is distinct from them if it stands under a rule that distinguishes it from every other apprehension, and makes one way of combining the manifold necessary." (B236/A190). The particular rule of succession for the change of state of the log is something that can only be discovered empirically, for instance by taking into account the position of the log in the river in my various perceptions of it, the direction therein in which I perceived the river to be flowing, etc. However, that objective cognition presupposes *that there is* some such rule is what the principle of causality tells us. This knowledge is *a priori*, according to Kant. One notable aspect of such a rule, which we will return to later, is that Kant seems to require that it be deterministic in the sense of making possible the perfect *prediction* of an event from what has immediately preceded it. In Kant's

[25]This is a variation on Kant's example of the ship (A191-193/B236-238).

words: "there must therefore lie in that which in general precedes an occurrence the condition for a rule, in accordance with which this occurrence always and necessarily follows" (B238–239/A193–194).

The concept of something that is an object *for us*—i.e. something we have achieved objective cognition of—presupposes a determination of the manifold in accordance with the principle of causality and the other synthetic a priori principles, for Kant (B198). On the other hand, and importantly, these principles are only regulative with respect to experience as a whole. By this is meant that from a methodological point of view we are required to investigate nature in accordance with them (for otherwise the objective cognition of anything would be impossible). However it does not follow from this that the objects of our inquiries are already determined in accordance with these principles in advance of our investigations, or that we can know a priori that any particular endeavour to attain objective cognition will be successful (cf. A509/B537). A particularly striking example of the failure to obtain objective cognition in this sense is provided by quantum theory, which we will begin to discuss in the next section.

5 The Situation in Quantum Mechanics

To begin with, consider the following 'classical' scenario.[26] A medium-sized object is launched towards a diaphragm into which an opening, or 'slit', has been made that is large enough to allow the object to pass through, but small enough so that it invariably deflects the object to some degree as it does so. After passing through the slit the object eventually impacts upon a further screen. Assume that the diaphragm is movable (e.g. let it be attached to the rest of the apparatus by springs), so that when the object collides with the edges of the slit as it passes through the diaphragm, the latter recoils slightly. We would like to describe the state of the object immediately after its interaction with the diaphragm. Assume that the (centre-of-mass) positions and momenta of both the object and the diaphragm just prior to the interaction are known, but that the precise shape and size of the launched object are unknown, so that we cannot calculate in advance what the respective positions and momenta of the object and diaphragm will be after the collision. Once they do collide, however, we can then measure the momentum of the diaphragm by observing the amount by which it recoils, which will allow us to (via the law of conservation of momentum) determine what the momentum of the launched object is immediately after the collision. Similarly, since the common centre of mass of the combined system remains at rest, we can determine the object's position by measuring the position of the slit.

In classical physics, which is adequate to describe the motion of macroscopic objects like the one in the imagined scenario, a precise determination of momentum and position is sufficient to completely characterise the state of a particle (or collection of particles) *vis á vis* its variable parameters at any one time. And from the precise characterisation of the state of a particle at any one time, one can then precisely infer the state of the particle at

[26]The following example is adapted from Bacciagaluppi (2015, p. 381), as is my discussion of the analogous single-slit example involving quantum phenomena which follows. The quantum example is of course originally due to Bohr (1935). In his paper, Bohr actually discusses (successively) multiple variations of the single-slit experiment. The one discussed here and in Bacciagaluppi (2015) is the particular variation discussed on pp. 698–699: "In an arrangement suited for measurements of the momentum of the first diaphragm, it is further clear that even if we have measured this momentum before the passage of the particle through the slit, we are after this passage still left with a *free choice* whether we wish to know the momentum of the particle or its initial position relative to the rest of the apparatus".

all other times, assuming the system is isolated or closed (Hughes, 1989, §2.1).[27] Thus through precisely determining the state of the object immediately after its impact with the diaphragm, we are able to predict the object's subsequent trajectory with certainty.

This is in spite of the fact that the diaphragm—our measuring instrument—disturbs the motion of the object as the latter impacts upon it. For in classical theory this is in principle unproblematic. In every case of classical measurement, either the interaction of the measuring instrument with the object of inquiry can be made negligible for the purposes of the analysis, or (as above) we can use physical theory to abstract away from this interaction in such a way as to allow us to precisely determine the object's various state parameters. Indeed for precisely this reason, after we have measured the momentum of the *diaphragm*—which will in general disturb *it*—we can subsequently measure the diaphragm's position, and then use physical theory to abstract away from the interaction involved in our previous momentum measurement to determine the position of the object which has just passed through. In this way we are able to simultaneously ascribe *both* position and momentum parameters—i.e. a complete state description—to the launched object (Bai & Stachel, 2017).

In the words of the previous section, in every case classical theory allows us to transition from the subjective conditions (represented by the diaphragm in the above example) under which we perceive an object, to an objective description of that object wherein these subjective conditions no longer explicitly appear. They nevertheless remain implicit in the sense that our description of the object presupposes that it has been determined by us to be such as we describe it (either directly or perhaps only indirectly) through a process of measurement in space and time. This process of determination is characteristic of *all* objective cognition, as we have seen, for Kant.

