Does protective measurement imply the reality of the wave function?

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Abstract

Recently the first protective measurement has been realized in experiment [Nature Phys. 13, 1191 (2017)], which can measure the expectation value of an observable from a single quantum system. This raises an important and pressing issue of whether protective measurement implies the reality of the wave function. If the answer is yes, this will improve the influential PBR theorem [Nature Phys. 8, 475 (2012)] by removing auxiliary assumptions, and help settle the issue about the nature of the wave function. In this paper, we demonstrate that this is indeed the case. It is shown that a ψ-epistemic model and quantum mechanics have different predictions about the variance of the result of a Zeno-type protective measurement with finite N.

By a conventional projective measurement on a single quantum system, we obtain one of the eigenvalues of the measured observable, and the expectation value of the observable can be obtained only as the statistical average of eigenvalues for an ensemble of identically prepared systems. Thus the meaning of the expectation value of an observable, as well as the meaning of the wave function, is usually regarded as statistical. On the other hand, it has been discovered that by a protective measurement [1,2], we can measure the expectation value of an observable from a single quantum system. Recently the first protective measurement has been realized in experiment [3]. This raises an important and pressing issue of whether protective measurement implies the reality of the wave function for a single quantum system [4-10].

At first sight, protective measurement (PM) seems to provide a strong argument supporting the reality of the wave function [4]; the expectation values of observables and the wave function should be the property of a single quantum system, since they can be measured by PMs only from a single quantum system. However, it has been realized that things are more complicated [10]. The complication lies mainly in the protection procedure.

Take the Zeno-type protection scheme as an example. In a Zeno-type PM, one makes frequent projective measurements of an observable O, of which the measured state $|\psi\rangle$ is a nondegenerate eigenstate, in a time interval $[0, \tau]$. For instance, O is measured in $[0, \tau]$ at times $t_n = (n/N)\tau, n = 1, 2, ..., N$, where N is an arbitrarily large number. These projective measurements protect the measured state from being changed. At the same time, one makes a projective measurement of an observable A in the interval $[0, \tau]$, which is described by the usual interaction Hamiltonian $H_I = g(t)PA$, where P is the conjugate momentum of the pointer variable, and $g(t)$ represents the time-dependent coupling strength of the interaction, which is a smooth function normalized to $\int dtg(t) = 1$ during the measurement interval τ, and $g(0) = g(\tau) = 0$. The result of the measurement is the expectation value of the observable A, $\langle A\rangle$ [1,2,8].

It can be seen that in a Zeno-type PM there is also an ensemble of identically copies of the measured system, which is prepared by the protection procedure, namely the frequent projective measurements, when the protection is successful. Thus, it is possible that the expectation value of the measured observable is also obtained as the ensemble average of the eigenvalues of the measured observable as for conventional projective measurements. Indeed, it has been suggested that this may be realized by a certain mechanism in a ψ-epistemic (ontological) model [10]. The model assumes that the observable A of the measured system has a definite value at any time, which is one of the eigenvalues of A. The suggested mechanism is as follows. When each projective measurement of O results in the state of the measured system being in $|\psi\rangle$, it also randomizes the value of A and make it be a_i with probability p_i, where a_i is an eigenvalue of A, and $p_i = | < a_i|\psi > |^2$ is the corresponding Born probability. Then the measured system shifts the pointer by a_i/N after the follow-up measurement of A. In the end, the total pointer shift, denoted by Δx, will be the expectation value of A:

$$\Delta x = \lim_{N\to\infty} \sum_i n_i a_i/N = \sum_i p_i a_i = \langle A\rangle. \tag{1}$$

The above ψ-epistemic model shows that the result of a Zeno-type PM, the expectation value of the measured observable, may be generated from the eigenvalues of the observable for the ensemble of identically copies of the measured system, which is prepared by the protection systems in the PM. However, it still needs to see whether all predictions of the model are consistent with quantum mechanics for a Zeno-type PM with finite N. In this paper, we will first give a general ontological model of Zeno-type PM, and then analyze whether the predictions of a ψ-epistemic (ontological) model, especially the prediction about the variance of the measurement result, can be consistent with quantum mechanics.

