Time Evolution of Energy Density, EoS Parameter and the Density Parameters for Matter and Dark Energy in Brans-Dicke Gravity

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Abstract

The present study has been conducted in the framework of Brans-Dicke theory, using FRW metric with flat space, to determine the time dependence of the Brans-Dicke parameter (ω), energy density and the equation of state parameter of the cosmic fluid in an universe expanding with acceleration, following a phase of decelerated expansion. For this purpose, a scale factor has been so chosen that the deceleration parameter, obtained from it, changes its sign from positive to negative as time goes on. Considering the total pressure to be due to the entity named dark energy, the time dependence of energy parameters for matter and dark energy have been determined. Time dependence of the equation of state (EoS) parameter for dark energy has also been studied. Time variations of all these parameters have been shown graphically. It is quite evident from the present study that the dependence of the scalar field upon the scale factor plays a very important role in governing the time evolution of the cosmological quantities mentioned above. This model has an inherent simplicity in the sense that it allows one to determine the time evolution of dark energy without involving any self interaction potential or cosmological constant in the formulation.

Keywords: Scalar Field, Brans-Dicke Theory, Dark Energy, Density Parameters, Energy density, Equation of State (EoS) Parameter, Cosmology.

1. Introduction

The accelerated expansion of the universe is one of the most interesting and important phenomena in the field of cosmology that have been obtained through astrophysical observations [1-5]. An exotic form of energy, with a negative pressure, has been found to be responsible for the accelerated expansion of the universe. This energy is known as dark energy (DE). In the fields of physics and astronomy, an extensive research is now taking place, throughout the world, on DE. A number of models have been proposed to explain this accelerated expansion of the universe, following a phase of deceleration. In most of the models, DE is represented by cosmological constant [6].
The present article is based on Brans-Dicke theory of gravitation. The Brans-Dicke (BD) theory is characterized by a scalar field ϕ and a dimensionless coupling parameter ω that govern the dynamics of space-time geometry. It can be regarded as a natural extension of the general theory of relativity which is obtained in the limit of an infinite ω and a constant value

of the scalar field ϕ [7]. The BD theory of gravity can be regarded as one of the most important theories, among all prevalent alternative theories of gravitation, which have very successfully explained the early and late time behaviours of the universe and solved the problems of inflation [8]. As an extension of the original BD theory, a generalized version was proposed, where the coupling parameter ω is regarded as a function of the scalar field ϕ [9-11]. Several models regarding the expanding universe have been built up on the basis of this theory [7, 12-15].

In the present study, BD field equations (for a spatially flat, homogeneous and isotropic space-time geometry) have been used to determine time dependent expressions of the coupling parameter (ω), equation of state (EoS) parameter (γ) and energy density (ρ). Time dependence of ω was previously studied by many groups, using scale factors that have power law dependence upon time [7, 14, 15]. These scale factors actually lead to constant deceleration parameters. According to observations, the universe has made a transition from a phase of decelerated expansion to a phase of accelerated expansion, implying that the deceleration parameter has a dependence upon time [12, 16]. Taking this fact into consideration, we have used a scale factor (a) such that the deceleration parameter changes sign from positive to negative at some point in the history of the universe. We have used power law dependence of ω upon ϕ and also ϕ upon a. Time dependence of the EoS parameter, obtained from the present study, is consistent with the results obtained from other studies based on general relativity with anisotropic space-time [17, 18]. Time variation of energy density, obtained from the present model, is consistent with the results obtained from other studies anisotropic dark energy models [17, 18]. A theoretical model has been proposed to determine the time variation of density parameters corresponding to dark energy and matter. The results from this model are in agreement with those obtained from a study where an interaction between scalar field and matter was considered [19, 20].