Things are more interesting when we come to the case of quantum phenomena. Let the object now be a quantum object—a photon, for example—which passes through the slit in a diaphragm on its way to an eventual impact with a photographic plate. Similarly as in our previous example, we would like to objectively describe its state immediately after it has collided with the slit. In this case, however, quantum mechanics' well-known 'uncertainty'—or as Bohr preferred to call it: 'indeterminacy'—relation for position and momentum precludes us from ascribing determinate values to these quantities simultaneously. For according to this relation, as the indeterminacy in the position of an object approaches zero, the indeterminacy in its momentum approaches infinity, and vice versa. Expressed in terms of our example, this means that if we choose to precisely measure the momentum of the diaphragm (so as to ascribe a determinate momentum to the object that has just passed though it), then the consequent disturbance of the diaphragm will be such as to make impossible a further precise determination of the object's position. That is, the diaphragm's displacement consequent upon the momentum measurement of it will have been, in contrast to the situation in classical theory, "uncontrollable" (Bohr, 1935, p. 698)—or at any rate not controllable enough—and we will be unable to account for this displacement and abstract away from it as we did in the classical case for the purposes of a subsequent position determination of the object. The situation will be similar if we first choose to measure the diaphragm's position (and thus the position of the object); through this choice we will have precluded ourselves from precisely determining the object's momentum.

[27] Note, however, the caveat in §2.4.

6 Conceptual Indeterminacy

Bohr expresses the significance of the limitation imposed by the indeterminacy relations as follows:

> Indeed we have in each experimental arrangement suited for the study of proper quantum phenomena not merely to do with an ignorance of the value of certain physical quantities, but with the impossibility of defining these quantities in an unambiguous way (Bohr, 1935, p. 699).[28]

That is, the significance of the indeterminacy relations is, for Bohr at any rate, not epistemic in the sense that one presupposes the quantum object in the above example to be perfectly determinate in itself with respect to all of its ascribable physical parameters, but yet not completely knowable by us.[29] What is being expressed here, rather, is that their significance is epistemic in a different sense; a better word would be *conceptual*.

From the point of view of the previous section, we can understand this as follows.[30] Consider the result of some experiment, say the mark on a photographic plate, or the particular situation of a pointer measuring the momentum of the diaphragm in the above example. The pointer and the mark are in themselves classical objects; they can each be described as having definite spatiotemporal coordinates, and as causally interacting in a definite way with their surroundings. However our aim is to go beyond these phenomena and describe the particular situation of the pointer (or the mark) as having arisen through its interaction with some independently existing object. Our goal is to 'get at' this object as it exists independently of the 'subjective conditions' associated with our experimental apparatus. We do this by eliminating the interaction between the apparatus and object from our description of the latter.

For a Kantian, in order to describe something objectively—i.e. as an object existing independently of us that we can nevertheless have possible experience of—it must be determinable in space and time in accordance with the synthetic a priori principles. Earlier I mentioned causality as an example of a 'dynamical' principle of this kind. In addition to the dynamical principles there are also what Kant calls *mathematical* principles (CPR, B198-B294).[31] According to the latter, anything that appears to us must be apprehended as having, determinately, both an extensive (length, breadth, etc.) and an intensive magnitude (i.e. a degree). The dynamical principles, in contrast, are not principles for the apprehension but for the connection of appearances in time. They state, first, that all change presupposes something permanent; second, that all change must occur according to the law

[28]An earlier statement expressing an essentially identical viewpoint can be found in Bohr (1928, p. 580) and is discussed in Cuffaro (2010, pp. 311–312).

[29]This is, incidentally, the viewpoint normally associated with (the young) Heisenberg, although see the discussion in Frappier (2017, pp. 92-94) for a contrary view.

[30]I consider Bohr's views on quantum mechanics to have substantial elements in common with Kant's philosophy, and I will be pointing out some of these elements as we proceed. However my goal in this paper is not specifically to present an argument for this interpretation of Bohr, as I take that to have already been established by the work of numerous others (some of whom have been referred to above). Indeed, even those who have identified pragmatism as the more dominant influence on Bohr's thought now acknowledge that the influence on him of Kantianism was nevertheless substantial (e.g. Camilleri, 2017; Faye, 2017; Folse, 2017). In any case, as I alluded to in the introduction to this paper, and in agreement with Folse (2017), I do not see pragmatism as incompatible with Kantianism when the latter is construed broadly in the sense of a research program (cf. Bitbol, 2017).