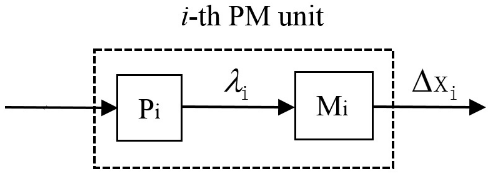

Figure 1: An ontological model of Zeno-type PM

A Zeno-type PM is composed of N identical units, each of which contains a protection system and a measuring system. In each unit such as the i-th unit (see Figure 1), when the protection is successful, the wave function of the measured system, which is prepared by the protection system P_i, is still the initial wave function $|\psi\rangle$. According to the ontological models framework [11,12], on which the PBR theorem is based, this pure state $|\psi\rangle$ corresponds to a probability distribution $p(\lambda_i|P_i)$, where λ_i is the ontic state of the measured system. Moreover, when the measured system interacts with the measuring system M_i, its the ontic state, λ_i, determines the probability $p(a|\lambda_i, M_i)$ of different pointer shifts a of the measuring system M_i. Then, the pointer shift generated by the i-th unit, Δx_i, has a probability distribution

$$p(\Delta x_i = a|P_i, M_i) = \int_\Omega p(a|\lambda_i, M_i)p(\lambda_i|P_i)d\lambda_i. \tag{2}$$

Since all units are identical, the statistical properties of every random variable Δx_i should be the same. In particular, we have $E(\Delta x_i) = E(\Delta x_j)$ and $Var(\Delta x_i) = Var(\Delta x_j)$ for any i and j, where $E(\cdot)$ is the expectation value, and $Var(\cdot)$ is the variance. This further means that the expectation value of the total pointer shift Δx after the Zeno-type PM is

$$E(\Delta x) = \sum_i E(\Delta x_i) = N \times E(\Delta x_1). \tag{3}$$

Since the total pointer shift is $\langle A\rangle$ for a Zeno-type PM of observable A, we have $E(\Delta x_i) = \langle A\rangle/N$ for any i. Moreover, since each measuring system is designed to make the pointer shift be proportional to the measuring time, we have $\Delta x_i \propto 1/N$ and may write $\Delta x_i = A_i/N$, where A_i is another random variable independent of N. Then we have $E(A_i) = \langle A\rangle$ for any i.

When all units are prepared independently, it is natural to assume that the random variables Δx_i or A_i are statistically independent. This assumption of preparation independence is similar to that of the PBR theorem [12].[1]

[1] I will drop this assumption later.

Under this assumption, we have $E(A_iA_j) = E(A_i)E(A_j)$ and

$$Var(\Delta x) = \sum_i Var(\frac{A_i}{N}) = N \times \frac{Var(A_1)}{N^2} = \frac{Var(A_1)}{N}, \tag{4}$$

where $Var(A_1) = Var(A_i)$ for any $i \neq 1$, and it is a quantity independent of N.

In a ψ-epistemic model, since $\langle A \rangle$ is not a property of the measured system but generated by a random process, we have $Var(A_1) \neq 0$ no matter how small the variance is. Note that $Var(A_1)$ is not necessarily equal to $Var(A) \equiv \langle A^2 \rangle - \langle A \rangle^2$, but $Var(A_1)$ should be proportional to $Var(A)$, as when $Var(A) = 0$, $Var(\Delta x) = 0$. Since the total pointer shift after a Zeno-type PM indicates the measurement result, Eq. (4) also gives the variance of the result of the Zeno-type PM. This can then be compared with the predictions of quantum mechanics.

In addition, in a ψ-epistemic model, the initial position of the pointer, whose wave function is a wavepacket with a certain width, is not definite but random. A good measuring system is required to satisfy the condition that the pointer shift during a measurement is independent of the initial position of the pointer. Then the variance of the final position of the pointer is

$$Var(x_f) = Var(x_i) + Var(\Delta x) = Var(x_i) + \frac{Var(A_1)}{N}, \tag{5}$$

where $x_f = x_i + \Delta x$ is the final position of the pointer, x_i is the initial position of the pointer, and $Var(x_i)$ is its variance. Note that the independence of the random variables x_i and Δx also means that $Var(A_1)$ does not depend on $Var(x_i)$.[2] In other words, in a particular ψ-epistemic model, the value of $Var(A_1)$ is the same for any value of $Var(x_i)$. In particular, the value of $Var(x_i)$ can be set to be negligible compared with $Var(A_1)$. In this case, the initial position of the pointer is almost definite.

In the following, we will demonstrate that a ψ-epistemic model cannot explain the Zeno-type PM, and in particular, the above prediction of the model, namely Eq. (4), is inconsistent with quantum mechanics. For the above Zeno-type PM, we set $g(t_i) = 1/\tau$ for any i for convenience of analysis. Then the state of the combined system immediately before $t_1 = \tau/N$ is given (up to the second order) by

[2]This result can also be obtained by noticing that the initial position x_i is not a frame-independent quantity, while the pointer shift Δx is. For example, consider another reference frame S' in which we have $x_f^{'} = x_i^{'} + \Delta x^{'}$ and $x_i^{'} \neq x_i$. Since $\Delta x^{'} = \Delta x$ for any x_i and $x_i^{'}$ (in the non-relativistic domain), Δx is independent of x_i.