2. Metric and Field Equations

In the generalized Brans-Dicke theory, the action is expressed as,

$$S = \int d^4x\sqrt{-g}\left(\frac{\phi R}{16\pi G} + \frac{\omega(\phi)}{\phi} g^{\mu\nu}\, \partial_\mu\phi\partial_\nu\phi + L_m\right) \tag{01}$$

Here, g is the determinant of the tensor metric $g^{\mu\nu}$, R is the Ricci scalar, L_m is the Lagrangian of matter, φ is the Brans-Dicke scalar field and ω is a dimensionless Brans-Dickeparameter.In generalized BD theory, this coupling parameter ω is considered to be a function of φ.

The line element for a non-flat universe, in the framework of the Friedmann-Robertson-Walker (FRW) cosmology, can be expressed in polar coordinates as,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\xi^2\right] \tag{02}$$

Here $a(t)$ is the scale factor of the universe, t is the cosmic time, r is the radial component, k is the curvature parameter, θ and ξ are the two polar coordinates.

The gravitational field equations of Brans-Dicke theory, for a universe filled with a perfect fluid and described by FRW space-time with scale factor $a(t)$ and spatial curvature k, are given by,

$$3\frac{\dot{a}^2+k}{a^2}+3\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}-\frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2}=\frac{\rho}{\phi} \tag{03}$$

$$2\frac{\ddot{a}}{a}+\frac{\dot{a}^2+k}{a^2}+\frac{\omega(\phi)}{2}\frac{\dot{\phi}^2}{\phi^2}+2\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi}+\frac{\ddot{\phi}}{\phi}=-\frac{\gamma\rho}{\phi} \tag{04}$$

The wave equation for the scalar field (ϕ), in generalized Brans-Dicke theory of gravity, where ω is a time dependent parameter, is expressed as,

$$\frac{\ddot{\phi}}{\phi}+3\frac{\dot{a}\dot{\phi}}{a\phi}=\frac{\rho}{\phi}\frac{(1-3\gamma)}{2\omega+3}-\frac{\dot{\phi}}{\phi}\frac{\dot{\omega}}{2\omega+3} \tag{05}$$

Combining equations (3), (4) and (5), and taking $k=0$ (for flat space), one obtains,

$$\dot{\omega}+\left(2\frac{\ddot{\phi}}{\dot{\phi}}+6\frac{\dot{a}}{a}-\frac{\dot{\phi}}{\phi}\right)\omega-6\left(\frac{\dot{a}^2}{a^2}+\frac{\ddot{a}}{a}\right)\frac{\phi}{\dot{\phi}}=0 \tag{06}$$

In the above expressions, $\gamma(\equiv P/\rho)$ is the *equation of state* (EoS) parameter for the cosmic fluid, which is treated here as a function of time.

3. Theoretical Model

In the present study we have used the following empirical expressions of ϕ and ω. They have been assumed to have power-law dependence upon a and φ respectively.

$$\phi=\phi_0\left(\frac{a}{a_0}\right)^n \tag{07}$$
$$\omega=\omega_0\left(\frac{\phi}{\phi_0}\right)^m \tag{08}$$

The expression of ϕ in equation (7) has been taken from some earlier studies in this regard [12, 16]. The reason for choosing the empirical expression of ω in equation (8) is the fact that this dimensionless coupling constant in the generalized Brans-Dicke theory is regarded as a function of the scalar field [12].
Using the expressions (7) and (8) in equation (6), one obtains,

$$m\omega n+(n+4-2q)\omega-\frac{6}{n}(1-q)=0 \tag{09}$$

Here, q is the deceleration parameter.
Taking $\omega=\omega_0$ and $q=q_0$ in equation (9), at $t=t_0$, we get,

$$m=\frac{6(1-q_0)}{n^2\omega_0}-\frac{n+4-2q_0}{n} \tag{10}$$

Combining equations (3) with (7) and taking $k=0$ (for flat space), one gets,

$$\rho=\phi H^2\left(3+3n-\frac{\omega}{2}n^2\right) \tag{11}$$

Replacing all parameters in equation (11) by their values at $t=t_0$, one obtains,

$$\omega_0 = \frac{2}{n^2 {H_0}^2}\left(3{H_0}^2 + 3n{H_0}^2 - \rho_0/\phi_0\right) \tag{12}$$