[31]The mathematical principles are the *Axioms of Intuition* and *Anticipations of Perception*; the dynamical principles are the *Analogies of experience* and the *Postulates of empirical thought as such*.

of cause and effect; third, that all substances that are perceived as simultaneous are in mutual interaction.[32] An objective description is one that is determined according to *both* sets of principles. Together, they assert that the determination of any appearance as an object of possible experience must be such that at a determinate instant in time, it has a determinate extent (constrained by the mathematical principles) and hence a determinate position in space, and that there is a law (subject to the dynamical principles) by which it dynamically interacts with its surroundings in and through time. In the context of our example of the slit, one can interpret this as signifying that any description of quantum objects that purports to pick out an object of possible experience for us must be such as to ascribe to that object both a determinate position and a determinate momentum parameter. But according to the indeterminacy relations, it is impossible in principle to describe the particle's momentum with any degree of precision without a corresponding loss of precision with regards to its spatial coordinates. The upshot of all of this is that we cannot complete our description of the object according to the Kantian criteria for objects of possible experience. And yet these are *necessary* criteria, for Kant, in the sense that, as we have seen, objective knowledge is impossible for us without them. We may 'cheat' by ascribing only indeterminate values of position, momentum, etc. to the 'object' of our investigations, but the resultant 'unsharply defined'[33] description can as a result never be an object *for us*—i.e. it can never be an object that we can possibly experience and thus never be real for us in that sense. We can consider it merely as a noumenon, or abstract object.[34]

For a Kantian, the situation thus seems hopeless. An objective description just is one in which we determine something according to both spatiotemporal and dynamical criteria—we simply have no other choice. And yet these criteria cannot fulfil their intended function in the quantum domain, for a determination of one necessarily excludes a determination of the other in the sense of the indeterminacy relations. Further, there is a different (though related) sense in which they mutually exclude one another as well, which stems from the so-called 'wave-particle duality' of quantum phenomena. As Bohr points out (Bohr, 1928, p. 581), in the equation expressing this duality: $E\tau = I\lambda = h$, Planck's constant (h) relates quantities that are incompatible from a classical point of view. That is, in the first relation, E (energy) is associated with the concept of a particle given with definite spatiotemporal coordinates, while τ (the period of vibration) is associated with a wave-train "of unlimited extent", not conceptualisable with respect to definite space-time coordinates. Likewise, respectively, for I (momentum) and λ (wavelength). Bohr's point is that it is inconsistent from a classical point of view to describe an object as being, in accordance with the above relations, both given at some definite spatiotemporal location and of unlimited extent in space and time. However a violation of the indeterminacy relations in the description of a quantum object would imply (Cuffaro, 2010, p. 311) a precise determination of that object with respect to both of the above parameters, which would entail the simultaneous applicability of these mutually exclusive 'pictures' of the object, which cannot be. Thus not only do the classical—or Kantian for our intents and purposes—criteria mutually exclude one another *vis á vis* their determinability. Even if we could ascribe both spatiotemporal and dynamical attributes to the object, the resulting object would be self-contradictory.

Ironically it is the uncertainty relations which save us, at least to some extent. They guarantee that we can nevertheless achieve a unified—albeit abstract—description of quantum phenomena by 'patching together' the mutually exclusive dynamical and spatiotemporal

[32]Here, I only consider the *Analogies*, as the *Postulates* are not directly relevant for our discussion.
[33]Bohr (Cf. 1928, p. 582).
[34]See Cuffaro (2010, p. 313).

descriptions that result from our various experiments. As Bohr puts it:

> The apparently incompatible sorts of information about the behaviour of the object under examination which we get by different experimental arrangements can clearly not be brought into connection with each other in the usual way, but may, as equally essential for an exhaustive account of all experience, be regarded as "complementary" to each other (Bohr, 1937, p. 291).

The uncertainty relations guarantee that a dynamical description can never contradict a spatiotemporal description—that the two can be used in a complementary way in our description of an abstract quantum object—for any experiment intended to *determinately* establish the object's spatiotemporal coordinates *can tell us nothing* about its dynamical parameters, and vice versa.

> the proper rôle of the indeterminacy relations consists in assuring quantitatively the logical compatibility of apparently contradictory laws which appear when we use two different experimental arrangements, of which only one permits an unambiguous use of the concept of position, while only the other permits the application of the concept of momentum ... (Bohr, 1937, p. 293).

We are not licensed, however, to take the next step and ascribe physical reality to this 'patched together' object of our descriptions, for the object is not real but abstract, and its classical statiotemporal and dynamical attributes are idealisations.

For one seeking to defend the Kantian metaphysical and epistemological framework, including the Kantian criteria for objective cognition, and in particular the law of causality, this is good news, at least in one sense. It is true that the indeterminacy relations imply that in general it is impossible to predict with certainty the future behaviour of a quantum object from a complete characterisation of its present state, i.e that the latter in general does not contain "the condition for a rule, in accordance with which [a subsequent] occurrence always and necessarily follows" (B238–239/A193–194). Nevertheless this does not represent a falsification of Kant's views. For quantum state descriptions are merely abstract objects from a Kantian point of view and thus are not objects of possible experience for us. It is only the concept of the latter which presupposes that it be determinable in accordance with the principle of causality and other synthetic a priori principles. And it is only for such objects of possible experience that the perfect prediction of its subsequent states from a determinate description of the object is implied.