$$\begin{aligned} e^{-\frac{i}{\hbar}\frac{1}{N}PA}\left|\psi\right\rangle\left|\phi(x_0)\right\rangle &= \sum_i c_i \left|a_i\right\rangle \left|\phi(x_0+\frac{1}{N}a_i)\right\rangle \\ &= \left|\psi\right\rangle \left|\phi(x_0+\frac{1}{N}\langle A\rangle)\right\rangle \\ &\quad +\frac{A-\langle A\rangle}{N}\left|\psi\right\rangle \left|\phi'(x_0+\frac{1}{N}\langle A\rangle)\right\rangle \\ &\quad +\frac{Var(A)}{2N^2}\left|\psi\right\rangle \left|\phi''(x_0+\frac{1}{N}\langle A\rangle)\right\rangle, \end{aligned} \tag{6}$$

where $|\phi(x_0)\rangle$ is the initial pointer wavepacket centered in position x_0, $|a_i\rangle$ are the eigenstates of A, and c_i are the expansion coefficients. Note that the second term in the r.h.s of the formula is orthogonal to the measured state $|\psi\rangle$. Then the branch of the state of the combined system after $t_1 = \tau/N$, in which the projective measurement of O results in the state of the measured system being in $|\psi\rangle$ (i.e. the protection is successful), is given by

$$\begin{aligned} \left|\psi\right\rangle\left\langle\psi\right|e^{-\frac{i}{\hbar}\frac{1}{N}PA}\left|\psi\right\rangle\left|\phi(x_0)\right\rangle &= \left|\psi\right\rangle \left|\phi(x_0+\frac{1}{N}\langle A\rangle)\right\rangle \\ &\quad +\frac{Var(A)}{2N^2}\left|\psi\right\rangle \left|\phi''(x_0+\frac{1}{N}\langle A\rangle)\right\rangle. \end{aligned} \tag{7}$$

Finally, the branch of the state of the combined system after τ (i.e. after N such measurements), in which each projective measurement of O results in the state of the measured system being in $|\psi\rangle$, is

$$\begin{aligned} \left|t=\tau\right\rangle &= \left|\psi\right\rangle\left|\phi(x_0+\langle A\rangle)\right\rangle \\ &\quad +\frac{Var(A)}{2N}\left|\psi\right\rangle\left|\phi''(x_0+\langle A\rangle)\right\rangle. \end{aligned} \tag{8}$$

Since the modulus squared of the amplitude of this branch approaches one when $N \to \infty$, this state will be the state of the combined system after the PM.

Suppose the initial pointer wavepacket is a Gaussian wavepacket. Then we can calculate the variance of the final position distribution of the pointer, which is (up to the first order)

$$Var(\rho_f) = Var(\rho_i) + \frac{Var(\rho_i)}{N}Var(A)(k_1 + k_2 Var(\rho_i)), \tag{9}$$

where the first term in the r.h.s of the equation, $Var(\rho_i)$, is the variance of the initial position distribution of the pointer, k_1 and k_2 are numerical constants related to the Gaussian wavepacket. When the variance of the initial position distribution of the pointer is negligible, namely $Var(\rho_i) \approx 0$, the variance of the final position distribution of the pointer, which is then represented by the second term in the r.h.s of the above equation, will be the variance of the measurement result.

Now we can compare the predictions of a ψ-epistemic model and quantum mechanics for the variance of the result of an N-unit Zeno-type PM, namely Eq. (4) and Eq. (9). When the initial position of the pointer is almost definite, quantum mechanics predicts that the variance of the measurement result is $\frac{Var(\rho_i)}{N}Var(A)(k_1+k_2Var(\rho_i))$, while a ψ-epistemic model will predict that the variance of the measurement result is $\frac{Var(A_1)}{N}$. Since in this case $Var(\rho_i) \approx 0$ and it is negligible compared with $Var(A_1)$, the two predictions are very different.[3] Note again that $Var(A_1)$ does not depend on $Var(x_i)$ or $Var(\rho_i)$.

Let's summarize the above proof with all underlying assumptions clearly stated and examined. The proof, like the proof of the PBR theorem [12], is based on the ontological models framework [11,12]. The first assumption is about the existence of the underlying state of reality. It says that if a quantum system is prepared such that quantum mechanics assigns a pure state to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is represented by a mathematical object, λ. In the above ontological model of Zeno-type PM, this means that in each unit i when the protection is successful and the wave function of the measured system is a pure state $|\psi\rangle$, the measured system has an ontic state, denoted by λ_i, which is prepared by the protection system P_i and input to the measuring system M_i. This assumption is necessary for an analysis of the ontological status of the wave function, since if there are no any underlying ontic states, it will be meaningless to ask whether or not the wave functions describe them.