Using equation (12) in equation (10) one obtains,

$$m = \frac{3(1-q_0){H_0}^2}{3{H_0}^2+3n{H_0}^2-\rho_0/\phi_0} - \frac{n+4-2q_0}{n} \tag{13}$$

Equations (12) and (13) show that both ω_0 and m depends upon the parameter n, which determines the time dependence of ϕ, as per equation (7).
Using equations (7) and (11) in (4), for $k = 0$ (zero spatial curvature) and writing $\frac{\ddot{a}}{a} = -qH^2$ one obtains the following expression of the equation of state parameter,

$$\gamma = \frac{2q-1-0.5\omega n^2-n-n^2+nq}{3+3n-0.5\omega n^2} \tag{14}$$

The value of the equation of state parameter at the present time is therefore given by,

$$\gamma_0 = \frac{2q_0-1-0.5\omega_0 n^2-n-n^2+nq_0}{3+3n-0.5\omega_0 n^2} \tag{15}$$

For the expression of ω in equation (8), the values of m and ω_0 should be taken from equations (10) and (12) respectively. The value of ω, required for the expressions of ρ and γ, in equations (11) and (14) respectively, should be taken from equation (8).
To determine the time dependence of several cosmological parameters ($\phi, \omega, \rho, \gamma$), following empirical expression of the scale factor has been used.

$$a = a_0 Exp\left[\alpha\left\{(t/t_0)^\beta - 1\right\}\right] \tag{16}$$

This scale factor has been so chosen that, the deceleration parameter, calculated from it, changes sign from positive to negative, as a function of time. This signature flip indicates a transition of the universe from a phase of decelerated expansion to a phase of accelerated expansion, in accordance with several recent studies based on astrophysical observations [12]. We have taken $a_0 = 1$ for all calculations.
Here α, β should have the same sign to ensure an increase of the scale factor with time. Using equation (16), the Hubble parameter (H) and the deceleration parameter (q) are obtained as,

$$H = \frac{\dot{a}}{a} = \frac{\alpha\beta}{t_0}\left(\frac{t}{t_0}\right)^{\beta-1} \tag{17}$$

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{1-\beta}{\alpha\beta}\left(\frac{t}{t_0}\right)^{-\beta} \tag{18}$$

For $0 < \beta < 1$ and $\alpha > 0$ one finds that, $q \to +\infty$ as $t \to 0$ and $q \to -1$ as $t \to \infty$, showing clearly a signature flip of q with time.
Taking $H = H_0$ and $q = q_0$, at $t = t_0$, one gets,

$$\alpha = \frac{H_0 t_0}{1-H_0 t_0(1+q_0)} \tag{19}$$

$$\beta = 1 - H_0 t_0 (1 + q_0) \tag{20}$$

The values of the constants α and β can be determined from equations (19) and (20) respectively.
It is often necessary to express the evolution of cosmological quantity as a function of the redshift parameter (z) where $z = \frac{a_0}{a} - 1$. Using equation (16) one gets the following expression as a relation between redshift parameter and time.

$$z = \frac{a_0}{a} - 1 = Exp\left[-\alpha\left\{(t/t_0)^{\beta} - 1\right\}\right] - 1 \tag{21}$$

The total pressure (P) of the entire matter-energy content of the universe is contributed by dark energy because, the whole matter content (dark matter + baryonic matter) is regarded as pressureless dust [12, 21]. Thus we can write,

$$P = \gamma\rho = \gamma_D \rho_D \tag{22}$$

Here, γ_D is the equation-of-state parameter for dark energy and ρ_D is the density of dark energy.
We propose the following empirical relationship between γ_D and γ.

$$\gamma_D = A\,\gamma t^{\mu} \tag{23}$$

Here A and μ are constants.
Using equations (22) and (23) we can write,

$$\Omega_D = \frac{\rho_D}{\rho} = \frac{\gamma}{\gamma_D} = A^{-1} t^{-\mu} \tag{24}$$