There is, nevertheless, a potential problem, if not for Kant's framework as a whole then at least for the principle of causality. For as we saw, the principle of causality is, in addition to being presupposed by the concept of objective cognition, also (arguably precisely for this reason) a regulative principle for the purposes of the investigation of nature as a whole. But if objective cognition in the Kantian sense is typically excluded in quantum mechanics, and if moreover the doctrine of complementarity allows us to continue to do physics in spite of this, then it seems that we can do away with causality as a regulative principle even if, strictly speaking, it is not contradicted by quantum mechanics. As Schlick put it in 1931:

> The principle of causality does not directly express a fact to us, say, about the regularity of the world, but it constitutes an imperative, a precept to seek regularity, to describe events by laws. Such a direction is not true or false but is good or bad, useful or useless. And what quantum physics teaches us is just this:

> that the principle is *bad*, useless, impracticable within the limits precisely laid down by the principle of indeterminacy. Within those limits it is impossible to seek for causes. Quantum mechanics actually teaches us this, and thus gives a guiding thread to the activity that is called investigation of nature, an opposing rule against the causal principle. (1962 [1931], p. 285).

In response, one might note that Schlick's construal of the situation regarding the principle of causality in quantum mechanics only applies to the quantum state description itself. While it is true that one cannot simultaneously attach determinate spatiotemporal and dynamical attributes to this description, it remains the case on the viewpoint expressed here that we anyhow require the Kantian categories and principles in order to 'patch together' this very description. That is, they are necessary for the very interpretation of the results of the measurements involving mutually exclusive experimental arrangements from which we construct our description of the quantum object. In this sense they do in fact continue to function at least as methodological *tools* for the investigation of nature. The epistemological lesson driven home by quantum mechanics, one might say, is that epistemic primacy–the conditions under which we can assert that we have cognised some object—does not imply ontological primacy. In other words the necessary principles according to which we cognise something as an object cannot for that reason be construed as the necessary conditions for the possibility of objects *in themselves*.

But how, one might ask, do we actually proceed in this 'patching together' of information about the object which we glean from our mutually exclusive experiments upon it? The indeterminacy relations show us only that this patching together cannot lead to contradictions, but it still remains for us to understand just how these ordinary concepts may continue to be used to augment our knowledge.

We will consider this challenge in the next section, and we will see there that in answering it there will be more to say as a consequence regarding the principle of causality and the Kantian viewpoint in general.

7 Grete Hermann and the Relative Context of Observation

A response to Schlick's challenge as well as a thoroughgoing analysis of the relevance of quantum mechanics for Kant's theoretical philosophy was provided by Grete Hermann in her 1935 essay on the 'Natural-Philosophical Foundations of Quantum Mechanics' (Hermann, 2017b). As compared to the other important philosophers of the period who commented on quantum mechanics, Hermann is comparatively little known. Today she is perhaps best known for having anticipated, by roughly thirty years, John Bell's seminal criticism of von Neumann's purported proof of the impossibility of 'hidden variables' in quantum mechanics, i.e. of parameters not described by quantum mechanics which if taken under consideration would provide a more determinate description of a quantum system's state (see Seevinck, 2017). Hermann received her doctorate in mathematics in 1925 under the supervision of the eminent mathematician and theoretical physicist Emmy Noether at the university of Göttingen. During her time in Göttingen she studied philosophy with Leonard Nelson, who eventually was to serve as the examiner for her dissertation, and for whom she was a private assistant from the period following her successful defense up until Nelson's death in 1927 (Hansen-Schaberg, 2017). Nelson was the leader of the Neo-Friesian school of Neo-Kantianism, a movement dedicated to the revival (but not without modification) of the views of the Neo-Kantian philosopher, Jakob Friedrich Fries. Hermann also considered herself a Friesian. Indeed a likely part of the reason her work remained relatively unknown amongst philosophers of quantum mechanics is her decision to publish her analyses in the obscure Neo-Friesian journal *Abhandlugen der Fries'schen Schule*. The few meagre excerpts of her long essay that later appeared in the more important journal *Die Naturwissenschaften* (Hermann, 1999) do not do justice to her work and indeed present a misleading impression of it.

Like all neo-Kantians, Fries sought to retain what he took to be the essential features of Kant's critical philosophy while amending certain of its details. Fries' principal emendation of Kant was with regard to Kant's method. In particular Fries did not consider it legitimate to, as Kant had done (e.g. in his Transcendental Deduction), derive the a priori principles for theoretical cognition from a priori bases. For Fries, in contrast, one must begin with a conceptual analysis of actual experience in order to discover the naturally necessary forms implicit therein (Beiser 2014, §1.10, Leary 1982, p. 226). Fries nevertheless concurred with Kant with regards to the *content* of critical philosophy. His categories and principles for objective cognition ultimately do not differ from Kant's.