The second assumption is that when the measured system interacts with the measuring system, the ontic state of the measured system determines the probability of different pointer shifts of the measuring system. In the above ontological model of Zeno-type PM, the ontic state of the measured system, λ_i, determines the probability $p(a|\lambda_i, M)$ of different pointer shifts a of the measuring system M_i. This assumption can be regarded as an extension of the original assumption of the ontological models framework, which says that when a measurement is performed, the probability of different results is determined by the ontic state of the measured system, along with the properties of the measuring device. In order to investigate whether an ontological model is consistent with the empirical predictions of quantum mechanics, we must have a rule of connecting the underlying ontic states

[3] This conclusion also holds true for a general initial pointer wavepacket, for which quantum mechanics predicts that the variance of the measurement result is also negligible compared with $Var(A_1)$ when the initial position of the pointer is almost definite.

with the results of measurements, such as this assumption.

The above two assumptions are the basic assumptions of an ontological model of Zeno-type PM. Based on these assumptions, we have shown that a ψ-epistemic model of N-unit Zeno-type PM cannot yield the same predictions as quantum mechanics about the variance of the measurement result. In a ψ-epistemic model, the expectation value of the measured observable, such as $\langle A \rangle$, is not a property of the measured system. Then the pointer shift of each PM unit is not $\langle A \rangle / N$, but another quantity fluctuating around $\langle A \rangle / N$, such as a_i/N, where a_i is an eigenvalue of A. The total pointer shift after the PM, which represents the measurement result, is then a random variable. Even if the expectation value of this variable can be exactly $\langle A \rangle$, its variance is shown to be inconsistent with the predictions of quantum mechanics. Quantum mechanics predicts that the variance of the result of the PM is proportional to the width of the initial pointer wavepacket, and it can be arbitrarily close to zero for finite N. While the ψ-epistemic model predicts that the variance is independent of the width of the initial pointer wavepacket, and it cannot be arbitrarily close to zero for finite N.

By comparison, in a ψ-ontic model, the expectation value of the measured observable, such as $\langle A \rangle$, is a property of the measured system. As a result, the pointer shift of each PM unit is exactly $\langle A \rangle / N$, and the random process that exists in a ψ-epistemic model does not exist in a ψ-ontic model. Moreover, the variance of the final measurement result is determined by the post-measurement wave function (e.g. via the dynamical collapse of the wave function in collapse theories), and it depends on the width of the initial pointer wavepacket by the time evolution of the wave function. Thus a ψ-ontic model may yield the same predictions as quantum mechanics about the variance of the measurement result.

The above proof, like the proof of the PBR theorem [12], also resorts to an additional assumption of preparation independence that leads to one of the main results, Eq. (4). It says that when all N units of the Zeno-type PM are prepared independently, the pointer shift generated by each unit, Δx_i, are statistically independent. However, this assumption can be removed without influencing our proof of the inconsistency between a ψ-epistemic model and quantum mechanics. The reason is that whether all Δx_i are statistically independent, the total pointer shift, Δx, is independent of the initial position of the pointer, x_i, and their variances, $Var(\Delta x)$ and $Var(x_i)$, are also independent of each other. It is the independence that leads to the inconsistency between a ψ-epistemic model and quantum mechanics.

In fact, even if assuming that the total pointer shift, Δx, depends on the initial position of the pointer, x_i, and when $Var(x_i)$ or $Var(\rho_i)$ is close to zero, $Var(\Delta x)$ is also close to zero, we can also prove the ψ-ontic view. The reason is that under this assumption, when $Var(\rho_i)$ is arbitrarily close to zero or the initial pointer wavepacket approaches the δ function, the variance of the pointer shift of each unit, $Var(\Delta x_i)$, must be also arbitrarily close to zero. Since $E(\Delta x_i) = \langle A \rangle / N$, this means that the pointer shift of each unit must be arbitrarily close to $\langle A \rangle / N$. Then the probability $p(a|\lambda_i, M_i)$ of different pointer shifts a will approach a δ function, namely $\delta(a - \langle A \rangle / N)$. As a result, the ontic state of the measured system, λ_i, will determine the definite pointer shift $\langle A \rangle / N$ in the limit. Then, $\langle A \rangle$ will be a property of the measured system (whose wave function is a pure state, $|\psi\rangle$). Since a wave function can be constructed from the expectation values of a sufficient number of observables, the wave function $|\psi\rangle$ is also a property of the measured system. This proves the reality of the wave function.

In conclusion, we have demonstrated that protective measurement implies the reality of the wave function. When considering only conventional projective measurements, auxiliary assumptions are needed to prove the reality of the wave function. For example, the PBR theorem is based on an additional assumption of preparation independence [12]. Our new proof in terms of protective measurements does not rely on auxiliary assumptions, and it may help settle the issue about the nature of the wave function.

Acknowledgments

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