Here, Ω_D denotes the density parameter for dark energy.
Taking $\Omega_D = \Omega_{D0}$ at $t = t_0$, in equation (24), we get,

$$A = \frac{1}{\Omega_{D0}\, {t_0}^{\mu}} \tag{25}$$

Using equation (25) in (24) we get,

$$\Omega_D = \Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu} \tag{26}$$

Using equation (26), the density parameter for matter can be expressed as,

$$\Omega_m = \frac{\rho_m}{\rho} = \frac{\rho - \rho_D}{\rho} = 1 - \Omega_D = 1 - \Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu} \tag{27}$$

In deriving equation (27), we have used the fact that the universe is composed mainly of dark energy and matter, considering all other forms of energy to be negligibly small [27, 28].
Using equations (26) and (27), the expressions for densities of dark energy and matter can be respectively written as,

$$\rho_D = \rho\Omega_D = \rho\Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu} \tag{28}$$

$$\rho_m = \rho\Omega_m = \rho\left[1 - \Omega_{D0} \left(\frac{t}{t_0}\right)^{-\mu}\right] \tag{29}$$

Using equation (25) in (23), the equation-of-state parameter for dark energy (γ_D) becomes,

$$\gamma_D = \frac{\gamma}{\Omega_D} = \frac{\gamma}{\Omega_{D0}}\left(\frac{t}{t_0}\right)^{\mu} \tag{30}$$

Using equation (21), the expressions of Ω_D and Ω_m can be written in terms of redshift (*z*) as,

$$\Omega_D = \Omega_{D0}\left(1 + \frac{1}{\alpha} ln \frac{1}{z+1}\right)^{-\mu/\beta} \tag{31}$$

$$\Omega_m = 1 - \Omega_{D0}\left(1 + \frac{1}{\alpha} ln \frac{1}{z+1}\right)^{-\mu/\beta} \tag{32}$$

The values of Ω_{D0} is close to 0.7 according to several astrophysical observations [27, 28].
As per the history of evolution of density parameters of the universe, there was a time in the recent past when $\Omega_D = \Omega_m = 0.5$ and, the corresponding z value is lying somewhere in the range of $0 < z < 1$, as obtained from recent studies [19, 20]. That phase of the universe might be same as (or close to) the one when there was a transition from decelerated expansion to an accelerated expansion of the universe because it is the dark energy that is known to be responsible for the accelerated expansion. Taking z_c to denote the value of the redshift parameter at which the universe had $\Omega_D = \Omega_m$, we get the following expression of μ from equations (31) and (32).

$$\mu = \frac{\beta\, ln(2\Omega_{D0})}{ln\left[1+\frac{1}{\alpha}ln\left(\frac{1}{z_c+1}\right)\right]} \tag{33}$$

In Brans-Dicke theory, the scalar field (ϕ) is the reciprocal of the gravitational constant (*G*) [12, 22]. Hence, using equation (7), we can write,

$$G = \frac{1}{\phi} = \frac{1}{\phi_0}\left(\frac{a}{a_0}\right)^{-n} \tag{34}$$

According to some studies, the gravitational constant increases with time [23-25]. In the expression of G in equation (34), the scale factor (a) is an increasing function of time. For negative values of the parameter n, G would be an increasing with time. Therefore, in the present study we have used only negative values of the parameter *n*.

Using equation (34), we get,

$$\frac{\dot{G}}{G} = -n\frac{\dot{a}}{a} = -nH \tag{35}$$

Equation (35) suggests that the value of the parameter *n* can be estimated from the experimental observations of the fractional rate of change of the gravitational constant ($\dot{G}/G$), obtained from various studies [26].
The values of different cosmological parameters used in this article are:

$H_0 = \frac{72\frac{Km}{s}}{Mpc} = 2.33 \times 10^{-18} sec^{-1}$, $q_0 = -0.55$, $\rho_0 = 9.9 \times 10^{-27} Kgm^{-3}$, Ω_{D0}=0.7

$\varphi_0 = \frac{1}{G_0} = 1.498 \times 10^{10} Kgs^2m^{-3},\ t_0 = 1.4 \times 10^{10} Years = 4.42 \times 10^{17} s$