Given Fries' emphasis on empirical knowledge, it is no surprise that he was greatly interested in the sciences. Fries made contributions to the philosophies of chemistry, biology, and mathematics. But his principal interest was psychology, which for him was the proper empirical basis to build the critical philosophy upon. German philosophy at this time was dominated by the ideas of Absolute Idealism, and philosophers of this period were not held in high regard by the German scientific community. Fries, in contrast, was very well respected and read by his scientific contemporaries. He had a substantial influence, for example, on M. J. Schleiden, the founder of modern cytology (Leary, 1982, p. 221). The neo-Friesian Leonard Nelson was no different in this regard. Nelson was very knowledgeable in mathematics, physics, and law, and maintained a friendship and professional relationship with the eminent mathematician David Hilbert and his school (Leal, 2017, pp. 20, 23).

It was much the same for Grete Hermann (Paparo, 2017, pp. 44-46). Hermann became interested in quantum theory in 1933 or perhaps earlier. As a neo-Kantian and neo-Friesian, she had been deeply interested in assertions such as Heisenberg's that "Because all experiments are subject to the laws of quantum mechanics, the invalidity of the causal law is definitively determined by quantum mechanics" (Heisenberg 1927, as translated by Rynin in Schlick 1962 [1931], p. 281). At some time in 1933 she completed a manuscript (never published) entitled 'Determinism and Quantum Mechanics' (Hermann, 2017a), which included penetrating criticisms of both the impossibility proof of von Neuman (mentioned previously) as well as of a similar argument of Dirac's. She had apparently attended a lecture of Heisenberg's shortly before completing this manuscript (Heckmann, 2017, p. 221), and then written it shortly thereafter, sending it both to Copenhagen, where it was read at least by Bohr, Heisenberg, and von Weizsäcker, as well as to Dirac. It is unknown whether Dirac responded to Hermann. However in a letter to her dated December 17, 1933 from her fellow Neo-Friesian Gustav Heckmann (who was in exile from the Nazi regime in Denmark at the time) we learn that:

> They take your paper absolutely and completely *seriously* and in the days while he is still here, H[eisenberg] together with Bohr and a student of H[eisenberg]'s, Weizsäcker, wanted to jointly draw up an answer to you. ... Otherwise he thinks, as you do, that you must still learn more physics; study the 4 papers by Bohr—he has told you about them. Hei[senberg] speaks with such imprecise phil[osophical] terminology that it would make a Friesian climb the walls. If he

> understood transc[endental] idealism, then he would surely realise the possibility of finding the key also to solving the philos[ophical] difficulties arising from quantum m[echanics]. ... You and Dora will always find an open door and an open ear with Heisenberg. Use it yet, ere you knock at the harder door made of older wood, Bohr. ... It makes me very happy that you have acquired the considerable regard of Bohr and Hei[senberg] with your paper. Get onto them with the transc[endental] id[ealism]! (Heckmann, 2017, pp. 221–222).

Shortly afterwards, in 1934, Hermann travelled to Leipzig to participate in a seminar run by Heisenberg which brought together a number of eminent physicists to debate and discuss the new developments in quantum theory (Soler, 2017, p. 64). Hermann remained in Leipzig for some time, continuing her discussions with Heisenberg and von Weizsäcker, and at the end of this period published her essay describing her conclusions regarding quantum theory's significance for Kantian philosophy (Hermann, 2017b). Her essay was reviewed glowingly one year later by von Weizsäcker (1936). Heisenberg later devoted a chapter of his scientific autobiography to the discussions with Hermann, remarking there that "We had the feeling that we had all learned a good deal about the relationship between Kant's philosophy and modern science" (Heisenberg, 1971, p. 124).

In her 1935 essay (Hermann, 2017b), Hermann begins by noting that the emergence of quantum theory has "shaken" (p. 239) the idea that a priori principles discoverable through critical philosophical analysis lie at the foundations of natural science. Ultimately she will argue that the challenge presented by quantum theory to the critical philosophy can be met. However it cannot be met, she argues, by simply recapitulating a purely philosophical deduction of these principles.

> For, even if the physical development of the theory is not sufficient to put the foundations of the thus achieved knowledge of nature into the sharp light of awareness, still the scientific progress that has been obtained in these theories precisely through the willingness to abandon or revise old familiar concepts provides the guarantee that new and fruitful points of view have been introduced here into research. Only their philosophical interpretation and elaboration will produce clarity concerning both the philosophical arguments for the a-prioricity of natural-philosophical principles and the objections to them arising from the side of physics. (Hermann, 2017b, p. 240).

She notes that this philosophical elaboration and interpretation cannot take the form suggested by physicists such as Born, i.e. of attempting to disprove quantum theory by empirical means so as to restore principles, such as that of causality, to their former unexceptioned status (Born, 1929). For the question at issue is precisely to what extent such principles are a priori and as such amenable to a purely philosophical analysis, albeit one informed by quantum theory. Additionally, with respect to the principle of causality in particular, it would be pointless, Hermann argues, to seek to save this principle for philosophy by abandoning the criterion by which causal connections in nature can be known, i.e. the criterion of prediction. She writes:

> One who wished to brush this off with the excuse that, while the *knowledge* of the causes determining the processes is limited, the *existence* of such causes is not put in doubt, removes the law of causality from the realm of the principles governing natural knowledge into that of mysticism. Where it is impossible in

principle to decide what falls under a given concept in nature, the statement *that* anything falls under it also loses its meaning. (Hermann, 2017b, p. 242).

She nevertheless cautions against the use of the criterion of prediction in the "positivistically distorted form" (ibid.) employed by certain philosophers of her day. She rather directs attention to the way the criterion of prediction is used in physics, with a view to providing a philosophical analysis of the sense in which an in-principle limit is set for it within quantum mechanics. Hermann, like Bohr, does not consider the indeterminacy relations of the theory to signify a kind of necessary ignorance with regard to certain properties of objects that despite this are possessed by those objects in themselves. Rather, as for Bohr, their significance for Hermann is definitional or *conceptual* in the sense that the simultaneous subsumption of our description of a quantum object under the wave and particle pictures of phenomena is only possible via a limited application of these two pictures to the phenomena (Hermann, 2017b, p. 246).

Precisely because a quantum object cannot, according to these relations, be simultaneously described as having a determinate position and momentum, however, it cannot be the case that its future motion is determined by these state parameters as it would be for a classical object. This suggests the question of whether there may be further parameters, not described by quantum theory, which in fact precisely determine the quantum object's motion. Such a question is meaningful and legitimate, she argues, and moreover verifiable in principle (2017b, p. 254). Further, the various arguments purporting to show the impossibility of such parameters are all, as she shows, lacking.[35] Indeed, "there can be only one sufficient reason for abandoning as fundamentally useless the further search for the causes of an observed process: *that one already knows these causes*." (2017b, p. 254).

Somewhat surprisingly, however, Hermann maintains that quantum mechanics itself provides the resources with which to identify such causes. She notes that an interpretation of the particular situation of a measurement pointer as a statement about the current state of a quantum object presupposes a (classical) theory of the interaction that has taken place between the quantum object and the measuring apparatus.[36] For instance, in the above example of the diaphragm and slit we appeal to the conservation law for momentum in order to interpret the reading which we get upon measuring the diaphragm as asserting a fact about the object which has passed through it.[37] But in the inference made possible by such a theory, Hermann argues, the reading of the pointer is "explained as the necessary effect that the system to be measured has imposed on the instrument in the process of measurement" (Hermann, 2017b, p. 255). Because of this there is no need, Hermann claims, to seek for the physical features overlooked by quantum mechanics which would make possible a causal explanation of the measurement pointer's particular situation. For a (classical) causal explanation for this particular measurement result (of momentum) is already provided by quantum mechanics in the way just mentioned. Likewise, the classical

[35] Notable is her analysis (2017b, pp. 251-253) of von Neumann's impossibility proof, which as I mentioned earlier, anticipates the objections of Bell (see Seevinck, 2017). Note that the argument presented in her 1935 paper is mostly a reproduction the argument presented in her earlier unpublished paper (Hermann, 2017a). The latter presentation is arguably clearer.

[36] Compare this with the statements attributed to Einstein in Heisenberg (1971, pp. 63-64).

[37] In her exposition, Hermann actually employs the example of a microscope. It is sufficient for illustrating her essential point, however, to continue with the example of the diaphragm and slit which we have used above. For more on the particulars of Hermann's microscope example, see Filk (2017); Frappier (2017). For a comparison of the two examples and an explanation of why they illustrate an essentially similar point, see Bacciagaluppi (2017).

theory of interaction appealed to in this case also provides a classical causal explanation for the particular value of momentum we ascribe to the *quantum object* subsequent to its interaction with the diaphragm.

Note, however, that due to the uncontrollable displacement of the diaphragm consequent upon a measurement of its momentum, we can as a result no longer determine the quantum object's position immediately after its interaction with the slit, and therefore cannot include a determination of that object's position in a causal explanation of the reading of the measurement pointer. But as Soler (2017) notes, in this way Hermann's analysis seems to depart from a strictly Kantian conception of the law of causality. Hermann, that is, appears to identify the particular value of momentum possessed by the quantum object as the cause of the particular reading of the measurement pointer set up to measure the momentum of the diaphragm. Recall, however, that for Kant a causal process is one in which the appearances of an object are connected in time. But the appearance of an object just is an appearance which can be given an objective description. And an objective description just is one in which *both* dynamical and spatiotemporal parameters can be ascribed to the object.[38] Thus the causal explanations that Hermann claims can be reconstructed from our different observations of phenomena do not seem to be causal in a full Kantian sense. As Soler puts it:[39]

> Hermann's interpretation in no way allows the conjugate variables to be simultaneously measured, and therefore in no way allows the reconstitution of the continuous trajectory of an object. It is precisely on the basis of the possibility of gaining access to such continuous trajectories that classical physicists conceived of causality. For them, causal behaviour meant that the values of two conjugate variables of an object at a given time (position *and* momentum equated with *the* cause) univocally determined the subsequent trajectory (position *and* momentum at a later time equated with *the* effect). Here one can readily attack Hermann's conclusions by claiming that the concept of causality involved is very different from (or at least cannot be identified with) the classical, Kantian concept of causality. (p. 65).