FIGURES

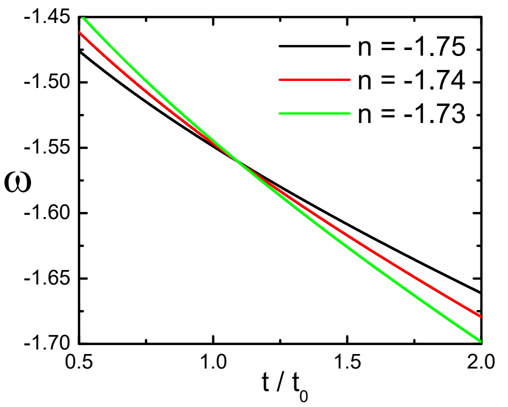

Figure 1: Plots of Brans-Dicke parameter versus time for three different values of the parameter *n*.

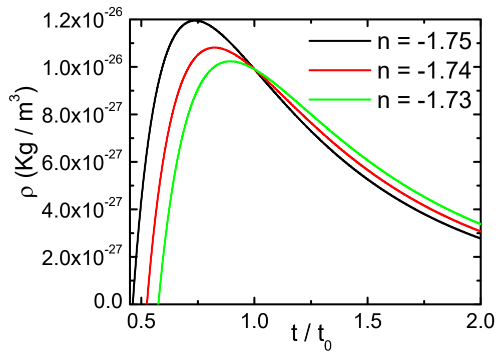

Figure 2: Plots of energy density versus time for three different values of the parameter *n*.

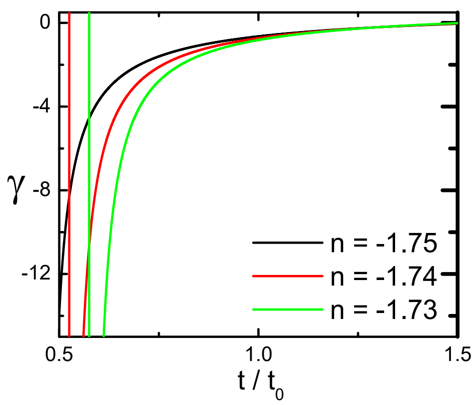

Figure 3: Plots of the equation of state parameter versus time for three different values of the parameter *n*.

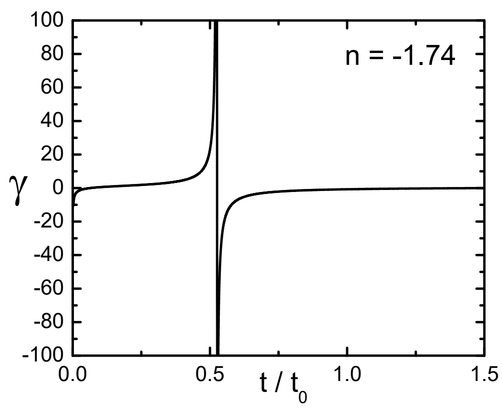

Figure 4: Plot of the equation of state parameter versus time, for $n = -1.74$, on a larger time scale.

<u>FIGURES</u>

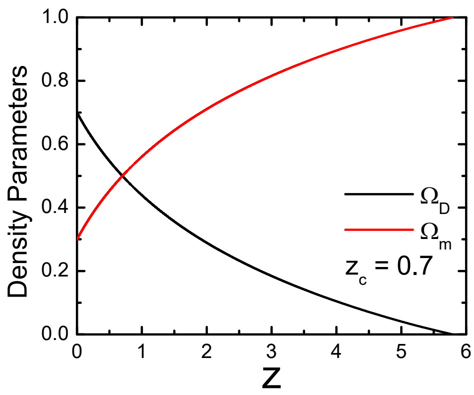

Figure 5: Plots of density parameters, for matter and dark energy, for $z_c = 0.7$, against the redshift parameter.