One might, however, attempt to defend Hermann's viewpoint—or anyway one not too distant from Hermann's—in the following way. Note that I do not claim—i.e. it is not clear to me—that Hermann would agree to be so defended, but neither can I identify anything in her writings which would contradict what I say in this and the following three paragraphs. In the particular scenario associated with the one-slit experiment that we have been discussing, we have, the reader will recall, the following components. First, we have a measurement pointer connected to a diaphragm. Into the diaphragm there has been cut a narrow slit through which an object is to pass. The object collides with the walls of the slit as it does so. After being deflected slightly by the slit in this way, it then travels toward a second measurement apparatus, connected to a second pointer, where it is perhaps then measured again. Note that the first and the second pointers are themselves classical objects. Each of them can be described as having a determinate position in space and as interacting in a determinate way with its surroundings. Thus the description of any particular reading of one of these pointers is a classical description of that reading.

After the initial collision, there arises the appearance of a particular reading of the first pointer. We explain this particular reading through the postulation of an object that has

[38] Cf. Cuffaro (2010, p. 313).

[39] Bacciagaluppi (2017, p. 140) makes a similar point, but in relation to Bohr.

passed through the slit in the diaphragm and that has consequently obtained a particular value of momentum as a result (which we infer from the reading). We then subject this object to a further measurement of momentum, and there thus arises the appearance of a second pointer reading. This second pointer reading, like the first, can be classically described. The second reading, moreover, can be explained as the necessary effect of the first. Indeed, from a description of the first pointer reading one can predict what the reading of the second pointer will be with certainty, and vice versa: from a description of the second pointer reading we know what the first pointer reading must have been.[40]

We have not, however, objectively described the quantum object which links these appearances together, of course—from the Kantian point of view it is only a noumenal or abstract object, as we saw earlier. For this reason it is impossible to predict the result of *any* measurement we may choose to perform after the initial collision with certainty. Nevertheless, from a Kantian point of view we can say that for the given event—the reading of the second pointer—we have determined a cause, i.e. the reading of the first pointer, from which it must always and necessarily follow;[41] i.e. given the first momentum determination, the second momentum determination had to have been what it was and vice versa. In this sense we have given a 'retrospective' causal explanation, and moreover one that is in accord with the basic tenets of Kantianism.

Now one might object that we have in this way only linked together the series of *appearances*[42], albeit appearances which can be objectively described (i.e. we can provide classical descriptions of both measurement pointers). But do we not seek to get at the *reality* responsible for these appearances? And if so do we not then demand a causal description which includes an objective determination of the quantum object that has given rise to them? Perhaps. But for a Kantian the causal law in fact just is a law for connecting together the series of appearances such as the one described above in time. It does not demand that the noumenal ground of this causal sequence also be known. It is precisely for this reason that Kant is able to limit the reach of his theoretical philosophy in order to make room for his practical philosophy (B566-567).

In any case, Hermann takes her analysis to have shown that the concept of causality is both compatible with quantum mechanics and indeed that it is presupposed by it (2017b, p. 263) in the sense that we appeal to a theory describing the object's prior interaction with our measuring instrument in order to make our momentum (or position) ascriptions to the object. Unlike in classical mechanics, however, it is not the case in quantum mechanics that the complete quantum mechanical state description of an object enables us to predict the outcome of any experiment we may wish to perform on it with certainty. Nevertheless, Hermann argues, this does not excuse us from providing a criterion according to which a causal claim can be checked. This criterion can only be the criterion of prediction, for Hermann. But as we saw above our causal claims may only be checked 'indirectly' in this sense: From the particular reading of a measurement pointer we first infer backwards (via a theory of interaction) to its cause, and from this we then ascribe to the object of interest a particular value for some theoretical quantity, which then can be used to predict what the result of a further measurement upon the object will be.

The significance of this severing of the principle of causality from the criterion of 'perfect

[40]These are simply expressions of the ideas that quantum measurements are repeatable and that any quantum measurement process can also be thought of as a state preparation procedure (cf. Hermann, 2017b, p. 272).

[41]Cf. CPR, B238—239/A193–194.

[42]Cf. Hermann (2017b, p. 262)

prediction'—in the sense of being able to determinately predict the result of any measurement performed upon the object—is, for Hermann, this. The (Kantian) principle of causality says only that for every event there is a cause which can be found from which it always and necessarily follows. Classical physics, however, adds to this principle the idea that given a cause, we can always infer in advance what its particular effects will be. The reason, according to Hermann, that classical physics makes this conflation is that in the domain of classical physics it is always possible to provide a description of an object that is such that it does not refer to the *particular* conditions under which it is perceived (although even in classical physics our descriptions nevertheless do refer implicitly to the conditions under which an object *can be* perceived as such; i.e. in space and time). In quantum mechanics, in contrast, although objective description is still possible in a sense, it is only objective insofar as it is relativised to a particular experimental context.[43] Hermann writes:

> Quantum mechanics requires us to resolve this merging of different natural-philosophical principles—to drop the assumption of the absolute character of the knowledge of nature and to handle the principle of causality independently of it. Consequently, it has not refuted the law of causality, but has clarified it and freed it from other principles that are not necessarily connected with it. (Hermann, 2017b, p. 254).