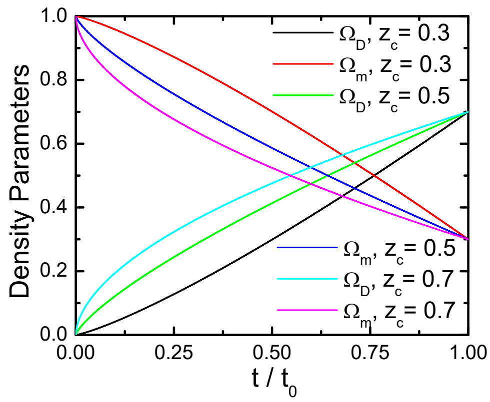

Figure 6: Plots of density parameters, for matter and dark energy, as functions of time, for three different values of z_c.

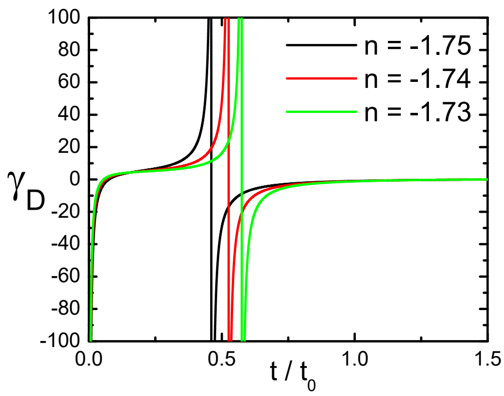

Figure 7: Plots of the equation of state (EoS) parameter for dark energy versus time for three different values of the parameter n.

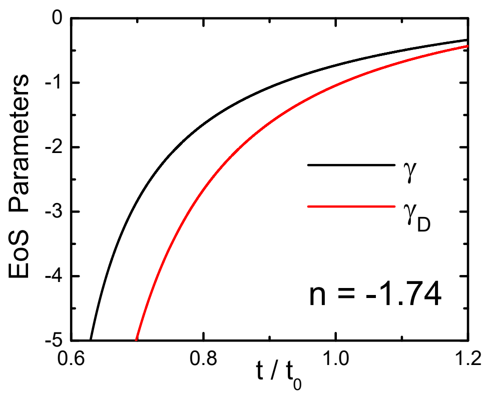

Figure 8: Plots of EoS parameters versus time, for total energy (γ) and dark energy (γ_D). For $n = -1.74$, $\gamma_0 = -0.73$ and $\gamma_{D0} = -1.04$.

4. Results and Discussions

Figure 1 shows the variation of the Brans-Dicke parameter (ω) as a function of time, for three different values of the parameter n. Here, ω has a negative value and it becomes more negative with time. For more negative values of n, it decreases less rapidly. This nature of ω, which is gradually becoming more negative with time, is also obtained from other studies on Brans-Dicke theory [13, 29]. The time dependence of ω, obtained from the present study, is based on a scale factor which leads to a time dependent deceleration parameter (that shows a signature flip with time from positive to negative), unlike some recent studies [14, 15] that are based on time independent deceleration parameters. The value of ω_0 is found to be close to -1.55 which is consistent with the range provided in the conclusion of a recently published article based on Kantowski-Sachs space-time [15].

Figure 2 shows the variation of energy density (ρ) with time, for three different values of the parameter n. Here, ρ is found to rise steeply to a peak and then decreases at a slower rate. More negative values of n cause quicker attainment of the peak and also a larger peak value of energy density. It falls more steeply for more negative values of n.

Figure 3 shows the time dependence of the equation of state (EoS) parameter (γ) for three different values of the parameter n. These curves show a steep rise initially and then they gradually become asymptotic to a small negative value. Saturation is reached faster for less negative values of n. The value of γ at the present time ($t = t_0$) is -0.73 which is in agreement with the ranges of values obtained from astrophysical observations, as provided in earlier studies [17, 18]. These curves are similar to those obtained from dark energy models based on LRS Bianchi type-V metric in the framework of Einstein's general theory of relativity.