This brings us to the core of Hermann's view, which we can elucidate again by means of the example of the diaphragm and slit.[44] First note that immediately after the object has passed through the diaphragm in this example, we have, according to Hermann, a choice. We can measure the momentum of the diaphragm, in which case we can use the conservation theorem for momentum in order to determine the launched object's momentum. Alternately we can measure the diaphragm's position so as to determine the position of the launched object. Finally, according to Hermann, we can choose not to perform any measurement at all. In this case the result is what we would now call an 'entangled' state of the object and diaphragm (Hermann, 2017b, p. 258). She writes:

> The coexistence of these different possibilities now evidently means that, depending on how one procures one's knowledge of the observed system, or, as we can say for this, depending on the relevant observational context, one can obtain different wave functions for the same system and for the same instant—namely for the [object] at the time immediately after the collision with the [diaphragm]. Thus the quantum-mechanical characterisation, unlike the classical one, does not pertain to the physical system still somehow 'in itself', and this means here: independently of which observation one uses to procure one's knowledge of it. (Hermann, 2017b, pp. 258-259).

The idea that an objective description of phenomena presupposes a reference to the conditions under which it is apprehended by us—what Hermann calls the 'relative context of observation'—is precisely the core doctrine of transcendental idealism as we have explicated it above.[45] The point is simply that what can be described as an object by us must refer in that description to the conditions under which it can be apprehended as such by us. In

[43]Cf. Healey (Forthcoming).

[44]The example Hermann utilises is actually that of a microscope (see fn. 37 above).

[45]In identifying the idea of the relative context of observation as the core aspect of Hermann's philosophy of natural science I am in full agreement with Crull (2017).

the classical case these conditions are simply the conditions under which an object can be apprehended in general: space and time. Since these conditions are the same for all rational cognisers, then in classical physics we may claim to, in a sense, cognise the object 'in itself'. But even here this is never absolutely true (at least not for a Kantian) as we saw in §2. In the quantum case, in contrast, these conditions are in fact the particular conditions under which an object has been actually apprehended in the context of a particular experiment. In quantum mechanics, that is, our conception of an object *includes* the conditions under which it has *actually* been determined.

What quantum mechanics teaches us, finally, to answer the question with which we began this section, is not that the philosophical principles at the basis of critical philosophy have been shown to be false, invalid, or useless. Rather, through our experience with quantum mechanics we realise, according to Hermann, that the consequences of the critical philosophy run deeper than we would have been led to believe on the basis of a merely philosophical consideration alone. On the basis of the critical philosophy alone (for example through considering Kant's antinomies) we are led to understand that no one mode of description can 'get at' reality as it exists behind the phenomena. In this way it is possible, for a Kantian, to have the theoretical sciences co-exist with the practical sciences (e.g. psychology). Quantum mechanics, on the other hand, teaches us that this 'splitting of truth', as Hermann calls it, reaches into physical theory itself—that even from within physical theory no one unique perspective can achieve full objectivity (Hermann, 2017b, pp. 276-278).

8 Summary and Conclusion

In §2 I introduced Kant's doctrine of transcendental idealism and explicated what is meant by a conceptual interpretation of this doctrine. In §3 I then described how this doctrine was motivated, for Kant, by his struggle to provide first principles for metaphysical cognition in the period leading up to the publication of the first Critique. In §4 I then discussed how Kant's search for the first principles of metaphysical cognition was transformed, during the critical period, into the search for the first principles of synthetic a priori cognition, and I focused in particular on Kant's conception of the principle of causality. In §5 I described the challenges faced by Kant's view which arise from the emergence of quantum theory. Then in §6 I argued that causality in the Kantian sense strictly speaking remains valid within quantum mechanics, but that there is nevertheless a worry that it ceases to be relevant as a regulative principle for the investigation of nature. In §7 I finally considered the views of Grete Hermann, and in particular I considered her response to the worry just mentioned as well as her understanding of the general situation regarding Kant's philosophy *vis á vis* quantum theory. Throughout I have presupposed an epistemic—or rather conceptual—understanding of Kant's transcendental idealism.

In addition to being more charitable to Kant and more consistent with his published writings (Allison, 2004), transcendental idealism understood epistemically is neither an artificial nor an arbitrary doctrine. For it can be motivated both by the questions that concerned Kant in his own intellectual development, and by the fact that these very questions come to the fore in the confrontation of quantum theory with experience. Indeed, as I have argued above, they were precisely the questions which concerned both physicists such as Bohr whose views contained Kantian influences, as well as professed neo-Kantians such as Hermann, in the crucial initial stages of the debate over the interpretation of the theory.

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