Figure 4 shows a single plot EoS parameter, for $n = -1.74$, on a longer time scale. It shows a pattern that is very much similar to the one depicted in one of the earlier studies based on dark energy models in anisotropic Bianchi type-I (B-I) space-time, in the framework of Einstein's general theory of relativity [17].

The plots in the figures 2-4 show very clearly that the time evolution of energy density and EoS parameter, obtained here for FRW space-time in the framework of Brans-Dicke theory, is quite in agreement with the results obtained from anisotropic space-time in the framework of general relativity.

Figure 5 shows the variations of density parameters, for dark energy and matter, as functions of the redshift parameter. Here, the value of μ has been taken to be -0.557, causing $z_c = 0.7$, implying that the dark energy took over at around $z = 0.7$. These plots are almost identical in nature to the plots obtained from some studies [19, 20], which were carried out on premises completely different from the present study.

Figure 6 shows the plots of density parameters, for dark energy and matter, as functions of time. Here we have three pairs (Ω_m and Ω_D) of density parameters, corresponding to three different values of the parameter μ, connected to three different values of z_c.

Figure 7 shows the time variation of the EoS parameter for dark energy (γ_D) for three different values of the parameter n. These plots are very much similar to the ones obtained from studies based on anisotropic space-time in the framework of general relativity [17, 18]. Plots with less negative values of n take longer time to attain saturation.

Figure 8 shows the time variations of EoS parameters for total energy and dark energy. Here γ_D has a more negative value compared with γ. Their difference decreases with time, as the proportion of dark energy increases in the universe. For $n = -1.74$ the values of γ_0 and γ_{D0} are respectively -0.73 and -1.04 respectively. These values are quite consistent with the ranges of values obtained from several astrophysical observations described in some studies regarding dark energy and time varying EoS parameter [17-20].

The parameter n, in this model, plays an important role in governing the time evolution of various cosmological quantities. Here ω_0 is a function of n and, the parameter m depends upon both ω_0 and n. Taking $n = -1.74$ we get, $\left(\frac{\dot{G}}{G}\right)_{t=t_0} = 1.28 \times 10^{-10}\, Yr^{-1}$ from equation (35). This is smaller than the maximum possible value, i.e. $4 \times 10^{-10} Yr^{-1}$, as predicted by S. Weinberg [30].

5. Concluding Remarks

The present study is based on a spatially flat, homogeneous and isotropic space-time, in the framework of Brans-Dicke theory. Here, the time evolution of Brans-Dicke parameter (ω), energy density (ρ) and EoS parameter (γ) have been determined from the field equations. Unlike many other studies, no cosmological constant and self interaction potential has been used in the formulation of the theory. The choice of the scale factor, that ensures a signature flip of the deceleration parameter with time, is likely to give more credibility to the results of this model. Without involving any self interaction potential (V_ϕ) and cosmological constant (Λ), the time evolutions of density parameters for matter and dark energy have been determined and these results are sufficiently consistent with those obtained through much more rigorous calculations [19, 20]. The parameter μ, in this model, governs the time dependence of Ω_D, Ω_m and γ_D. It has been shown that μ can be determined by using information from astrophysical observations regarding the time or redshift (z) at which $\Omega_D = \Omega_m$ in the recent past of the expanding universe. In this model, the parameter n, which actually controls the rate of change of gravitational constant with time (as per eqn. 34), determines the nature of time variation of some cosmological quantities (ω, ρ and γ) mentioned above. It indicates that the dependence of the scalar field $(\phi \equiv 1/G)$ upon the scale factor (a) plays a very important role in cosmic expansion. The dependence of ω upon φ is controlled by ω_0 and m and, both of them are functions of n. Positive values of n leads to large negative values of γ_{D0}, contrary to its range of values close to -1, obtained from recent astrophysical observations [19]. This result is in favour of using negative values of n, implying an increase of gravitational constant with time, as per equation (34). This behaviour of gravitational constant is also obtained from other studies [23-25]. Keeping in view all these facts, one may think of improving this model by choosing a better relation, between ϕ and a, than the ansatz of equation (7), to have more than one tunable parameter like n controlling the time variation of the scalar field (ϕ).

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